ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理
Chinese Physics B

Nonlinear ion-acoustic solitary waves in an electron-positron-ion plasma with relativistic positron beam

Ridip Sarma, Amar P Misra, Nirab C Adhikary

Citation: Chin. Phys. B . 2018, 27(10): 105207. **doi:** 10.1088/1674-1056/27/10/105207

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Upstream ion wave excitation in an ion-beam-plasma system

Kai-Yang Yi(弋开阳), Jin-Xiu Ma(马锦秀), Zi-An Wei(卫子安), Zheng-Yuan Li(李政元)

Chin. Phys. B . 2018, 27(5): 055201. **doi:** 10.1088/1674-1056/27/5/055201

Dynamics of laser beams in inhomogeneous electron—positron—ion plasmas

Cheng Li-Hong, Tang Rong-An, Du Hong-E, Xue Ju-Kui

Chin. Phys. B . 2015, 24(7): 075201. **doi:** 10.1088/1674-1056/24/7/075201

Parametric instabilities in single-walled carbon nanotubes

He Cai-Xia, Jian Yue, Qi Xiu-Ying, Xue Ju-Kui

Chin. Phys. B . 2014, 23(2): 025202. **doi:** 10.1088/1674-1056/23/2/025202

Effects of bi-kappa distributed electrons on dust-ion-acoustic shock waves in dusty superthermal plasmas

M. S. Alam, M. M. Masud, A. A. Mamun

Chin. Phys. B . 2013, 22(11): 115202. **doi:** 10.1088/1674-1056/22/11/115202

The effects of relativistic broadening and frequency down-shift on electron–cyclotron emission measurements in EAST

Liu Yong,Han Xiang,Ti Ang,Wang Yu-Min,Ling Bi-Li,Hu Li-Qun,Gao Xiang

Chin. Phys. B . 2012, 21(4): 045201. **doi:** 10.1088/1674-1056/21/4/045201

Nonlinear ion–acoustic solitary waves in an electron–positron–ion plasma with relativistic positron beam

Ridip Sarma[1], Amar P Misra[2], and Nirab C Adhikary[3,†]

[1] Department of Mathematics, University of Science and Technology, Techno City, Khanapara, Baridua, 9th Mile, Ri-Bhoi, Meghalaya, India
[2] Department of Mathematics, Siksha Bhavana, Visva-Bharati University, Santiniketan-731235, West Bengal, India
[3] Physical Sciences Division, Institute of Advanced Study in Science and Technology, Vigyan Path, Paschim Boragaon, Garchuk, Guwahati-781035, Assam, India

(Received 22 February 2018; revised manuscript received 24 May 2018; published online 27 September 2018)

The propagation characteristics of nonlinear ion–acoustic (IA) solitary waves (SWs) are studied in thermal electron–positron–ion plasma considering the effect of relativistic positron beam. Starting from a set of fluid equations and using the reductive perturbation technique, we derive a Korteweg–de Vries (KdV) equation which governs the evolution of weakly nonlinear IA SWs in relativistic beam driven plasmas. The properties of the IA soliton are studied, and it is shown that the presence of relativistic positron beam significantly modifies the characteristics of IA solitons.

Keywords: electron–positron–ion plasma, relativistic effect, ion–acoustic wave, homogeneous balance method

PACS: 52.35.Fp, 52.35.Mw, 52.35.Sb, 52.27.Ny **DOI:** 10.1088/1674-1056/27/10/105207

1. Introduction

The nonlinear propagation of solitary waves and shocks (SWS) in multicomponent plasmas has been widely studied for understanding the electrostatic disturbances in various plasma environments.[1–5] The study of electron–positron–ion (EPI) plasmas has been growing extensively due to their presence in the early universe, in the active galactic nuclei, in the pulsar magnetosphere, ionosphere, in the solar atmospheres, etc.[6–11] It is well known that when positrons are introduced into an electron–ion plasma, the response of the electrostatic waves changes significantly in comparison to the usual two component plasmas.[5,12,13] Therefore, it is worthwhile to study the nonlinear wave propagation in electron–positron–ion plasmas for understanding the dynamical behaviour of astrophysical and laboratory plasmas.[14–20] In the latter, the authors have shown that the nonlinear waves in the plasmas having positrons behave differently. Misra *et al.*[5] reported that the nonlinear mode of propagation and interaction in an electron–positron plasma exhibits a remarkable feature which is quite distinctive from the linear mode. Also, in some earlier works, Popel *et al.*[14] have found that the maximum amplitude of ion–acoustic (IA) solitons could be considerably reduced when a substantial fraction of hot positrons was present in the system. Interestingly, in a plasma system when a source of beam is inserted, it makes the system prone to a well-known streaming instability. Recently, Shan *et al.*[20] showed that due to the increased speed of positron beam and super-thermal electrons, the region of existence, amplitude, and width of solitons can be modified. Again, very recently, Shan *et al.*[21] found that with the variation in positron beam speed and positron population, and nonlinearities of the plasma system can be predominantly changed. Therefore, the options to create intense sources of positrons capable of fuelling extensive antimatter related research need to be developed, and this gap or requirement motivated us to start our investigation on ion–acoustic solitary waves (IASWs) in EPI plasmas with a relativistic positron beam impact which is a new kind of study not reported elsewhere as far as our knowledge is concerned.

