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Small amplitude double layers in an electronegative dusty plasma with q-distributed electrons*

Zhong-Zheng Li(李中正)[1], Juan-Fang Han(韩娟芳)[2], Dong-Ning Gao(郜东宁)[2], and Wen-Shan Duan(段文山)[2,†]

[1] *Physics of Department, Gansu Normal University for Nationalities, Hezuo* 747000, *China*

[2] *College of Physics and Electronic Engineering and Joint Laboratory of Atomic and Molecular Physics of Northwest Normal University, Lanzhou* 730070, *China*

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The small amplitude dust ion-acoustic double layers in a collisionless four-component unmagnetized dusty plasma system containing nonextensive electrons, inertial negative ions, Maxwellian positive ions, and negatively charged static dust grains are investigated theoretically. Using the pseudo-potential approach and reductive perturbation method, an energy integral equation for the system has been derived and its solution in the form of double layers is obtained. The results appear that the existence regime of the double layer is very sensitive to the plasma parameters, e.g., electron nonextensivity, negative-to-positive ion number density ratio etc. It has been observed that for the selected set of parameters, the system supports rarefactive, (compressive) double layers depending upon the degree of nonextensivity of electrons.

Keywords: dusty plasmas, double layers

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1. Introduction

The field of dusty plasma physics has become a hot point of interest in recent years, due to its wide application in space and laboratory dusty plasma environments.[1–5] Many phenomena and mechanism of the processes are relevant to the knowledge of plasma wave in studying of the dusty plasmas, such as the plasma parameters measurement,[6] diagnosis,[7] wave heating,[8] and communication,[9] etc. There are different types of acoustic modes propagating in dusty plasmas depending on their time scales.[10–13] One of these waves is the dust ion-acoustic double layers (DLs), which is a nonlinear structure consisting of two oppositely charged parallel layers.

In the past few years, the ion-acoustic DL has been a topic of significant interest because of its relevance in cosmic applications,[14–17] confinement of plasma in tandem mirror devices,[18] ion heating in linear turbulence heating devices,[19] etc. Using the pseudo-potential approach and reductive perturbation method, several authors[20–23] have studied ion-acoustic DLs in different plasma systems. Electronegative plasmas, i.e., plasmas containing an appreciable amount of negative ions, are found in plasma processing reactors,[24] neutral beam sources,[25] the cometary comae,[26] and the upper region of Titan's,[27] etc. Therefore, the importance of negative ion plasmas to the field of plasma physics is growing day by day. Ghebache and Tribeche[28] investigated small amplitude DLs in an electronegative plasma with nonextensive electrons. It has been observed that the ion-acoustic wave phase velocity, in different concentration of negative and positive ions, decreases as the non-extensive parameter q increases. Shaukat and Nadia[29] investigated small amplitude DLs in a warm electronegative plasma with trapped kappa distributed electrons. They have found that the small amplitude ion-acoustic DLs are significantly modified by the variation of various parameters such as the electron trapping efficiency and the ratio of the mass of negative ion to positive ion.

It is well known that the Maxwell distribution is taken to be valid for the macroscopic ergodic equilibrium state. Space plasma observations clearly indicate the presence of ion and electron populations that are far away from their thermodynamic equilibrium.[30–35] A new statistical approach, namely nonextensive statistics, is proposed to study the cases where Maxwell distribution is deemed to be inappropriate. The rapidly growing interest to survey the effect of nonextensivity of plasma particles on any nonequilibrium plasma system is due to the fact that such effects of nonextensivity of plasma constituents are quite common in astrophysical and cosmological scenarios. After the rudimentary concept of nonextensive entropy proposed by Renyi[36] and subsequently proposed by Tsallis,[37] the nonextensive behavior of electrons and ions has been successfully employed in plasma physics.[38–40] Liu *et al.*[41,42] show that when $q < 1$, the plasma described by the q-distribution contains a plentiful supply of superthermal particles; when $q > 1$, it contains a large number of low velocity particles. Recently, Ema *et al.*[43] and Li *et al.*[44] investigated the propagations of dust ion-acoustic solitary waves in a multi-

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†Corresponding author. E-mail: duanws@nwnu.edu.cn

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component dusty plasma with nonextensive distributed electrons and inertial negative ions. Their analysis supports the existence of either compressive or rarefactive solitary waves depending on the plasma parameters.

