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Synchronization performance in time-delayed random networks induced by diversity in system parameter*

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Synchronization rhythm and oscillating in biological systems can give clues to understanding the cooperation and competition between cells under appropriate biological and physical conditions. As a result, the network setting is appreciated to detect the stability and transition of collective behaviors in a network with different connection types. In this paper, the synchronization performance in time-delayed excitable homogeneous random networks (EHRNs) induced by diversity in system parameters is investigated by calculating the synchronization parameter and plotting the spatiotemporal evolution pattern, and distinct impacts induced by parameter-diversity are detected by setting different time delays. It is found that diversity has no distinct effect on the synchronization performance in EHRNs with small time delay being considered. When time delay is increased greatly, the synchronization performance of EHRN degenerates remarkably as diversity is increased. Surprisingly, by setting a moderate time delay, appropriate parameter-diversity can promote the synchronization performance in EHRNs, and can induce the synchronization transition from the asynchronous state to the weak synchronization. Moreover, the bistability phenomenon, which contains the states of asynchronous state and weak synchronization, is observed. Particularly, it is confirmed that the parameter-diversity promoted synchronization performance in time-delayed EHRN is manifested in the enhancement of the synchronization performance of individual oscillation and the increase of the number of synchronization transitions from the asynchronous state to the weak synchronization. Finally, we have revealed that this kind of parameter-diversity promoted synchronization performance is a robust phenomenon.

Keywords: synchronization, time delay, excitable homogeneous random network, diversity

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1. Introduction

Synchronization phenomena are ubiquitous in nature and can often be observed in various realistic systems. Since the realization of synchronization between coupled chaotic oscillators carried out by Pecora and Carroll in 1990,[1] the problems of synchronization have become one of the most important issues in nonlinear science, and have attracted great attention during the last 20 years.[2,3] Theoretically, the synchronization realization is classified as complete synchronization, weak synchronization, lag synchronization, phase synchronization, and generalized synchronization.[4–9] Furthermore, many researchers thought that the synchronization approach can be used to estimate unknown parameters[10] and enhance secure communication.[11–13] Experimental studies have shown that synchronous oscillations can emerge in neuronal networks and brain systems, and are associated with some specific and important physiological functions.[14–20] Besides the synchronization approach on finite oscillators under coupling, the stability and transition of synchronization in networks are more appreciated and attractive to discovery some important mechanism in biological tissue and systems. As the concepts of "small-world"[21] and "scale-free"[22] have been proposed by Watts and Barabási, remarkable advances have been achieved in a lot of fields related to complex networks in recent years. Now, synchronization in complex networks has become one of the central topics under investigation due to their extensive applications.[23–27]

Time delay and diversity are inevitable in neuronal networks and brain systems.[28,29] Time delays exist due to the finite propagation velocities in the conduction of signals along neuron axons and the finite response period to external forcing or internal signal processing. More importantly, these unignorable delays play a key role in determining the electrical activities of neurons and self-organization in these two systems as well. In recent years, lots of works have been undertaken on the neuronal networks and brain systems with time delays, and several amazing phenomena have been discovered.[30–45] For example, Dhamala investigated the enhancement of neural synchrony by time delay.[30] Significantly, Wang found that

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time delays can enhance the coherence of spiral waves,[33] induce multiple stochastic resonances,[34] and synchronization transitions,[35,36] and can cause synchronous bursts.[37] Moreover, Yu demonstrated the synchronization transitions in time-delayed neuronal networks with hybrid synapses.[39,40] Ma exposed that an appropriate setting in time delay embedded in autapse can enhance spiral waves to regulate the collective behaviors in neuronal networks.[44,45] Besides, we revealed time delay and long-range connection induced synchronization transitions in Newman–Watts small-world neuronal networks, and discussed the mechanisms underlying the spatiotemporal patterns obtained in time-delayed Newman–Watts small-world neuronal networks.[41] It is well known that neurons are diverse in terms of morphology and function in neuronal networks and brain systems. Several diversity induced surprising phenomena are identified.[46–49] People have found that diversity can sustain pattern formation in sub-excitable media,[46] can induce resonance,[47] and irregular collective behavior,[48] and can cause synchronization transitions in small-world neuronal networks.[49] In our previous work,[50] time delay-induced synchronization transitions in excitable homogeneous random networks (EHRNs) have been studied. However, how the diversity in system parameters impacts on the synchronization performance in time-delayed EHRNs is still unclear. This is the task we aim to explore in the present paper.

In this paper, by calculating the synchronization parameter and plotting the spatiotemporal evolution pattern, we systematically investigate the synchronization performance in time-delayed EHRNs induced by diversity in system parameters. The remainder of paper is organized as follows. Section 2 introduces the mathematical model and the order parameter. The impact of parameter-diversity on synchronization performance in time-delayed EHRNs is studied in Section 3. Section 4 presents the parameter-diversity promoted synchronization performance in time-delayed EHRNs. In Section 5, we discuss the universality of parameter-diversity promoted synchronization performance in time-delayed EHRNs. Finally, we give the conclusion in the last section.

