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Modulating quantum Fisher information of qubit in dissipative cavity by coupling strength*

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By using the non-Markovian master equation, we investigate the effect of the cavity and the environment on the quantum Fisher information (QFI) of an atom qubit system in a dissipation cavity. We obtain the formulae of QFI for two different initial states and analyze the effect of the atom–cavity coupling and the cavity–reservoir coupling on the QFI. The results show that the dynamic behavior of the QFI is obviously dependent on the initial atomic states, the atom–cavity coupling, and the cavity–reservoir coupling. The stronger the atom–cavity coupling, the quicker the QFI oscillates, and the slower the QFI decreases. In particular, the QFI will tend to be a stable value rather than zero if the atom–cavity coupling is large enough. On the other hand, the smaller the cavity–reservoir coupling, the stronger the non-Markovian effect, and the slower the QFI decays. In other words, choosing the best parameter can improve the accuracy of the parameter estimation. In addition, the physical explanation of the dynamic behavior of the QFI is given by means of the QFI flow.

Keywords: quantum Fisher information, parameter estimation, dissipative cavity

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1. Introduction

Quantum Fisher information (QFI), which was originally introduced by Fisher in 1925, plays an important role in quantum estimation theory and quantum information theory, and has been widely studied.[1] The QFI is used to describe the probability distribution of a threshold value of the parameter estimation.[2] The parameter estimation in probability distribution is a very basic and essential content in information theory. Since the quantum measurement is found to be probabilistic, parameter estimation in probability distribution is gradually applied to the quantum field. The quantum Cramér–Rao (QCR) theorem shows that the limit of the accuracy of the parameter estimation is determined by the reciprocal of QFI.[2,3] Therefore, how to increase the QFI has now become a key issue for research. The definition of quantum Fisher information is based on the symmetric logarithmic derivative operator and this form is the largest of all possible Fisher information (i.e., this QFI takes advantage of all possible information). In this paper, we will use the symmetric logarithmic derivative (SLD) method to calculate the QFI.[2,3]

Recently, more attention has been paid to the QFI,[4–9] and QFI has also been widely applied in other quantum information tasks, such as entanglement detection,[10] non-Markovian description and determination,[11,12] and investigation of the uncertainty relations.[13–15] The researches on the QFI mentioned are focused on the qubit in a bosonic environment.[4] However, in this work, we study the QFI of an atom in a dissipation cavity by using the dressed-state and the arbitrary state, and its physical significance is given. It is found that the QFI has different tendency when the coupling strength is different. The QFI monotonously decreases and eventually disappears in the case that the atom–cavity is weakly coupled. When the atom is strongly coupled with the cavity, the QFI will repeatedly oscillate. Furthermore, we also analyze the dynamic behavior of QFI by using the QFI flow.[4,16]

The rest of this paper is organized as follows. In Section 2, we give a model of an atom in a dissipation cavity and the reduced density matrix of the atom. In Section 3, we introduce the QFI and the QFI flow. In Section 4, we discuss the influence of the cavity and the environment on the QFI. Finally, we end with a brief summary of important results in Section 5.

2. Physical model

In this paper, we consider an atom qubit interacting with a cavity, and the cavity is coupled to a bosonic environment.[17] The total Hamiltonian is given by

$$H = \frac{1}{2}\omega_0\sigma_z + \omega_0 a^\dagger a + \Omega\left(a\sigma_+ + a^\dagger\sigma_-\right) + \sum_k \omega_k c_k^\dagger c_k + \left(a^\dagger + a\right)\sum_k g_k\left(c_k^\dagger + c_k\right), \quad (1)$$

where $a^\dagger(a)$ is the creation (annihilation) operator of the cavity, σ_i $(i = \pm, z)$ is the atomic operator,[18] ω_0 is the atomic

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Bohr frequency, Ω is the atom–cavity coupling constant, $c_k^\dagger$ (c_k) is the creation (annihilation) operator of the reservoir, and g_k is the cavity–reservoir coupling strength.

Using the second order of the time convolutionless (TCL) expansion[19] which neglects the atomic spontaneous emission and the Lamb shifts, and assuming one initial excitation and a reservoir at zero temperature, the non-Markovian master equation for the density operator $R(t)$ in the dressed-state basis $\{|E_{1+}\rangle, |E_{1-}\rangle, |E_0\rangle\}$ is

$$\begin{aligned}\dot{R}(t) &= -\mathrm{i}[H_{\mathrm{ac}}, R(t)] \\ &\quad + \gamma(\omega_0+\Omega,t)\left(\frac{1}{2}|E_0\rangle\langle E_{1+}|R(t)|E_{1+}\rangle\langle E_0| \right. \\ &\quad \left. -\frac{1}{4}\{|E_{1+}\rangle\langle E_{1+}|, R(t)\}\right) \\ &\quad + \gamma(\omega_0-\Omega,t)\left(\frac{1}{2}|E_0\rangle\langle E_{1-}|R(t)|E_{1-}\rangle\langle E_0| \right. \\ &\quad \left. -\frac{1}{4}\{|E_{1-}\rangle\langle E_{1-}|, R(t)\}\right), \end{aligned} \tag{2}$$