The aim of this study is to investigate the nature of small amplitude DLs in a collisionless four-component unmagnetized dusty plasma system consisting of nonextensive electrons, inertial negative ions, Maxwellian positive ions, and negatively charged static dust grains. We are interested to apply the Sageev pseudo-potential approach and the reductive perturbation method, because the qualitative behaviors of double-layers are most easily seen. Moreover, we aim to investigate the effects of the compositional parameters such as Mach number, non-extensive parameter, negative-to-positive ion number density ratio, etc. on the formation of the double layers.

The paper is organized as follows. In Section 2, we present the mathematical model in an electronegative dusty plasma with q-distributed electrons and the generalized Sageev pseudo-potential is derived. Using the reductive-perturbation method, the Gardner equation has been derived, the DL solution of the Gardner equation has been obtained in Section 3. A discussion is given in Section 4. Finally, Section 5 contains a conclusion of the present study.

2. Basic equations and Sageev pseudo-potential

We consider a collisionless four-component unmagnetized dusty plasma system containing nonextensive electrons, inertial negative ions, Maxwellian positive ions, and negatively charged static dust grains. In such a system where the electrostatic force plays a major role, so the forces coupled with other components of the system can be ignored. Thus, the equilibrium condition reads $n_{\rm p0}-n_{\rm n0}-n_{\rm e0}-Z_{\rm d}n_{\rm d0}=0$, where $n_{\rm e0}$, $n_{\rm p0}$, $n_{\rm n0}$, and $n_{\rm d0}$ are the unperturbed number densities of the electron, positive ion, negative ion, and immobile dust, respectively, and $Z_{\rm d}$ is the number of electrons residing onto the dust grain surface. The number densities of q-distributed electrons, and Maxwellian positive ions are,

$$n_{\rm e}=n_{\rm e0}\left[1+(q-1)\frac{e\psi}{k_{\rm B}T_{\rm e}}\right]^{(q+1)/2(q-1)}, \tag{1}$$

$$n_{\rm p}=n_{\rm p0}\exp\left(-\frac{e\psi}{k_{\rm B}T_{\rm p}}\right). \tag{2}$$

The one-dimensional dimensionless equations of the motion of negative ions are:

$$\frac{\partial n_{\rm n}}{\partial t}+\frac{\partial}{\partial x}(n_{\rm n}u_{\rm n})=0, \tag{3}$$

$$\frac{\partial u_{\rm n}}{\partial t}+u_{\rm n}\frac{\partial u_{\rm n}}{\partial x}=\frac{\partial \psi}{\partial x}, \tag{4}$$

$$\frac{\partial^2\psi}{\partial x^2}=\mu_{\rm e}[1+(q-1)\psi]^{(q+1)/2(q-1)}+\mu_{\rm n}n_{\rm n}-{\rm e}^{-\sigma\psi}+\mu_{\rm d}, \tag{5}$$

where q is the nonextensive parameter characterizing the degree of nonextensivity, $u_{\rm n}$ refers to the velocity of the negative ion fluid normalized by $C_{\rm n}=(k_{\rm B}T_{\rm e}/m_{\rm n})^{1/2}$, $n_{\rm n}$ refers to the number density of the negative ion normalized by $n_{\rm p0}$, and ψ is the electrostatic wave potential normalized by $k_{\rm B}T_{\rm e}/e$. The time variable t is normalized by $\omega_{\rm pn}^{-1}=(m_{\rm n}/4\pi e^2 n_{\rm p0})^{1/2}$, the space variable x is normalized by $\lambda_{Dn}=(k_{\rm B}T_{\rm e}/4\pi e^2 n_{\rm p0})^{1/2}$, where $k_{\rm B}$ is the Boltzmann constant, $T_{\rm e}$ is the electron temperature, and e is the magnitude of the electron charge. We have defined the parameters arising in Eq. (5): $\mu_{\rm e}=n_{\rm e0}/n_{\rm p0}$ (electron-to-positive ion number density ratio), $\mu_{\rm n}=n_{\rm n0}/n_{\rm p0}$ (negative-to-positive ion number density ratio), $\sigma=T_{\rm e}/T_{\rm p}$ (electron-to-positive ion temperature ratio), and $\mu_{\rm d}=Z_{\rm d}n_{\rm d0}/n_{\rm p0}$ (dust grains-to-positive ion number density ratio multiplied by $Z_{\rm d}$).