2. Mathematical model and order parameter

In this paper, we consider EHRNs with time delays. The Bär–Eiswirth[51] model is adopted for local dynamics. The evolution of the studied network dynamics is described by the following equations:

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -\frac{1}{\varepsilon}u_i(t)[u_i(t)-1][u_i(t)-\frac{v_i(t)+b_i}{a}] + D\sum_{j=1}^{N} A_{i,j}[u_j(t-\tau)-u_i(t)], \quad (1)$$

$$\frac{\mathrm{d}v_i(t)}{\mathrm{d}t} = f[u_i(t)] - v_i(t), \quad (2)$$

where $i = 1,2,\ldots,N$ represents the position of excitable node in the network. The function $f[u(t)]$ takes the form: $f[u(t)] = 0$ for $u(t) < 1/3$; $f[u(t)] = 1 - 6.75u(t)[u(t)-1]^2$ for $1/3 \le u(t) \le 1$; and $f[u(t)] = 1$ for $u(t) > 1$. Here variables u_i and v_i describe the activator and the inhibitor of the ith node, respectively. The small relaxation parameter ϵ represents the time ratio between the activator u and the inhibitor v. The dimensionless parameters a and b denote the activator kinetics of the local dynamics, and D is the coupling intensity, which decides the interaction strength between linking nodes. The system parameters are kept throughout this paper as $a = 0.84$, $b = 0.07$, $\varepsilon = 0.04$, and $D = 0.5$, under which the local cell can follow a typically excitable dynamics. As EHRN is considered in the present paper, we adopt identical degree $k = 3$ for each node (i.e., each node in the network couples to 3 other nodes, and the bidirectional symmetric couplings are chosen randomly). $A_{i,j}$ is the adjacency matrix element, and is defined as $A_{i,j} = A_{j,i} = 1$ if there is a connection linking nodes i and j, and $A_{i,j} = A_{j,i} = 0$ otherwise. Variable τ is the time delay in information transmission, and is an important adjustable parameter.

Diversity is introduced in the system parameter b (i.e., the values of b_i for each excitable node in time-delayed EHRNs are different). It satisfies the following Gaussian distribution:

$$\langle b_i \rangle = b, \quad \langle (b_i - b)(b_j - b) \rangle = \sigma^2 \delta(i-j). \quad (3)$$

Here σ is the standard deviation of Gaussian probability distribution of the system parameter b, which represents the strength of parameter-diversity in the network. By manipulating the standard deviation σ, we can control the parameter-diversity of time-delayed EHRNs. The above dynamical equations are integrated by the forward Euler integration scheme with time step $\Delta t = 0.001$, and the random initial condition is used in the numerical simulation (i.e., the initial variables $u_i(t=0)$ and $v_i(t=0)$ are randomly given between 0 and 1).

In order to quantitatively investigate the synchronization performance in time-delayed EHRNs induced by the diversity in system parameters, the synchronization parameter R is used.[52,53] It is numerically calculated as

$$R = \frac{\langle \bar{u}(t)^2 \rangle - \langle \bar{u}(t) \rangle^2}{\frac{1}{N}\sum_{i=1}^{N} \left[\langle u_i(t)^2 \rangle - \langle u_i(t) \rangle^2 \right]}, \quad (4)$$

where

$$\bar{u}(t) = \frac{1}{N}\sum_{i=1}^{N} u_i(t). \quad (5)$$

The angular brackets denote the average over time. It is indicated from Eq. (4) that the larger the synchronization parameter R is, the more synchronization is realized in the network. The value of R close to zero indicates that the states of individual cells are significantly different and the whole network oscillates asynchronously at all. While a value of

R close to unity represents that all nodes can oscillate synchronously and complete synchronization is achieved in the network. Consequently, the synchronization parameter R is an excellent indicator to reveal the synchronization performance in time-delayed EHRNs and the related transition. To guarantee the statistical accuracy with respect to the network structure and the random initial condition, $N_{\mathrm{ALL}} = 100$ independent samples are performed for each set of parameters in the simulation. We will calculate the average of synchronization parameters corresponding to these N_{ALL} independent samples, i.e.,

$$\bar{R} = \frac{1}{N_{\mathrm{ALL}}} \sum_{i=1}^{N_{\mathrm{ALL}}} R_i. \tag{6}$$

In the following, the synchronization parameter R will be used to identify the synchronization performance of each oscillation in time-delayed EHRNs. Moreover, $\bar{R}$ will be utilized as an order parameter to investigate the synchronization performance in time-delayed EHRNs for a given set of system parameters.