where $H_{\mathrm{ac}} = \frac{1}{2}\omega_0\sigma_z + \omega_0 a^\dagger a + \Omega(a\sigma_+ + a^\dagger\sigma_-)$, $|E_{1\pm}\rangle = (|1g\rangle \pm |0e\rangle)/\sqrt{2}$ are the eigenstate of H_{ac} with one total excitation, with energy $\omega_0/2 \pm \Omega$, and $|E_0\rangle = |0g\rangle$ is the ground state, with energy $-\omega_0/2$. The time-dependent decay rates for $|E_{1-}\rangle$ and $|E_{1+}\rangle$ are $\gamma(\omega_0-\Omega,t)$ and $\gamma(\omega_0+\Omega,t)$, respectively.

If the reservoir at zero temperature is modeled with a Lorentzian spectral density[20,21]

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_1-\omega)^2+\lambda^2}, \tag{3}$$

where the parameter λ defines the spectral width of the coupling, which is connected to the reservoir correlation time τ_{R} by $\tau_{\mathrm{R}} = \lambda^{-1}$, and the parameter γ_0 is related to the relaxation time scale τ_{S} by $\tau_{\mathrm{S}} = \gamma_0^{-1}$. $\lambda > 2\gamma_0$ is called a weak coupling regime or a Markvian regime. In this regime, the relaxation time is greater than the reservoir correlation time. Supposing the spectrum peaks at the frequencies of the states $|E_{1-}\rangle$, i.e., $\omega_1 = \omega_0 - \Omega$, the decay rates for the two dressed states $|E_{1\pm}\rangle$ are respectively expressed as[20] $\gamma(\omega_0-\Omega,t) = \gamma_0(1-\mathrm{e}^{-\lambda t})$ and $\gamma(\omega_0+\Omega,t) = \frac{\gamma_0\lambda^2}{4\Omega^2+\lambda^2}\left[1+(\frac{2\Omega}{\lambda}\sin 2\Omega t - \cos 2\Omega t)\mathrm{e}^{-\lambda t}\right]$.

We can acquire the density matrix elements of the atom–cavity at all times from Eq. (2)

$$\begin{aligned} &R_{11}(t) = A_{11}^{11}R_{11}(0),\ R_{12}(t) = A_{12}^{12}R_{12}(0),\ R_{13}(t) = A_{13}^{13}R_{13}(0), \\ &R_{22}(t) = A_{22}^{22}R_{22}(0),\ R_{23}(t) = A_{23}^{23}R_{23}(0), \\ &R_{33}(t) = A_{33}^{11}R_{11}(0) + A_{33}^{22}R_{22}(0) + A_{33}^{33}R_{33}(0), \end{aligned} \tag{4}$$

where $R_{ij}(0)(i,j=1,2,3)$ is the density matrix element of the initial state, and

$$\begin{aligned} &A_{11}^{11} = \mathrm{e}^{-\frac{1}{2}I_+},\ A_{12}^{12} = \mathrm{e}^{-2\mathrm{i}\Omega t}\mathrm{e}^{-\frac{1}{4}(I_+ + I_-)},\ A_{13}^{13} = \mathrm{e}^{-\mathrm{i}(\omega_0+\Omega)t}\mathrm{e}^{-\frac{1}{4}I_+}, \\ &A_{22}^{22} = \mathrm{e}^{-\frac{1}{2}I_-},\ A_{23}^{23} = \mathrm{e}^{-\mathrm{i}(\omega_0-\Omega)t}\mathrm{e}^{-\frac{1}{4}I_-}, \\ &A_{33}^{11} = 1 - A_{11}^{11},\ A_{33}^{22} = 1 - A_{22}^{22},\ A_{33}^{33} = 1, \end{aligned} \tag{5}$$

where

$$\begin{aligned} I_- &= \gamma_0 t + \frac{\gamma_0}{\lambda}(\mathrm{e}^{-\lambda t} - 1), \\ I_+ &= \frac{\gamma_0\lambda^2}{4\Omega^2+\lambda^2}\left[t - \frac{4\Omega\mathrm{e}^{-\lambda t}\sin(2\Omega t)}{4\Omega^2+\lambda^2} \right. \\ &\quad \left. + \frac{(\lambda^2-4\Omega^2)(\mathrm{e}^{-\lambda t}\cos(2\Omega t) - 1)}{\lambda(4\Omega^2+\lambda^2)}\right]. \end{aligned} \tag{6}$$

By taking a partial trace of the atom–cavity density matrix over the cavity degree of freedom, the atomic reduced operator $\rho(t)$ is given by[22]

