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Hierarchical and probabilistic quantum information splitting of an arbitrary two-qubit state via two cluster states*

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Based on non-maximally entangled four-particle cluster states, we propose a new hierarchical information splitting protocol to probabilistically realize the quantum state sharing of an arbitrary unknown two-qubit state. In this scheme, the sender transmits the two-qubit secret state to three agents who are divided into two grades with two Bell-state measurements, and broadcasts the measurement results via a classical channel. One agent is in the upper grade and two agents are in the lower grade. The agent in the upper grade only needs to cooperate with one of the other two agents to recover the secret state but both of the agents in the lower grade need help from all of the agents. Every agent who wants to recover the secret state needs to introduce two ancillary qubits and performs a positive operator-valued measurement (POVM) instead of the usual projective measurement. Moreover, due to the symmetry of the cluster state, we extend this protocol to multiparty agents.

Keywords: cluster state, hierarchical quantum information splitting, positive operator-valued measurement (POVM)

PACS: 03.65.Aa, 03.67.Dd, 03.67.Hk, 03.67.Mn **DOI:** 10.1088/1674-1056/27/11/110302

1. Introduction

Following the development of quantum computing and quantum algorithms,[1] quantum cryptography has gradually become one of the most important research topics. Quantum secret sharing is one of the more significant branches of quantum cryptography. There are two kinds of quantum secret sharing protocols: sharing classic information and sharing quantum information. Sharing quantum information is also called the quantum information splitting (QIS) protocol. Hillery *et al.*[2] first proposed a quantum information splitting protocol in 1999. The QIS is used to share secret quantum information from one sender to several distant agents through quantum teleportation, via shared entangled states and classical communication channel. Every agent can recover the secret quantum information with the help of other agents. Subsequently, the quantum information splitting protocols of an arbitrary single-particle state,[3,4] two-particle state[5,6] and some other types of protocols[7,8] and applications[9,10] have successfully been developed.

However, all of these protocols have focused on the symmetry scenario; that is, agents in the agreement have the same authority to recover the quantum secret message. Gottesman[11] first pointed out that a more applicable protocol should include an asymmetrical case where the agent authority to recover a secret state is different. In 2010, Wang *et al.*[12] successfully realized a hierarchical quantum information protocol (HQIS) for distributing an arbitrary qubit to three agents via the four-qubit entangled state proposed by Yeo and Chua.[13] The three agents in that agreement have different authorities on the recovery of the quantum secret message. The agent in the upper grade only needs one of the other two agents to work together, while the agent in the lower grade needs the cooperation with two agents. In 2010, Wang *et al.*[14] proposed a scheme with six-photon cluster state, where a boss transmits quantum secret to five distant agents who are divided into two grades. In 2011, Wang *et al.*[15] proposed a multiparty HQIS protocol with a multipartite-entanglement channel and classical communication. In 2014, an HQIS of an arbitrary two-qubit state via two four-qubit cluster states was proposed by Xu *et al.*[16] Some of these protocols have been implemented by using the maximally entangled state. However, an initially maximally entangled state may easily evolve into a non-maximally entangled or mixed state due to the inevitable environment effect in the actual communication. Therefore, Peng *et al.*[17,18] proposed a hierarchical and probabilistic information splitting protocol that is based on the non-maximally entangled state and realized the transmission of arbitrary single-qubit. Subsequently, with an unknown eight-qubit cluster state. Bai *et al.*[19] implemented a hierarchical and probabilistic QIS. In their protocol, the sender can distribute an arbitrary single-qubit state to seven distant agents who are divided into two grades.

According to Xu *et al.*'s protocol,[16] we propose a hierarchical and probabilistic quantum information splitting protocol via non-maximally entangled states. In our protocol, two four-qubit cluster states are shared by four participants, forming a quantum channel. The sender transmits an arbi-

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trary two-qubit state to three agents with two Bell measurements and broadcasts measurement results via classical channel. Here, we assume that the quantum channel is noiseless and that the classical channel is certified. Bob with high authority can successfully recover the quantum secret message by his cooperation with Charlie or David. Charlie and David have low authorities. Any one of them needs the cooperation with two agents to recover the secret. In addition, the agent needs to introduce two auxiliary particles and performs CNOT operation. Eventually, the positive operator-valued measurement (POVM)[20] operation is performed to distinguish non-orthogonal states to obtain the secret state. Moreover, due to the symmetry of the four-qubit cluster state, we further propose a hierarchical and probabilistic quantum information splitting protocol where there are m agents with high authority and n agents with low authority. If an agent with high authority recovers the secret state, then he or she only needs the help from all the other agents in the upper grade and one of the agents in the lower grade. However, if an agent with low authority recovers the secret state, then he or she needs the cooperation with all agents.

2. Hierarchical and probabilistic QIS of arbitrary two-qubit state via four-qubit cluster states among three agents

Suppose that there are four legitimate participants Alice, Bob, Charlie, and David in the communication. Sender Alice wants to teleport an unknown two-qubit state

$$|\delta\rangle_{xy} = (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{xy}, \tag{1}$$

where α, β, γ, and δ are arbitrary with $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. To share this unknown state, the sender Alice prepares two two-qubit entangled states

$$\begin{aligned} |\psi\rangle_{12} &= a|0\rangle(|0\rangle + |1\rangle) + b|1\rangle(|0\rangle - |1\rangle), \\ |\psi\rangle_{56} &= c|0\rangle(|0\rangle + |1\rangle) + d|1\rangle(|0\rangle - |1\rangle), \end{aligned} \tag{2}$$

and four single qubits whose initial states are all in the state $|0\rangle$. Alice then performs CNOT operations to entangle these single qubits into the entangled states. After that, these two entangled states become

$$\begin{aligned} |\psi\rangle_{1234} &= (a|0\rangle|\varphi^0\rangle + b|1\rangle|\varphi^1\rangle)_{1234}, \\ |\psi\rangle_{5678} &= (c|0\rangle|\varphi^0\rangle + d|1\rangle|\varphi^1\rangle)_{5678}, \end{aligned} \tag{3}$$

where a, b, c, and d should be nonzero real numbers and satisfy $a^2 + b^2 = 1/2$ and $c^2 + d^2 = 1/2$. Here,

$$\begin{aligned} |\varphi^0\rangle &= |000\rangle + |011\rangle, \\ |\varphi^1\rangle &= |100\rangle - |111\rangle. \end{aligned} \tag{4}$$