Some of the earlier works considered electron-ion and EPI plasmas, however, most of them were non relativistic in nature.[3,22] But when the particle velocities are comparable to the speed of light, relativistic effects should be taken into account. Many researchers in due course of time exhibited the importance of relativistic effects in the modification of plasma wave dynamics.[23–29] In the ionospheric plasma system,[30] there is always a possibility of its interaction with highly intense charged particle beam coming from outer space. Therefore, in such conditions, the solitary wave in nonrelativistic plasmas may interact with relativistic charged particle beam. Based on all such observations, our brief communication has heralded on a model of EPI plasma to know the various effects of relativistic positron beam on soliton dynamics.

In the present paper, we investigate the propagation characteristics of ion–acoustic solitons in a relativistic beam driven plasma, and employ a new potent mathematical formalism known as the homogeneous balance method probably first generated by Wang *et al.* to show the solution of a

†Corresponding author. E-mail: iasst@yahoo.co.in

© 2018 Chinese Physical Society and IOP Publishing Ltd

Korteweg–de Vries (KdV) equation.[32] Here, Wang *et al.* introduced a new direct method called the homogeneous balance method to look for travelling wave solutions of nonlinear evolution equations with variable coefficients. It is seen that the method presents a wider applicability for handling many kinds of nonlinear evolution equations, such as high-dimensional equations, variable-coefficient equations, and differential-difference equations. The method is based on the homogeneous balance principle and linear ordinary differential equation (LODE) theory. Being concise and straightforward, this method has been applied to handle nonlinear partial differential equations in a direct manner with no requirement for initial/boundary conditions or initial trial function at the outset.

This paper is structured as follows. Section 2 describes the basic equations governing the plasma dynamics. The nonlinear evolution equation in the form of a Korteweg–de Vries equation is derived by using the reductive perturbation technique. In this section, the homogeneous balance method has been employed to execute the salient features of solitons. Section 3 analyzes thoroughly the results and the discussion parts. Finally, we conclude the results in Section 4.

2. Basic equations governing the plasma dynamics

We consider the nonlinear propagation of IASWs in an unmagnetized plasma consisting of thermal electrons and ions with a flow of relativistic positron beam. The basic equations governing the plasma dynamics can be written as

$$\frac{\partial n_{\rm i}}{\partial t}+\frac{\partial}{\partial x}\left(n_{\rm i}v_{\rm i}\right)=0, \tag{1}$$

$$\frac{\partial v_{\rm i}}{\partial t}+v_{\rm i}\frac{\partial v_{\rm i}}{\partial x}+\frac{1}{m_{\rm i}n_{\rm i}}\frac{\partial p_{\rm i}}{\partial x}=\frac{-q_{\rm i}}{m_{\rm i}}\frac{\partial \varphi}{\partial x},\ \text{where}\ q_{\rm i}=e, \tag{2}$$

$$\left(\frac{\partial}{\partial t}+v_{\rm i}\frac{\partial}{\partial x}\right)p_{\rm i}+3p_{\rm i}\frac{\partial v_{\rm i}}{\partial x}=0, \tag{3}$$

$$\frac{\partial n_{\rm e}}{\partial t}+\frac{\partial}{\partial x}\left(n_{\rm e}v_{\rm e}\right)=0, \tag{4}$$

$$\frac{\partial v_{\rm e}}{\partial t}+v_{\rm e}\frac{\partial v_{\rm e}}{\partial x}+\frac{1}{m_{\rm e}n_{\rm e}}\frac{\partial p_{\rm e}}{\partial x}=\frac{-q_{\rm e}}{m_{\rm e}}\frac{\partial \varphi}{\partial x},\ \text{where}\ q_{\rm e}=-e, \tag{5}$$

$$\left(\frac{\partial}{\partial t}+v_{\rm e}\frac{\partial}{\partial x}\right)p_{\rm e}+3p_{\rm e}\frac{\partial v_{\rm e}}{\partial x}=0, \tag{6}$$

$$\frac{\partial n_{\rm b}}{\partial t}+\frac{\partial}{\partial x}\left(n_{\rm b}v_{\rm b}\right)=0, \tag{7}$$

$$\left(\frac{\partial}{\partial t}+v_{\rm b}\frac{\partial}{\partial x}\right)(\gamma v_{\rm b})+\frac{1}{m_{\rm b}n_{\rm b}}\frac{\partial p_{\rm b}}{\partial x}=\frac{-q_{\rm b}}{m_{\rm b}}\frac{\partial \varphi}{\partial x}, \tag{8}$$

$$\left(\frac{\partial}{\partial t}+v_{\rm b}\frac{\partial}{\partial x}\right)p_{\rm b}+3p_{\rm b}\frac{\partial}{\partial x}(\gamma v_{\rm b})=0,\ \text{where}\ q_{\rm b}=e, \tag{9}$$

and the Poisson's equation

$$\frac{\partial^2\varphi}{\partial x^2}=-4\pi e\left(n_{\rm i}+n_{\rm b}-n_{\rm e}\right), \tag{10}$$

where n_j, v_j, p_j, q_j, and m_j respectively denote the number density, the fluid velocity, the thermal pressure, the charge, and the mass of j-th species particles ($j=$ i, e, and b respectively stand for ions, electrons, and positron beams). Furthermore, $k_{\rm B}$ is the Boltzmann constant, φ is electrostatic potential, γ is the relativistic factor, and c_1 is the ratio between the speed of light in vacuum c and the IA speed $c_{\rm s}$.