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{21}(t) & \rho_{22}(t) \end{pmatrix}, \tag{7}$$

where

$$\begin{aligned} \rho_{11}(t) &= \frac{1}{2}(R_{11}(t) + R_{12}(t) + R_{21}(t) + R_{22}(t) + 2R_{33}(t)), \\ \rho_{12}(t) &= \frac{1}{\sqrt{2}}(R_{13}(t) - R_{23}(t)), \\ \rho_{21}(t) &= \frac{1}{\sqrt{2}}(R_{31}(t) - R_{32}(t)), \\ \rho_{22}(t) &= \frac{1}{2}(R_{11}(t) - R_{12}(t) - R_{21}(t) + R_{22}(t)). \end{aligned} \tag{8}$$

3. Quantum Fisher information

3.1. Quantum Fisher information

The QFI indicates the sensitivity of the state to the change of the parameter. Let ϕ denote a single parameter to be estimated, and the QFI is defined as[3]

$$F_\phi = \mathrm{Tr}(\rho_\phi L_\phi^2) = \mathrm{Tr}(\partial_\phi\rho_\phi L_\phi), \tag{9}$$

where L_ϕ is symmetric logarithmic derivative (SLD) for the parameter ϕ, which is a Hermitian operator determined by

$$\partial_\phi\rho_\phi = \frac{1}{2}\{\rho_\phi, L_\phi\}, \tag{10}$$

where $\partial_\phi \equiv \frac{\partial}{\partial\phi}$ and $\{\cdot,\cdot\}$ denotes the anticommutator.

An essential feature of the QFI is that we can obtain the achievable lower bound of the mean square error of unbiased estimators for the parameter ϕ through the quantum Cramér–Rao theorem[23]

$$V_{\mathrm{ar}}(\phi) \geq \frac{1}{\nu F_\phi}, \tag{11}$$

where $V_{\mathrm{ar}}(\cdot)$ denotes the variance, ϕ denotes the unbiased estimator, and ν denotes the number of repeated experiments. In the following, we use this method to calculate the QFI of the atom coupled to the dissipation cavity.

In order to understand the dynamic behavior of QFI, we introduce the QFI flow, which is defined as the change rate of the QFI by[16]

$$I_\phi = \frac{\partial F_\phi}{\partial t}. \tag{12}$$

It is well known that $I_\phi < 0$ represents the energy and information flow from the system to the environment, and $I_\phi > 0$ represents the energy and information flow from the environment to the system.

3.2. Example

Example 1 We construct a complete orthogonal basis for a three-state system with the dressed-state basis $\{|E_{1+}\rangle, |E_{1-}\rangle, |E_0\rangle\}$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} e^{i\phi} \sin\frac{\theta}{2}|E_{1+}\rangle - \frac{1}{\sqrt{2}} e^{i\phi} \sin\frac{\theta}{2}|E_{1-}\rangle + \cos\frac{\theta}{2}|E_0\rangle, \tag{13}$$

where $\theta \in [0,\pi)$ and $\phi \in [0,2\pi)$ are parameters which may be regarded as encoding the amplitude and phase information, respectively.

Inserting Eq. (13) into Eq. (7), the reduced density matrix of the atom is given by

$$\rho'(t) = \begin{pmatrix} \rho'_{11}(t) & \rho'_{12}(t) \\ \rho'_{21}(t) & \rho'_{22}(t) \end{pmatrix}, \tag{14}$$

where the matrix elements are

$$\begin{aligned}
\rho'_{11}(t) &= \left[1 - \frac{1}{4}\left(A_{11}^{11} - A_{12}^{12} - A_{21}^{21} + A_{22}^{22}\right)\right] \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}, \\
\rho'_{12}(t) &= \frac{1}{2}\left(A_{13}^{13} + A_{23}^{23}\right) e^{-i\phi} \sin\frac{\theta}{2}\cos\frac{\theta}{2}, \\
\rho'_{21}(t) &= \frac{1}{2}\left(A_{31}^{31} + A_{32}^{32}\right) e^{i\phi} \sin\frac{\theta}{2}\cos\frac{\theta}{2}, \\
\rho'_{22}(t) &= \frac{1}{4}\left(A_{11}^{11} - A_{12}^{12} - A_{21}^{21} + A_{22}^{22}\right) \sin^2\frac{\theta}{2}.
\end{aligned} \tag{15}$$

From Eqs. (9) and (14), we obtain the QFI of the parameter ϕ

$$F_\phi = \frac{1}{4}\left(A_{13}^{13} + A_{23}^{23}\right)\left(A_{31}^{31} + A_{32}^{32}\right) \sin^2\theta. \tag{16}$$

Example 2 We consider a simple example to calculate the QFI. We use the standard basis $|e\rangle \equiv (1,0)^T$ and $|g\rangle \equiv (0,1)^T$, corresponding to the ground state and excited state, respectively. Consider an arbitrary single-qubit state

$$|\psi_2\rangle = \cos\frac{\theta}{2}|e\rangle + e^{i\phi} \sin\frac{\theta}{2}|g\rangle, \tag{17}$$

where θ and ϕ refer to the polar and azimuth angles on the Bloch sphere, respectively. Here, two parameters θ and ϕ in Eq. (17) are assumed to be unitary encoded.

