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Generation of sustained optimal entropy squeezing of a two-level atom via non-Hermitian operation*

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We investigate the entropy squeezing of a two-level atom in the Jaynes–Cummings model, and provide a scheme to generate the sustained optimal entropy squeezing of the atom via non-Hermitian operation. Our results show that the squeezing degree and the persistence time of entropy squeezing of atomic polarization components greatly depend on the non-Hermiticity intensity in non-Hermitian operation. Especially, under a proper choice of non-Hermiticity parameters, the sustained optimal entropy squeezing of the atom can be generated even though the atom is initially prepared in a no entropy squeezing state.

Keywords: entropy squeezing, non-Hermitian dynamics, quantum optics, Jaynes–Cummings model

PACS: 42.50.–p, 03.67.Lx, 03.67.–a **DOI:** 10.1088/1674-1056/27/11/114207

1. Introduction

The precision measurement of observable quantities in quantum physical experiments is greatly restricted by quantum uncertainty of quantum systems. Hence breaking through the restriction of quantum uncertainty caused by quantum noises and realizing quantum physical experiments with ultra-low quantum noise is a strong expectation for physical scientists. As we know, the suppression of quantum noises in quantum systems via squeezing effects is one of the most outstanding achievements in modern quantum optics over the last few years. Therefore, many squeezing effects have been investigated, such as the atomic squeezing. It should be pointed out that quantum information entropy is an accurate measure of the atomic squeezing, and one of our authors has defined the entropy squeezing (ES) for a two-level atom by using quantum information entropy according to entropic uncertainty relation based on two conjugate observables,[1] which overcomes the limitation of the definition of squeezing via the variance of the system observable according to Heisenberg uncertainty relation. Furthermore, an optimal squeezing state corresponds to a quantum state with minimum quantum noise, and sustained squeezing can provide long enough time for scientists to make use of the squeezing resource to perform quantum information processing, quantum computation, and quantum communication. According to most of previous works,[2–4] available squeezing effects at most possess either optimal squeezing degree or long squeezing time, but not both. As far as we know, it is difficult to prepare the squeezing resource which possesses not only optimal squeezing degree but also long persistence time. Hence there is a significant subject in the research field of squeezing effects: generating squeezing effects which not only have optimal squeezing degree but also possess persistent squeezing time. More recently, some investigations on the sustained optimal entropy squeezing (SOES) of the atom have been reported.[5,6]

On the other hand, the study of non-Hermitian (NH) systems recently receives increasing attentions. In the last two decades, many theories of NH systems with real and complex spectra have been investigated. Bender *et al.* proposed the parity–time reversal (PT-) symmetric Hamiltonians whose energy spectra are also real and positive,[7] proved that the time evolution of PT-symmetric system becomes unitary by introducing an inner-product structure with a C-symmetric operator,[8] and demonstrated that the evolution time of NH systems can even be made arbitrarily small without violating the time–energy uncertainty principle.[9] After Ref. [9], Günther *et al.* proposed that some NH systems can be reinterpreted as a subsystem of a larger conventional quantum mechanics system in a higher-dimensional Hilbert space governed by a Hermitian Hamiltonian.[10] Lee *et al.* found that the no-signalling principle can be violated when performing the local PT-symmetric operation on one of the entangled particles.[11] Chen *et al.* proved that the concurrence, Bell's inequality, and quantum steering of a bipartite entangled state can be increased under the local NH operation.[12] The framework for the NH formalism of Hamiltonians has been proposed by Brody *et al.*[13] and Sergi *et al.*[14] With the framework, Sergi *et al.* and Zloshchastiev *et al.* studied NH dynamics of a two-level system,[14] comparison and uni-

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fication of NH and Lindblad approaches,[15] time correlation functions for NH systems,[16] and stability of pure states in NH systems.[17] Other researchers studied the non-Hermiticity of quantum systems in weak measurement,[18] quantum thermodynamics,[19] characteristics of NH Hamiltonians,[20] quantum Fisher information,[21,22] quantum entropy and quantum entropic uncertainty relation,[23,24] and so on. These research results show that the NH approach has powerful control on quantum systems and its quantum states. As far as we know, few of researchers make use of the NH approach as an efficient method of generating SOES. In this paper, we study the generation of SOES of a two-level atom in the Jaynes–Cummings (JC) model via NH operation. Through adjusting the non-Hermiticity parameter, we discuss the effect of NH operation on the atomic ES, and provide a scheme to realize the generation of the atomic SOES via a proper NH operation.