The collective effects in electron–positron plasmas can be studied if the electron–positron time scale is much longer than the electron–positron plasma effects, which is typically inverse of the plasma frequency. In a relativistic regime, the processes of creation and annihilation of electron–positron pairs become important. Before studying the nonlinear properties of the electron–positron plasma, it is necessary to take a look at the electron–positron pair annihilation time. Based on the theory of Svensson[32] concerning the electron- positron pair equilibria in relativistic plasma, to neglect the annihilation effect, the following inequality must be satisfied:

$$\omega_{p,j}^{-1}\ll\tau_{\rm ann}, \tag{11}$$

where $\omega_{p,j}^{-1}$ ($j=$ e and b respectively stand for electrons and positron beam) is the inverse of the plasma frequency equal to $\sqrt{4\pi e^2 n_{j0}/m}$ and $\tau_{\rm ann}$ is the annihilation time. The annihilation time $\tau_{\rm ann}$ in the relativistic regime can be expressed by the following relation:

$$\tau_{\rm ann}=\frac{4}{3}\frac{1}{n_j\sigma_\tau c}\left(\frac{E_{\rm Th}^2}{1/4+\ln\left(2\aleph E_{\rm Th}+1\right)}\right)\ \text{for}\ E_{\rm Th}\gg 1, \tag{12}$$

where σ_τ (6.65×10^{-25} cm^2) is the electron Thomson cross section, and $E_{\rm Th}\left(k_{\rm B}T_j/(m_jc^2)\right)$ is the normalised thermal energy. Also, $\aleph=\mathrm{e}^{-\aleph_{\rm E}}\equiv0.5615$ with $\aleph_{\rm E}\,(\approx0.5772)$ is the Euler's constant. Combining Eqs. (11) and (12) and using $n_{\rm e0}=n_{\rm p0}=n_0$, the pair annihilation condition becomes

$$\left(\frac{E_{\rm Th}^2}{1/4+\ln\left(2\aleph E_{\rm Th}+1\right)}\right)\gg 2.6\times10^{-19}n_0^{1/2}. \tag{13}$$

For some illustration purposes, we have chosen the number density value $n_0=10^4$ cm^{-3}, which is typically found in astrophysical environment (viz. ionospheric auroral region). By using the number density value into Eq. (13), one can obtain $0.229003\gg2.6\times10^{-15}$, which indicates that the electron–positron pair annihilation can be ignored in our plasma model Eqs. (1)–(10).

Now, we normalize the physical quantities as follows: $T=t\omega_{\rm pi}, X=x\omega_{\rm pi}/c_{\rm s}, V_j=v_j/c_{\rm s}, N_j=n_j/n_{j0}, \Phi=e\varphi/k_{\rm B}T_{\rm e}$, $P_j=p_j/n_{j0}k_{\rm B}T_j$, where $\omega_{\rm pi}=\sqrt{4\pi n_{\rm e0}e^2/(k_{\rm B}T_{\rm e})}$ is the ion–plasma oscillation frequency, $c_{\rm s}=\sqrt{\kappa_{\rm B}T_{\rm e}/m_{\rm i}}$ is the ion–acoustic speed, n_{j0} is the unperturbed number density, and T_j the temperature of j-th species particles. Thus, we recast Eq. (1) to Eq. (10) in dimensionless forms as

$$\frac{\partial N_{\rm i}}{\partial T}+\frac{\partial}{\partial X}\left(N_{\rm i}V_{\rm i}\right)=0, \tag{14}$$

$$\frac{\partial V_i}{\partial T}+V_i\frac{\partial V_i}{\partial X}+\frac{\sigma}{N_i}\frac{\partial P_i}{\partial X}=\frac{-\partial\Phi}{\partial X},\ \text{where}\ \sigma=\frac{T_i}{T_e}, \tag{15}$$

$$\left(\frac{\partial}{\partial T}+V_i\frac{\partial}{\partial X}\right)P_i+3P_i\frac{\partial V_i}{\partial X}=0, \tag{16}$$

$$\frac{\partial N_e}{\partial T}+\frac{\partial}{\partial X}(N_eV_e)=0, \tag{17}$$

$$\beta_1N_e\left(\frac{\partial V_e}{\partial T}+V_e\frac{\partial V_e}{\partial X}\right)+\frac{\partial P_e}{\partial X}=N_e\frac{\partial\Phi}{\partial X},\ \text{where}\ \beta_1=\frac{m_e}{m_i}, \tag{18}$$

$$\left(\frac{\partial}{\partial T}+V_e\frac{\partial}{\partial X}\right)P_e+3P_e\frac{\partial V_e}{\partial X}=0, \tag{19}$$

$$\frac{\partial N_b}{\partial T}+\frac{\partial}{\partial X}(N_bV_b)=0, \tag{20}$$

$$\beta_2N_b\left(\frac{\partial}{\partial T}+V_b\frac{\partial}{\partial X}\right)V_b'+\alpha\frac{\partial P_b}{\partial X}=-N_b\frac{\partial}{\partial X}, \tag{21}$$

where $V_b'=V_b\left(1+V_b^2/(2C_1^2)\right)$, $\beta_2=m_b/m_i$, $\alpha=T_b/T_e$, and $c_1=c/c_s$,

$$\frac{\partial P_b}{\partial T}+V_b\frac{\partial P_b}{\partial X}+3P_b\frac{\partial V_b'}{\partial X}=0, \tag{22}$$

where $V_b'=V_b\left(1+V_b^2/(2C_1^2)\right)$, or $V_b'=V_b+V_b^3/(2C_1^2)$, and

$$\frac{\partial^2\Phi}{\partial X^2}=N_e(1+\mu_b)-N_b\mu_b-N_i,\ \text{where}\ \mu_b=\frac{n_{b0}}{n_{i0}}. \tag{23}$$