With Eqs. (7) and (17), the atom reduced density matrix $\rho(t)$ is as

$$\rho''(t) = \begin{pmatrix} \rho''_{11}(t) & \rho''_{12}(t) \\ \rho''_{21}(t) & \rho''_{22}(t) \end{pmatrix}, \tag{18}$$

and the matrix elements are

$$\begin{aligned}
\rho''_{11}(t) &= \left[1 - \frac{1}{4}\left(A_{11}^{11} - A_{12}^{12} - A_{21}^{21} + A_{22}^{22}\right)\right] \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}, \\
\rho''_{12}(t) &= \frac{1}{2}\left(A_{13}^{13} - A_{23}^{23}\right) e^{-i\phi} \sin\frac{\theta}{2}\cos\frac{\theta}{2}, \\
\rho''_{21}(t) &= \frac{1}{2}\left(A_{31}^{31} - A_{32}^{32}\right) e^{i\phi} \sin\frac{\theta}{2}\cos\frac{\theta}{2}, \\
\rho''_{22}(t) &= \frac{1}{4}\left(A_{11}^{11} - A_{12}^{12} - A_{21}^{21} + A_{22}^{22}\right) \cos^2\frac{\theta}{2}.
\end{aligned} \tag{19}$$

Using Eqs. (9) and (17), we obtain the QFI of the parameter ϕ

$$F'_\phi = \frac{1}{4}\left(A_{13}^{13} - A_{23}^{23}\right)\left(A_{31}^{31} - A_{32}^{32}\right) \sin^2\theta. \tag{20}$$

4. Discussion and results

Now we use Eqs. (16) and (20) to calculate the QFI of the parameter ϕ. Because the atom is coupled to the dissipation cavity, both of the atom–cavity coupling constant and the cavity–reservoir coupling strength can affect the dynamic behavior of the QFI on the parameter ϕ. We study the effect of the cavity–reservoir coupling strength on the QFI under both Markovian and non-Markovian regimes. In the meantime, we also study the effect of the atom–cavity coupling constant on the QFI under weak and strong coupling.

Let us begin with the first example by taking the dressed-state $|\psi_1\rangle = \frac{1}{\sqrt{2}} e^{i\phi} \sin\frac{\theta}{2}|E_{1+}\rangle - \frac{1}{\sqrt{2}} e^{i\phi} \sin\frac{\theta}{2}|E_{1-}\rangle + \cos\frac{\theta}{2}|E_0\rangle$ of the atom coupled to the dissipation cavity. We first concentrate on $\theta = \frac{\pi}{2}$. Figure 1 shows the QFI dynamics of the atom coupled to dissipation cavity with the dressed-state $|\psi_1\rangle$ in the Markovian ($\lambda = 5\gamma_0$) and the non-Markovian ($\lambda = 0.05\gamma_0$) regimes.

As we know, $\lambda > 2\gamma_0$ represents the Markovian regime of the reservoir, $\lambda < 2\gamma_0$ represents the non-Markovian regime of the reservoir, $\Omega > 2\gamma_0$ represents the strong atom–cavity coupling, and $\Omega < 2\gamma_0$ represents the weak atom–cavity coupling. As plotted in Fig. 1(a), the reservoir is Markovian ($\lambda = 5\gamma_0$), the atom has a weak coupling with the cavity ($\Omega = 0.05\gamma_0$), and F_ϕ decreases monotonously with time and quickly approaches zero. In Fig. 1(b), $\lambda = 5\gamma_0$ (the Markovian reservoir), but $\Omega = 3\gamma_0$, that is, the atom–cavity coupling is strong, and F_ϕ oscillates damply with time. Then with the increase of the time, F_ϕ again rises to 0.75 from zero. After several cycles of oscillation, F_ϕ ultimately decays to zero. Comparing Figs. 1(a) and 1(b), we know that the energy and information can be swapped effectively between the atom and the cavity when $\Omega = 3\gamma_0$ and $\lambda = 5\gamma_0$. Thus F_ϕ will oscillate significantly and finally decay to zero due to the dissipation of the Markovian reservoir. However, F_ϕ will tend to a stable value when $\Omega = 20\gamma_0$, as shown in the inset of Fig. 2(b). Figure 1(c) indicates the dynamic behavior of F_ϕ in the non-Markovian regime ($\Omega = 0.05\gamma_0$) and the weak atom–cavity

coupling ($\lambda = 5\gamma_0$). F_ϕ also reduces monotonously with time and vanishes only in the asymptotic limit $t \to \infty$. Comparing Figs. 1(a) and 1(c), it is seen that the decay rate of F_ϕ in the former is obviously larger than in the latter. As we can see from Fig. 1(d), when $\lambda = 0.05\gamma_0$ (in the non-Markovian regime) and $\Omega = 3\gamma_0$ (the strong atom–cavity), F_ϕ will tend to be a stable value instead of zero after many cycles of oscillation. Comparing Figs. 1(b) and 1(d), it is found that in the same atom–cavity coupling ($\Omega = 3\gamma_0$), due to the memory and feedback effect of the non-Markovian reservoir, the oscillation time in Fig. 1(d) is larger than that in Fig. 1(b), and the stable value in Fig. 1(d) is greater than that in Fig. 1(b).