2. Entropy squeezing and non-Hermitian dynamics

In this section, we will briefly review the definition of ES and the dynamics of NH systems. ES is defined by Ref. [1]. As we know, the information entropy of operators S_α ($\alpha = x, y, z$) for a two-level atom is

$$H(S_\alpha) = -\sum_{i=1}^{2} P_i(S_\alpha) \ln P_i(S_\alpha), \tag{1}$$

where $P_i(S_\alpha) = \langle \varphi_{\alpha i}|\rho(t)|\varphi_{\alpha i}\rangle$ ($i = 1, 2$) is the probability distribution for two possible outcomes of measurements of operators S_α for an arbitrary quantum states $\rho(t)$, and $|\varphi_{\alpha i}\rangle$ are eigenvalues of S_α. Using an atomic density operator $\rho(t)$ and Eq. (1), we can directly obtain expressions of the information entropy of S_α

$$\begin{aligned} H(S_x) &= -\left[\frac{1}{2} + \mathrm{Re}\,\rho_{21}(t)\right] \ln \left[\frac{1}{2} + \mathrm{Re}\,\rho_{21}(t)\right] \\ &\quad - \left[\frac{1}{2} - \mathrm{Re}\,\rho_{21}(t)\right] \ln \left[\frac{1}{2} - \mathrm{Re}\,\rho_{21}(t)\right], \\ H(S_y) &= -\left[\frac{1}{2} + \mathrm{Im}\,\rho_{21}(t)\right] \ln \left[\frac{1}{2} + \mathrm{Im}\,\rho_{21}(t)\right] \\ &\quad - \left[\frac{1}{2} - \mathrm{Im}\,\rho_{21}(t)\right] \ln \left[\frac{1}{2} - \mathrm{Im}\,\rho_{21}(t)\right], \\ H(S_z) &= -\rho_{22}(t) \ln \rho_{22}(t) - \rho_{11}(t) \ln \rho_{11}(t), \end{aligned} \tag{2}$$

where $\rho_{ij}(t)$ ($i, j = 1, 2$) are matrix elements of $\rho(t)$. $H(S_x)$, $H(S_y)$, and $H(S_z)$ satisfy the entropic uncertainty relation

$$\delta H(S_x)\delta H(S_y) \geq \frac{4}{\delta H(S_z)}, \tag{3}$$

where $\delta H(S_\alpha) \equiv \exp[H(S_\alpha)]$ is regarded as the entropy fluctuation (mathematically). If the information entropy $H(S_\alpha)$ ($\alpha = x$ or y) satisfies the condition

$$E(S_\alpha) \equiv \delta H(S_\alpha) - \frac{2}{\sqrt{\delta H(S_z)}} < 0, \tag{4}$$

one can say that the entropy fluctuation $\delta H(S_\alpha)$ ($\alpha = x$ or y) in the component S_α of the atomic dipole is squeezed in entropy, and $E(S_\alpha)$ ($\alpha = x$ or y) can be called the ES factor whose range is $1 - \sqrt{2} \leq E(S_\alpha) \leq 2 - \sqrt{2}$. $E(S_\alpha) > 0$ means that there is no ES, while $E(S_\alpha) < 0$ represents that there exists ES, and squeezing degree increases with the value of $E(S_\alpha)$ decreasing. The value of the optimal ES is $E(S_\alpha) = 1 - \sqrt{2}$.