3. Time delay-induced synchronization transitions in EHRNs without diversity

In this part, we discuss time delay induced synchronization transitions in EHRNs without parameter-diversity. Figure 1 shows the dependence of the order parameter $\bar{R}$ on the time delay τ in EHRNs for $\sigma = 0.0$. System size is set as $N = 100$. In Fig. 1 three distinct parameter regions of time delay are discovered. As $1.0 \leq \tau \leq 2.2$ (i.e., domain I in Fig. 1), the order parameters $\bar{R}$ are all around zero. This means that time delay has no effect on synchronization performance in EHRNs, the time-delayed EHRNs are all in the asynchronous states (called the asynchronous region). When τ is in the parameter region of $2.3 \leq \tau \leq 2.8$ (i.e., domain II in Fig. 1, indicated by the blue rectangle), the order parameters increase rapidly. It indicates that time delay can improve the synchronous performance of EHRN, and induce the synchronization transition from an asynchronous state to weak synchronization (called the transition region). Consequently, the first critical value $\bar{R}_{\mathrm{c1}}$, which is obtained at $\tau = 2.3$ (located in the beginning of the transition region), can be used to identify the asynchronous state. As time delay $\tau \geq 2.9$ (i.e., domain III in Fig. 1), the order parameters $\bar{R}$ are all around unity. This means that time delay can induce complete synchronization in EHRNs. The time delayed EHRNs in this parameter region are all in complete synchronization states (called the synchronous region). Consequently, the second critical value $\bar{R}_{\mathrm{c2}}$, which is obtained at $\tau = 2.9$ (located in the beginning of the synchronous region), can be used to identify the complete synchronization state. In Fig. 1, we have discovered three distinct parameter regions, i.e., asynchronous region (domain I for $1.0 \leq \tau \leq 2.2$), transition region (domain II for $2.3 \leq \tau \leq 2.8$), and synchronous region (domain III for $\tau \geq 2.9$). Furthermore, the two critical values $\bar{R}_{\mathrm{c1}}$ and $\bar{R}_{\mathrm{c2}}$ are exposed to identify the asynchronous state, the weak synchronization, and the complete synchronization in EHRNs, when the synchronization parameter of each oscillation satisfies $R < \bar{R}_{\mathrm{c1}}$, $\bar{R}_{\mathrm{c1}} \leq R \leq \bar{R}_{\mathrm{c2}}$, and $R > \bar{R}_{\mathrm{c2}}$, respectively.

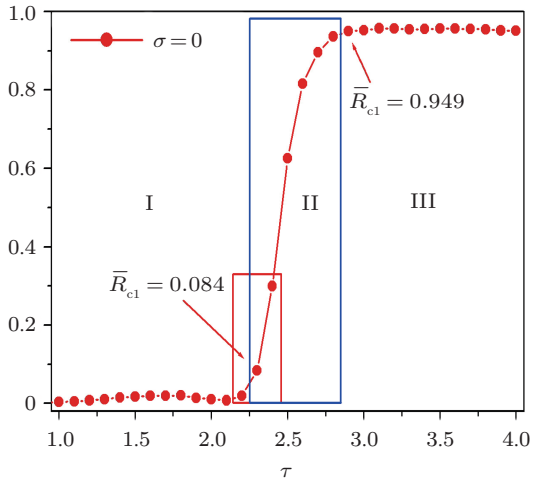


Fig. 1. (color online) The dependence of the order parameter $\bar{R}$ on the time delay τ in excitable homogeneous random networks (EHRNs) without parameter-diversity (i.e., $\sigma = 0.0$). System parameters are chosen as $a = 0.84$, $b = 0.07$, $\epsilon = 0.04$, $D = 0.5$, and $N = 100$. Here the order parameter $\bar{R}$ is calculated by the formula $\bar{R} = (1/N_{\mathrm{ALL}}) \sum_{i=1}^{N_{\mathrm{ALL}}} R_i$, in which R_i is the synchronization parameter of each oscillation in time-delayed EHRNs, and $N_{\mathrm{ALL}} = 100$ is the total number of independent samples performed for each set of parameters. Three distinct parameter regions, i.e., asynchronous region (domain I for $1.0 \leq \tau \leq 2.2$), transition region (domain II for $2.3 \leq \tau \leq 2.8$, indicated by the blue rectangle), and synchronous region (domain III for $\tau \geq 2.9$) are discovered. $\bar{R}_{\mathrm{c1}}$ and $\bar{R}_{\mathrm{c2}}$ represent the two critical values, which can be used to identify the asynchronous state, the weak synchronization, and the complete synchronization in EHRNs. The red rectangle indicates the parameter region of time delay (i.e., $2.2 \leq \tau \leq 2.4$), in which appropriate parameter-diversity can promote the synchronization performance in time-delayed EHRNs.

4. The impacts of parameter-diversity on synchronization performance in time-delayed EHRNs

Here, we use space–time plots to firstly investigate the impact of parameter-diversity on synchronization performance in EHRNs for small time delay. Figure 2 displays the space–time plots of u for time delay $\tau = 1.0$ with different diversities σ. In the white regions, the nodes fire, while in the black ones they are quiescent. Time passes from left to right. Initially, in the absence of parameter-diversity (shown by Fig. 2(a) for $\sigma = 0.0$), most of the nodes in the network oscillate asynchronously and the zigzag fronts are observed. This means that the time-delayed EHRN is in the asynchronous state. Now we introduce diversity in the system parameter b (i.e., the parameter-diversity is introduced in the network). Figure 2(b) shows the result obtained at diversity $\sigma = 0.1$, an irregular spatiotemporal evolution pattern is observed. As diversity is further increased (shown by Fig. 2(c) for $\sigma = 0.4$), a similar disordered pattern is gained, the time-delayed EHRN is still in the asynchronous state. It is shown from Fig. 2 that there

is no significant change in synchronization performance, the time-delayed EHRNs are all in the asynchronous states. This indicates that parameter-diversity has no distinct effect on synchronization performance in EHRNs with small time delay being considered.