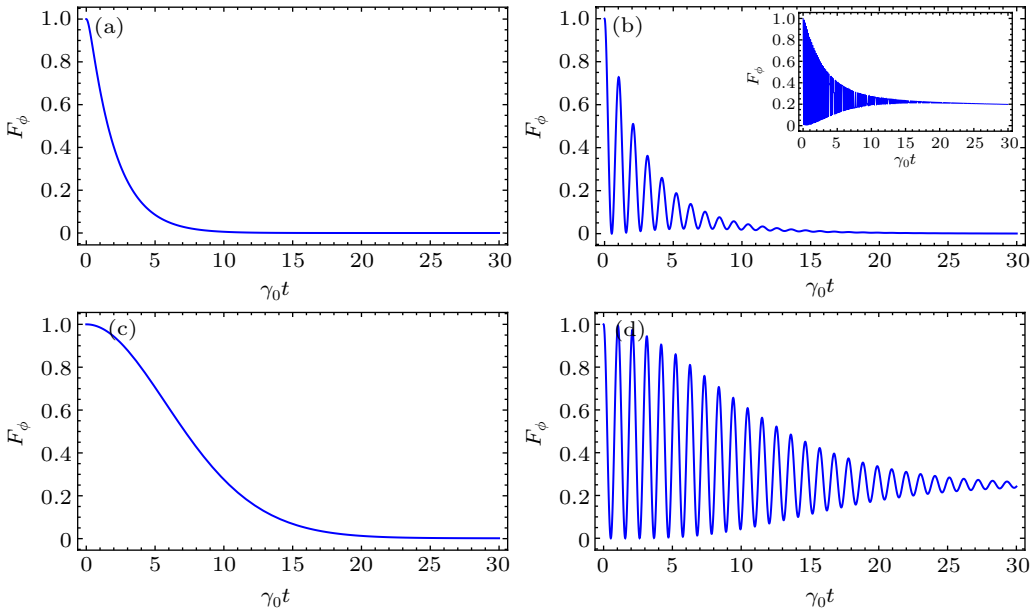


Fig. 1. (color online) F_ϕ as a function of the dimensionless quantity $\gamma_0 t$ for the initial state $|\psi_1\rangle$. Here, $\theta = \frac{\pi}{2}$, and (a) $\lambda = 5\gamma_0$, $\Omega = 0.05\gamma_0$; (b) $\lambda = 5\gamma_0$, $\Omega = 3\gamma_0$; (c) $\lambda = 0.05\gamma_0$, $\Omega = 0.05\gamma_0$; and (d) $\lambda = 0.05\gamma_0$, $\Omega = 3\gamma_0$. The inset in (b) shows the case of $\Omega = 20\gamma_0$.

In Fig. 2, we plot the QFI flow as a function of γ_0t with the initial state $|\psi_1\rangle$ in the Markovian and non-Markovian regimes for different numbers of Ω. From Fig. 2(a), we can find that I_ϕ changes rapidly to -0.4 from zero, then again rises to zero when $\lambda = 5\gamma_0$ and $\Omega = 0.05\gamma_0$. That is, I_ϕ is always less than zero. This shows that the energy and information quickly flow to the reservoir from the atom so that F_ϕ quickly decreases to zero. This result is consistent with Fig. 1(a). In Fig. 2(b), I_ϕ decreases very fast to -3.0 from zero then again rises very fast to $+2.3$, and then becomes damping oscillation and close to zero. Because $I_\phi < 0$ indicates the energy and information flow out from the atom and $I_\phi > 0$ represents the energy and information flow in the atom, F_ϕ quickly decreases when I_ϕ changes from $0 \to -0.5 \to 0$, and F_ϕ again rises quickly to a certain value when I_ϕ changes from $0 \to 2.3 \to 0$, then F_ϕ oscillates to zero, as shown in Fig. 1(b). In addition, comparing Figs. 2(b) and 2(d), it can be seen that the trend of QFI flow is similar. The difference is the value of λ. In particular, the smaller the value of λ is, the strong the non-Markovian effect is, and the slower the evolution of I_ϕ decays. By comparing Figs. 2(a) and 2(b), Fig. 2(c) and 2(d), we can see that the larger the value of Ω is, the stronger the cavity–reservoir coupling is, and the faster the QFI flow oscillates and recovers. These phenomena can be understood as the reverse flow of energy and information from the reservoir to the system. As a result, QFI is linked to the flow of information exchanged between the system and the environment.