According to Refs. [13] and [14], NH Hamiltonians can be decomposed into Hermitian and anti-Hermitian parts,

$$H_{\mathrm{NH}} = H_+ + H_-, \tag{5}$$

where $H_\pm = \pm H_\pm^\dagger$ and $H_- = -\mathrm{i}\Gamma$. $\Gamma = \Gamma^\dagger$ is usually referred to the decay rate operator. The Schrödinger equation for an arbitrary quantum state $|\varphi(t)\rangle$ is

$$\frac{\partial}{\partial t}|\varphi(t)\rangle = -\frac{\mathrm{i}}{\hbar} H_{\mathrm{NH}}|\varphi(t)\rangle = -\frac{\mathrm{i}}{\hbar} H_+|\varphi(t)\rangle - \frac{1}{\hbar}\Gamma|\varphi(t)\rangle, \tag{6}$$

and the time-evolution operator is directly obtained by time integral of Eq. (6):

$$U_{\mathrm{NH}} = \exp\left(-\frac{\mathrm{i}}{\hbar}\int_0^t H_{\mathrm{NH}}\mathrm{d}t'\right) = \exp\left(-\frac{\mathrm{i}}{\hbar} H_{\mathrm{NH}} t\right). \tag{7}$$

Given an initial state $\rho(0) = |\varphi(0)\rangle\langle\varphi(0)|$, the density operator $\rho'(t) = |\varphi(t)\rangle\langle\varphi(t)|$ can also be expressed as

$$\rho'(t) = U_{\mathrm{NH}}\rho(0)U_{\mathrm{NH}}^\dagger. \tag{8}$$

We can see from Eqs. (7) and (8) that, in general, U_{NH} is a non-unitary evolution operator, and $\rho'(t)$ is a non-normalized density operator. Although the NH Hamiltonian H_{NH} possesses the non-Hermiticity which is an unconventional property of quantum mechanics, we have to investigate ES factors $E(S_\alpha)$ in conventional quantum mechanics, which requires the procedure of renormalization to ensure that the density matrix is trace-preserving. Hence, we renormalize the density operator $\rho'(t)$:

$$\rho(t) = \frac{\rho'(t)}{\mathrm{Tr}\rho'(t)} = \frac{U_{\mathrm{NH}}\rho(0)U_{\mathrm{NH}}^\dagger}{\mathrm{Tr}\left[U_{\mathrm{NH}}\rho(0)U_{\mathrm{NH}}^\dagger\right]}. \tag{9}$$

Then the normalized $\rho(t)$ of a NH system could be used to investigate ES.

3. Physical model

We consider a two-level atom with transition frequency ω_{a} coupled to a single-mode cavity field with frequency ω_{f}, and we perform a NH operation on the atom. The total Hamiltonian H_{tot} of the system is

$$H_{\mathrm{tot}} = H_{\mathrm{JC}} + H_{\mathrm{NH}} \otimes I_{\mathrm{f}}, \tag{10}$$

where $I_{\rm f}$ is an identity matrix for the cavity field. In the absence of the NH operation, our system can be described by the JC model Hamiltonian $H_{\rm JC}$ (in the rotating-wave approximation and $\hbar = 1$):

$$H_{\rm JC} = \omega_{\rm a} S_z + \omega_{\rm f} a^+ a + g(aS_+ + a^+ S_-), \tag{11}$$

where g is coupling strength, a^+ and a are the creation and annihilation operators of the cavity field, and $S_\pm = S_x \pm {\rm i} S_y$ are atomic operators. In this paper, we consider the resonant case ($\omega_{\rm a} = \omega_{\rm f}$), and the atom and the cavity field are initially prepared in

$$\begin{aligned} |\psi_{\rm a}(0)\rangle &= \cos\left(\frac{\theta}{2}\right)|+\rangle + \sin\left(\frac{\theta}{2}\right)|-\rangle, \\ |\alpha\rangle &= \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \end{aligned} \tag{12}$$

where θ is the superposition coefficient, $\alpha = \sqrt{\overline{n}}$, and $\overline{n}$ is average photon number. By solving the evolution equation, we obtain the atomic reduced density operator $\rho_{\rm a}$, and its matrix elements are given as