To obtain the travelling wave solutions of Eqs. (3)–(5) that are stationary in a frame moving with a velocity M, we suppose that all the dependent variables depend on $\xi=\varepsilon(x-Mt)$, M being the Mach number normalized to ion-acoustic speed $C_{\rm n}$. Then equations (3) and (4) reduce to

$$n_{\rm n}=\frac{1}{\sqrt{2\psi/M^2}}. \tag{6}$$

Now, substituting the density expression (6) into the Poisson Eq. (5), we obtain

$$\frac{{\rm d}^2\psi}{{\rm d}\xi^2}=\mu_{\rm n}\frac{1}{\sqrt{2\psi/M^2}}+{\rm e}^{-\sigma\psi}-\mu_{\rm e}[(1+(q-1)\psi)]^{(q+1)/2(q-1)}-\mu_{\rm d}. \tag{7}$$

Multiplying both sides of Eq. (7) by ${\rm d}\psi/{\rm d}\xi$ and integrating once with respect to ψ and imposing the boundary condition ${\rm d}\psi/{\rm d}\xi\to 0$ as $\xi\to\pm\infty$, we obtain the energy integral form

$$\frac{1}{2}\left(\frac{{\rm d}\psi}{{\rm d}\xi}\right)^2+V(\psi)=0 \tag{8}$$

with

$$V(\psi)=\mu_{\rm n}M^2\left[1-\left(1+\frac{2\psi}{M^2}\right)^{1/2}\right]+\frac{1}{\sigma}(1-{\rm e}^{-\sigma\psi})+\frac{2\mu_{\rm e}}{3q-1}[1-(1+(q-1)\psi)^{(3q-1)/2(q-1)}]-\mu_{\rm d}\psi, \tag{9}$$

where $V(\psi)$ is a Sagdeev like potential. For the existence of small amplitude DLs, the following Sagdeev potential conditions must be satisfied:

(i) $V(\psi)=0$ at $\psi=0$ and $\psi=\psi_m$.

(ii) ${\rm d}V(\psi)/{\rm d}\psi=0$ at $\psi=0$ and $\psi=\psi_m$.

(iii) ${\rm d}^2V(\psi)/{\rm d}\psi^2<0$ at $\psi=0$ and $\psi=\psi_m$.

3. Derivation of double layers solution

In this Section, we present nonlinear analysis on the DLs solution obtained by the reductive perturbation method.

First, the reductive perturbation method is used to derive the KdV equation[43]

$$\frac{\partial \psi_1}{\partial \tau} + A\psi_1 \frac{\partial \psi_1}{\partial \xi} + B\frac{\partial^3 \psi_1}{\partial \xi^3} = 0, \tag{10}$$

where

$$A = \frac{V_p^3}{2\mu_n}\left[\frac{\mu_e}{4}(q+1)(q-3) - \frac{3\mu_n}{V_p^4} + \sigma^2\right],$$
$$B = \frac{V_p^3}{2\mu_n}, \quad V_p = \sqrt{\frac{\mu_n}{\mu_e(q+1)/2 + \sigma}}.$$

The independent variables are scaled as $\xi = \varepsilon^{1/2}(x - V_p t)$ and $\tau = \varepsilon^{3/2}t$, where V_p is the phase speed of the waves and ε is a small parameter characterizing the strength of the nonlinearity. We note that at $A = 0$ there is a critical value q_c, $A > 0$ for $q > q_c$, and $A < 0$ for $q < q_c$ as shown in Fig. 1. Figure 1(a) is the dependence of q_c on σ and μ_n. Figure 1(b) is the phase diagram: one region is $A > 0$, the other is $A < 0$. However, the nonlinear term vanishes at $q = q_c$ and is not valid near $q = q_c$ for the KdV equation, which makes the solution amplitude large enough to break down the validity of the reductive perturbation method. As is well known, the Gardner equation is valid for q near its critical value q_c.