As the second example, we consider an arbitrary single-qubit state $|\psi_2\rangle = \cos\frac{\theta}{2}|e\rangle + e^{i\phi}\sin\frac{\theta}{2}|g\rangle$ in the standard basis. In order to analyze the effect of the atom–cavity coupling Ω in the Markovian ($\lambda = 5\gamma_0$) and non-Markovian ($\lambda = 0.05\gamma_0$) regimes, we plot the evolution of F_ϕ at $\theta = \frac{\pi}{2}$ in Fig. 3. We can observe that in Fig. 3(a), the reservoir is Markovian ($\lambda = 5\gamma_0$), the atom is a weak coupling with the cavity ($\Omega = 0.05\gamma_0$), and F_ϕ rises to 0.08 from zero then oscillates damply to zero. Beside this, this figure also shows that with the increase of $\gamma_0 t$, the value of F_ϕ drops considerably, which implies that the accuracy of the estimate is higher. In Fig. 3(b), it can be found that when $\lambda = 5\gamma_0$ (the Markovian reservoir) and $\Omega = 3\gamma_0$, that is, the atom–cavity coupling is strong, F_ϕ will oscillate and increase to 1.0 from zero. In particular, when $\Omega = 20\gamma_0$, F_ϕ will damply oscillate to the stable value of 0.25 instead of zero, as shown in the inset of Fig. 3(b). Comparing Figs. 3(a) and 3(c), we can see that their F_ϕ dynamics are similar. The difference is the maximum of F_ϕ. The latter has a larger maximum and tends to zero more slowly. As shown in Figs. 3(b) and 3(d), the trend of F_ϕ is similar, except that the former eventually reaches a stable value of 0.25, and the latter disappears for the

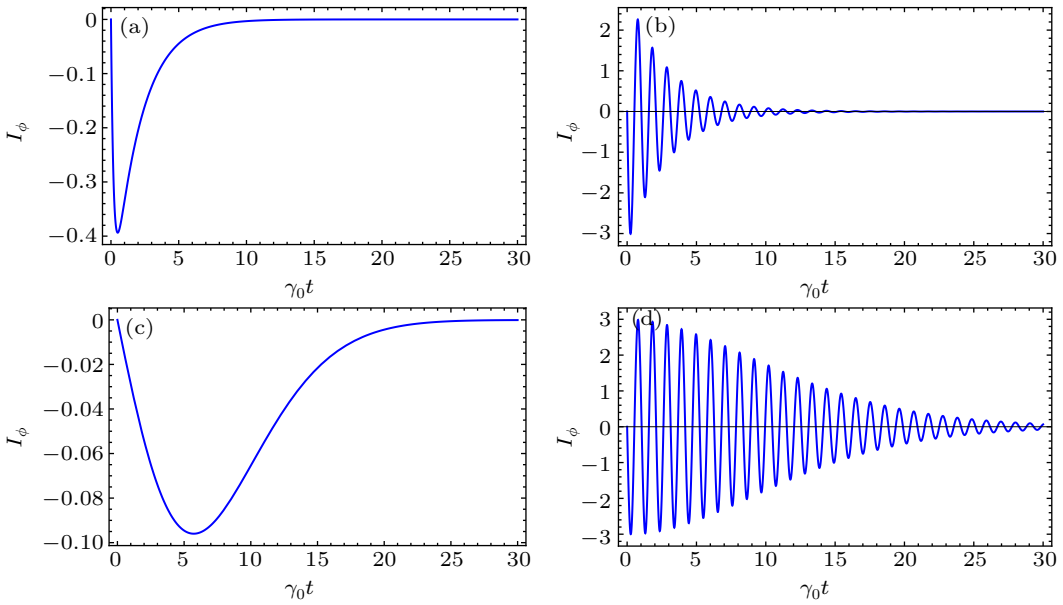


Fig. 2. (color online) The effect of the atom–cavity coupling Ω and the cavity–reservoir coupling λ on the QFI flow (I_ϕ) versus $\gamma_0 t$. Here, $\theta = \frac{\pi}{2}$, and (a) $\lambda = 5\gamma_0$, $\Omega = 0.05\gamma_0$; (b) $\lambda = 5\gamma_0$, $\Omega = 3\gamma_0$; (c) $\lambda = 0.05\gamma_0$, $\Omega = 0.05\gamma_0$; and (d) $\lambda = 0.05\gamma_0$, $\Omega = 3\gamma_0$.