In order to derive the evolution equation for the propagation of IASWs in a relativistic positron beam plasma, we use the reductive perturbation technique in which the independent variables are stretched as $\lambda\zeta=\varepsilon^{1/2}(X-T)$, $\tau=\varepsilon^{3/2}T$, where λ is the phase velocity and ε is a small nonzero constant measuring the weakness of the dispersion.

$$\frac{\partial}{\partial X}=\varepsilon^{1/2}\frac{\partial}{\partial\zeta},\ \frac{\partial}{\partial T}=\varepsilon^{3/2}\frac{\partial}{\partial\tau}-\varepsilon^{1/2}\frac{\partial}{\partial\zeta},$$
$$\text{and}\ \gamma=\left(1-\frac{V_b^2}{c'^2}\right)^{-1/2}=1+\frac{1}{2}\frac{V_b^2}{c'^2}. \tag{24}$$

Now, using the expansions, $N_e=1+\varepsilon N_{e1}+\varepsilon^2N_{e2}+\ldots,N_i=1+\varepsilon N_{i1}+\varepsilon^2N_{i2}+\ldots,N_b=1+\varepsilon N_{b1}+\varepsilon^2N_{b2}+\ldots,V_e=\varepsilon V_{e1}+\varepsilon^2V_{e2}+\ldots,V_i=\varepsilon V_{i1}+\varepsilon^2V_{i2}+\ldots,V_b=V_{b0}+\varepsilon V_{b1}+\varepsilon^2V_{b2}+\ldots,P_b=1+\varepsilon P_{b1}+\varepsilon^2P_{b2}+\ldots$, and $\Phi=\varepsilon\Phi_1+\varepsilon^2\Phi_2+\ldots$, and by following the standard procedure and substituting the values of N_e, N_i, N_b, V_e, V_i, V_b, P_b, and Φ in Eqs. (14)–(23), and then equating the coefficients of first order and second order in ε, we get together the dispersion relation for the phase velocity $\sqrt{3\sigma+3\alpha\gamma_b/(\alpha\gamma_b+(1+\alpha\gamma_b)\mu_b)}$, where $\gamma_b=\left(1+3V_b'^2\right)$, $V_b'=V_{b0}/2C_1$, and eliminating the second-order quantities, we obtain the following KdV equation for the nonlinear propagation of IASWs in a thermal electron–ion plasma with a relativistic positron beam.

$$\frac{\partial\Phi_1}{\partial\tau}+\frac{p}{q}\Phi_1\frac{\partial\Phi_1}{\partial\zeta}+\frac{1}{q}\frac{\partial^3\Phi_1}{\partial\zeta^3}=0,$$
$$\frac{\partial\Phi_1}{\partial\tau}+A\Phi_1\frac{\partial\Phi_1}{\partial\zeta}+B\frac{\partial^3\Phi_1}{\partial\zeta^3}=0,\ A=\frac{p}{q}\wedge B=\frac{1}{q}, \tag{25}$$

$$p=\frac{\left(3\lambda^2+7\sigma\right)}{\left(\lambda^2-3\sigma\right)^2}-\frac{(1+\mu_b)\left(2\lambda^2\beta_1+6\right)}{\left(\lambda^2\beta_1-3\right)^3}$$
$$+\mu_b\left(\frac{\gamma_b\beta_2\left(\lambda-2c_1V_b'\right)^2+\gamma_b\left(\beta_2\left(\lambda-2c1V_b'\right)^2-3\alpha\right)-6V_b\left(\frac{\lambda}{c_1}-2Vb'\right)\left(\beta_2\left(\lambda-2c_1V_b'\right)^2-3\alpha\right)+9\alpha\gamma_b^2}{\gamma_b^3\left(\beta_2\left(\lambda-2c1V_b'\right)^2-3\alpha\right)^3}\right),$$
$$q=\frac{2\lambda}{\left(\lambda^2-3\sigma\right)^2}+(1+\mu_b)\frac{2\lambda\beta_1}{\left(\lambda^2\beta_1-3\right)^2}+\frac{2\beta_2\left(\lambda-2c1V_b'\right)\mu_b}{\gamma_b\left(\beta_2\left(\lambda-2c_1V_b'\right)^2-3\alpha\right)^3}.$$

Now, we apply the transformation

$$\Phi(\zeta,\tau)=\Phi(\xi),\ \text{where}\ \xi=\xi-V\tau. \tag{26}$$

Then, we have

$$-V\Phi'+A\Phi'+B\Phi'''=0,\ \text{where}\ \Phi'=\frac{\partial\Phi}{\partial\xi}. \tag{27}$$

Integrating it with respect to ξ once yields

$$M-V+\frac{A}{2}\Phi^2+B\Phi''=0, \tag{28}$$

where M is an integration constant.

In order to get the soliton solution of Eq. (25), we use the homogeneous balance method which needs to assume a solution (ξ) in the form

$$\Phi(\xi)=\sum_{N=0}^{\infty}a_NQ(\xi)^N, \tag{29}$$

where $Q(\xi)$ is the solution of the equation $Q'(\xi)=Q^2(\xi)-Q(\xi)$.