$$\begin{aligned} \rho_{\rm a22} &= \sum_{n=0}^{\infty}|P_n|^2\left[\cos^2\left(\frac{\theta}{2}\right)\cos^2(\sqrt{n+1}gt)\right. \\ &\quad \left. + \sin^2\left(\frac{\theta}{2}\right)\sin^2(\sqrt{n}gt)\right], \\ \rho_{\rm a21} &= \frac{1}{2}\sin(\theta)\sum_{n=0}^{\infty}\left[|P_n|^2\cos(\sqrt{n+1}gt)\cos(\sqrt{n}gt)\right. \\ &\quad \left. + P_{n+1}P_{n-1}^*\sin(\sqrt{n+1}gt)\sin(\sqrt{n}gt)\right] \\ &\quad + {\rm i}\sum_{n=0}^{\infty}\left[\cos^2\left(\frac{\theta}{2}\right)P_nP_{n-1}^*\cos(\sqrt{n+1}gt)\sin(\sqrt{n}gt)\right. \\ &\quad \left. - \sin^2\left(\frac{\theta}{2}\right)P_{n+1}P_n^*\sin(\sqrt{n+1}gt)\cos(\sqrt{n}gt)\right], \\ \rho_{\rm a11} &= 1-\rho_{\rm a22}, \\ \rho_{\rm a12} &= \rho_{\rm a21}^*. \end{aligned} \tag{13}$$

We perform a NH operation on the atom, and choose a special scheme of a two-level system introduced by Ref. [17] to describe our NH operation:

$$H_+ = -\omega\sigma_x, \quad \Gamma = \lambda\sigma_z, \tag{14}$$

where ω and λ are assumed to be real-valued and λ represents the intensity of the non-Hermiticity. According to Eq. (5), the NH Hamiltonian is

$$H_{\rm NH} = -\omega\begin{pmatrix} {\rm i}\widetilde{\lambda} & 1 \\ 1 & -{\rm i}\widetilde{\lambda}\end{pmatrix}, \tag{15}$$

where $\widetilde{\lambda} \equiv \lambda/\omega$. In this paper, we set that ω^{-1} is a scaling constant, and then $\widetilde{\lambda}$ has the same sign as the non-Hermiticity parameter λ. The eigenvalues of $H_{\rm NH}$ are $E_\pm = \pm\sqrt{\omega^2 - \lambda^2} = \pm\sqrt{1-\widetilde{\lambda}^2}$. It is easy to determine that when $\lambda \in (-1,1)$, $H_{\rm NH}$ is a PT-symmetric Hamiltonian with real eigenvalues, while $\lambda \in (-\infty,-1)\cup(1,\infty)$, $H_{\rm NH}$ is a PT-symmetric broken NH Hamiltonian with complex eigenvalues, and $\lambda = \pm 1$ are usually regard as exceptional points[25] which are the points where eigenvalues switch from real to complex values. According to Eq. (9), we obtain the renormalized reduced density operator $\rho(t)$ of the atom under the NH operation

$$\rho(t) = \frac{U_{\rm NH}\rho_{\rm a}U_{\rm NH}^\dagger}{{\rm Tr}\left(U_{\rm NH}\rho_{\rm a}U_{\rm NH}^\dagger\right)}, \tag{16}$$

and matrix elements of $\rho(t)$ are given as

$$\begin{aligned} \rho_{11}(t) &= \frac{1}{\lambda_1^2 T}\left\{\lambda_1^2\rho_{\rm a11}\cosh^2(\lambda_1 t)\right. \\ &\quad + \left[1+\lambda_1^2\rho_{\rm a11} + {\rm i}\widetilde{\lambda}(\rho_{\rm a12}-\rho_{\rm a21})\right]\sinh^2(\lambda_1 t) \\ &\quad \left. - \lambda_1\left[\widetilde{\lambda}\rho_{\rm a11} + \frac{1}{2}{\rm i}\,(\rho_{\rm a12}-\rho_{\rm a21})\right]\sinh(2\lambda_1 t)\right\}, \\ \rho_{12}(t) &= \frac{1}{\lambda_1^2 T}\left[\lambda_1^2\rho_{\rm a12}\cosh^2(\lambda_1 t)\right. \\ &\quad + \left({\rm i}\widetilde{\lambda} - \widetilde{\lambda}^2\rho_{\rm a12} + \rho_{\rm a21}\right)\sinh^2(\lambda_1 t) \\ &\quad \left. + \frac{1}{2}{\rm i}\lambda_1(1-2\rho_{\rm a11})\sinh(2\lambda_1 t)\right], \\ \rho_{22}(t) &= 1-\rho_{11}(t), \\ \rho_{21}(t) &= \rho_{12}^*(t), \end{aligned} \tag{17}$$

where $T = {\rm Tr}(U_{\rm NH}\rho_{\rm a}U_{\rm NH}^\dagger)$ and $\lambda_1 = \sqrt{\widetilde{\lambda}^2 - 1}$.