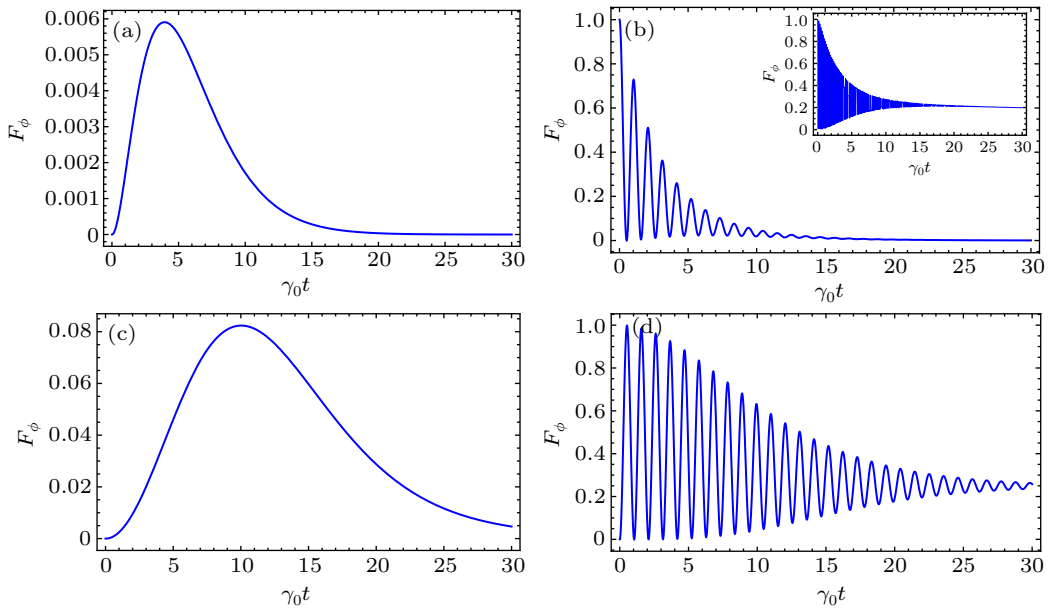


Fig. 3. (color online) F_ϕ as a function of the dimensionless quantity $\gamma_0 t$ for the initial state $|\psi_2\rangle$. Here, $\theta = \frac{\pi}{2}$, and (a) $\lambda = 5\gamma_0$, $\Omega = 0.05\gamma_0$; (b) $\lambda = 5\gamma_0$, $\Omega = 3\gamma_0$; (c) $\lambda = 0.05\gamma_0$, $\Omega = 0.05\gamma_0$; and (d) $\lambda = 0.05\gamma_0$, $\Omega = 3\gamma_0$. The inset in (b) shows the case of $\Omega = 20\gamma_0$.

smaller Ω; but when $\Omega = 20\gamma_0$, F_ϕ will tend to 0.25. In Fig. 3, with the increase of Ω, the oscillating frequency of F_ϕ will become quick and the decay of F_ϕ will become slow. But in the weak coupling, all QFIs will eventually be reduced to zero in a short time, as shown in Figs. 3(a) and 3(c). However, in the strong coupling regime, all F_ϕ will quickly oscillate and tend to a stable value of 0.25 in the end.

In Fig. 4, we plot the effect of the atom–cavity coupling Ω on the I_ϕ dynamics in the Markovian ($\lambda = 0.05\gamma_0$) and non-Markovian ($\lambda = 5\gamma_0$) regimes. In Fig. 4(a), it can be found that when $\lambda = 0.05\gamma_0$ (the Markovian regime) and $\Omega = 3\gamma_0$ (i.e., the weak atom–cavity coupling), as the time t increases, I_ϕ will increase from zero to 0.0025 and then oscillate to zero. This shows that the atom first obtains the energy and information from the cavity by the atom–cavity, and then this energy and information will disappear due to the cavity dissipation. This is corresponding to the oscillation of F_ϕ, see Fig. 3(a). In addition, it can be found that I_ϕ quickly oscillates to zero in the strong coupling in Fig. 4(b). This is accounting for the oscillation of F_ϕ, see Fig. 3(b), because the cavity has memory effect, and some of the missing information can be returned to the system of the cavity. In Fig. 4(c), I_ϕ oscillates slowly

to zero. Comparing Figs. 4(a) and 4(c), we can see that the larger the value of Ω, the slower the decay of I_ϕ to zero. In Fig. 4(d), I_ϕ quickly oscillates and decays to zero, which indicates that the information of the system and environment are quickly interflowing.

By analyzing the two examples, we can give a physical explanation of the above results. In the Markovian (non-Markovian) regime and with small Ω, F_ϕ will obviously decrease to zero, but F_ϕ will be delayed in the non-Markovian regime, as shown in Figs. 1(a), 1(c), 3(a), and 3(c). However, when Ω is very large, regardless of the Markovian or non-Markovian regime, F_ϕ will oscillate damply to the stable value, as shown in Figs. 1(b), 1(d), 3(b), and 3(d). Furthermore, there is difference for the second state $|\psi_2\rangle$ due to the memory and feedback effect of the non-Markovian environment. This difference is the change of F_ϕ from 0 to a maximum and then again to zero. By studying I_ϕ, we can understand about the change of F_ϕ. The QFI flow is positive or negative, which is consistent with the increase or decrease of Figs. 2 and 4. In the weak coupling regime ($\Omega = 0.05\gamma_0$), for both Markovian and non-Markovian regimes, I_ϕ is outward, corresponding to the decay of F_ϕ, as shown in Figs. 2(a), 2(c), 4(a), and 4(c). However, due to the interaction with the environment, I_ϕ first changes from zero to maximum, as shown in Figs. 2(a) and 2(c). In the strong coupling regime ($\Omega = 3\gamma_0$), I_ϕ is inward and outward, as shown Figs. 2(b), 2(d) and Figs. 4(b), 4(d).

