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Sonic horizon dynamics of ultracold Fermi system under elongated harmonic potential*

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We study the phenomena of the sonic horizon in an ultracold atomic Fermi system in an elongated harmonic trap. Based on the one-dimensional Gross–Pitaevskii equation model and variational method combined with exact derivation approach, we derive an analytical formula which describes the occurrence of the sonic horizon and the associated Hawking radiation temperature. Using a pictorial demonstration of the key physical quantities we identify the features reported in prior numerical studies of a three-dimensional (3D) ultracold atomic system, proving the applicability of the theoretical model presented here.

Keywords: sonic horizon, ultracold Fermi system, Gross–Pitaevskii equation

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1. Introduction

For a long time, black hole related problems have been a fascinating subject in macro-world physics fundamental problem scrutiny. With their close analog in ultracold atomic systems, black hole related physics has recently drawn special attention. As a key concept, a black hole has its origin in astrophysics. General relativity predicts that a sufficiently compact mass can deform spacetime to form a black hole: a region of spacetime exhibiting such a strong gravitational effect that even light cannot escape from inside it. The event horizon is a boundary in spacetime beyond which events cannot affect an outside observer, and for a black hole, it is the boundary of the region from which escape is impossible. One of the particular features of the black holes that are predicted by Hawking is the blackbody radiation due to quantum effects near the event horizon. However, Hawking radiation from normal black holes is extremely weak so the experimental investigation of the phenomenon would seem to be virtually impossible, or would depend on the highly unlikely discovery of a small black hole near the Earth.

The quantum fluid supplies the necessary scenario where black hole related physics can be investigated in a practical and controllable way.[1–7] As pointed out by Unruh,[8] a quantum fluid could be used to form an analogue black hole: sonic black hole,[9,10] with the formation of the boundary which marked transition from subsonic flow to supersonic flow. This is analogous to a black-hole event horizon and is called sonic horizon in such setting. As a quantum fluid, a Bose–Einstein condensate (BEC) is a promising candidate for realizing a sonic black hole. Recent experiments of accelerating an elongated BEC under a step like potential also demonstrated the formation of a sonic black hole, and self-amplifying Hawking radiation and spontaneous Hawking radiation were detected.[11,12] Besides, an alternative approach to creating a sonic black hole in a BEC is to trigger condensate expansion by changing the interaction which can be controlled by the Feshbach resonance. Additionally, other intriguing methodologies for the study of sonic black hole require specific attention, for example, a similar technique was employed in observing "Bose–Novae" jets and bursts in a collapsing condensate.[13] Recently, a scheme of the quantum simulation of traversable wormhole spacetimes in a BEC by using the Feshbach resonance has been proposed.[14] In this paper, our investigation is based on the method of tuning the inter-particle nonlinear interaction.

In the previous work,[15] the sonic horizon phenomenon was studied for oscillating Bose–Einstein condensates in an isotropic harmonic potential based on the Gross–Pitaevskii equation model[16,17] and the variational method, where the analytical formula for the criteria and lifetime of the formation of the sonic horizon were derived, and the derived analytical results matched very well with the results obtained by a numerical simulation. Taking the advantage of the flexible modulation of nonlinear interaction strength, it is interesting to investigate similar phenomena in a degenerate Fermion system and inspect sonic black hole-related problems[18] in the Bardeen–Cooper–Schrieffer (BCS) to BEC crossover.[19,20] In this paper, we use the variational method combined with an exact derivation approach to study the evolution of the sonic horizon in a quasi-one-dimensional ultracold degenerate Fermion

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system. Across the BCS–BEC crossover regime, we derive the criterion formula for the occurrence of the sonic horizon and give the analytical expression for the associated Hawking radiation temperature. The results exhibit identical features compared with those obtained by the study based on a numerical method.[21]

This paper is organized as follows: the next section presents the theoretical model, together with the methodology for the calculation of the results for the sonic horizon formation. This is followed by a discussion. The last section presents conclusive remarks.

