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On the usefulness of an assisted driving Hamiltonian for quantum adiabatic evolution*

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In our recent work we showed, by investigating the initialization of some unusual forms of assisted driving Hamiltonians, that the addition of an assisted driving Hamiltonian is not always useful in quantum adiabatic evolution. These unusual forms are those that are not the relatively fixed ones that are widely used in the literature. In this paper, we continue this study, providing further evidence for the validity of the conclusion above by researching some relatively more complex forms of assisted driving scheme, which generalize the ones studied in our previous work.

Keywords: quantum adiabatic evolution, assisted driving Hamiltonian, quantum computing

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In around the year 2000, quantum adiabatic evolution was proposed by Farhi *et al.*[1] as an alternative quantum computation model. Since then, this new model has attracted a lot of attention. Various aspects of quantum adiabatic computation have been widely investigated. For example, whether quantum adiabatic algorithms can efficiently solve nondeterministic polynomial time (NP) problems[2–5] has been actively researched in the past few years. Quantum pattern recognition for two-qubit systems is another algorithm that has been designed by quantum adiabatic evolution (see Ref. [6]). Recently, a lower bound of the required computation time for the quantum adiabatic algorithm in Ref. [6] was estimated by means of an analytical solution of a time-dependent Hamiltonian.[7] Another important aspect of quantum adiabatic computation which attracts a lot of attention is its experimental implementation. For the latest developments in this direction, one can refer to the related literature for more detail, for example, Refs. [8] and [9]. In particular, recently in Ref. [9], noise-induced adiabaticity in a nuclear magnetic resonance system was experimentally realized, and the adiabatic Hadamard gate and entangled state were demonstrated. It was found that the noise will not destroy, but rather induce, the adiabatic process, contrary to what one would intuitively expect. In Ref. [10], the authors gave concrete reasons showing why quantum adiabatic computation and its specific implementation, D-Wave computers, are so attractive.

The idea behind quantum adiabatic computation can be recalled as follows. We use the Hamiltonian $H(t)$ to characterize a time-dependent quantum system, and let this quantum system evolve according to the Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H(t)|\psi(t)\rangle, \tag{1}$$

in which $|\psi(t)\rangle$ is the simultaneous state of the system. At the beginning of the adiabatic evolution, the ground state of the initial Hamiltonian $H(0) = H_{\mathrm{i}}$ is easy to prepare, and after evolution of the system, the ground state of the final Hamiltonian $H(T) = H_{\mathrm{f}}$ encodes the solution to the problem. Rewriting the Hamiltonian $H(t)$ as $\tilde{H}(s) = H(\frac{t}{T})$, one can obtain

$$\tilde{H}(s) = (1-s)H_{\mathrm{i}} + sH_{\mathrm{f}} \tag{2}$$

for the simultaneous quantum system Hamiltonian. By the adiabatic theorem,[11] the running time of the quantum adiabatic computation with the Hamiltonian above can be bounded. Also, it is known that if one wants to improve speedup over classical algorithms by using quantum adiabatic evolution, one cannot take the above parameter s to be t/T, where t is the time and T is the total running time of the quantum algorithm. Instead, one has to spend most of the time near the point where the gap function of the simultaneous system Hamiltonian has its minimum.

It is easy to see that the simple linear form of the quantum Hamiltonian in Eq. (2) can be generalized to the nonlinear forms. For example, besides the main Hamiltonian as shown in Eq. (2), an assisted driving scheme can also be considered, as will be shown more clearly soon. In fact, there is no reason not to try nonlinear quantum Hamiltonians which involve terms that are not linear combinations of the initial and final

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Hamiltonians. For this, the only requirement is that the adiabaticity condition holds. That is that, as just stated, at the beginning of the adiabatic evolution, the quantum system stays in the ground state of the initial Hamiltonian, and after adiabatic evolution, the quantum system ends in the ground state of the final Hamiltonian. In Ref. [12], the authors declared that by adding an assisted driving item to the main system Hamiltonian, it is possible that the algorithmic failures can be converted into successes for the problems considered there. It was found in Ref. [13] that, when adding a transitionless quantum driving term in the local adiabatic quantum search, the success rate of the quantum algorithm could exactly reach 1 with arbitrary evolution time, and the reason for the drastic decrease in the evolution time was that the driving Hamiltonian increased the lowest eigenvalues of the quantum system Hamiltonian to a corresponding maximum value. In Refs. [15]–[17], we found that for the quantum search[14] with the final state being orthogonal to the initial state, only when an assisted driving Hamiltonian is used can the effectiveness of the corresponding quantum adiabatic algorithm be guaranteed. From these, it seems that the additional driving Hamiltonian plays a vital role in the efficiency of the quantum adiabatic computation. It can also be seen that, in the related literature where assisted driving schemes are used, they all have a relatively fixed form. For examples, one can refer to those studied in Refs. [15]–[18].