Alice then sends particles 2 and 6 to Bob, particles 3 and 7 to Charlie, and particles 4 and 8 to David. Alice keeps particles 1 and 5 in her hand. Consequently, the four participants share two four-particle cluster states, thereby forming the quantum channel required by the protocol. The state of the whole system is written as

$$\begin{aligned} &|\delta\rangle_{xy} \otimes |\psi\rangle_{1234} \otimes |\psi\rangle_{5678} \\ &= (\alpha ac|0000\rangle|\varphi^0\rangle|\varphi^0\rangle + \alpha ad|0001\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \alpha bc|0100\rangle|\varphi^1\rangle|\varphi^0\rangle + \alpha bd|0101\rangle|\varphi^1\rangle|\varphi^1\rangle \\ &\quad + \beta ac|0010\rangle|\varphi^0\rangle|\varphi^0\rangle + \beta ad|0011\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \beta bc|0110\rangle|\varphi^1\rangle|\varphi^0\rangle + \beta bd|0111\rangle|\varphi^1\rangle|\varphi^1\rangle \\ &\quad + \gamma ac|1000\rangle|\varphi^0\rangle|\varphi^0\rangle + \gamma ad|1001\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \gamma bc|1100\rangle|\varphi^1\rangle|\varphi^0\rangle + \gamma bd|1101\rangle|\varphi^1\rangle|\varphi^1\rangle \\ &\quad + \delta ac|1010\rangle|\varphi^0\rangle|\varphi^0\rangle + \delta ad|1011\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \delta bc|1110\rangle|\varphi^1\rangle|\varphi^0\rangle \\ &\quad + \delta bd|1111\rangle|\varphi^1\rangle|\varphi^1\rangle)_{x1y5234678}. \end{aligned} \tag{5}$$

Subsequently, Alice should perform two joint measurements on her pair qubits $(x,1)$ and $(y,5)$ by using the Bell basis $\{\Phi^{\pm}, \Psi^{\pm}\}$, respectively. The four Bell states are given by

$$\begin{aligned} |\Phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \\ |\Psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \end{aligned} \tag{6}$$

It is known that Alice can obtain one of the 16 kinds of measurement results with equal possibility. The 16 kinds of measurement results and the corresponding joint states for the three agents after the measurement are summarized in Table 1. Alice needs to broadcast the measurement results to all her agents through a classical channel. In this way, the quantum information is spilt into the entangled states shared with the three agents. According to the quantum non-cloning theorem, only one agent's qubits can collapse into the secret state. As a result, all three agents are able to successfully recover the secret quantum message with the cooperation of other agents.

Case 1 Agent Bob with high authority is assigned to recover Alice's secret state. Without loss of generality, we assume that Alice's measurement result is $|\Psi^+\Phi^-\rangle_{x1y5}$, the states of the other agents will collapse into the following state:

$$\begin{aligned} &|\phi^{10}\rangle_{234678} \\ &= \Big[\frac{1}{2}|0000\rangle(\gamma ac|00\rangle - \delta ad|01\rangle + \alpha bc|10\rangle - \beta bd|11\rangle) \\ &\quad + \frac{1}{2}|1010\rangle(\gamma ac|00\rangle - \delta ad|01\rangle - \alpha bc|10\rangle + \beta bd|11\rangle) \\ &\quad + \frac{1}{2}|0101\rangle(\gamma ac|00\rangle + \delta ad|01\rangle + \alpha bc|10\rangle + \beta bd|11\rangle) \\ &\quad + \frac{1}{2}|1111\rangle(\gamma ac|00\rangle + \delta ad|01\rangle \\ &\quad - \alpha bc|10\rangle - \beta bd|11\rangle)\Big]_{374826}. \end{aligned} \tag{7}$$

Table 1. Relationships between Alice's measurement results (AR) and the corresponding joint states (JS) shared with the other agents.