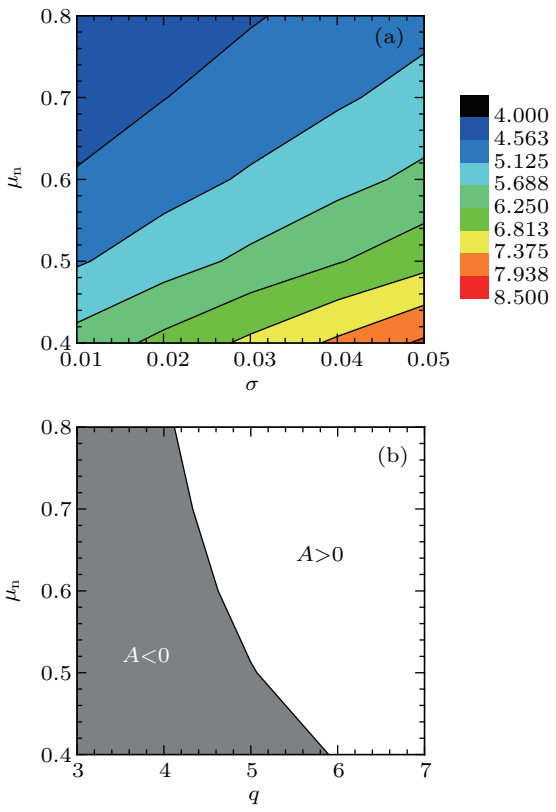


Fig. 1. (color online) (a) Variation of q_c with σ and μ_n, where q_c is the critical value of nonextensive index q above ($A > 0$) or below ($A < 0$). (b) The phase diagram of A with q and μ_n, where $\sigma = 0.01$. The other parameter used is $\mu_e = 0.05$.

Second, if q is around its critical value q_c, the reductive perturbation method is used to derive the higher order nonlinear equation, namely the Gardner equation[43]

$$\frac{\partial \psi_1}{\partial \tau} + sA_q B\psi_1 \frac{\partial \psi_1}{\partial \xi} + \alpha B{\psi_1}^2 \frac{\partial \psi_1}{\partial \xi} + B\frac{\partial^3 \psi_1}{\partial \xi^3} = 0, \tag{11}$$

where

$$\alpha = \frac{15\mu_n}{2V_p^6} - \frac{\mu_e}{16}(q+1)(q-3)(3q-5) - \frac{1}{2}\sigma^3,$$
$$A_q \simeq \frac{\mu_e(q_c+1)}{2} - \frac{3\mu_e}{\mu_n}\left(\frac{\mu_e(q_c+1)}{2} + \sigma\right),$$

$\varepsilon \simeq |q - q_c|$, and $s = -1$ for $q < q_c$ and $s = 1$ for $q > q_c$. Here, the independent variables are scaled as $\xi = \varepsilon(x - V_p t)$ and $\tau = \varepsilon^3 t$.

Before analyzing the stationary solution of the Gardner equation, we first introduce a transformation $\zeta = \xi - U_0\tau$, which allows us to write Eq. (11), under the steady state condition, as

$$\frac{1}{2}\left(\frac{\mathrm{d}\psi_1}{\mathrm{d}\zeta}\right)^2 + V(\psi_1) = 0, \tag{12}$$

where the pseudo-potential $V(\psi_1)$ is given by

$$V(\psi_1) = -\frac{U_0}{2B}\psi_1^2 + \frac{sA_q}{6}\psi_1^3 + \frac{\alpha}{12}\psi_1^4. \tag{13}$$

Applying the boundary conditions (i)–(iii), we obtain the value of ψ_{1m} and the nonlinear structure speed U_0 as

$$\psi_{1m} = -\frac{sA_q}{\alpha}, \quad U_0 = -\frac{BA_q^2}{6\alpha}. \tag{14}$$

Using these values, equation (12) becomes

$$\frac{1}{2}\left(\frac{\mathrm{d}\psi_1}{\mathrm{d}\zeta}\right)^2 + \frac{\alpha}{12}\psi_1^2(\psi_1 - \psi_{1m})^2 = 0. \tag{15}$$

The DL solutions can be obtained through the integration of Eq. (15) as follows:

$$\psi_1 = \frac{\psi_{1m}}{2}\left[1 - \tanh\left(\sqrt{-\frac{\alpha}{24}}\psi_{1m}\zeta\right)\right]. \tag{16}$$

Therefore,

$$\psi = \varepsilon\psi_1. \tag{17}$$

From Eq. (16), it can be easily inferred that the existence of ion-acoustic DL depends on the physical parameters, the value of coefficients α must be negative. It is obvious that compressive (rarefactive) DLs exist if $q > q_c$ ($q < q_c$).