Now substituting the value of Eq. (29) into Eq. (28), we balance the leading order of the nonlinear term with that of the linear term. This manipulation evaluates the exact numbers for N of the series obtained as $N=2$. Correspondingly, a series with $N=2$ terms become

$$\Phi(\xi)=a_0+a_1Q+a_2Q^2, \tag{30}$$

where a_0, $a_1\wedge a_2$ are constants to be determined such that $a_2\neq0$.

Here $Q(\xi)=1/(1\mp e^{\xi})$ satisfies the following equation

$$Q'(\xi)=Q^2(\xi)-Q(\xi). \tag{31}$$

From Eqs. (29) and (30), we obtain

$$\Phi^2(\xi) = a_2^2 Q^4 + 2a_2 a_1 Q^3 + \left(a_1^2 + 2a_2 a_0\right) Q^2 + 2a_1 a_0 Q + a_0^2, \quad (32)$$

$$\Phi''(\xi) = (Q-1)\, Q\left[(-1+2Q)\, a_1 + 2Q\,(-2+3Q)\, a_2\right]. \quad (33)$$

By substituting Eqs. (30), (32), and (33) into Eq. (28) and rearranging all coefficients of Q^0, Q^1, Q^2, and so on, and then using Mathematica 10 we get the values of a's, M, and Ω as $\Omega a_0 = 0, a_1 = 0, a_2 = -12B/A, M = 0, = \sqrt{V/B}$. Also taking the stability condition into account we obtain

$$\Phi(\xi) = \frac{3V}{A}\operatorname{sech}^2\left(\frac{\sqrt{V}}{2\sqrt{B}}\xi\right), \quad \text{where} \quad \xi = \zeta - V\tau,$$

$$\Phi(\xi) = \Phi_{\mathrm{m}}\operatorname{sech}^2\left(\frac{\xi}{\Delta}\right), \quad (34)$$

where $\Phi_{\mathrm{m}} = 3V/A$ and $\Delta = 2\sqrt{B}/\sqrt{V}$, which is the well-known solitary wave solution of the KdV Eq. (25).

3. Results and discussion

In what follows, we study the effects of the relativistic positron beam and other plasma parameters on the profiles of IASWs. The effects of nonlinearity become significant when the amplitude of the wave is sufficiently large. Nonlinear structures such as solitons and shocks are formed due to relative competition between nonlinearity, dispersion, and dissipation. When the effect of dissipation is small compared to nonlinearity and dispersion, the system can be supportive for the formation of solitons. The existence of a soliton solution with finite amplitude (Eq. (25)) depends on the behavior of the product AB, where A is the nonlinear term and B represents that of dispersion in Eq. (25). The solitary wave becomes unstable when $AB > 0$ and gives rise to a compressive solitary structure. So, some interesting results can be expected when we are examining positron beam velocity, positron beam concentration, ion and positron temperature, and mass ratio along with its amplitude, width, and dispersion relation in our electron–positron–ion model. We have considered an external positron beam which is relativistic in nature under the effect of adiabatic responses of the positron beam to contribute additional sources of energy to the nonlinear behaviour of the usual electron positron ion plasma dynamical system.

We numerically examine in Fig. 1 the variation of the phase velocity λ, plotted with respect to the positron concentration μ_{b} along with variable ion temperature σ. The other plasma parameters are $\alpha = 0.2$, $c_1 = 10^3$, and $V_{\mathrm{b}}' = 0.2$. Figure 1 shows that the dispersion properties of the present model have strong dependence on the plasma temperature. At elevated ion temperature σ, dispersion is triumphant and consequently the medium becomes more favourable for the propagation of compressive solitary waves, and the phase velocity increases as the ion temperature σ increases. The findings in Fig. 2, where variation of the phase velocity λ is plotted with respect to the positron concentration μ_{b} along with variable positron temperature α with other plasma parameters as $c_1 = 10^3$, $V_{\mathrm{b}}' = 0.2$, and $\sigma = 0.1$, are in similar lines with the outcomes in Fig. 2. In Fig. 3, the variation of the phase velocity λ is plotted with respect to the positron concentrationμ_{b} along with variable relativistic factor V_{b}' with other plasma parameters as $c_1 = 10^3$, $\alpha = 0.2$, and $\sigma = 0.1$. Here, from the investigation, it can be observed that the phase velocity λ is influenced by the relativistic effect V_{b}' even if V_{b}' is very small or considered fixed as shown in Figs. 1 and 2. When the ion temperature σ and positron temperature α are kept constant, phase velocity λ increases as V_{b}' increases. Thus from the present investigation it can be emphasized that the behaviour of ion acoustic solitary waves propagating in a relativistic electron–positron–ion plasma model is strongly dependent on the ion and positron temperature, the mass ratio, and the relativistic effect.

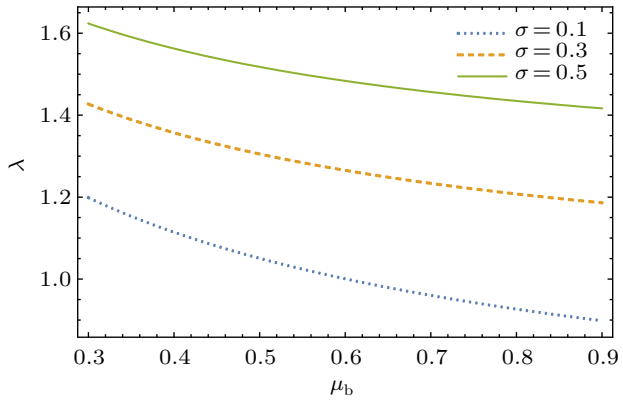


Fig. 1. (color online) The phase velocity λ is plotted with respect to the positron concentration μ_{b} along with variable ion temperature σ, where σ is 0.1, 0.3, and 0.5 for blue dotted, red dashed, and green lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_{\mathrm{b}}' = 0.2$, and $\alpha = 0.2$.