AR	JS
$\|\Phi^+\Phi^+\rangle_{x1y5}$	$\|\phi^1\rangle_{234678} = \frac{1}{2}(\alpha ac\|\varphi^0\rangle\|\varphi^0\rangle + \beta ad\|\varphi^0\rangle\|\varphi^1\rangle + \gamma bc\|\varphi^1\rangle\|\varphi^0\rangle + \delta bd\|\varphi^1\rangle\|\varphi^1\rangle)$
$\|\Phi^+\Phi^-\rangle_{x1y5}$	$\|\phi^2\rangle_{234678} = \frac{1}{2}(\alpha ac\|\varphi^0\rangle\|\varphi^0\rangle - \beta ad\|\varphi^0\rangle\|\varphi^1\rangle + \gamma bc\|\varphi^1\rangle\|\varphi^0\rangle - \delta bd\|\varphi^1\rangle\|\varphi^1\rangle)$
$\|\Phi^+\Psi^+\rangle_{x1y5}$	$\|\phi^3\rangle_{234678} = \frac{1}{2}(\alpha ad\|\varphi^0\rangle\|\varphi^1\rangle + \beta ac\|\varphi^0\rangle\|\varphi^0\rangle + \gamma bd\|\varphi^1\rangle\|\varphi^1\rangle + \delta bc\|\varphi^1\rangle\|\varphi^0\rangle)$
$\|\Phi^+\Psi^-\rangle_{x1y5}$	$\|\phi^4\rangle_{234678} = \frac{1}{2}(\alpha ad\|\varphi^0\rangle\|\varphi^1\rangle - \beta ac\|\varphi^0\rangle\|\varphi^0\rangle + \gamma bd\|\varphi^1\rangle\|\varphi^1\rangle - \delta bc\|\varphi^1\rangle\|\varphi^0\rangle)$
$\|\Phi^-\Phi^+\rangle_{x1y5}$	$\|\phi^5\rangle_{234678} = \frac{1}{2}(\alpha ac\|\varphi^0\rangle\|\varphi^0\rangle + \beta ad\|\varphi^0\rangle\|\varphi^1\rangle - \gamma bc\|\varphi^1\rangle\|\varphi^0\rangle - \delta bd\|\varphi^1\rangle\|\varphi^1\rangle)$
$\|\Phi^-\Phi^-\rangle_{x1y5}$	$\|\phi^6\rangle_{234678} = \frac{1}{2}(\alpha ac\|\varphi^0\rangle\|\varphi^0\rangle - \beta ad\|\varphi^0\rangle\|\varphi^1\rangle - \gamma bc\|\varphi^1\rangle\|\varphi^0\rangle + \delta bd\|\varphi^1\rangle\|\varphi^1\rangle)$
$\|\Phi^-\Psi^+\rangle_{x1y5}$	$\|\phi^7\rangle_{234678} = \frac{1}{2}(\alpha ad\|\varphi^0\rangle\|\varphi^1\rangle + \beta ac\|\varphi^0\rangle\|\varphi^0\rangle - \gamma bd\|\varphi^1\rangle\|\varphi^1\rangle - \delta bc\|\varphi^1\rangle\|\varphi^0\rangle)$
$\|\Phi^-\Psi^-\rangle_{x1y5}$	$\|\phi^8\rangle_{234678} = \frac{1}{2}(\alpha ad\|\varphi^0\rangle\|\varphi^1\rangle - \beta ac\|\varphi^0\rangle\|\varphi^0\rangle - \gamma bd\|\varphi^1\rangle\|\varphi^1\rangle + \delta bc\|\varphi^1\rangle\|\varphi^0\rangle)$
$\|\Psi^+\Phi^+\rangle_{x1y5}$	$\|\phi^9\rangle_{234678} = \frac{1}{2}(\alpha bc\|\varphi^1\rangle\|\varphi^0\rangle + \beta bd\|\varphi^1\rangle\|\varphi^1\rangle + \gamma ac\|\varphi^0\rangle\|\varphi^0\rangle + \delta ad\|\varphi^0\rangle\|\varphi^1\rangle)$
$\|\Psi^+\Phi^-\rangle_{x1y5}$	$\|\phi^{10}\rangle_{234678} = \frac{1}{2}(\alpha bc\|\varphi^1\rangle\|\varphi^0\rangle - \beta bd\|\varphi^1\rangle\|\varphi^1\rangle + \gamma ac\|\varphi^0\rangle\|\varphi^0\rangle - \delta ad\|\varphi^0\rangle\|\varphi^1\rangle)$
$\|\Psi^+\Psi^+\rangle_{x1y5}$	$\|\phi^{11}\rangle_{234678} = \frac{1}{2}(\alpha bd\|\varphi^1\rangle\|\varphi^1\rangle + \beta bc\|\varphi^1\rangle\|\varphi^0\rangle + \gamma ad\|\varphi^0\rangle\|\varphi^1\rangle + \delta ac\|\varphi^0\rangle\|\varphi^0\rangle)$
$\|\Psi^+\Psi^-\rangle_{x1y5}$	$\|\phi^{12}\rangle_{234678} = \frac{1}{2}(\alpha bd\|\varphi^1\rangle\|\varphi^1\rangle - \beta bc\|\varphi^1\rangle\|\varphi^0\rangle + \gamma ad\|\varphi^0\rangle\|\varphi^1\rangle - \delta ac\|\varphi^0\rangle\|\varphi^0\rangle)$
$\|\Psi^-\Phi^+\rangle_{x1y5}$	$\|\phi^{13}\rangle_{234678} = \frac{1}{2}(\alpha bc\|\varphi^1\rangle\|\varphi^0\rangle + \beta bd\|\varphi^1\rangle\|\varphi^1\rangle - \gamma ac\|\varphi^0\rangle\|\varphi^0\rangle - \delta ad\|\varphi^0\rangle\|\varphi^1\rangle)$
$\|\Psi^-\Phi^-\rangle_{x1y5}$	$\|\phi^{14}\rangle_{234678} = \frac{1}{2}(\alpha bc\|\varphi^1\rangle\|\varphi^0\rangle - \beta bd\|\varphi^1\rangle\|\varphi^1\rangle - \gamma ac\|\varphi^0\rangle\|\varphi^0\rangle + \delta ad\|\varphi^0\rangle\|\varphi^1\rangle)$
$\|\Psi^-\Psi^+\rangle_{x1y5}$	$\|\phi^{15}\rangle_{234678} = \frac{1}{2}(\alpha bd\|\varphi^1\rangle\|\varphi^1\rangle + \beta bc\|\varphi^1\rangle\|\varphi^0\rangle - \gamma ad\|\varphi^0\rangle\|\varphi^1\rangle - \delta ac\|\varphi^0\rangle\|\varphi^0\rangle)$
$\|\Psi^-\Psi^-\rangle_{x1y5}$	$\|\phi^{16}\rangle_{234678} = \frac{1}{2}(\alpha bd\|\varphi^1\rangle\|\varphi^1\rangle - \beta bc\|\varphi^1\rangle\|\varphi^0\rangle - \gamma ad\|\varphi^0\rangle\|\varphi^1\rangle + \delta ac\|\varphi^0\rangle\|\varphi^0\rangle)$

To recover Alice's secret state, Charlie and David should make two single-qubit measurements on their particles and transmit the measurement results to Bob via a classical channel. Obviously, if Charlie and David measure their particles by using the basis $\{|0\rangle, |1\rangle\}$, then their results are always correlated with each other. Thus, Bob can deduce the Charlie's (David's) results from David's (Charlie's) results. This shows that only one of the other two agents is needed to inform Bob of his measurement results. Suppose that Charlie or David's measurement result is $|01\rangle$, then Bob's state will collapse into

$$\frac{1}{2}(\gamma ac|00\rangle + \delta ad|01\rangle + \alpha bc|10\rangle + \beta bd|11\rangle)_{26}. \quad (8)$$

According to the measurement results of the three participants, Bob performs the corresponding operation on particles 2 and 6 in his hand. In this case, Bob performs the operation shown in Table 2.

Table 2. Relationships between Charlie or David's measurement results (CDR) and Bob's operations (BO).

CDR	BO
$\|00\rangle$	$I \otimes Z$
$\|01\rangle$	$I \otimes I$
$\|10\rangle$	$Z \otimes Z$
$\|11\rangle$	$Z \otimes I$

After the above operation, Bob's state eventually becomes

$$\frac{1}{2}(\gamma ac|00\rangle + \delta ad|01\rangle + \alpha bc|10\rangle + \beta bd|11\rangle)_{26}. \quad (9)$$

If Alice's measurement result is not $|\Psi^+\Phi^-\rangle_{x1y5}$, then Bob's state will eventually transform into one of the following four states with Pauli operations based on the measurement results:

$$\begin{aligned}
&\frac{1}{2}(\alpha ac|00\rangle + \beta ad|01\rangle + \gamma bc|10\rangle + \delta bd|11\rangle)_{26},\\
&\frac{1}{2}(\beta ac|00\rangle + \alpha ad|01\rangle + \delta bc|10\rangle + \gamma bd|11\rangle)_{26},\\
&\frac{1}{2}(\gamma ac|00\rangle + \delta ad|01\rangle + \alpha bc|10\rangle + \beta bd|11\rangle)_{26},\\
&\frac{1}{2}(\delta ac|00\rangle + \gamma ad|01\rangle + \beta bc|10\rangle + \alpha bd|11\rangle)_{26}. \quad (10)
\end{aligned}$$

To demonstrate the realization of the protocol in detail, we still make the above assumptions; that is, after a series of operations, Bob's state is as shown in Eq. (9). It is obvious that Bob's state is different from the secret qubits sent by Alice. To successfully obtain Alice's secret state, Bob needs to take the initial state of two auxiliary qubits m and n as initial state $|00\rangle_{mn}$ and performs two CNOT operations, with qubits 2 and 6 used as the controlled qubits while the auxiliary qubits m and n as the target qubits. Now, Bob's state becomes

$$\frac{1}{2}(\gamma ac|0000\rangle + \delta ad|0101\rangle + \alpha bc|1010\rangle + \beta bd|1111\rangle)_{26mn}. \quad (11)$$