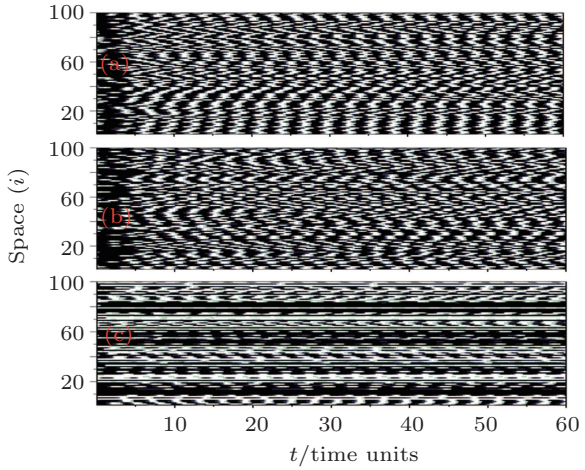


Fig. 2. (color online) The space–time plots of u for time delay $\tau = 1.0$ with different diversities σ in EHRNs. (a) $\sigma = 0.0$ ($R = 0.001$, asynchronous state); (b) $\sigma = 0.1$ ($R = 0.002$, asynchronous state); and (c) $\sigma = 0.4$ ($R = 0.004$, asynchronous state). The figures are plotted in greyscale from black (lowest value at 0.0) to white (highest value at 1.0), and this greyscale will be used throughout this paper. Time passes from left to right.

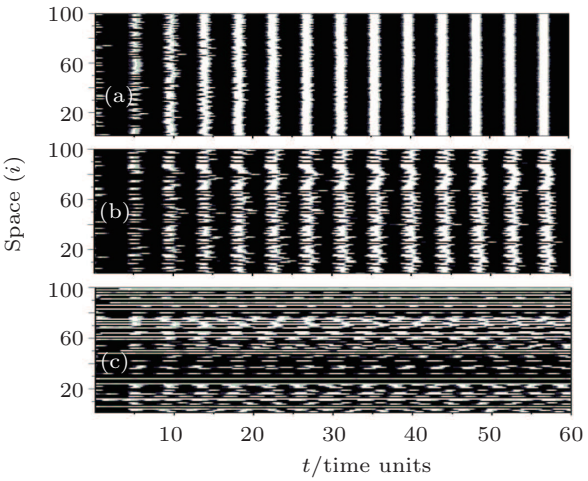


Fig. 3. The space–time plots of u for time delay $\tau = 4.0$ with different diversities σ in EHRNs. (a) $\sigma = 0.0$ ($R = 0.983$, complete synchronization); (b) $\sigma = 0.1$ ($R = 0.596$, weak synchronization), and (c) $\sigma = 0.4$ ($R = 0.010$, asynchronous state).

Now we study the impact of parameter-diversity on synchronization performance in EHRNs for large time delay. Figure 3 displays the space–time plots of u for time delay $\tau = 4.0$ with different diversities σ. As $\sigma = 0.0$, all nodes in the network fire simultaneously and damp to their rest state together (shown by Fig. 3(a)). This indicates that complete synchronization can be achieved in EHRN for large time delay when parameter-diversity is absent. As diversity in parameter b is introduced (shown by Fig. 3(b) for $\sigma = 0.1$), some nodes in the network execute synchronous oscillation, while others fire irregularly. A weak synchronization state of time-delayed EHRN is detected. When parameter-diversity is further increased (shown by Fig. 3(c) for $\sigma = 0.4$), most of the nodes in the network oscillate asynchronously, and an irregular spatiotemporal pattern is observed. The time-delayed EHRN is in the asynchronous state. It is shown from Fig. 3 that the synchronization performance in EHRN degenerates remarkably as parameter-diversity is increased for large time delay. This means that diversity in system parameters is harmful for synchronization performance in EHRNs when time delay is increased greatly.

Finally, we discuss the impact of parameter-diversity on the synchronization performance in EHRNs for moderate time delay. Figures 4(a)–4(c) show the space–time plots of u for time delay $\tau = 2.3$ with different diversities σ. Without parameter-diversity, the nodes in the network oscillate asynchronously and the time-delayed EHRN is in the asynchronous state (shown by Fig. 4(a) for $\sigma = 0.0$). As diversity in parameter b is introduced (shown by Fig. 4(b) for $\sigma = 0.1$), to our surprise, some nodes in the network can carry out synchronous oscillation. These synchronous nodes almost can fire simultaneously and damp to their rest state together. Consequently, a weak synchronization state of time-delayed EHRN is observed. This indicates that the synchronization performance in EHRN can be promoted by appropriate parameter-diversity for moderate time delay. The parameter-diversity that induced the synchronization transition from the asynchronous state to the weak synchronization is exposed. However, the weak synchronization state of time-delayed EHRN degenerates as parameter-diversity is further increased (shown by Fig. 4(c) for $\sigma = 0.4$). The time-delayed EHRN finally falls into the asynchronous oscillation again. From Fig. 4 we can conclude that, by setting moderate time delay, appropriate parameter-diversity can promote the synchronization performance in EHRNs, and can induce the synchronization transition from the asynchronous state to the weak synchronization.

