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Controlling of entropic uncertainty in open quantum system via proper placement of quantum register*

Ying-Hua Ji(嵇英华)[1,2], Qiang Ke(柯强)[1,2], and Ju-Ju Hu(胡菊菊)[1,2,†]

[1] College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang 330022, China
[2] Key Laboratory of Photoelectronics and Telecommunication of Jiangxi Province, Nanchang 330022, China

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We investigate the dynamical behaviors of quantum-memory-assisted entropic uncertainty and its lower bound in the amplitude-damping channel. The influences of different placement positions of the quantum register on the dynamics of quantum coherence, quantum entanglement, and quantum discord are analyzed in detail. The numerical simulation results show that the quantum register should be placed in the channel of the non-Markovian effect. This option is beneficial to reduce the entropic uncertainty and its lower bound. We also find that this choice does not change the evolution of the quantum coherence and quantum entanglement, but changes the dynamical process of the quantum discord of the system. These results show that quantum coherence, quantum entanglement, and quantum discord are different quantum resources with unique characteristics and properties, and quantum discord can play a key role in reducing the uncertainty of quantum systems.

Keywords: quantum-memory-assisted, entropic uncertainties, quantum register, open quantum system

PACS: 03.65.Ta, 05.40.Ca, 03.65.Yz **DOI:** 10.1088/1674-1056/27/10/100302

1. Introduction

In recent years, quantum entanglement, quantum discord, and quantum coherence have been widely used as useful physical resources in quantum information processing.[1,2] In fact, quantum coherence is a fundamental quantum resource, and both quantum entanglement and quantum discord of the quantum system are closely related to quantum coherence. The research on quantum entanglement has received a great deal of attention and been widely investigated. At present, the theory of entanglement has been improved, and the experimental verification is quite complete, which has been widely applied in practice.[3–5] Quantum discord has also been studied extensively, which is more widespread than quantum entanglement in quantum systems.[6,7] Despite the fundamental importance of quantum coherence, the rigorous definition and effective measurement of quantum coherence have only recently been studied systematically.[8,9] The coherence of quantum systems is the basis of non-classical correlation, thus there should be some relation between quantum coherence and quantum correlation. Reference [10] proved the relation between quantum coherence and quantum discord in quantum systems, and described the condition of quantum coherence transformed into quantum discord. The research result obtained by Streltsov *et al.* showed that the necessary condition for the entanglement between two systems was that one of the quantum systems must have coherence.[11]

To describe the uncertainty of the quantum system, Heisenberg first proposed the uncertainty principle.[12] Subsequently, Kennard and Robertson extended the principle to a standard deviation given by[13,14]

$$\Delta Q \cdot \Delta R \geqslant \frac{1}{2} |\langle \psi | [Q,R] | \psi \rangle|^2, \tag{1}$$

for two arbitrary non-commuting observables Q and R in a quantum state $|\psi\rangle$ of the system to be measured, where $\Delta X = \sqrt{\langle \psi | X^2 | \psi \rangle - \langle \psi | X | \psi \rangle^2}$ denotes the variance of operator X in quantum state $|\psi\rangle$, and $[Q,R] = QR - RQ$ stands for the commutator. When $Q = x$ and $R = p$, one has $\Delta x \cdot \Delta p \geq \hbar/2$, which is Heisenberg's uncertainty relation.

The Shannon entropy can be used to measure the uncertainty of a physical measurement. For this reason, Hirschman *et al.* used Shannon entropy to measure the uncertainty in quantum mechanics, and first proposed the entropic uncertainty relation between position and momentum.[15] Deutsch *et al.* extended it to any pair of non-mechanical quantities and proposed a more generalized classical entropic uncertainty relation.[16,17] Subsequently, Maassen, Uffink, and Kraus improved Deutsch's result to[18]

$$H(Q) + H(R) \geqslant -\log_2 c, \tag{2}$$

where $H(Q)\,(H(R))$ is the Shannon entropy of the probabilities of measurement outcomes of the observable $Q(R)$. The parameter $c = \max_{mn} |\langle \psi_m | \varphi_n \rangle|^2$ is the maximal overlap of the observables Q and R with $|\psi_m\rangle$ and $|\varphi_n\rangle$ being the eigenstates of Q and R, respectively.