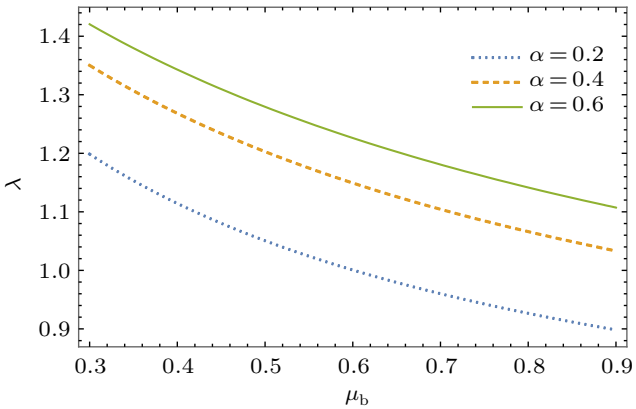


Fig. 2. (color online) The phase velocity λ is plotted with respect to the positron concentration μ_{b} along with variable positron temperature α, where α is 0.2, 0.4, and 0.6 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_{\mathrm{b}}' = 0.2$, and $\sigma = 0.1$.

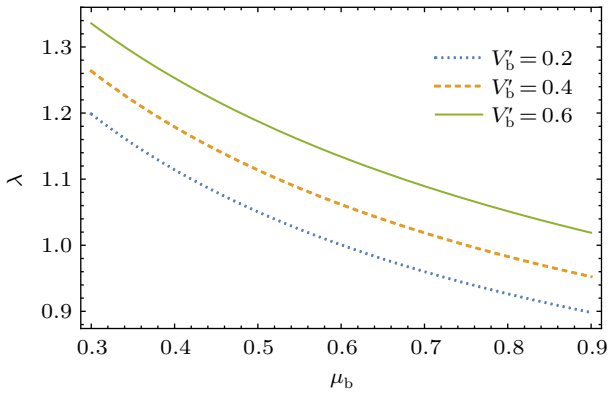


Fig. 3. (color online) The phase velocity λ is plotted with respect to the positron concentration μ_b along with variable relativistic factor V_b', where V_b' is 0.2, 0.4, and 0.6 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $\alpha = 0.2$, and $\sigma = 0.1$.

Next, we examine the properties of the soliton solution, namely its amplitude and width for different values of the system parameters. The results are shown in Figs. 4–9. Figure 4 represents the variation of the width plotted with respect to the positron concentration μ_b along with the increase in positron temperature, and as the positron temperature increases, the width increases. At higher positron temperature, i.e., at $\alpha = 0.4$ and $\alpha = 0.6$, it can be observed that the level of the increase in width is greater than that at lower temperature. Figure 5 exhibits the variation in width plotted along with variable positron temperature α. It shows that the width decreases with the increase in the positron concentration. On the other hand, in Fig. 6, the variation of width is plotted along with variable ion temperature σ. The results obtained in Figs. 5 and 6 seem to differ with the results obtained in non-relativistic cases.[17] It may be due to the fact that in relativistic electron–positron–ion plasma, increasing the positron density enhances the depth of the potential well, resulting in narrower solitons. Also, when the relativistic factor V_b' is kept constant, the dispersion effect starts to dominate over the nonlinearity with increasing values of ion temperature ratio σ. Since the width of the soliton is inversely proportional to the dispersion effect, the decrease in the ion temperature ratio σ results in the increase of the width. Figures 7, 8, and 9 seem to agree quite well when compared with the results obtained from Figs. 4, 5, and 6. In Figs. 8 and 9, it can be observed that the amplitude increases with increasing μ_b. The physical reason behind this is may be due to the consideration of an external positron beam relativistic in nature under the effect of adiabatic responses, which contributes additional sources of energy to the nonlinear behaviour of the usual electron positron ion plasma dynamical system. This similar observation is also congruent with the result exhibited in Fig. 7. Also, an interesting fact can be observed in Fig. 9, that is, the graphs of the three regions ($\mu_b = 0.5$, 0.7, and 0.9) tend to come closer to each other at higher ion temperature. This result is quite opposite to the character as observed for positron temperature in Fig. 8. This seems to be a quite interesting result, in view of its application in further study of astrophysical plasma regions.

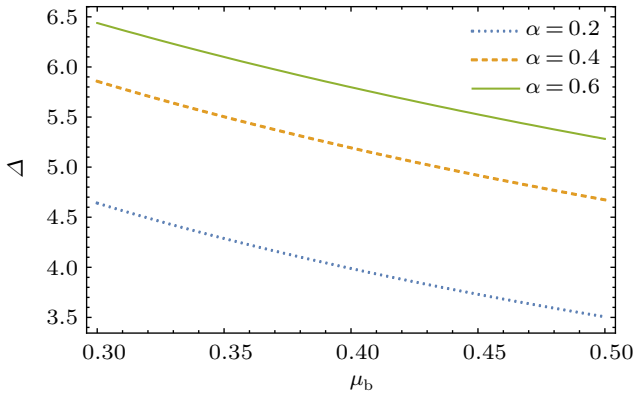


Fig. 4. (color online) The soliton width Δ is plotted with respect to the positron concentration μ_b along with variable positron temperature α, where α is 0.2, 0.4, and 0.6 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_b' = 0.2$, $\sigma = 0.1$, $\beta_1 = 0.005$, and $\beta_2 = 0.0054$.