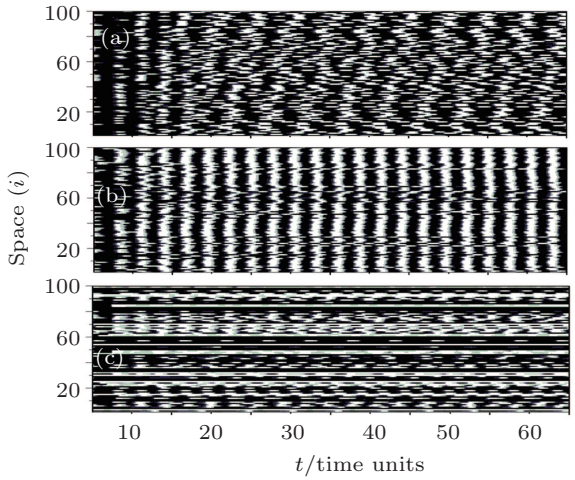


Fig. 4. The space–time plots of u for time delay $\tau = 2.3$ with different diversities σ in EHRNs. (a) $\sigma = 0.0$ ($R = 0.013$, asynchronous state); (b) $\sigma = 0.1$ ($R = 0.629$, weak synchronization), and (c) $\sigma = 0.4$ ($R = 0.009$, asynchronous state).

To further verify the above visual assessments and to quantitatively investigate the impacts of parameter-diversity on synchronization performance in time-delayed EHRNs, the dependence of the order parameter $\bar{R}$ on the diversity σ with different time delays τ is exhibited in Fig. 5.

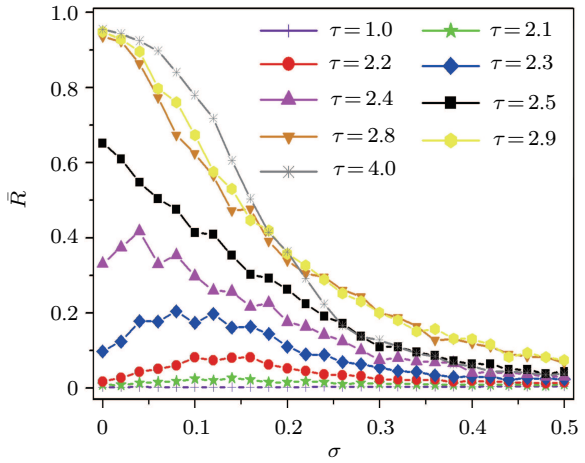


Fig. 5. (color online) The dependence of the order parameter $\bar{R}$ on the diversity σ with different time delays τ in EHRNs.

It is shown from Fig. 5 that the order parameters $\bar{R}$ are all around zero as time delay is small (such as $\tau = 1.0$ and $\tau = 2.1$). This indicates that parameter-diversity has no distinct effect on the synchronization performance in EHRNs with small time delay being considered. When time delay is increased greatly (such as $\tau = 2.5$, $\tau = 2.8$, $\tau = 2.9$, and $\tau = 4.0$), the order parameter $\bar{R}$ decreases remarkably as the diversity σ is increased, which means that diversity in system parameters is harmful for synchronization performance in time-delayed EHRNs in this case. However, by setting moderate time delay (such as $\tau = 2.2$, $\tau = 2.3$, and $\tau = 2.4$), which is located around the border between the asynchronous region and the transition region (indicated by the red rectangle in Fig. 1), the order parameter $\bar{R}$ initially increases, then passes through a maximum, and finally deceases to zero. This implies that the synchronization performance in EHRNs can be promoted by appropriate parameter-diversity for moderate time delay, and there is an optimal diversity for supporting the synchronization performance in time-delayed EHRNs. Now the impact of parameter-diversity on synchronization performance in time-delayed EHRNs has been exposed clearly, and the above visual assessments have been confirmed.

Moreover, the cross phenomenon between the curves $\tau = 4.0$ and $\tau = 2.8, 2.9$ in Fig. 5 is exposed. We consider this is induced by the following reason. From Fig. 1 we can find that time delay can induce complete synchronization in EHRNs without parameter-diversity when τ is in the synchronous parameter region (see domain III in Fig. 1). As complete synchronization is achieved in time-delayed EHRN (see the spatiotemporal evolution pattern shown in Fig. 3(a) for time delay $\tau = 4.0$ and diversity $\sigma = 0.0$), all excitable nodes in the network can excite simultaneously and can damp to their rest state together, permanently oscillate just as a single cell. It is well known that the excitable dynamics is non-oscillatory, the single excitable node can oscillate if and only if it is driven by a certain oscillation source. Once time delay is contained in the excitable dynamics, the time-delayed feedback can serve as the oscillation source to sustain the permanent oscillation of excitable dynamics. However, due to the existence of the refractory period of excitable dynamics, there must be a minimum time delay $\tau_{\min}$ at a given set of system parameters. The specific local excitable dynamics at this given set of system parameters can emerge when time delay is not less than $\tau_{\min}$.