In our recent work,[18] we have thoroughly studied quantum adiabatic evolution with the following generalized form of system Hamiltonian:

$$\tilde{H}(s) = (1-s)H_{\rm i} + sH_{\rm f} + s(1-s)H_{\rm m}, \tag{3}$$

where $H_{\rm m}$ is an assisted driving scheme which is only turned on during the quantum computation. Specifically in the quantum search problem, the initial and final Hamiltonians can be set as follows:

$$H_{\rm i} = I - |\varphi_0\rangle\langle\varphi_0|, \quad H_{\rm f} = I - |\varphi_1\rangle\langle\varphi_1|. \tag{4}$$

The task is to evolve the initial state $|\varphi_0\rangle$ to the final state $|\varphi_1\rangle$, and thus estimate the efficiency of the resulting quantum adiabatic algorithm. It was found that for the special kind of search problem where the two quantum states $|\varphi_0\rangle$ and $|\varphi_1\rangle$ are orthogonal to each other,[18] the additional item $H_{\rm m}$ could not be arbitrary to ensure the effectiveness of the quantum adiabatic algorithm. However, as just stated, it seems that there does not exist any other constraint for designing $H_{\rm m}$ except that it should be turned off at the beginning and the end of the quantum adiabatic computation. Note here that by saying effectiveness, we mean that the quantum adiabatic evolution will not fail for the problem, i.e., the running time of the resulting quantum adiabatic algorithm is finite.

In this paper, we continue the study of this special kind of search problem in the framework of quantum adiabatic evolution with the system Hamiltonian in the form constructed in Eq. (3). The difference with our previous work is that here we investigate a certain form of the assisted driving Hamiltonian $H_{\rm m}$, which can be thought as a natural generalization of the ones studied in Ref. [18]. Our result can thus be considered as a piece of complementary work which will strengthen our main earlier conclusion.

Now we turn to the main part of the work here. Consider the following assisted driving Hamiltonian:

$$H_{\rm m} = x|\varphi_0\rangle\langle\varphi_0| + y|\varphi_0^{\perp}\rangle\langle\varphi_0^{\perp}|, \tag{5}$$

in which the parameters x and y are constants. It is easy to see that if $x = 1$ and $y = 0$, or $x = 0$ and $y = 1$, the assisted driving scheme $H_{\rm m}$ just degenerates to the ones investigated in Ref. [18]. With these special cases of assisted driving items added to the main quantum Hamiltonian, it was found previously that both of the resulting quantum adiabatic evolutions would fail totally for the special kind of search problem. This tells us the important fact that the design of the assisted driving Hamiltonian cannot be arbitrary in quantum adiabatic computation. Here we will see that this conclusion still holds for the assisted driving Hamiltonian with the form specified in Eq. (5), which would make the conclusion above more persuasive.

To see this, as in Ref. [18], by writing $|\varphi_1\rangle = A|\varphi_0\rangle + B|\varphi_0^{\perp}\rangle$, we can get the two-dimensional matrix form of the total system Hamiltonian in Eq. (3) with the extra driving scheme constructed in Eq. (5)

$$\tilde{H}(s) = \begin{pmatrix} sB^2 + s(1-s)x & -sAB \\ -sAB & 1 - sB^2 + s(1-s)y \end{pmatrix},$$

where $A = |\langle\varphi_0|\varphi_1\rangle|$ and $B = \sqrt{1-A^2}$. From this, we can also easily obtain the two eigenvalues of the matrix and thus the gap between them as

$$g(s) = \sqrt{G(s)}, \tag{6}$$

in which

$$\begin{aligned} G(s) \;=\; & 1 + (2y - 2x - 4B^2)s(1-s) \\ & - 4B^2(y-x)s^2(1-s) + (x-y)^2 s^2 (1-s)^2. \end{aligned} \tag{7}$$

For our main purpose here, below we shall take into consideration whether the minimum value of the function $g(s)$ is zero or not.