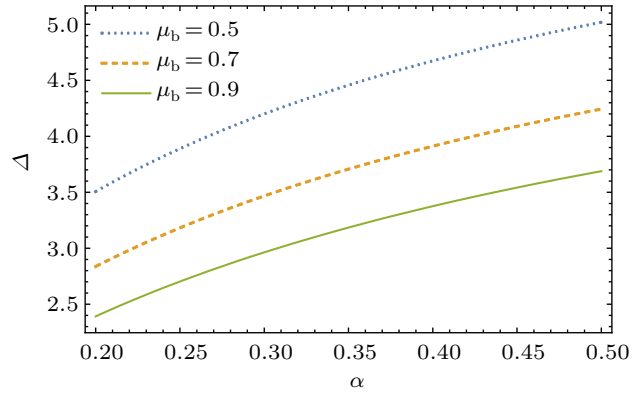


Fig. 5. (color online) The soliton width Δ is plotted with respect to the positron temperature α along with variable positron concentration μ_b, where μ_b is 0.5, 0.7, and 0.9 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_b' = 0.2$, $\sigma = 0.1$, $\beta_1 = 0.005$, and $\beta_2 = 0.0054$.

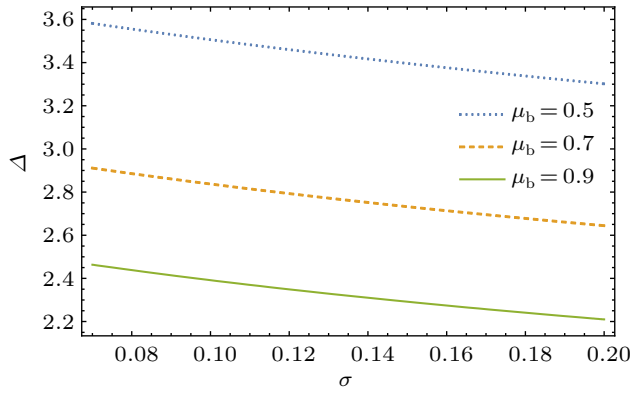


Fig. 6. (color online) The soliton width Δ is plotted with respect to the ion temperature σ along with variable positron concentration μ_b, where μ_b is 0.5, 0.7, and 0.9 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_b' = 0.2$, $\alpha = 0.2$, $\beta_1 = 0.005$, and $\beta_2 = 0.0054$.

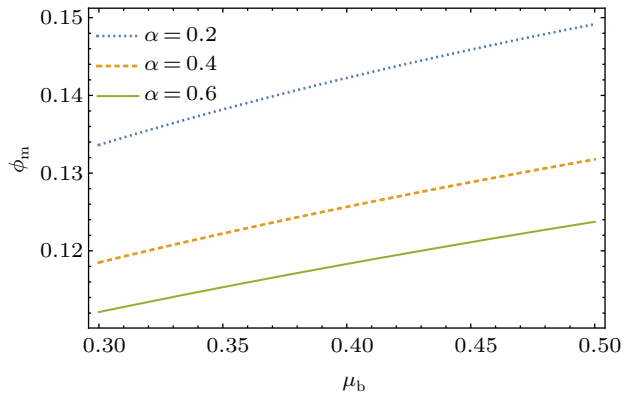


Fig. 7. (color online) The soliton amplitude ϕ_m is plotted with respect to the positron concentration μ_b along with variable positron temperature α, where α is 0.2, 0.4, and 0.6 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_b' = 0.2$, $\sigma = 0.1$, $\beta_1 = 0.005$, and $\beta_2 = 0.0054$.

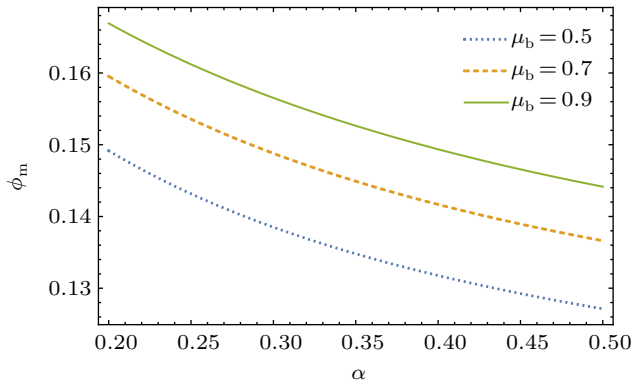


Fig. 8. (color online) The soliton amplitude ϕ_m is plotted with respect to the positron temperature α along with variable positron concentration μ_b, where μ_b is 0.5, 0.7, and 0.9 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_b' = 0.2$, $\sigma = 0.1$, $\beta_1 = 0.0050$, and $\beta_2 = 0.0054$.