2. Methods and results

2.1. The one-dimensional GPE model and variational ansatz

To study the formation of a sonic horizon in a one-dimensional ultracold degenerate Fermi gas in the BCS–BEC crossover regime, we use the one-dimensional GPE incorporating polytropic approximation.[22] We firstly consider a quasi-one-dimensional Fermi gas in an elongated harmonic potential $V(x,y,z) = (1/2)m(\omega_x x^2 + \omega_y y^2 + \omega_z z^2)$ ($\omega_x \ll \omega_y, \omega_z$). The system is effectively one-dimensional along the x direction. The formation of the sonic horizon can be initiated by expanding the system along the x direction. One way to do this is to enforce an abrupt change in the system's inter-particle scattering length via the Feshbach resonance experimental technique. Under certain experimental settings, the sonic horizon can be formed during the system's dynamical evolution process. This is done by solving the one-dimensional time-dependent GPE for the system in an elongated harmonic trap. The one-dimensional GPE used here can be expressed as[23]

$$\mathrm{i}\hbar\frac{\partial\psi(x,t)}{\partial t} = \left[-\frac{\hbar^2\partial^2}{2m\partial x^2} + \frac{m}{2}\Omega^2x^2 + g(t)|\psi|^{2\gamma}\right]\psi(x,t), \quad (1)$$

where $\Omega = \omega_x$. The system is initially in the stationary symmetric ground state $\psi_0 = \psi(x/\sigma_0, 0)$ with a scattering length corresponding to the nonlinear interaction strength $g(t \le 0) = g_0 = 4\pi\hbar^2 a_\mathrm{i}/m$. At time $t = 0$, using the Feshbach resonance technique, the inter-particle scattering length is abruptly changed to a new value $g(t > 0) = g = 4\pi\hbar^2 a_\mathrm{f}/m$ (with a_i, a_f being the initial atomic interaction and changed atomic interaction respectively), and the system starts to evolve with the wave function $\psi(x,t) = \psi(x/\sigma(t),t)$ ($\sigma(t=0) = \sigma_0$).

We use a variational method to obtain a quantitative description of the dynamical evolution of the system. According to the analytical results from prior work,[23–25] the bright soliton solution can be obtained from Eq. (1), so the ansatz for the wave function $\psi(x/\sigma(t),t)$ is set to $\mathrm{sech}^{1/\gamma}(x/\sigma(t),t)$. We consider the action $S = \int \mathrm{d}t\,\mathrm{d}\boldsymbol{r}\mathscr{L}$, where the Lagrangian density $\mathscr{L}$ is expressed as

$$\mathscr{L} = \psi^*\left[\mathrm{i}\hbar\frac{\partial}{\partial t} + \frac{\hbar^2\nabla^2}{2m} - \frac{m}{2}\Omega^2x^2 - \frac{1}{\gamma+1}g(t)|\psi|^{2\gamma}\right]\psi. \quad (2)$$

We assume that the system is initially in the ground state which corresponds to $x = 0$. The ansatz chosen for $\psi(x/\sigma(t),t)$ takes the following form

$$\psi(x/\sigma(t),t) = \frac{C_0}{\sigma^{1/2}(t)}\mathrm{sech}^{1/\gamma}\left(\frac{x}{\sigma(t)}\right)\exp(-\mathrm{i}\beta(t)x^2), \quad (3)$$

where C_0 is normalization constant. Substituting the ansatz in Eq. (3) into Eq. (1), and by setting the coefficient of term $\mathrm{i}\psi$ of Eq. (1) to zero, we get the imaginary part:

$$\beta(t) = -\frac{m\dot{\sigma}(t)}{2\hbar\sigma(t)}. \quad (4)$$

We will use Eq. (4) in the calculation of the variation of the action S which will be used for the quantitative analysis of the dynamical mode of the system.