Equation (11) can be rewritten as

$$\frac{1}{8}(|K_1\rangle|Q_1\rangle + |K_2\rangle|Q_2\rangle + |K_3\rangle|Q_3\rangle + |K_4\rangle|Q_4\rangle)_{26mn}, \quad (12)$$

where

$$
\begin{aligned}
|K_1\rangle &= \gamma|00\rangle + \delta|01\rangle + \alpha|10\rangle + \beta|11\rangle,\\
|K_2\rangle &= \gamma|00\rangle + \delta|01\rangle - \alpha|10\rangle - \beta|11\rangle,\\
|K_3\rangle &= \gamma|00\rangle - \delta|01\rangle + \alpha|10\rangle - \beta|11\rangle,\\
|K_4\rangle &= \gamma|00\rangle - \delta|01\rangle - \alpha|10\rangle + \beta|11\rangle,\\
|Q_1\rangle &= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle,\\
|Q_2\rangle &= ac|00\rangle + ad|01\rangle - bc|10\rangle - bd|11\rangle,\\
|Q_3\rangle &= ac|00\rangle - ad|01\rangle + bc|10\rangle - bd|11\rangle,\\
|Q_4\rangle &= ac|00\rangle - ad|01\rangle - bc|10\rangle + bd|11\rangle. \qquad (13)
\end{aligned}
$$

From Eq. (12), Bob can obtain the state $|K_i\rangle (i = 1,2,3,4)$, provided that the states $|Q_i\rangle (i = 1,2,3,4)$ are obtained via appropriate measurements on auxiliary particles. Unfortunately, the states $|Q_i\rangle (i = 1,2,3,4)$ are nonorthogonal in general. As a result, they cannot be differentiated deterministically. To distinguish the non-orthogonal states, we can only measure with POVM. The form of POVM is

$$
\begin{aligned}
&O_1 = \frac{1}{\omega}|M_1\rangle\langle M_1|,\ \ O_2 = \frac{1}{\omega}|M_2\rangle\langle M_2|,\ \ O_3 = \frac{1}{\omega}|M_3\rangle\langle M_3|,\\
&O_4 = \frac{1}{\omega}|M_4\rangle\langle M_4|,\ \ O_5 = I - \frac{1}{\omega}\sum_{i=1}^{4}|M_i\rangle\langle M_i|, \qquad (14)
\end{aligned}
$$

with

$$
\begin{aligned}
|M_1\rangle &= \frac{1}{\sqrt{\varepsilon}}\left(\frac{1}{ac}|00\rangle + \frac{1}{ad}|01\rangle + \frac{1}{bc}|10\rangle + \frac{1}{bd}|11\rangle\right),\\
|M_2\rangle &= \frac{1}{\sqrt{\varepsilon}}\left(\frac{1}{ac}|00\rangle + \frac{1}{ad}|01\rangle - \frac{1}{bc}|10\rangle - \frac{1}{bd}|11\rangle\right),\\
|M_3\rangle &= \frac{1}{\sqrt{\varepsilon}}\left(\frac{1}{ac}|00\rangle - \frac{1}{ad}|01\rangle + \frac{1}{bc}|10\rangle - \frac{1}{bd}|11\rangle\right),\\
|M_4\rangle &= \frac{1}{\sqrt{\varepsilon}}\left(\frac{1}{ac}|00\rangle - \frac{1}{ad}|01\rangle - \frac{1}{bc}|10\rangle + \frac{1}{bd}|11\rangle\right),\\
\varepsilon &= \frac{1}{(ac)^2} + \frac{1}{(ad)^2} + \frac{1}{(bc)^2} + \frac{1}{(bd)^2} = \frac{1}{4(abcd)^2}, \qquad (15)
\end{aligned}
$$

where I is an identity operator, and ω is a parameter relating to a, b, c, and d and should be able to insure that O_5 is a positive operator; that is, ω satisfies the inequality $\omega \geq 16\max\{(bd)^2,(ac)^2,(bc)^2,(ad)^2\}$. After performing the above POVM operation on his auxiliary qubits m and n, Bob can obtain O_i $(i = 1,2,3,4)$ with equal probability

$$
\langle T|O_i|T\rangle = \frac{1}{64}\langle Q_i|O_i|Q_i\rangle = \frac{1}{4\omega\varepsilon},\ \ (i = 1,2,3,4). \qquad (16)
$$

Subsequently, Bob performs appropriate unitary operations to recover the secret qubits. However, with the probability of $1 - 1/\omega\varepsilon$, Bob may obtain the value of O_5, which means that Bob cannot infer the state of qubits m and n.

This discussion is based on the assumption that Alice's measurement result is $|\Psi^+\Phi^-\rangle_{x1y5}$ and Charlie or Bob's measurement result is $|01\rangle$. Taking all of the cases into account, we can work out the success probability of this scheme to be

$$
\frac{1}{4\omega\varepsilon} \times 4 \times 4 \times 16 = \frac{16a^2c^2b^2d^2}{\omega} \times 16. \qquad (17)
$$

Case 2 The agent with low authority is able to recover the secret state. In this protocol, both Charlie and David have low power to recover the secret state. Here, we take Charlie as an example. Equation (7) can now be rewritten as