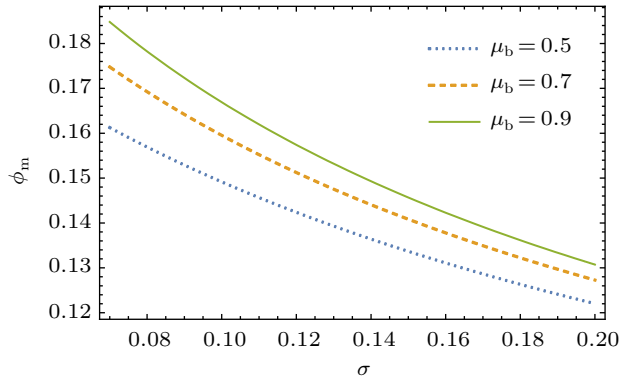


Fig. 9. (color online) The soliton solution is plotted with respect to the ion temperature σ along with variable positron concentration μ_b, where μ_b is 0.5, 0.7, and 0.9 for blue dotted, red dashed, and green bold lines, respectively, and other plasma parameters are $c_1 = 10^3$, $V_b' = 0.2$, $\alpha = 0.2$, $\beta_1 = 0.0050$, and $\beta_2 = 0.0054$.

4. Conclusion

We have investigated the propagation characteristics of ion–acoustic solitary waves in an unmagnetized thermal electron–ion plasma with a flow of relativistic positron beam. Using the reductive perturbation technique, the KdV equation is derived which governs the evolution of weakly nonlinear ion–acoustic solitons in relativistic beam driven plasmas. The effects of the plasma parameters, namely the ion to electron temperature ratio, the beam to electron temperature ratio, and the relativistic parameter due to the flow of positron beam on the profiles of the ion–acoustic solitons, are studied. It is found that the propagation characteristics of ion–acoustic solitons are significantly modified by these plasma parameters. To conclude, the results should be useful for understanding the properties of ion–acoustic solitons that may be relevant in laboratory and space plasmas.

Acknowledgment

One of the authors (Amar P Misra) acknowledges support from UGC-SAP (DRS, Phase III) with Sanction order No. F.510/3/DRS-III/2015(SAPI), and UGC-MRP with F. No. 43-539/2014 (SR) and FD Diary No. 3668.

References

[1] Zhao M, Shan X, Niu S and Chen X 2017 *Chin. Phys. B* **26** 093103
[2] Gan C Y, Xiang N and Yu Z 2016 *Chin. Phys. Lett.* **33** 085203
[3] Adhikary N C, Deka M K, Dev A N and Sarmah J 2014 *Phys. Plasmas* **21** 083703
[4] Ye C E and Zhang W G 2010 *Acta Phys. Sin.* **59** 5229 (in Chinese)
[5] Misra A P and Roy Chowdhury A 2003 *Chaos, Solitons and Fractals* **15** 801
[6] Abdelwahed H G, El-Shewy E K and Mahmoud A A 2017 *Chin. Phys. Lett.* **34** 035202
[7] Alam M S, Hafez M G, Talukder M R and Ali M H 2017 *Chin. Phys. B* **26** 095203
[8] Dev A N 2017 *Chin. Phys. B* **26** 025203
[9] Jian M J, Rong Y J and Ying L C 2012 *Acta Phys. Sin.* **61** 020206 (in Chinese)
[10] Surko C M, Leventhal M and Passner A 1989 *Phys. Rev. Lett.* **62** 901
[11] Surko C M and Murphy T J 1990 *Phys. Fluids A* **2** 1372
[12] Greaves R G, Tinkle M D and Surko C M 1994 *Phys. Plasmas* **1** 1439
[13] Sarri G, Dieckmann M E, Kourakis I, Di Piazza A, Reville B, Keitel C H and Zepf M 2015 *J. Plasma Phys.* **81** 455810401
[14] Popel S I, Vladimirov S V and Shukla P K 1995 *Phys. Plasmas* **2** 716
[15] Nejoh Y N 1996 *Phys. Plasmas* **3** 1447
[16] Saleem H 2006 *Phys. Plasmas* **13** 34503
[17] Esfandyari-Kalejahi A, Akbari-Moghanjoughi M and Saberian E 2010 *Plasma Fusion Res.* **5** 045
[18] Chatterjee P, Ghosh U, Roy K, Muniandy S V, Wong C S and Sahu B 2010 *Phys. Plasmas* **17** 122314
[19] Chatterjee P, Saha T, Muniandy S V, Wong C S and Roychoudhury R 2010 *Phys. Plasmas* **17** 012106
[20] Shan S A, El-Tantawy S A and Moslem W M 2013 *Phys. Plasmas* **20** 082104
[21] Shan S A and El-Tantawy S A 2016 *Phys. Plasma S* **23** 072112
[22] Adhikary N C, Mishra A P, Deka M K and Dev A N 2017 *Phys. Plasmas* **24** 073703
[23] Dev A N, Deka M K, Sarma J, Saikia D and Adhikary N C 2016 *Chin. Phys. B* **25** 105202
[24] Gill T S, Singh A, Kaur H, Saini N S and Bala P 2007 *Phys. Lett. A* **361** 364
[25] Gill T S, Bains A S and Saini N S 2009 *Can. J. Phys.* **87** 861
[26] Han J, Du S and Duan W 2008 *Phys. Plasmas* **15** 112104
[27] Shah A, Haque Q and Mahmood S 2011 *Astrophys. Space Sci.* **335** 529
[28] Hafez M G, Talukder M R and Sakthivel R 2016 *Indian J. Phys.* **90** 603
[29] Hafez M G and Talukder M R 2015 *Astrophys. Space Sci.* **359** 27
[30] Gsponer A 2004 arXiv: physics/0409157v3[physics.plasm-ph]
[31] Wang M L 1995 *Phys. Lett. A* **199** 169
[32] Svensson R 1982 *Astrophys. J.* **258** 335