4. Results and discussion

In this section, we numerically analyze the impact of different plasma parameters on the small amplitude ion-acoustic DL structures in an electronegative plasma. It is known that the conditions (i)–(iii) of the Sagdeev potential must be satisfied for DLs. Now, condition (iii) gives the lower limit of Mach number for the existence of DLs as $M > M_C$ where

$$M_C = \sqrt{\frac{\mu_n}{\mu_e(q+1)/2+\sigma}}, \tag{18}$$

which is the critical Mach number below which no DL is possible. In this present model, the Mach number M is normalized by C_n and the changes in M_C depend upon the different parameters in the system. It should be mentioned here that as the value of M depends upon a specific normalization, some care should be taken to interpret the results physically, and so the true Mach number is defined by the ratio M/M_C from which the reference speed C_n disappears.[17]

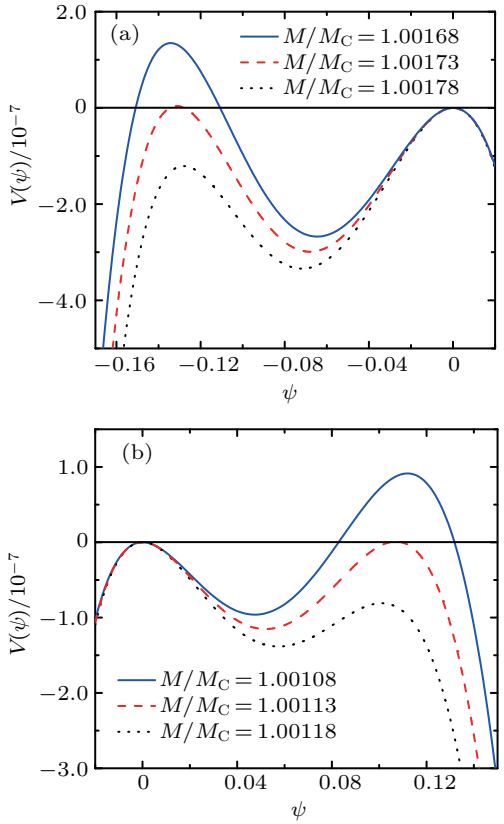


Fig. 2. (color online) Variation of Sagdeev-like potential profile $V(\psi)$ for DL structures against ψ for different values of M/M_C. (a) $q = 4.2$ and (b) $q = 5.0$. Here, $\mu_n = 0.6$, $\mu_e = 0.05$, and $\sigma = 0.01$.

Thus the existence condition, $M > M_C$ for DLs, reduces to the condition that the true Mach number $M/M_C > 1$. Henceforth, the true Mach numbers have been used for plotting different figures. $V(\psi)$ are plotted for different values of M/M_C in Fig. 2, keeping other parameters fixed. The existence of negative potential solitary waves is observed for $M/M_C < 1.00173$ and at this critical point $M/M_C = 1.00173$, formation of DL is observed above which there does not exist any kind of rarefactive solitary wave or DL, as shown in Fig. 2(a). The existence of compressive solitary waves is observed for $M/M_C < 1.00113$ and at this critical point $M/M_C = 1.00113$, the formation of compressive DL is observed above which there does not exist any kind of negative potential solitary wave or DL, as shown in Fig. 2(b).