$$
\begin{aligned}
|\phi^{10}\rangle_{234678} = {}& \frac{1}{4}|{+}{+}\rangle\{[|{+}{+}\rangle(\gamma ac|{+}{+}\rangle - \delta ad|{+}{-}\rangle + \alpha bc|{-}{+}\rangle - \beta bd|{-}{-}\rangle)]\\
&+ [|{+}{-}\rangle(\gamma ac|{+}{-}\rangle - \delta ad|{+}{+}\rangle + \alpha bc|{-}{-}\rangle - \beta bd|{-}{+}\rangle)] + [|{-}{+}\rangle(\gamma ac|{-}{+}\rangle - \delta ad|{-}{-}\rangle + \alpha bc|{+}{+}\rangle - \beta bd|{+}{-}\rangle)]\\
&+ [|{-}{-}\rangle(\gamma ac|{-}{-}\rangle - \delta ad|{-}{+}\rangle + \alpha bc|{+}{-}\rangle - \beta bd|{+}{+}\rangle)]\} + \frac{1}{4}|{+}{-}\rangle\{[|{+}{+}\rangle(\gamma ac|{+}{+}\rangle + \delta ad|{+}{-}\rangle + \alpha bc|{-}{+}\rangle + \beta bd|{-}{-}\rangle)]\\
&+ [|{+}{-}\rangle(\gamma ac|{+}{-}\rangle + \delta ad|{+}{+}\rangle + \alpha bc|{-}{-}\rangle + \beta bd|{-}{+}\rangle)] + [|{-}{+}\rangle(\gamma ac|{-}{+}\rangle + \delta ad|{-}{-}\rangle + \alpha bc|{+}{+}\rangle + \beta bd|{+}{-}\rangle)]\\
&+ [|{-}{-}\rangle(\gamma ac|{-}{-}\rangle + \delta ad|{-}{+}\rangle + \alpha bc|{+}{-}\rangle + \beta bd|{+}{+}\rangle)]\} + \frac{1}{4}|{-}{+}\rangle\{[|{+}{+}\rangle(\gamma ac|{+}{+}\rangle - \delta ad|{+}{-}\rangle - \alpha bc|{-}{+}\rangle + \beta bd|{-}{-}\rangle)]\\
&+ [|{+}{-}\rangle(\gamma ac|{+}{-}\rangle - \delta ad|{+}{+}\rangle - \alpha bc|{-}{-}\rangle + \beta bd|{-}{+}\rangle)] + [|{-}{+}\rangle(\gamma ac|{-}{+}\rangle - \delta ad|{-}{-}\rangle - \alpha bc|{+}{+}\rangle + \beta bd|{+}{-}\rangle)]\\
&+ [|{-}{-}\rangle(\gamma ac|{-}{-}\rangle - \delta ad|{-}{+}\rangle - \alpha bc|{+}{-}\rangle + \beta bd|{+}{+}\rangle)]\} + \frac{1}{4}|{-}{-}\rangle\{[|{+}{+}\rangle(\gamma ac|{+}{+}\rangle + \delta ad|{+}{-}\rangle - \alpha bc|{-}{+}\rangle - \beta bd|{-}{-}\rangle)]\\
&+ [|{+}{-}\rangle(\gamma ac|{+}{-}\rangle + \delta ad|{+}{+}\rangle - \alpha bc|{-}{-}\rangle - \beta bd|{-}{+}\rangle)] + [|{-}{+}\rangle(\gamma ac|{-}{+}\rangle + \delta ad|{-}{-}\rangle - \alpha bc|{+}{+}\rangle - \beta bd|{+}{-}\rangle)]\\
&+ [|{-}{-}\rangle(\gamma ac|{-}{-}\rangle + \delta ad|{-}{+}\rangle - \alpha bc|{+}{-}\rangle - \beta bd|{+}{+}\rangle)]_{264837}\}, \qquad (18)
\end{aligned}
$$

where

$$
\begin{aligned}
|+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),\\
|-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \qquad (19)
\end{aligned}
$$

Bob and David perform single-particle measurements on their pair qubits (2,6) and (4,8) by using the basis $\{|+\rangle,|-\rangle\}$, respectively. From Eq. (18), it can be seen that Charlie needs the cooperation of two agents to successfully recover the secret qubits. Without loss of generality, suppose that Bob's measurement result is $|-+\rangle$ and David's measurement result

is $|++\rangle$, then Charlie's quantum state will collapse into

$$\frac{1}{4}(\gamma ac|++\rangle - \delta ad|+-\rangle - \alpha bc|-+\rangle + \beta bd|--\rangle)_{37}. \quad (20)$$

Charlie performs two Hardamard transformations on pair qubits particles 3 and 7, then Bob's state is

$$\frac{1}{4}(\gamma ac|00\rangle - \delta ad|01\rangle - \alpha bc|10\rangle + \beta bd|11\rangle)_{37}. \quad (21)$$

To recover the secret state, Charlie then performs Z operation on qubits 7 and 3 separately. The state shown in Eq. (21) transforms into

$$\frac{1}{4}(\gamma ac|00\rangle + \delta ad|01\rangle + \alpha bc|10\rangle + \beta bd|11\rangle)_{37}. \quad (22)$$

Like Case 1, Charlie needs to introduce two auxiliary particles for CNOT operation in order to successfully obtain the secret state. Charlie also needs to perform a POVM operation with different coefficients. Finally, we can obtain the probability of success in this mode to be

$$\frac{1}{16\omega\varepsilon} \times 4 \times 16 \times 16 = \frac{16a^2c^2b^2d^2}{\omega} \times 16. \quad (23)$$

The relationship between the coefficient and the probability is shown with Fig. 1. No matter which kind of four-particle cluster prepared by Alice is set to be a quantum channel, we ultimately hope that the successful probability of splitting quantum information is maximum; that is, from Eqs. (23) and (17) the maximum can be obtained. In Fig. 1, the three curves correspond to different values of a and c; that is, the three curves represent different four-particle cluster states. It can be seen from the figure that no matter what kind the four-particle cluster is, the success probability of the scheme is inversely proportional to the value of ω. As mentioned previously, ω satisfies the inequality $\omega \geq 16\max\{(bd)^2,(ac)^2,(bc)^2,(ad)^2\}$. Thus, the scheme is an optimal strategy if $\omega = 16\max\{(bd)^2,(ac)^2,(bc)^2,(ad)^2\}$. In other words, the POVM is determined after the four-particle cluster has been identified. In particular, the probability is a maximum value of 1 when $a = c = 1/2$ and $\omega = 16\max\{(bd)^2,(ac)^2,(bc)^2,(ad)^2\}$. At this time, the four-particle cluster is in the maximally entangled state.

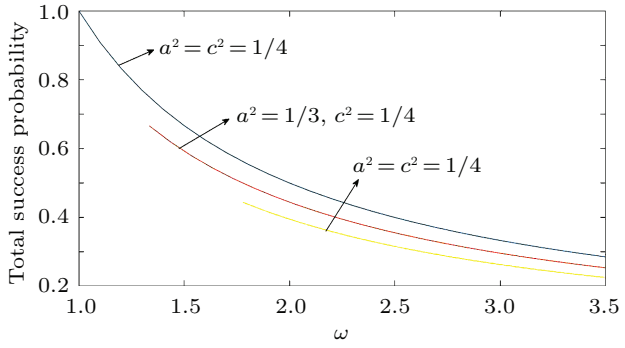


Fig. 1. (color online) Relationships between the value of ω and the total success probability P, which satisfies $P = (16a^2c^2b^2d^2)/\omega \times 16$ in the three cases, with coefficients satisfying $a^2 + b^2 = 1/2$ and $c^2 + d^2 = 1/2$.