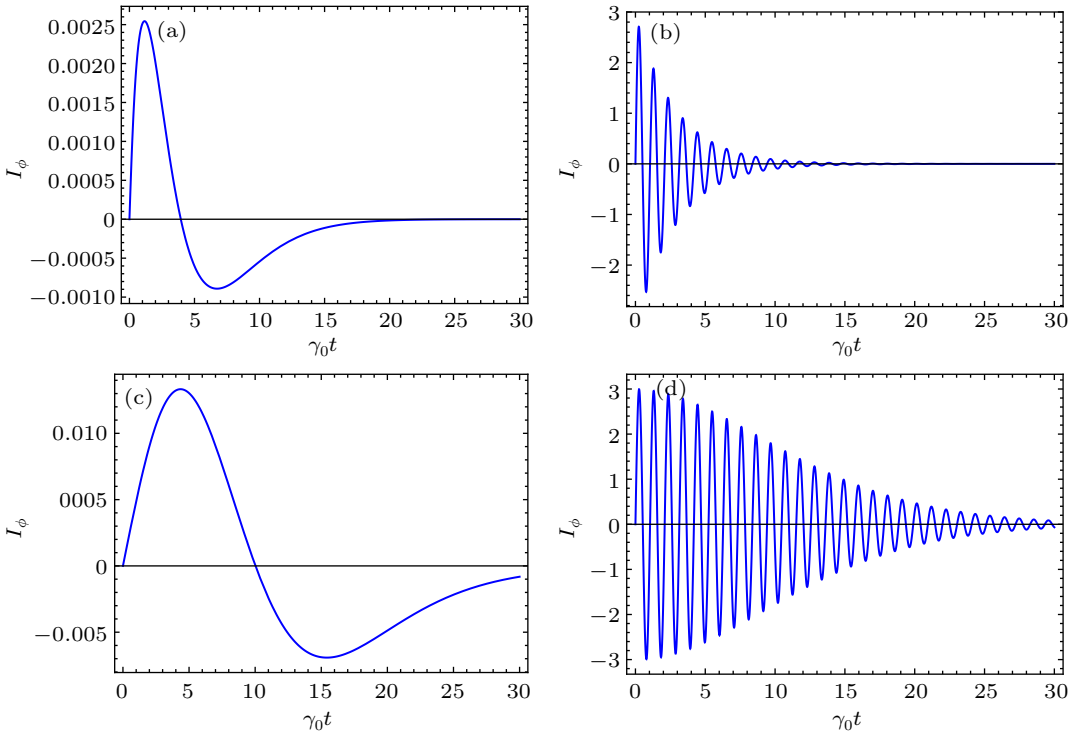


Fig. 4. (color online) The effect of the atom–cavity coupling Ω and cavity–reservoir coupling λ on the QFI flow versus $\gamma_0 t$. Here, $\theta = \frac{\pi}{2}$, and (a) $\lambda = 5\gamma_0$, $\Omega = 0.05\gamma_0$; (b) $\lambda = 5\gamma_0$, $\Omega = 3\gamma_0$; (c) $\lambda = 0.05\gamma_0$, $\Omega = 0.05\gamma_0$; and (d) $\lambda = 0.05\gamma_0$, $\Omega = 3\gamma_0$.

5. Conclusion

In conclusion, we have investigated the quantum Fisher information dynamics of the atom qubit in the dissipation cavity interacting with external environments by the TCL master equation method. We have examined two different states corresponding to two different representations: the first case is the QFI under the dressed-state basis, and the second is the QFI under an arbitrary single-qubit state. The results show that there is an obvious bias in the result of using different parameters. We demonstrate that in the weak coupling regime, the QFI about the parameter ϕ is monotonously decreased. When the strength of the atom–cavity coupling becomes stronger, the QFI will oscillate damply to the stable value of 0.25. We might consider more general parameterization than the canonical equation Eq. (9). In addition, we introduce the relationship between QFI flow and information to understand the changing trend of QFI. Thus, the QFI flow of negative value indicates that the information flows from the system to the environment, corresponding to the QFI of decay. The QFI flow of positive value means that information flows from the environment to the system, accounting for the QFI of revival. In other words, it is important to select the appropriate parameters to improve the accuracy of parameter estimation. In the future, it will be worthwhile to study the dynamical evolution of the QFI in different states, and it will also be an interesting topic to explore the estimation of multiple qubit to parameter.

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