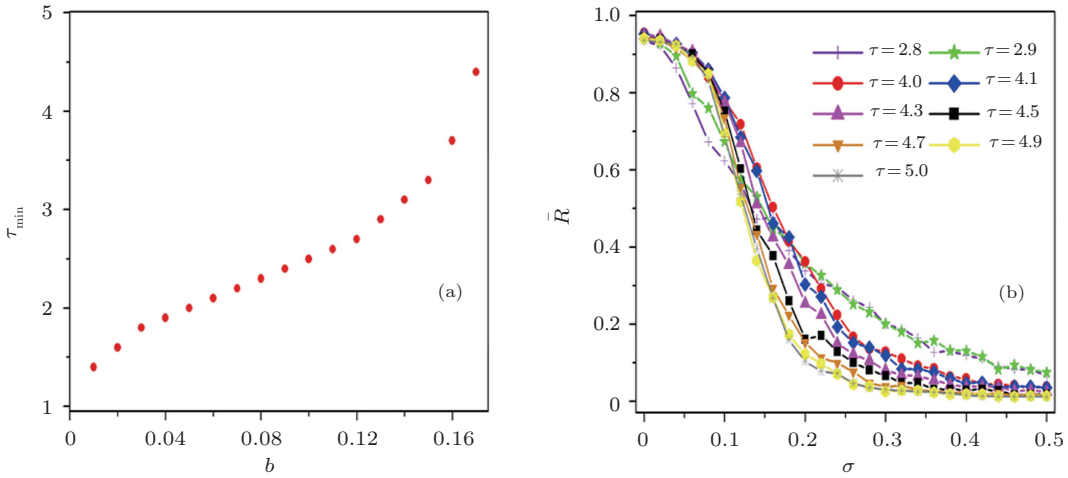


Fig. 6. (color online) (a) The dependence of the minimum time delay $\tau_{\min}$ for sustaining the specific local excitable dynamics on the system parameter b. (b) The dependence of the order parameter $\bar{R}$ on the diversity σ for other larger time delays τ.

Figure 6(a) reveals the dependence of the minimum time delay $\tau_{\min}$ for sustaining the specific local excitable dynamics on the system parameter b. It is shown that, for a given parameter b, there exists a corresponding minimum time delay $\tau_{\min}$. When the time delay contained in the excitable dynamics satisfies $\tau \geq \tau_{\min}$, the specific local excitable dynamics at this given parameter

b can be exhibited completely. Furthermore, as parameter b is increased, the corresponding minimum time delay $\tau_{\min}$ increases remarkably. This means that the larger parameter b is, the larger the minimum time delay $\tau_{\min}$ is needed to realize the specific local excitable dynamics.

As parameter-diversity is introduced in time-delayed EHRN, the values of b_i for each excitable node in the network will become diverse, and different minimum time delays are needed to realize the corresponding local excitable dynamics. In this case, the maximal $\tau_{\min}$ is the best choice to realize all distinct excitable dynamics. This means that the larger the time delay is, the more the excitable dynamics can realize in time-delayed EHRN. This will result in the degeneration of synchronization performance for large time delays. Consequently, the synchronization performance of EHRN with the same parameter-diversity will degenerate remarkably when the time delay is large. The cross phenomenon between the curves $\tau = 4.0$ and $\tau = 2.8, 2.9$ in Fig. 5 can be observed. Based on the above analysis, we can speculate that similar desynchronization phenomenon can also be observed for other larger time delays. To confirm our speculation, the dependence of the order parameter $\bar{R}$ on the diversity σ for other larger time delays τ (i.e., $\tau > 4.0$) is plotted in Fig. 6(b). It is shown that the order parameter $\bar{R}$ of EHRN with the same parameter-diversity decreases more abruptly as time delay τ increases. Our speculation is confirmed.

5. Parameter-diversity promoted synchronization performance in time-delayed EHRNs

Now we detailedly discuss the parameter-diversity promoted synchronization performance in time-delayed EHRNs. Fig. 7(a) displays the dependence of the synchronization parameter R on the diversity σ for moderate time delay $\tau = 2.3$. The blue and the green dashed lines represent the two critical synchronization parameters $\bar{R}_{c1}$ and $\bar{R}_{c2}$, which can be used to identify the asynchronous state, the weak synchronization, and the complete synchronization in time-delayed EHRNs. It is shown from Fig. 7(a) that, for some diversities σ, the synchronization parameters R can distribute in two different regions (located in the asynchronous state region and the weak synchronization region). This means that the bistability phenomenon can exist in time-delayed EHRNs. To visually test this result, the space-time plots of u obtained for time delay $\tau = 2.3$ and diversity $\sigma = 0.12$ are displayed in Fig. 8. The synchronization parameters corresponding to the spatiotemporal evolution patterns of Figs. 8(a) and 8(b) are $R = 0.007$ and $R = 0.569$, which indicates that the states of Figs. 8(a) and 8(b) are in the asynchronous state and weak synchronization, respectively. The bistability phenomenon, which contains the states of asynchronous state and weak synchronization, is visually confirmed in time-delayed EHRNs. Now we can conclude that the bistability phenomenon, which contains the states of asynchronous state and weak synchronization, can be observed in EHRNs for suitable diversity and time delay.