3. Hierarchical and probabilistic QIS of arbitrary two-qubit state via cluster states among multiparty agents

Due to the symmetry of the four-qubit cluster state, our protocol can be extended to multiparty agents. To illustrate this point more clearly, we first consider the case where there are two agents called Bob$_i$ ($i = 1,2$) with high authority and three agents called Charlie$_j$ (j = 1, 2, 3) with low authority. The sender Alice prepares two two-qubit entangled states in Eq. (2) and eight single qubits whose initial states are all in the state $|0\rangle$. Alice then performs some CNOT operations to entangle these single qubits into cluster states. These two entangled states then become

$$\begin{aligned} |\psi^0\rangle &= (a|000000\rangle + a|000111\rangle + b|111000\rangle \\ &\quad - b|111111\rangle)_{x_1x_2x_3x_4x_5x_6}, \\ |\psi^1\rangle &= (c|000000\rangle + c|000111\rangle + d|111000\rangle \\ &\quad - d|111111\rangle)_{y_1y_2y_3y_4y_5y_6}. \end{aligned} \quad (24)$$

Subsequently, Alice sends the particles x_{i+1} and y_{i+1} to Bob$_i$ (i = 1, 2), and the particles x_{j+3} and y_{j+3} to Charlie$_j$ (j = 1, 2, 3). Alice keeps the particles x_1 and y_1 in her hand. Consequently, the four participants share two six-qubit clusters, thus forming a quantum channel required by the protocol. The state of the whole system is written as

$$\begin{aligned} &|\delta\rangle_{xy} \otimes |\psi^0\rangle \otimes |\psi^1\rangle \\ &= (\alpha ac|0000\rangle|\varphi^0\rangle|\varphi^0\rangle + \alpha ad|0001\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \alpha bc|0100\rangle|\varphi^1\rangle|\varphi^0\rangle + \alpha bd|0101\rangle|\varphi^1\rangle|\varphi^1\rangle \\ &\quad + \beta ac|0010\rangle|\varphi^0\rangle|\varphi^0\rangle + \beta ad|0011\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \beta bc|0110\rangle|\varphi^1\rangle|\varphi^0\rangle + \beta bd|0111\rangle|\varphi^1\rangle|\varphi^1\rangle \\ &\quad + \gamma ac|1000\rangle|\varphi^0\rangle|\varphi^0\rangle + \gamma ad|1001\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \gamma bc|1100\rangle|\varphi^1\rangle|\varphi^0\rangle + \gamma bd|1101\rangle|\varphi^1\rangle|\varphi^1\rangle \\ &\quad + \delta ac|1010\rangle|\varphi^0\rangle|\varphi^0\rangle + \delta ad|1011\rangle|\varphi^0\rangle|\varphi^1\rangle \\ &\quad + \delta bc|1110\rangle|\varphi^1\rangle|\varphi^0\rangle \\ &\quad + \delta bd|1111\rangle|\varphi^1\rangle|\varphi^1\rangle)_{xx_1yy_1x_2x_3x_4x_5x_6y_2y_3y_4y_5y_6}, \end{aligned} \quad (25)$$

where

$$\begin{aligned} |\varphi^0\rangle &= |00000\rangle + |00111\rangle, \\ |\varphi^1\rangle &= |11000\rangle - |11111\rangle. \end{aligned} \quad (26)$$

To distribute $|\delta\rangle_{xy}$ in Eq. (1) to five agents, the sender Alice should perform two joint measurements on her pair qubits (x,x_1) and (y,y_1) by using the Bell basis $\{\Phi^{\pm},\Psi^{\pm}\}$, respectively. It is clear that Alice can obtain 16 kinds of measurement results. Like the protocol in the three agents, Alice should also broadcast the result to five agents.

Case 1 The agent with high authority is assigned to recover Alice's secret state. Here, we take Bob$_1$ as an example. Without loss of generality, we assume that Alice's measurement result is $|\Psi^+\Psi^+\rangle_{xx_1yy_1}$, the states of the other agents will collapse into the state

$$
\begin{aligned}
|\phi^1\rangle = \Big\{ & \frac{1}{4}|000000\rangle[|{+}{+}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle+\gamma bc|10\rangle+\delta bd|11\rangle)+|{+}{-}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle+\gamma bc|10\rangle-\delta bd|11\rangle)\\
& +|{-}{+}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle-\gamma bc|10\rangle-\delta bd|11\rangle)+|{-}{-}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle-\gamma bc|10\rangle+\delta bd|11\rangle)]\\
& +\frac{1}{4}|101010\rangle[|{+}{+}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle-\gamma bc|10\rangle-\delta bd|11\rangle)+|{+}{-}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle-\gamma bc|10\rangle+\delta bd|11\rangle)\\
& +|{-}{+}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle+\gamma bc|10\rangle+\delta bd|11\rangle)+|{-}{-}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle+\gamma bc|10\rangle+\delta bd|11\rangle)]\\
& +\frac{1}{4}|010101\rangle[|{+}{+}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle+\gamma bc|10\rangle-\delta bd|11\rangle)+|{+}{-}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle+\gamma bc|10\rangle+\delta bd|11\rangle)\\
& +|{-}{+}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle-\gamma bc|10\rangle+\delta bd|11\rangle)+|{-}{-}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle-\gamma bc|10\rangle-\delta bd|11\rangle)]\\
& +\frac{1}{4}|111111\rangle[|{+}{+}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle-\gamma bc|10\rangle+\delta bd|11\rangle)+|{+}{-}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle-\gamma bc|10\rangle-\delta bd|11\rangle)\\
& +|{-}{+}\rangle(\alpha ac|00\rangle-\beta ad|01\rangle+\gamma bc|10\rangle-\delta bd|11\rangle)\\
& +|{-}{-}\rangle(\alpha ac|00\rangle+\beta ad|01\rangle+\gamma bc|10\rangle+\delta bd|11\rangle)]\Big\}_{x_4y_4x_5y_5x_6y_6x_3y_3x_2y_2}. \qquad (27)
\end{aligned}
$$

To recover Alice's secret state, the agents with low authority should make two single-qubit measurements on their particles by using the basis $\{|0\rangle,|1\rangle\}$. Their measurement results are always correlated with each other. This shows that only one of these agents is needed to inform Bob_1 of his measurement results. Bob_2 with high authority should measure his particles by using the basis $\{|+\rangle,|-\rangle\}$. Suppose that the measurement result of the agent with low authority is $|00\rangle$ and Bob_2's is $|+-\rangle$, then the state of Bob_1 will be

$$\frac{1}{4}(\alpha ac|00\rangle-\beta ad|01\rangle+\gamma bc|10\rangle-\delta bd|11\rangle). \qquad (28)$$

Bob_1 performs Pauli operations on his particles according to other agents' measurement results. Using the same method, Bob_1 introduces two auxiliary qubits and performs a POVM operation with different coefficients according to others' measurement results. Finally, we can obtain the probability of success in this mode to be

$$\frac{1}{16\omega\varepsilon}\times 4\times 16\times 16=\frac{16a^2c^2b^2d^2}{\omega}\times 16. \qquad (29)$$