4. Generation of sustained optimal entropy squeezing via non-Hermitian operation

In this section, using atomic reduced density operators $\rho_{\rm a}$ and $\rho(t)$, we investigate the evolution of atomic ES factors $E(S_\alpha)$ ($\alpha = x$ or y) given by Eq. (4) with the help of numerical calculations. In following subsections, two cases for different initial states would be discussed. For comparison, we consider ES of the atom under two situations: the atom without NH operation and the atom under different strength NH operations.

4.1. Case I

In the first case, we consider that the atom is initially prepared in the excited state with $\theta = 0$, and the cavity field initially in a coherent state with $\overline{n} = 25$. For comparison, we briefly introduce the atomic ES in the absence of NH operation. The dynamics of ES factors $E(S_x)$ and $E(S_y)$ are shown in Figs. 1(a) and 1(b), respectively. We can see that there is no ES in the component S_x as $E(S_x) \geq 0$ all the time, while the component S_y exists ES in certain time ranges as $E(S_y) < 0$. Although ES appears in $E(S_y)$, it would gradually disappears.

It means that, in the absence of NH operation, the atomic ES in this case can not keep long time.

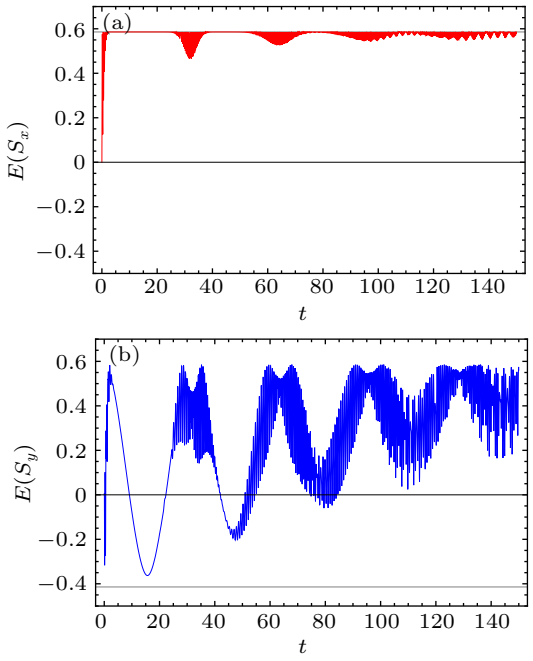


Fig. 1. (color online) The dynamics of ES factors $E(S_\alpha)$ of a two-level atom in the JC model without NH operation. The atom is initially in the excited state with $\theta = 0$, the cavity field in a coherent state with average photon number $\overline{n} = 25$, and the coupling strength $g = 1$. (a) $E(S_x)$ and (b) $E(S_y)$.

In the presence of NH operation, because the numerical result shows that there is no ES in the component S_x in this case, we only discuss the ES factor $E(S_y)$ plotted in Fig. 2. According to Eq. (15) and its explanation, we know that the non-Hermiticity parameter $\widetilde{\lambda} = \pm 1$ are usually considered as exceptional points where the NH Hamiltonian switches from PT-symmetric structure to PT-symmetric broken structure. When $\widetilde{\lambda}$ takes 0.9 in Fig. 2(a), comparing with Fig. 1(b), we can find that both the squeezing degree and the number of times of ES obviously increase. With $\widetilde{\lambda}$ increasing to 0.999, shown in Fig. 2(b), the number of times of ES decreases and the persistence time of the optimal ES remarkably becomes longer, but the optimal ES still periodically appears and disappears. When the value of $\widetilde{\lambda}$ is slightly over 1 such as $\widetilde{\lambda} = 1.00001$ in Fig. 2(c), it is obvious that the atomic ES is not only optimal but also sustained. Comparing Figs. 2(c) and 2(d), if $\widetilde{\lambda}$ is beyond the optimal parameter value, the squeezing degree of ES decreases with the value of $\widetilde{\lambda}$ increasing. It means that the relation between the squeezing degree of ES and the non-Hermiticity parameter $\widetilde{\lambda}$ is nonlinear. We can say that, under the proper choice of the non-Hermiticity parameter $\widetilde{\lambda}$, the atomic SOES can be generated.