2.2. Oscillation mode and the appearance of the sonic horizon

The Euler–Lagrangian equation for the action S for the Lagrangian Eq. (2) can be written as

$$m\ddot{\sigma}(t) = -\frac{\partial V(\sigma)}{\partial\sigma} \quad (5)$$

where $V(\sigma) = V_0(\sigma) + V_1(\sigma)$. Here

$$V_0(\sigma) = \frac{m\Omega^2}{2}\sigma^2 + \frac{\hbar^2}{2m}\frac{C_2}{C_1\sigma^2}, \quad (6)$$

$$V_1(\sigma) = \frac{2g(t)}{2}C_3(\sigma)(1/\sigma^2) \quad (7)$$

and

$$C_1 = \int_0^\infty x^2\,\mathrm{sech}^{2/\gamma}(x)\,\mathrm{d}x, \quad (8)$$

$$C_2 = \int_0^\infty\left(\frac{\mathrm{d}\,\mathrm{sech}^{2/\gamma}(x)}{\mathrm{d}x}\right)^2\mathrm{d}x, \quad (9)$$

$$C_3(\sigma) = \int_0^{\sqrt{1/\sigma}}(1/\sigma^2 - x^2))\,\mathrm{sech}^{2/\gamma}(x)\,\mathrm{d}x. \quad (10)$$

In the limit of the weak nonlinear interaction, equation (5) reduces to

$$m\ddot{\sigma}(t) = -\frac{\partial V_0(\sigma)}{\partial\sigma}. \quad (11)$$

Substituting Eq. (6) into Eq. (11), we solve for $\sigma(t)$ as follows:

$$\sigma^2(t) = A\sin(\omega t) + B, \quad (12)$$

where

$$A = \sqrt{\left(\frac{\sigma_0^2}{2} + \frac{\hbar^2}{2m^2\Omega^2\sigma_0^2}\right)^2 - \frac{C_2\hbar^2}{2C_1m^2\Omega^2}}, \quad (13)$$

$$B = \frac{\sigma_0^2}{2} + \frac{C_2\hbar^2}{2C_1m^2\Omega^2\sigma_0^2}, \quad (14)$$

$$\omega = 2\Omega. \tag{15}$$

For a relatively weak nonlinear interaction, the quantitative dynamical features of the system can be well demonstrated by the evolution of $\sigma(t)$. Using Eq. (12), we can see that the system's distribution width $\sigma(t)$ oscillates between a maximum value $\sqrt{B+A}$ and a minimum value $\sqrt{B-A}$. This dynamical feature will be used in the next subsection to derive a formula which describes the criteria for the occurrence of the sonic horizon and the associated thermal quantities, such as the Hawking radiation temperature.

2.3. Exact solution for the oscillation mode

We now try to derive the oscillation mode based on the exact solution of Eq. (1). As shown in prior work,[26] the exact solution of Eq. (1) can be expressed as

$$\psi(x,t) = v^{1/2}(x,t)\,\mathrm{e}^{\mathrm{i}\theta(x,t)} \tag{16}$$

where[26]

$$\theta(x,t) = \Phi(t)x^2 + \Gamma(t)x + \Omega(t). \tag{17}$$

From Eq. (21c) in Ref. [26], we have $\Gamma(t) = C_4/\sigma_1(t)$ (with σ_1 corresponds to σ in Ref. [26]), plugging into Eq. (17a) in Ref. [26], we have

$$\Phi(t) = \frac{\dot{\sigma}_1(t)}{4\sigma_1^3(t)}. \tag{18}$$

Substituting Eq. (18) into Eq. (16a) of Ref. [26], we obtain the equation for $\sigma_1(t)$ as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{\sigma}_1}{\sigma_1^3}\right) + 4\frac{\dot{\sigma}_1^2}{\sigma_1^4} + \frac{k}{\sigma_1^2} - \left(\frac{\dot{\sigma}_1}{2\sigma_1}\right)^2 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{\sigma}_1}{4\sigma_1}\right) = 0, \tag{19}$$

which is solvable with the following solution

$$\sigma_1(t) = \sqrt{A + B\sin\omega t}. \tag{20}$$

In dimensional form, equation (18) is $\Phi(t) = (m/4\hbar)(\dot{\sigma}_1/\sigma_1)$, comparing with Eq. (4), we have $\sigma_1(t) = \sigma^2(t)$ ($\sigma(t)$ of this article), comparing with Eq. (12), we obtain $\Phi(t) = -\beta(t)$. So for dilute case with $C_4 \ll 1$, the phase function of the exact solution here which demonstrates oscillation mode coincides with that obtained from variational analysis in previous subsection. This doubly confirm the existence of the oscillation mode.