The investigation of the entropic uncertainty principle is a hot topic in quantum optics and quantum informa-

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†Corresponding author. E-mail: jyh2006@jxnu.edu.cn
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tion. Recently, important advances on the entropic uncertainty principle have been made internationally. One of them is the quantum-memory-assisted entropic uncertainty relation (QMA EUR) proposed by Berta *et al.*,[19,20] i.e.,

$$S(Q|B)+S(R|B)\geq S(A|B)-\log_2 c, \tag{3}$$

where $S(A|B)=S(\rho_{AB})-S(\rho_B)$ is the conditional von Neumann entropy of ρ_{AB} and $S(\rho)$ is the von Neumann entropy with $S(\rho)=-\mathrm{tr}(\rho\log_2\rho)$, and $S(X|B)$ with $X\in(Q,R)$ is the conditional von Neumann entropy of the post-measurement state $\rho_{XB}=\sum_x(|\varphi_x\rangle\langle\varphi_x|\otimes I_B)\rho_{AB}(|\varphi_x\rangle\langle\varphi_x|\otimes I_B)$ after the quantum system A is measured, and I_B being an identity operator. Because quantum information is provided by the quantum register, the significance of QMA EUR lies in that the quantum information of the quantum memory system can help to reduce or eliminate the measurement uncertainty, which has been confirmed in recent experiments.[21,22]

At present, many researchers have studied the relation between quantum correlation, quantum coherence, and entropic uncertainty, and some quantum operations of quantum states are proposed for stable systems, quantum correlation of protection systems, and the reduction of entropic uncertainty,[23–29] for example, the behavior of quantum-memory assisted entropic uncertainty under different noises, and the relations between the quantum-memory-assisted entropic uncertainty principle with teleportation and entanglement witnessed,[30] and influence of quantum discord and classical correlation on the entropic uncertainty in the presence of quantum memory.[31]

It is well known that any environment can be classified as Markovian without the memory effect or non-Markvian with the memory effect. In fact, the realistic environment is strictly the non-Markvian type, and the Markvian environment is just an approximate result in most cases, i.e., in the case of weak coupling, the correlation time of the environment is far less than the associated relaxation time of the system itself. With the development of the experimental technology and the realistic demand of the quantum information technology, the strong coupling between subsystems and the environment has become more and more common in quantum state engineering, e.g., cavity QED system under strong coupling. In this case, the memory of the environment cannot be ignored; the feedback function of the environment on the system must be taken into account. Therefore, the investigation about the quantum system in the non-Markovian environment has important theoretical and practical significance for quantum information processing.

The characterization of non-Markovianity and the advantages of the memory effect is an active field of research.[32–35] Recently, Feng *et al.* found that the unit noises increase the amount of uncertainty in the Markovian environment.[36] The entropy uncertainty relation in the non-Markovian dissipative environment was investigated in Ref. [37]. In addition, some attention has been paid to the quantum entropic uncertainty of the system by weak measurement and measurement reversal.[38,39]

An interesting question is when two qubits are in different dissipation channels, which qubit should we choose as the quantum register to get better measurement accuracy? Under different choices, what are the dynamical features of quantum coherence, quantum entanglement, and quantum discord used as quantum resources? However, to the best of our knowledge, there has been no report on these questions in the literature. In this paper, the influence of different placement positions of the quantum register is analyzed in detail on the dynamics of quantum coherence, quantum entanglement, and quantum discord. The research results show that the quantum register should be placed in the channel of the non-Markovian effect. This option is beneficial to reducing the entropic uncertainty and its lower bound. The results also show that quantum discord can play a key role in reducing the uncertainty of quantum systems.

The paper is organized as follows. In Section 2, we present the dynamical evolution of the two-qubit system under independent reservoirs with Markovian and non-Markovian effects. In Section 3, we introduce the measure of coherence, quantum entanglement, quantum discord, and the quantum-memory-assisted entropic uncertainty of the two-qubit system respectively. In Section 4, based on a numerical simulation, we discuss the effects of the quantum register placed in different environments on the entropic uncertainty relation, coherence, entanglement, and quantum discord. Finally, a brief summary is given in Section 5.

2. Investigation model

We consider a system formed by two parts without interaction. Each part consists of a qubit A (or B) that is locally interacting respectively with a dissipative channel R_A (or R_B). The qubits A and B are initially entangled. Obviously, the model is governed by the following Hamiltonian:[40]

$$\begin{aligned}\hat{H}=&\frac{1}{2}\omega_{0A}\sigma_A^z+\frac{1}{2}\omega_{0B}\sigma_B^z+\sum_k\omega_k^A a_k^\dagger a_k\\&+\sum_j\omega_j^B b_j^\dagger b_j+\sum_{n=A,B}\left(\sigma_n^\dagger R_n+\sigma_n^- R_n^\dagger\right),\end{aligned} \tag{4}$$

where a_k ($a_k^\dagger$) is the annihilation (creation) operator of the k-th mode (of frequency ω_k) of the reservoir interacting with the first subsystem A. Similarly, b_j ($b_j^\dagger$) is the annihilation (creation) operator of the j-th mode (of frequency ω_j) of the reservoir interacting with the second subsystem B.