Now return to the context of the special quantum search problem, in which the overlap between the initial state and the final state is equal to zero, i.e., $A = 0$. In this case, the auxiliary gap function can be further simplified as

$$\begin{aligned} G(s) \;=\; & 1 + (2y - 2x - 4)s(1-s) \\ & - 4(y-x)s^2(1-s) + (x-y)^2 s^2 (1-s)^2. \end{aligned} \tag{8}$$

It is obvious that if $x = y = 0$, then we recover the results presented in a series of our previous works.[15–17] Another case is that when the parameters are set as $x = y \neq 0$, the minimum value of the gap function can reach zero at the time point

$s = 1/2$. This of course will lead to the breakdown of the quantum adiabatic evolution. For example, if $x = y = 1$, we see that one of the results in Ref. [18] can be regained. In the following discussions, we will thus assume that $x \neq y$ holds.

By denoting $w = y - x \neq 0$, we know that the squared gap function $G(s)$ can be re-expressed as

$$G(s) = 1 + (2w-4)s(1-s) - 4ws^2(1-s) + w^2s^2(1-s)^2, \quad (9)$$

which can be further rewritten as follows:

$$G(s) = (1-2s)^2 + 2ws(1-s)(1-2s) + w^2s^2(1-s)^2. \quad (10)$$

It can be clearly seen that

$$G(s) = [(1-2s) + ws(1-s)]^2. \quad (11)$$

For convenience, the auxiliary function

$$\bar{g}(s) = (1-2s) + ws(1-s), \quad (12)$$

is introduced so that it is easy to verify that

$$\bar{g}(0) = 1,\ \bar{g}(1) = -1. \quad (13)$$

Therefore, it is trivial to see that $\bar{g}(0)\bar{g}(1) < 0$ holds from the above expressions. By the intermediate value theorem, we know that there shall exist some point $s_0 \in (0,1)$ so that the minimum of $G(s)$ is zero, i.e., $G(s_0) = 0$. This definitely results in the failure of the quantum adiabatic evolution above. At this point, we can check again why the assisted driving Hamiltonian should have a relatively fixed form in the literature, i.e., $H_{\rm m} = |\varphi_0\rangle\langle\varphi_0^{\perp}| + |\varphi_0^{\perp}\rangle\langle\varphi_0|$, which is designed to avoid the uselessness of the quantum adiabatic computation for the search problem with zero overlap between the initial and final states of the system.

In this paper, we have studied quantum adiabatic evolution in which a certain form of assisted driving Hamiltonian is added to the usual main system Hamiltonian. It should be noted that here we just consider another kind of quantum search problem, in which the initial state is orthogonal to the final state. The main purpose is to discuss the usefulness of the assisted driving Hamiltonian beyond the widely used form for the quantum adiabatic evolution of this problem, i.e., to ascertain whether it is effective for the quantum computation. For this, we are only concerned with whether the minimum value of the gap function of the system Hamiltonian for the resulting quantum adiabatic evolution is zero or not, rather than whether it is a local or global type of quantum adiabatic computation being used, the difference between which was discussed in detail in Ref. [19]. In quantum adiabatic evolution with Grover speedup, this mainly concerns how to achieve improvement in the algorithmic performance over classical computation. In fact, when studying the Grover search problem by quantum adiabatic evolution, an assisted driving Hamiltonian can also be used to speed up the quantum computation from square root to $O(1)$ time complexity. See, for example, the early work of the authors of this paper,[16,17], that of Andrecut and Ali,[20] and the recent result in Ref. [13].

It is obvious that the assisted driving scheme here is the natural generalization of the ones investigated in our previous work. The main purpose there is to research the usefulness of some unusual forms of assisted driving Hamiltonian in the quantum adiabatic computation of a special kind of search problem.[18] In fact, it is not hard to see that considering other generalized combinational forms of assisted driving Hamiltonian and then discussing their usefulness in quantum adiabatic computation in the way presented here will cause no essential difficulty. For example, we can study the assisted driving scheme given by $H_{\rm m} = x|\varphi_0\rangle\langle\varphi_0^{\perp}| + y|\varphi_0^{\perp}\rangle\langle\varphi_0^{\perp}|$ or $H_{\rm m} = x|\varphi_0^{\perp}\rangle\langle\varphi_0| + y|\varphi_0\rangle\langle\varphi_0|$. However, here we omit this to avoid repetition and trivial details. Additionally, we see that the new method used here to verify that the minimum value of the gap function of the system Hamiltonian is zero can be used as an alternative way to derive the results that appeared in our previous work, but it will considerably simplify the calculations there. From the result presented here, we can further see why the assisted driving Hamiltonians studied in the related references should enjoy a unified form for the corresponding quantum adiabatic evolutions. We hope our conclusion can find its analogue in other types of problem in which the quantum adiabatic evolution computational model can be adopted.

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