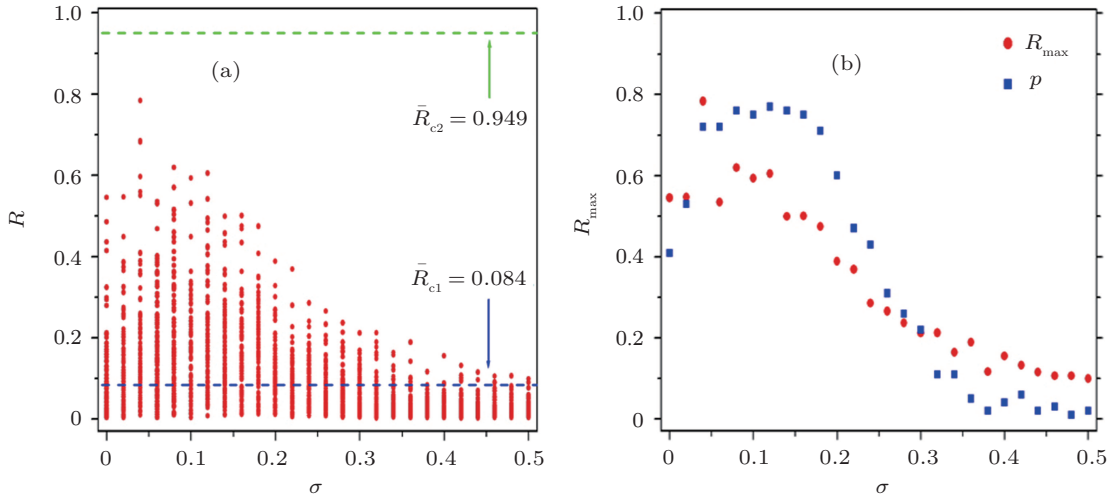


Fig. 7. (color online) (a) The dependence of the synchronization parameter R on the diversity σ for time delay $\tau = 2.3$ in EHRNs. The blue and the green dashed lines represent the two critical synchronization parameters $\bar{R}_{c1}$ and $\bar{R}_{c2}$, which can be used to identify the asynchronous state, the weak synchronization, and the complete synchronization in time-delayed EHRNs. (b) The dependence of the maximum synchronization parameter R_{max} (indicated by the left axis and shown by the red dots) and the proportion of weak synchronization p (indicated by the right axis and shown by the blue squares) on the diversity σ in time-delayed EHRNs. The proportion of weak synchronization p is defined as $p = (N_{WS}/N_{ALL})$, where N_{WS} is the number of weak synchronization counted in the $N_{ALL} = 100$ independent numerical samples performed for each set of parameters.

Moreover, from Fig. 7(a) we can also find that appropriate parameter-diversity can not only enhance the synchronization performance of individual oscillation in time-delayed EHRNs, but also can impact on the number of weak synchronization emerging in time-delayed EHRNs. To quantitatively verify the above two points, Fig. 7(b) displays the dependence of the maximum synchronization parameter R_{max} (indicated by the left axis and shown by the red dots) and the proportion of weak synchronization p (indicated by the right axis and shown by the blue squares) on the diversity

σ. Here, the proportion of weak synchronization p is defined as $p = N_{\rm WS}/N_{\rm ALL}$, where $N_{\rm WS}$ is the number of weak synchronization counted in the $N_{\rm ALL} = 100$ independent numerical samples performed for each set of parameters. It is shown from Fig. 7(b) that, as diversity σ is increased, the maximum synchronization parameter $R_{\rm max}$ can initially increase slightly, then passes through a maximum (the maximum $R_{\rm max}$ is 0.784, and is located at $\sigma = 0.04$), and finally deceases. This means that suitable parameter-diversity can enhance the synchronization performance of individual oscillation in time-delayed EHRNs. Furthermore, the proportion of weak synchronization p can initially increase significantly, then passes through a maximum (the maximum p is 0.77, and is located at $\sigma = 0.12$), and finally deceases. This indicates that suitable parameter-diversity can also remarkably increase the number of weak synchronizations emerging in time-delayed EHRNs (i.e., increase the number of synchronization transition from the asynchronous state to the weak synchronization). Now we can declare that the parameter-diversity promoted synchronization performance in time-delayed EHRNs is manifested in the enhancement of the synchronization performance of individual oscillation and the increase of the number of synchronization transitions from the asynchronous state to the weak synchronization.

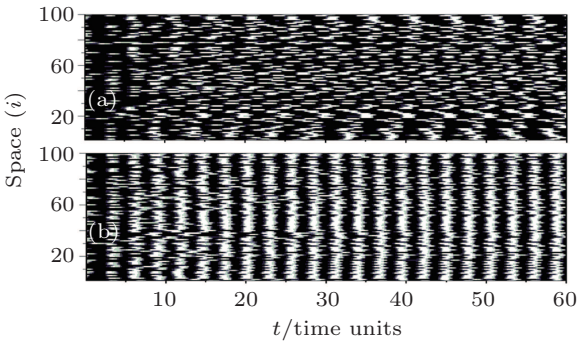


Fig. 8. The space–time plots of u for time delay $\tau = 2.3$ and diversity $\sigma = 0.12$ in EHRNs. The synchronization parameters corresponding to the spatiotemporal evolution patterns of panels (a) and (b) are $R = 0.007$ (asynchronous state) and $R = 0.569$ (weak synchronization), respectively.