When the environment is at zero-temperature and the qubit is initially in a general composite state of its two levels, the single-qubit reduced density matrix $\rho(t)$ in the qubit

basis $\{|0\rangle, |1\rangle\}$ has the form

$$\rho(t) = \begin{pmatrix} \rho_{11}(0)|h(t)|^2 & \rho_{12}(0)h(t) \\ \rho_{21}(0)h(t) & 1-\rho_{11}(0)|h(t)|^2 \end{pmatrix}, \tag{5}$$

with

$$\frac{\mathrm{d}}{\mathrm{d}t}h(t) = -\int_0^t \mathrm{d}\tau f(t-\tau)h(\tau), \tag{6}$$

where $f(t-\tau)$ denotes the two-point reservoir correlation function, which can be written as the Fourier transform of the spectral density $J(\omega)$

$$f(t-\tau) = \int_0^\infty \mathrm{d}\omega J(\omega)\mathrm{e}^{-\mathrm{i}(\omega-\omega_0)(t-\tau)}. \tag{7}$$

The environment is represented by a bath of harmonic oscillators. For the amplitude-damping channel, the effective spectral density $J_k(\omega)$ is taken as $(k = A, B)$,[41]

$$J_k(\omega) = \frac{1}{2\pi}\frac{\gamma_k\lambda_k^2}{(\omega_0-\delta_k-\omega_k)^2+\lambda_k^2}, \tag{8}$$

where γ_k is the coupling strength between the system and the environment; λ_k, defining the spectral width of the coupling, is connected to the reservoir correlation time τ_k by the relation $\tau_k = \lambda_k^{-1}$; a smaller λ_k indicates a longer correlation time and hence, more significant non-Markovianity. $\delta_k = \omega_0 - \omega_k^{\mathrm{c}}$ is the detuning between ω_k^{c} and ω_0, and ω_k^{c} is the center frequency of cavity k. It is worth noting that the effective coupling between the qubit and its environment decreases as the value of the detuning δ_k increases. According to the spectral density from Eq. (8) in the non-Markovian process, the function $h(t)$ can be expressed as[42]

$$\begin{aligned} h_k(t) &= \exp\left[-\frac{(\lambda_k - \mathrm{i}\delta_k)t}{2}\right] \\ &\times \left[\cosh\left(\frac{\Omega_k t}{2}\right) + \frac{\lambda_k - \mathrm{i}\delta_k}{\Omega_k}\sinh\left(\frac{\Omega_k t}{2}\right)\right], \end{aligned} \tag{9}$$

where $\Omega_k = \sqrt{(\lambda_k - \mathrm{i}\delta_k)^2 - 2\gamma_k\lambda_k}$.

The evolution of the reduced density matrix elements for a single qubit can be easily extended to the two-qubit system. Following the procedure presented in Ref. [43], we find that in the standard product basis $\{|00\rangle\ |01\rangle\ |10\rangle\ |11\rangle\}$, the diagonal elements of the reduced density matrix $\rho(t)$ for the two-qubit system can be written as

$$\rho_{11}(t) = \rho_{11}(0)|h_A(t)|^2|h_B(t)|^2, \tag{10}$$

$$\rho_{22}(t) = \rho_{22}(0)|h_A(t)|^2 + \rho_{11}(0)|h_A(t)|^2[1-|h_B(t)|^2], \tag{11}$$

$$\rho_{33}(t) = \rho_{33}(0)|h_B(t)|^2 + \rho_{11}(0)|h_B(t)|^2[1-|h_A(t)|^2], \tag{12}$$

$$\rho_{44}(t) = 1-[\rho_{11}(t)+\rho_{22}(t)+\rho_{33}(t)], \tag{13}$$

and the nondiagonal elements are

$$\rho_{12}(t) = \rho_{12}(0)|h_A(t)|^2 h_B(t), \tag{14}$$

$$\rho_{13}(t) = \rho_{13}(0)h_A(t)|h_B(t)|^2, \tag{15}$$

$$\rho_{14}(t) = \rho_{14}(0)h_A(t)h_B(t), \tag{16}$$

$$\rho_{23}(t) = \rho_{23}(0)h_A(t)h_B^*(t), \tag{17}$$

$$\rho_{24}(t) = \rho_{24}(0)h_A(t) + \rho_{13}(0)h_A(t)[1-|h_B(t)|^2], \tag{18}$$

$$\rho_{34}(t) = \rho_{34}(0)h_B(t) + \rho_{12}(0)h_B(t)[1-|h_A(t)|^2], \tag{19}$$

and $\rho_{jk}(t) = \rho_{kj}^*(t)$. $h_k(k = A, B)$ is determined by Eq. (9) and respectively relies on its reservoir correlation function. It should be pointed out that when we take $h_B = 1$, the system degenerates into a local single channel, that is, only qubit A is affected by the dissipative channel; when we take $h_A = 1$, the system also degenerates into a local single channel, but only qubit B is affected by the dissipative channel.