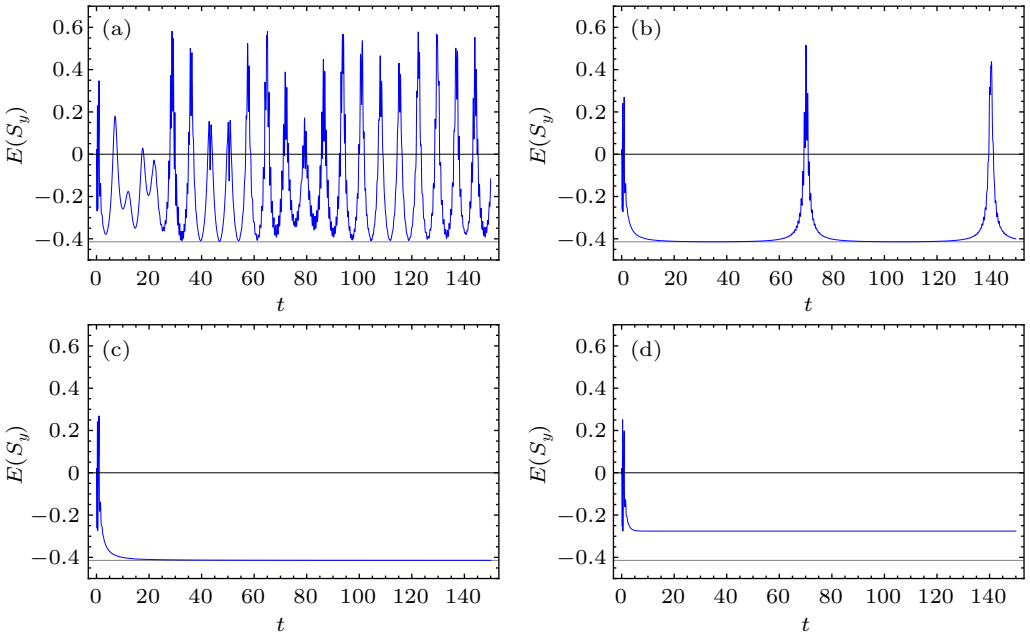


Fig. 2. (color online) The dynamics of the ES factor $E(S_y)$ of a two-level atom in the JC model with NH operation. The atom is initially in the excited state with $\theta = 0$, the cavity field in a coherent state with average photon number $\overline{n} = 25$, and the coupling strength $g = 1$. The NH operation with four different non-Hermiticity parameters $\widetilde{\lambda}$: (a) 0.9, (b) 0.999, (c) 1.00001, and (d) 1.1.

4.2. Case II

In the second case, we consider that the atom is initially prepared in a superposition state with $\theta = \pi/2$, and the cavity field initially in a coherent state with $\overline{n} = 36$. For comparison, the atomic ES in the absence of NH operation is briefly introduced. The dynamics of ES factors $E(S_x)$ and $E(S_y)$ are displayed in Figs. 3(a) and 3(b), respectively. It can be seen from Fig. 3 that $E(S_x)$ exists the optimal ES at the beginning, and

$E(S_x)$ and $E(S_y)$ exhibit alternate ES. However, the squeezing degree of alternate ES decreases with time, and gradually disappears. It means that, in the absence of NH operation, the optimal ES of the atom in this case can not keep sustained.

In the presence of NH operation, the dynamics of ES factors $E(S_x)$ and $E(S_y)$ are plotted in Fig. 4. $E(S_x)$ is depicted by Figs. 4(a), 4(c), and 4(e), while figures 4(b), 4(d), and 4(f) describe $E(S_y)$. It can be seen that the squeezing degree and the number of times of ES in the component S_x obviously decrease with the non-Hermiticity parameter $\widetilde{\lambda}$ increasing, and eventually disappear in the long time evolution. The ES factor $E(S_y)$ plotted in Figs. 4(b) and 4(d) shows that the number of times of ES decreases with $\widetilde{\lambda}$ increasing, but the optimal ES periodically shows up. Particularly, when $\widetilde{\lambda} = 1.00001$ shown in Fig. 4(f), $E(S_y)$ exhibits SOES. Moreover, comparing Figs. 4(a) and 4(f), we can find that the optimal ES which initially exists in the component S_x is completely transferred to the other component S_y, which is aroused by the proper NH operation which we performed on the atom.