2.4. Sonic horizon occurrence and Hawking radiation temperature

Using the dynamical evolution parameters derived in the previous subsections, we proceed by calculating the key physical quantities related to the occurrence of the sonic horizon. From Eq. (4), the fluid velocity of the system can be written as

$$v_0(x,t) = \frac{\hbar}{m}\nabla(-\beta(t)x^2) = \frac{\dot{\sigma}(t)}{\sigma(t)}x, \tag{21}$$

whose magnitude grows linearly with the distance from the center of the system $x = 0$. The sound velocity of the system in the weak interaction limit is derived by Wen[22] and takes the following form

$$\begin{aligned} c_\mathrm{s}(x,t) &= \sqrt{\frac{g(t)}{m}}|\psi(x,t)|^\gamma \\ &= \sqrt{\frac{g(t)}{m}}\frac{C_0}{\sigma^{1/2}(t)}\,\mathrm{sech}\left(x/\sigma(t)\right). \end{aligned} \tag{22}$$

The sonic horizon location $x = x_\mathrm{s}$ is determined by taking the root of the equation

$$(v_0(x,t) - c_\mathrm{s}(x,t))|_{x=x_\mathrm{s}} = 0. \tag{23}$$

Since $\sigma(t)$ is a periodic function of $\sin\omega t$ with period $T = 2\pi/\omega = \pi/\Omega$. Equation (23) only has a solution when $\dot{\sigma}(t)/\sigma > 0$, which means that x_s appears when $t \in (-T/4, T/4)$. We assume that the longitudinal dimensional size of the system is L_0 (L_0 is much greater than the radial dimension span of the system). Since $v_0(x,t)$ peaks at $(-T/4, T/4)$ and is at a minimum at $\pm T/4$, we assume that equation (23) can be solved when $x_\mathrm{s} = L_0$ at t_1 and t_2 where $-T/4 < t_1 < t_2 < T/4$. The sonic horizon $x = x_\mathrm{s}$ ($|x_\mathrm{s}| \le L_0$) will appear when $t_1 < t < t_2$. Specifically, for the sonic horizon x_s enters the system at time t_1 and stays in the system ($-L_0 < x_\mathrm{s} < L_0$) during $t_1 < t < t_2$, and then exits the system at time $t = t_2$ ($|x_\mathrm{s}| = L_0$). For sufficiently large L_0, $t_1 \to -T/4$ and $t_2 \to T/4$, and in one oscillation period T of the system, the lifespan of the sonic horizon is close to $T/2$.

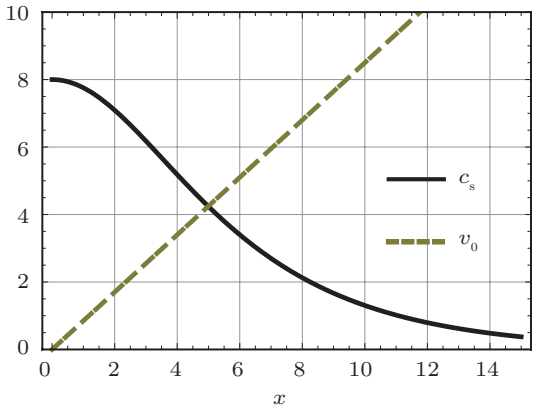


Fig. 1. (color online) Sound velocity c_s (solid line, formulae (22)) and fluid velocity v_0 (dashed line, Eq. (21)) in units of $(\hbar\Omega/m)^{1/2}$ versus x (in units of $(\hbar/m\Omega)^{1/2}$) at $\tilde{t} = \Omega t = 0.4$, with $a_\mathrm{i} = 200a_0$, $a_\mathrm{f} = 5a_\mathrm{i}$ (a_0 is the Bohr radius).