Case 2 An agent with low authority is to recover the secret state. Here, we take $\mathrm{Charlie}_1$ as an example. Equation (27) can now be rewritten as

$$
\begin{aligned}
|\phi^1\rangle = & \frac{1}{2^6}\{(|++++\rangle+|+--+\rangle+|-+-+\rangle+|----\rangle)[4(|++++\rangle+|+--+\rangle+|-+-+\rangle+|----\rangle)\\
& \times(\alpha ac|{+}{+}\rangle+\beta ad|{+}{-}\rangle+\gamma bc|{-}{+}\rangle+\delta bd|{-}{-}\rangle)\\
& +4(|+++-\rangle+|+--+\rangle+|-+--\rangle+|---+\rangle)(\alpha ac|{+}{-}\rangle+\beta ad|{+}{+}\rangle+\gamma bc|{-}{-}\rangle+\delta bd|{-}{+}\rangle)\\
& +4(|--+-\rangle+|-++-\rangle+|+---\rangle+|+++-\rangle)(\alpha ac|{-}{+}\rangle+\beta ad|{-}{-}\rangle+\gamma bc|{+}{+}\rangle+\delta bd|{+}{-}\rangle)\\
& +4(|++--\rangle+|+--+\rangle+|-++-\rangle+|--++\rangle)(\alpha ac|{-}{-}\rangle+\beta ad|{-}{+}\rangle+\gamma bc|{+}{-}\rangle+\delta bd|{+}{+}\rangle)]\\
& +(|-++-\rangle+|+++-\rangle+|---+\rangle+|-+--\rangle)[4(|++++\rangle+|+--+\rangle+|-+-+\rangle+|----\rangle)\\
& \times(\alpha ac|{+}{+}\rangle-\beta ad|{+}{-}\rangle+\gamma bc|{-}{+}\rangle-\delta bd|{-}{-}\rangle)\\
& +4(|+++-\rangle+|+--+\rangle+|-+--\rangle+|---+\rangle)(\alpha ac|{+}{-}\rangle-\beta ad|{+}{+}\rangle+\gamma bc|{-}{-}\rangle-\delta bd|{-}{+}\rangle)\\
& +4(|--+-\rangle+|-++-\rangle+|+---\rangle+|+++-\rangle)(\alpha ac|{-}{+}\rangle-\beta ad|{-}{-}\rangle+\gamma bc|{+}{+}\rangle-\delta bd|{+}{-}\rangle)\\
& +4(|++--\rangle+|+--+\rangle+|-++-\rangle+|--++\rangle)(\alpha ac|{-}{-}\rangle-\beta ad|{-}{+}\rangle+\gamma bc|{+}{-}\rangle-\delta bd|{+}{+}\rangle)]\\
& +(|--++\rangle+|-++-\rangle+|+--+\rangle+|++--\rangle)[4(|++++\rangle+|+--+\rangle+|-+-+\rangle+|----\rangle)\\
& \times(\alpha ac|{+}{+}\rangle+\beta ad|{+}{-}\rangle-\gamma bc|{-}{+}\rangle-\delta bd|{-}{-}\rangle)\\
& +4(|+++-\rangle+|+--+\rangle+|-+--\rangle+|---+\rangle)(\alpha ac|{+}{-}\rangle+\beta ad|{+}{+}\rangle-\gamma bc|{-}{-}\rangle-\delta bd|{-}{+}\rangle)\\
& +4(|--+-\rangle+|-++-\rangle+|+---\rangle+|+++-\rangle)(\alpha ac|{-}{+}\rangle+\beta ad|{-}{-}\rangle-\gamma bc|{+}{+}\rangle-\delta bd|{+}{-}\rangle)\\
& +4(|++--\rangle+|+--+\rangle+|-++-\rangle+|--++\rangle)(\alpha ac|{-}{-}\rangle+\beta ad|{-}{+}\rangle-\gamma bc|{+}{-}\rangle-\delta bd|{+}{+}\rangle)]\\
& +(|--++\rangle+|-++-\rangle+|+--+\rangle+|++--\rangle)[4(|++++\rangle+|+--+\rangle+|-+-+\rangle+|----\rangle)\\
& \times(\alpha ac|{+}{+}\rangle-\beta ad|{+}{-}\rangle-\gamma bc|{-}{+}\rangle+\delta bd|{-}{-}\rangle)\\
& +4(|+++-\rangle+|+--+\rangle+|-+--\rangle+|---+\rangle)(\alpha ac|{+}{-}\rangle-\beta ad|{+}{+}\rangle-\gamma bc|{-}{-}\rangle+\delta bd|{-}{+}\rangle)\\
& +4(|--+-\rangle+|-++-\rangle+|+---\rangle+|+++-\rangle)(\alpha ac|{-}{+}\rangle-\beta ad|{-}{-}\rangle-\gamma bc|{+}{+}\rangle+\delta bd|{+}{-}\rangle)\\
& +4(|++--\rangle+|+--+\rangle+|-++-\rangle+|--++\rangle)\\
& \times(\alpha ac|{-}{-}\rangle-\beta ad|{-}{+}\rangle-\gamma bc|{+}{-}\rangle+\delta bd|{+}{+}\rangle)]_{x_2y_2x_3y_3x_5y_5x_6y_6x_4y_4}. \qquad (30)
\end{aligned}
$$

Bob$_i$ ($i = 1, 2$) and Charlie$_j$ ($j = 2, 3$) perform single-particle measurements on their pair particles by using the basis $\{|+\rangle,|-\rangle\}$, respectively. From Eq. (30), it can be seen that Charlie$_1$ needs the cooperation with all the other agents to successfully recover the secret qubits. Without loss of generality, suppose that the measurement result of the agents with high authority is $|+-++\rangle$ and the measurement result of the agents with low authority is $|+-+-\rangle$, then Charlie$_1$'s quantum state collapses into

$$\frac{1}{16}(\alpha ac|++\rangle-\beta ad|+-\rangle+\gamma bc|-+\rangle-\delta bd|--\rangle). \quad (31)$$

Charlie$_1$ performs Hardamard transformations and Pauli operations on his particles according to other agents' measurement results. Using the same method, Charlie$_1$ introduces two auxiliary qubits and performs a POVM operation with different coefficients. Finally, we can obtain the probability of success in this mode to be

$$\frac{1}{16\times 16\omega\varepsilon}\times 4\times 16\times 16\times 16=\frac{16a^2c^2b^2d^2}{\omega}\times 16. \quad (32)$$

Now we suppose that there are m agents with high authority and n agents with low authority. Given that the protocols in three agents and five agents have been described in detail, we just give this protocol briefly. The sender Alice prepares two two-qubit entangled states in Eq. (2) and some single qubits whose initial states are all in the state $|0\rangle$. Alice then performs some CNOT operations to entangle these single qubits into cluster states

$$\begin{aligned}|\psi^0\rangle &= (a|\underbrace{00\ldots0}_{m+1}\underbrace{00\ldots0}_{n}\rangle+a|\underbrace{00\ldots0}_{m+1}\underbrace{11\ldots1}_{n}\rangle+b|\underbrace{11\ldots1}_{m+1}\underbrace{00\ldots0}_{n}\rangle\\ &\quad -b|\underbrace{11\ldots1}_{m+1}\underbrace{11\ldots1}_{n}\rangle),\\ |\psi^1\rangle &= (c|\underbrace{00\ldots0}_{m+1}\underbrace{00\ldots0}_{n}\rangle+c|\underbrace{00\ldots0}_{m+1}\underbrace{11\ldots1}_{n}\rangle+d|\underbrace{11\ldots1}_{m+1}\underbrace{00\ldots0}_{n}\rangle\\ &\quad -d|\underbrace{11\ldots1}_{m+1}\underbrace{11\ldots1}_{n}\rangle). \end{aligned} \quad (33)$$