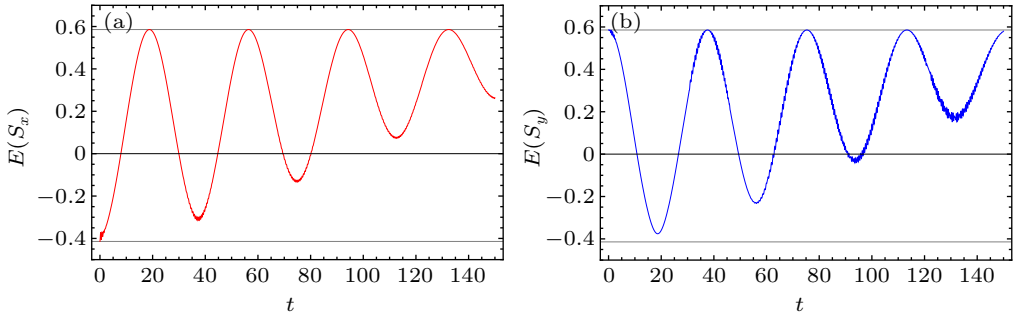


Fig. 3. (color online) The dynamics of ES factors $E(S_\alpha)$ of a two-level atom in the JC model without NH operation. The atom is initially in a superposition state with $\theta = \pi/2$, the cavity field in a coherent state with average photon number $\overline{n} = 36$, and the coupling strength $g = 1$. (a) $E(S_x)$ and (b) $E(S_y)$