Figure 1 shows the variation of c_s and v_0 (based on Eqs. (21) and (22)) with a spatial variable x. The x coordinate of the cross point of the two velocity curves is the location of the sonic horizon. Base on the formulation of Eqs. (21), (22), and (23) which do not depend on γ, we know that the location of the sonic horizon does not depend on γ. Furthermore, as discussed in prior work,[21] the dynamical evolution of the sonic horizon arising from an expanding system is indicated by the thermal radiation towards the center of the system. This radiation is the Hawking radiation, which is characterized by the Hawking temperature T_pc. T_pc depends on t (not on γ) by[21]

$$T_{\rm pc} = \frac{\hbar}{2\pi k_{\rm B}} \frac{\partial}{\partial x}(v_0 - c_{\rm s})\big|_{x_{\rm s}}$$
$$= \frac{\hbar}{2\pi k_{\rm B}} \left\{ \frac{\dot{\sigma}(t)}{\sigma(t)} + \left(\frac{m}{C_0^2 g(t)} \right)^{1/2} \frac{\dot{\sigma}^2}{\sigma^{5/2}} x_{\rm s}^2 \left[\left(\frac{m}{C_0^2 g(t)} \right) \frac{\dot{\sigma}^2}{\sigma} x_{\rm s}^2 - 1 \right]^{1/2} \right\}, \tag{24}$$

where equations (21) and (22) have been used, and $T_{\rm pc}$ does not depend on γ either. Figure 2 shows the variation of $T_{\rm pc}$ within a time range centered around $\pi/4\Omega$.

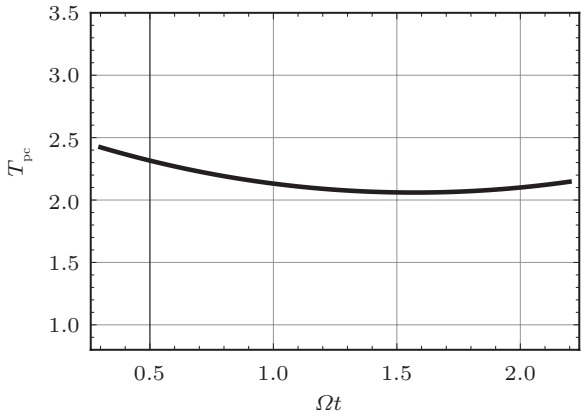


Fig. 2. Hawking radiation temperature $T_{\rm pc}$ (in units of $\hbar\Omega/2\pi k_{\rm B}$) versus Ωt, with $a_{\rm i} = 200a_0$, $a_{\rm f} = 5a_{\rm i}$.

3. Discussion

The variation of both the fluid velocity v_0 (ascending) and the sound velocity $c_{\rm s}$ (descending) with the location x (shown in Fig. 1) demonstrates the same 'universal' feature (ascending versus desending) shown in the numerical calculations by Kurita and Morinari (shown in Fig. 2 of prior work[21]). We may attribute this to a similar GPE theoretical model adopted for the quasi-one-dimensional system compared with the model adopted in the numerical analysis.[21] Moreover, the calculated Hawking temperature $T_{\rm pc}$ (shown in Fig. 2) demonstrates a similar nearly flat feature compared with that obtained by the numerical method (shown in Fig. 3 in Ref. [21]). Particularly, the analytical formulas for the sonic horizon location and the Hawking radiation temperature do not depend on the crossover power index γ. This demonstrates that the key sonic black hole quantities take the universal format in the BCS–BEC crossover regime, which indicates the same qualitative features regarding sonic horizon dynamics. Any minute quantitative difference during the BCS–BEC crossover appeared in the initial experimental setting, for example a different initial value of $\sigma(t)$ at $t = 0$ (σ_0), dose not impact the key qualitative features under the framework of the Gross–Pitaevskii equation model. Therefore, the theoretical results derived here are particularly useful in guiding experimental investigations of sonic black hole-related phenomena in quasi-one-dimensional ultracold atomic systems.

4. Conclusion

In this study, based on the one-dimensional Gross–Pitaevskii equation and the variational method combined with exact derivation approach, we calculated the evolution of the a quasi-one-dimensional, harmonically trapped ultracold Fermi system under the assumption of an abrupt change of scattering wavelength using Feshbach resonance technique. We derived a formula which describes the criteria for the occurrence of a sonic black hole and the associated Hawking radiation temperature. Our theoretical results show qualitative agreement with results obtained via the numerical method (for 3D isotropic BEC system with harmonic trapping potential) in prior work. The theoretical results presented here are useful to guide future experimental investigations into sonic black holes in quasi-one-dimensional ultracold systems.

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