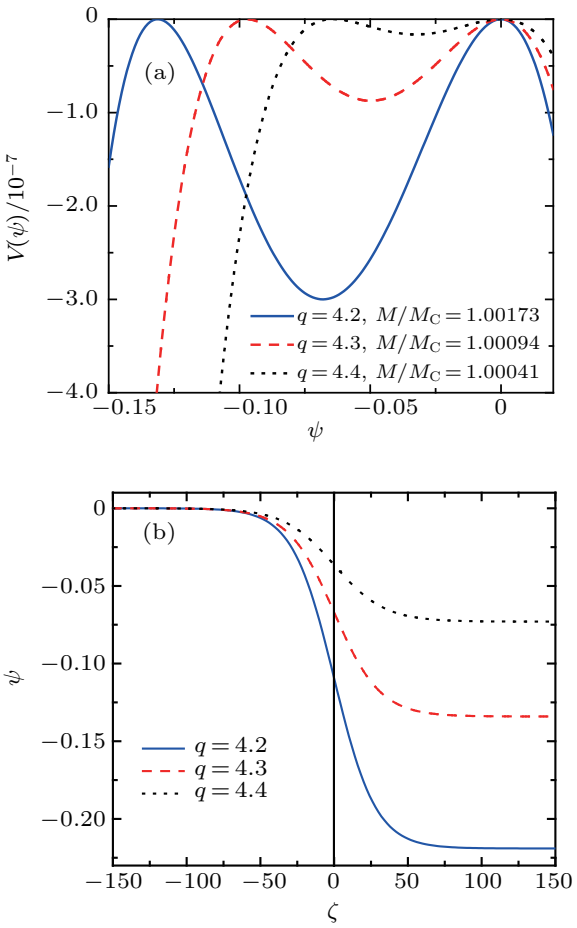


Fig. 3. (color online) (a) Variation of Sagdeev-like potential profile $V(\psi)$ for rarefactive DL structures against ψ for different values of q and M/M_C. (b) Variation of the electrostatic potential ψ for rarefactive DL structures against ζ for different values of q. Other parameters are the same as in Fig. 2.

In Figs. 3(a) and 3(b), the Sagdeev-like potential profile $V(\psi)$ and the corresponding potential profiles obtained from Eq. (17) are plotted, respectively, and they are showing the formation of rarefactive dust ion-acoustic DLs for different values of q and M/M_C. Again, $V(\psi)$ and the corresponding potential profiles are plotted in Figs. 4(a) and 4(b), respectively, and they are showing the formation of compressive DLs for different values of q and M/M_C. The following parameters $\mu_n = 0.6$, $\mu_e = 0.05$, and $\sigma = 0.01$ have been chosen. As q increases, the rarefactive dust ion-acoustic DL shrinks and, beyond a certain critical value 4.58, develops into a positive structure allowing therefore the existence of compressive dust ion-acoustic DLs. This means that our present dusty plasma

model can admit compressive as well as rarefactive small amplitude dust ion-acoustic DLs.

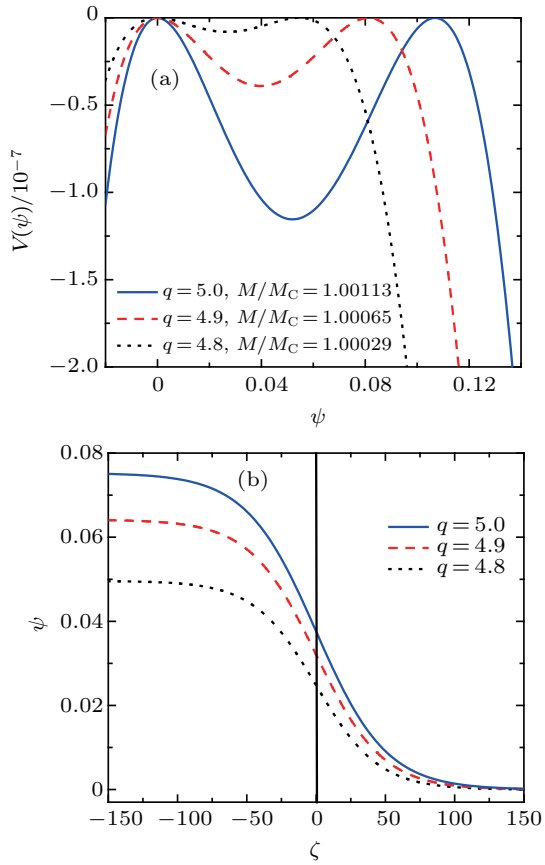


Fig. 4. (color online) (a) Variation of Sagdeev-like potential profile $V(\psi)$ for compressive DL structures against ψ for different values of q and M/M_C. (b) Variation of the electrostatic potential ψ for compressive DL structures against ζ for different values of q. Other parameters are the same as in Fig. 2.