The process of the protocol is similar to that of the protocol with five agents. In this multiparty protocol, let agents Bob$_1$, Bob$_2$, ..., and Bob$_m$ be in high grade, and the agents Charlie$_1$, Charlie$_2$, ..., and Charlie$_n$ be in low grade. Alice keeps the particles x_1 and y_1 in her hand. The whole system is

$$\begin{aligned}&|\delta\rangle_{xy}\otimes|\psi^0\rangle\otimes|\psi^1\rangle\\ &=(\alpha|00\rangle+\beta|01\rangle+\gamma c|10\rangle+\delta|11\rangle)_{xy}\\ &\otimes(a|\underbrace{00\ldots0}_{m+1}\underbrace{00\ldots0}_{n}\rangle+a|\underbrace{00\ldots0}_{m+1}\underbrace{11\ldots1}_{n}\rangle+b|\underbrace{11\ldots1}_{m+1}\underbrace{00\ldots0}_{n}\rangle\\ &\quad -b|\underbrace{11\ldots1}_{m+1}\underbrace{11\ldots1}_{n}\rangle)_{x_1x_2\ldots x_{(m+n+1)}}\\ &\otimes(c|\underbrace{00\ldots0}_{m+1}\underbrace{00\ldots0}_{n}\rangle+c|\underbrace{00\ldots0}_{m+1}\underbrace{11\ldots1}_{n}\rangle+d|\underbrace{11\ldots1}_{m+1}\underbrace{00\ldots0}_{n}\rangle\\ &\quad -d|\underbrace{11\ldots1}_{m+1}\underbrace{11\ldots1}_{n}\rangle)_{y_1y_2\ldots y_{(m+n+1)}}. \end{aligned} \quad (34)$$

Alice wants to teleport the state $|\delta\rangle_{xy}$ shown in Eq. (1). The sender Alice performs two Bell measurements on pair particles (x,x_1) and (y,y_1), respectively. If an agent with high authority recovers the secret state, then all of the other agents with high authority need to make two single-qubit measurements on their particles by using the basis $\{|+\rangle,|-\rangle\}$, and only one of the agents with low authority needs to make two single-qubit measurements on his or her particles by using the basis $\{|0\rangle,|1\rangle\}$. However, if an agent with low authority recovers the secret state, then all of the other agents need to make two single-qubit measurements on their particles by using the basis $\{|+\rangle,|-\rangle\}$. Whether an agent with high authority or an agent with low authority recovers the secret state or not, the probability of success is

$$\frac{1}{16\omega\varepsilon}\times 4\times 16\times 16=\frac{16a^2c^2b^2d^2}{\omega}\times 16. \quad (35)$$

4. Security and efficiency

Because of the no-cloning theorem and entanglement monogamy, the only way for an outside eavesdropper[21–25] to obtain the secret state is to adopt the intercept-resend attack. In particular, because which one of the agents will possess the secret state is previously undefined in the scheme, it is supposed that the eavesdropper intercepts all of the particles sent to the agents and resends fake particles to the agents. To keep the quantum correlation among the agents, the eavesdropper prepares two cluster states in Eq. (33), such as the sender in the protocol among the multiparty agents.

However, the quantum entanglement between the agents is destroyed. The agents can easily detect such an attack by performing local measurements on their particles. For example, they all perform the measurement by using the basis $\{|0\rangle,|1\rangle\}$. Under the attack by the eavesdropper, there is no correlation between the measurement results of the sender and the agents. However, in a no-eavesdropping case, where the measurement results of the sender and the agent with high authority are always correlated with each other, and the measurement results of the sender and the agents with lower authority are always anti-correlated with each other. Thus, the eavesdropping attack can always be detected in this way and a subset of entanglement channels will be sacrificed. As a matter of fact, most of entanglement-based quantum communication schemas need the users to utilize quantum correlations and sacrifice a subset of entanglement channels to check their security against an eavesdroppers' interceptions.

We can exploit the definition, and the total efficiency for quantum state sharing can be defined as

$$\eta=\frac{q_s}{q_t+b_t}, \quad (36)$$

where q_s represents the number of shared qubits to be achieved by the protocol. In this protocol, an arbitrary two-qubit is transmitted. In addition, q_t represents the number of qubits transmitted in the protocol; that is, the number of qubits sent by the sender to the agents at the beginning of the protocol. Meanwhile, b_t is the number of classic bits transmitted during the communication. In this protocol, b_t is the number of classic bits used for broadcasting the measurement results. Suppose that each agent has the same probability of recovering the secret message, then we will be able to calculate the average communication efficiency of the protocol among m agents with high authority and n agents with low authority from

$$\begin{aligned}&\frac{2}{2(m+n)+4+2+2(m-1)}\times\frac{m}{m+n}\\&+\frac{2}{2(m+n)+4+2m+2(n-1)}\times\frac{n}{m+n}\\&=\frac{1}{2m+2+n}\times\frac{m}{m+n}+\frac{1}{2m+2n+1}\times\frac{n}{m+n}.\end{aligned}\tag{37}$$

In our protocol, we need two non-maximally entangled states which can be prepared by some methods.[26,27] According to Ziman *et al.*'s research,[28] the POVM operation can also be realized. Thus, our protocol is feasible. Moreover, the operations we perform during communication are all the basic operations which are less complex.

5. Conclusions

The proposed protocol uses a non-maximally entangled state to implement a hierarchical and probabilistic quantum information splitting protocol, whereas Xu's protocol is implemented by using the maximally entangled state. Although the protocol of the maximally entangled state can recover the secret information successfully and deterministically, the maximally entangled state is easily influenced by the environment and becomes a non-maximally entangled state or a mixed state in practice. Compared with Peng *et al.*'s hierarchical and probabilistic quantum information splitting protocol, our proposed protocol achieves the sharing of an arbitrary two-qubit state. Moreover, our protocol can be extended to multiparty participants.

In general, we use a non-maximally entangled state to implement a hierarchical quantum information splitting protocol that shares arbitrary two qubits among three agents. In addition, different agents have different authorities for restoring the secret state. Bob with high authority only needs the cooperation of Charlie or David, while Charlie or David with low authority need the cooperation of the other two agents. In addition, the agent recovers the secret state by introducing two auxiliary particles through using the POVM operations. Due to the symmetry of the cluster state, we further extend the protocol into a multiparty protocol. When the agent with high authority is designed to recover the secret state, he will need the help of all the agents with high authority and one of the agents with low authority. However, the agent with low authority needs to cooperate with all of the agents to recover the secret state.

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