6. The universality of parameter-diversity promoted synchronization performance in time-delayed EHRNs

In this part, we test the universality of parameter-diversity promoted synchronization performance in time-delayed EHRNs. We first examine the results for other system sizes. Figure 9 displays the dependence of the order parameter $\bar{R}$ on the diversity σ for time delay $\tau = 2.3$ with different system sizes N. Although the synchronization performance in time-delayed EHRNs degenerates as the system size N is increased for the same σ, similar parameter-diversity promoted synchronization performance can also be observed. More importantly, the peaks of these order parameter curves appear at approximatively the same σ. This means that suitable diversity in system parameters is a key factor in promoting the synchronization performance in delayed EHRNs. Now we check the results in the presence of noise. White Gaussian noise is added to Eq. (1). Figure 10 exhibits the dependence of the order parameter $\bar{R}$ on the diversity σ for time delay $\tau = 2.3$ with different noise intensities $D_{\rm noise}$. The parameter-diversity promoted synchronization performance in time-delayed EHRNs can also be detected in the presence of noise, which further confirms the determinant of parameter-diversity in determining the synchronization performance in time-delayed EHRNs. Now we can conclude that the parameter-diversity promoted synchronization performance in time-delayed EHRNs is a robust phenomenon. The results revealed in this paper are universal.

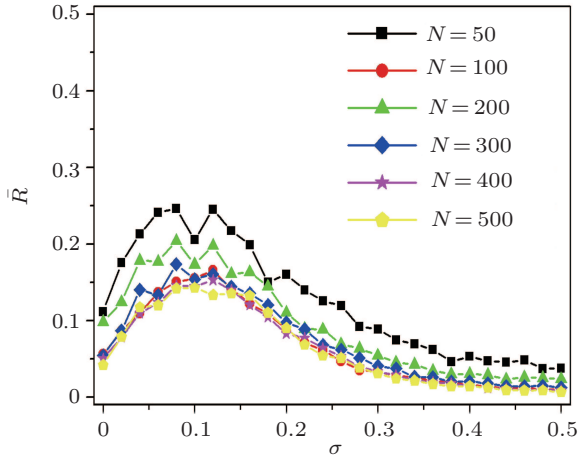


Fig. 9. (color online) The dependence of the order parameter $\bar{R}$ on the diversity σ for time delay $\tau = 2.3$ with different system sizes N in EHRNs.

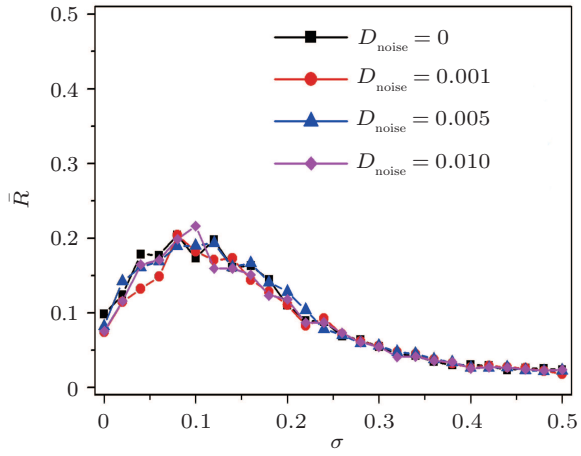


Fig. 10. (color online) The dependence of the order parameter $\bar{R}$ on the diversity σ for time delay $\tau = 2.3$ with different noise intensities $D_{\rm noise}$ in EHRNs.

7. Conclusion

In this paper, we systematically investigated the synchronization performance in time-delayed EHRNs induced by diversity in system parameters. Via calculating the synchronization parameter and plotting the spatiotemporal evolution

pattern, distinct impacts induced by parameter-diversity are detected by setting different time delays. Specifically, diversity has no distinct effect on synchronization performance in EHRNs with small time delay being considered, and the time-delayed EHRNs are all in the asynchronous states. When time delay is increased greatly, the synchronization performance of EHRN degenerates remarkably as diversity is increased. This means that diversity in system parameters is harmful for the synchronization performance in time-delayed EHRNs. However, by setting a moderate time delay (i.e., around the border between the asynchronous region and the transition region), appropriate diversity can promote the synchronization performance in EHRNs, and can induce the synchronization transition from the asynchronous state to the weak synchronization. Furthermore, the bistability phenomenon, which contains the states of asynchronous state and weak synchronization, is detected in EHRNs for suitable diversity and time delay. More importantly, we have exposed that the parameter-diversity promoted synchronization performance in time-delayed EHRN is manifested in the enhancement of the synchronization performance of individual oscillation and the increase of the number of synchronization transitions from the asynchronous state to the weak synchronization. Finally, we verify the universality of parameter-diversity promoted synchronization performance in time-delayed EHRNs. We have revealed that the parameter-diversity promoted synchronization performance in time-delayed EHRNs is robust to the system size and the external noise, which indicates that the results revealed in this paper are universal. Diversity in system parameters is exposed to be a key factor in determining the synchronization performance in time-delayed EHRNs.

As we know the synchronization oscillations in neuronal networks and brain systems are very important phenomena and are associated with some specific physiological functions. Time delays and diversities are inevitable in these two systems. A systematical investigation of the synchronization performance in time-delayed EHRNs induced by diversity in system parameter is expected to be useful both for theoretical understandings and practical applications. We do hope that our work will be a useful supplement to the previous contributions, and that it will have a useful impact in related fields.

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