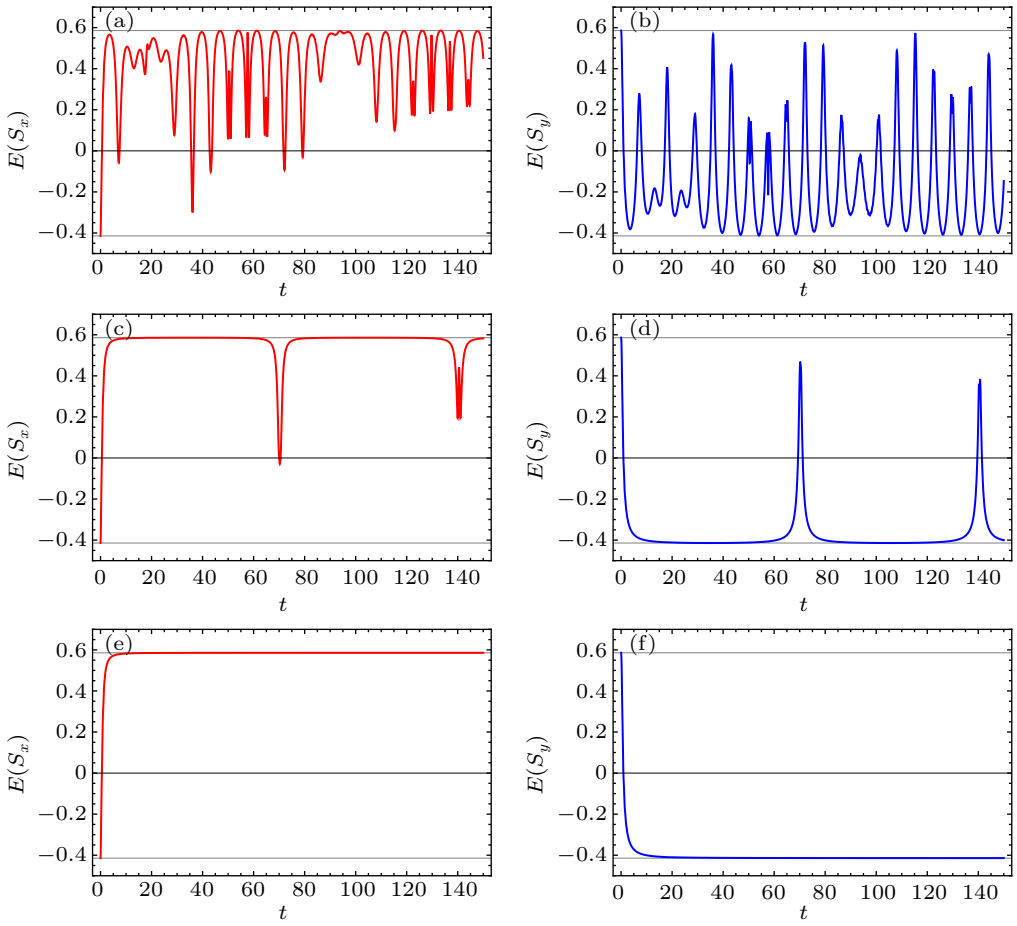


Fig. 4. (color online) The dynamics of ES factors $E(S_\alpha)$ of a two-level atom in the JC model with NH operation. The atom is initially in a superposition state with $\theta = \pi/2$, the cavity field in a coherent state with average photon number $\overline{n} = 36$, and the coupling strength $g = 1$. (a) $E(S_x)$ for $\widetilde{\lambda} = 0.9$, (b) $E(S_y)$ for $\widetilde{\lambda} = 0.9$, (c) $E(S_x)$ for $\widetilde{\lambda} = 0.999$, (d) $E(S_y)$ for $\widetilde{\lambda} = 0.999$, (e) $E(S_x)$ for $\widetilde{\lambda} = 1.00001$, and (f) $E(S_y)$ for $\widetilde{\lambda} = 1.00001$.

5. Discussion

It is well known that quantum noises in quantum systems can be suppressed by squeezing effects, an optimal squeezing state corresponds to a quantum state with minimum quantum noise, and sustained squeezing can provide long enough operation time for using the squeezing resource. Hence a good squeezing resource should possess both optimal squeezing degree and long persistence time. In Section 4, we have demonstrated that, under a proper NH operation, the atomic SOES can be generated. Here we give a possible physical explanation for the generation of SOES via the NH approach. As we all know, in conventional quantum mechanics, the Hamiltonian of a physical system requires the Hermiticity to ensure that the energy of the system is real, and that the time evolution is unitary. In order to give the NH system with non-unitary evolution a meaning in conventional quantum mechanics, Günther *et al.* proposed that the NH system can be Naimark-dilated and reinterpreted as a subsystem of a Hermitian system in a higher-dimensional Hilbert space, and the embedding of NH system into a higher-dimensional Hilbert space can be deemed as a strengthening of the wormhole analogy introduced by Ref. [9]. Therefore, if the information flow which represents the uncertainty of the system flows into a passage which is like a wormhole aroused by the non-Hermiticity of the operation, it is not strange that the entropy uncertainty of the system can be decreased.[23] Under the proper choice of the non-Hermiticity intensity, the entropy uncertainty even can be decreased to zero. It can be speculated that SOES also can be generated under the proper NH operation, because ES is defined by the entropic uncertainty relation and is measured by the quantum information entropy. Moreover, although the operation described by a NH Hamiltonian having the non-Hermiticity which is an unconventional property, we have to measure the atomic ES in conventional quantum mechanics, which requires renormalization of the density matrix to ensure trace preserving. It is worth pointing out that the similar requirement of a renormalization procedure can also appears in other operations, such as weak measurement. In other words, the non-Hermiticity of the Hamiltonian and the non-unitarity of the time evolution in NH operations are the main causes of the generation of the atomic SOES.

6. Conclusion

We investigated ES of a two-level atom prepared initially in two different states in the JC model, and provided a scheme to realize the generation of the atomic SOES via a proper NH operation. Firstly, in the absence of NH operation, the atomic ES discontinuously shows up and gradually disappears in the long time evolution. Secondly, in the presence of NH operation, ES of atomic polarization components greatly depends on the non-Hermiticity intensity in the operation. The NH operation not only increases the squeezing degree but also makes the persistence time longer, even generates the atomic SOES via the proper choice of the non-Hermiticity parameter. Our results provided a scheme to generate SOES. This is important for the precision measurement of physical observable and the ultra-low noise quantum communication, and it can be regarded as potential application in the realization of low noises quantum information processing.

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