The variation of ψ_m and M/M_C with q for different values of $\mu_n = 0.5$, 0.6, and 0.7 is shown in Fig. 5. Figure 5(a) shows that as q increases, the DL amplitude ψ_m increases and becomes positive beyond the critical value of the electron nonextensive index q_c. It is noted that the maximum amplitude difference of DLs for Sadeegv results and analytical results is negligible if q is close to q_c, while it cannot be neglected if $|q - q_c|$ is large enough. Figure 5(b) shows that when $q < q_c$, the true Mach number M/M_C decreases with the increasing q, while $q > q_c$, Mach number increases with the increasing q. This critical value is lowered as μ_n increases [$\mu_n = 0.5 \rightarrow q_c \sim 5.034$, $\mu_n = 0.6 \rightarrow q_c \sim 4.58$, $\mu_n = 0.7 \rightarrow q_c \sim 4.29$]. Here we have found that the nature (compressive or rarefactive) of the dust ion-acoustic DL depends sensitively on an interplay between the electron nonextensive index q and the relative fraction μ_n of negative ions present in our plasma model. It means that there is a critical value for the number of low velocity electrons in the system. When the number of low velocity electrons is lower than the critical value, the system supports rarefactive double layers, otherwise the system supports compressive double layers.

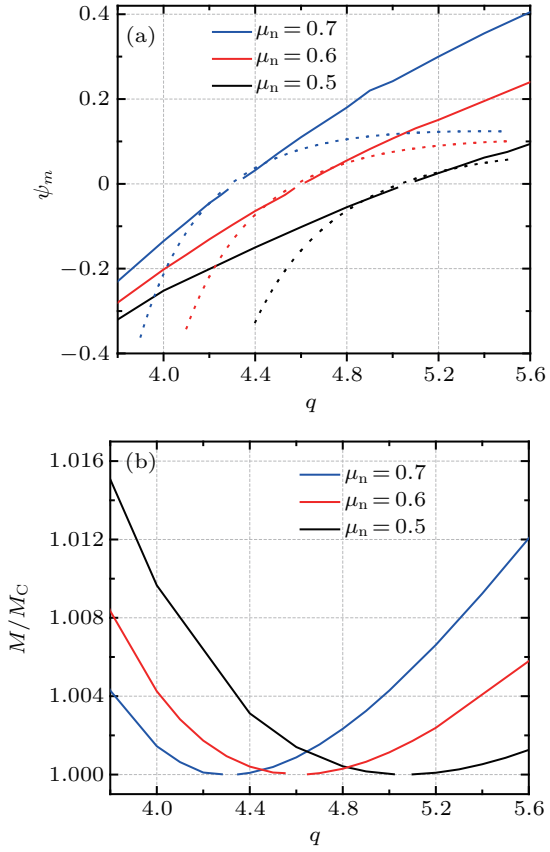


Fig. 5. (color online) (a) Variation of the amplitude ψ_m for DL structures against q for different values of μ_n. Here, the Sagdeev result is denoted by a solid line and the analytical expression is denoted by a doted line. (b) Variation of the real Mach number M/M_C for DL structures against q for different values of μ_n. Other parameters are the same as in Fig. 2.

5. Conclusion

In this paper we have investigated the nature of small amplitude DLs in a collisionless four-component unmagnetized dusty plasma system consisting of nonextensive electrons, inertial negative ions, Maxwellian positive ions, and negatively charged static dust grains. This study is based on applying the Sageev pseudo-potential approach and reductive perturbation method, because the qualitative behaviors of the admissible nonlinear dust ion-acoustic waves in the plasma are most easily seen. Our results show that in such a plasma DL structure, the amplitude and nature depend sensitively on the plasma parameters. In particular, it may be noted that due to the electron nonextensivity and/or the negative-to-positive ion number density ratio, our plasma model supports rarefactive(compressive) dust ion-acoustic DLs. Interestingly, one finds that as the electron nonextensivity q increases, the negative dust ion-acoustic DL shrinks and beyond a certain critical value q_c, develops into a positive structure allowing therefore the existence of compressive dust ion-acoustic DLs. This critical value is lowered as the number of negative ions becomes

important. We hope our results may be useful to explain the basic features of a nonlinear structure in the cometary comae, neutral beam sources, as well as the Earth sheet plasma region where such electronegative plasma can exist.

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