Without loss of generality, in the following simulations, we assume the initial states of the qubits as

$$\rho_\psi(r,\theta) = r|\psi(\theta)\rangle\langle\psi(\theta)| + \frac{1-r}{4}I_4, \tag{20}$$

where $|\psi(\theta)\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$, r is the purity of the initial state, and $0 \le r \le 1$; I_4 is the 4×4 identity matrix.

3. Characterization parameters

In view of the above derivations, we know that the quantum correlation is closely related to the initial quantum state, detuning, channel dissipative effect, and the coupling effect between qubit and channel. Suppose that the observed qubit A and quantum memory qubit B are initially prepared in the X-state, and let A and B independently pass through the noisy channels. For a two-qubit system which is described by the density operator, in the standard computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ the density operator can be given by

$$\rho_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \tag{21}$$

In order to investigate the properties of the QMA EUR, based on Eq. (21), we define the right-hand side of Eq. (3) as[44]

$$EB = S(A|B) - \log_2 c = S(\rho_{AB}) - S(\rho_B) - \log_2 c, \tag{22}$$

and the left-hand side of Eq. (3) as

$$EU = S(\rho_{\sigma_x B}) + S(\rho_{\sigma_z B}) - 2S(\rho_B). \tag{23}$$

As stated earlier, EU measures the accuracy of the measurement results, thus the higher the accuracy is, the smaller the value of this item is. EB is the lower limit of the entropic uncertainty, which can be used to measure the quality of an uncertainty relation.

To quantify the entanglement we adopt the concurrence defined by Wootters.[45] In the X-state, the concurrence can be easily calculated by

$$C^{(\mathrm{W})}(t) = 2\max\{0, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}\},$$

$$|\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}\}. \tag{24}$$

The coherence of the quantum system is measured by l_1-norm standard[46]

$$C^{(\mathrm{L})}(t) = \sum_{j\neq k} |\rho_{jk}|. \tag{25}$$

Quantum discord is defined as the following expression:[47]

$$C^{(Q)}(t) = I(\rho_{AB}) - CC(\rho_{AB}), \tag{26}$$

where $I(\rho_{AB})$ is the total correlation, which is defined by the mutual information

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) + \sum_{k=1}^{4} \lambda_k \mathrm{Log}_2 \lambda_k, \tag{27}$$

and $CC(\rho_{AB})$ is the classical correlation between two subsystems and is expressed as[48]

$$CC(\rho_{AB}) = \max_{\{B_k\}} [S(\rho_A) - S(\rho_{AB}|\rho_B)], \tag{28}$$

where $\{B_k\}$ is a set of von Neumann measurements performed on subsystem B locally, $S(\rho_{AB}|\{B_k\}) = \sum_k p_k S(\rho_k)$ is the quantum conditional entropy. In this paper, we adopt the expression in Ref. [49] to calculate $C^{(Q)}(t)$, i.e.,

$$C^{(Q)}(t) = \min(QD_1, QD_2), \tag{29}$$

$$QD_1 = H(\rho_{11}+\rho_{33}) + \sum_{k=1}^{4} \lambda_k \mathrm{Log}_2 \lambda_k + D_1, \tag{30}$$

$$QD_2 = H(\rho_{11}+\rho_{33}) + \sum_{k=1}^{4} \lambda_k \mathrm{Log}_2 \lambda_k + D_2, \tag{31}$$

where

$$D_1 = H\left(\frac{1+\sqrt{[1-2(\rho_{33}+\rho_{44})]^2 + 4(|\rho_{14}|+|\rho_{23}|)^2}}{2}\right), \tag{32}$$

$$D_2 = -\sum_{j=1}^{4} \rho_{jj} \mathrm{Log}_2 \rho_{jj} - H(\rho_{11}+\rho_{33}), \tag{33}$$

$$H(\rho_{11}+\rho_{33}) = -(\rho_{11}+\rho_{33})\mathrm{Log}_2(\rho_{11}+\rho_{33}) - (\rho_{22}+\rho_{44})\mathrm{Log}_2(\rho_{22}+\rho_{44}). \tag{34}$$

4. Results and discussions

4.1. Local single dissipative channel

Assume two channels in the studied model are independent, namely, one is a dissipative channel, and the other is a non-dissipative channel. First, we consider the case of the local single dissipative channel. A natural question is which channel should the quantum register be placed in?

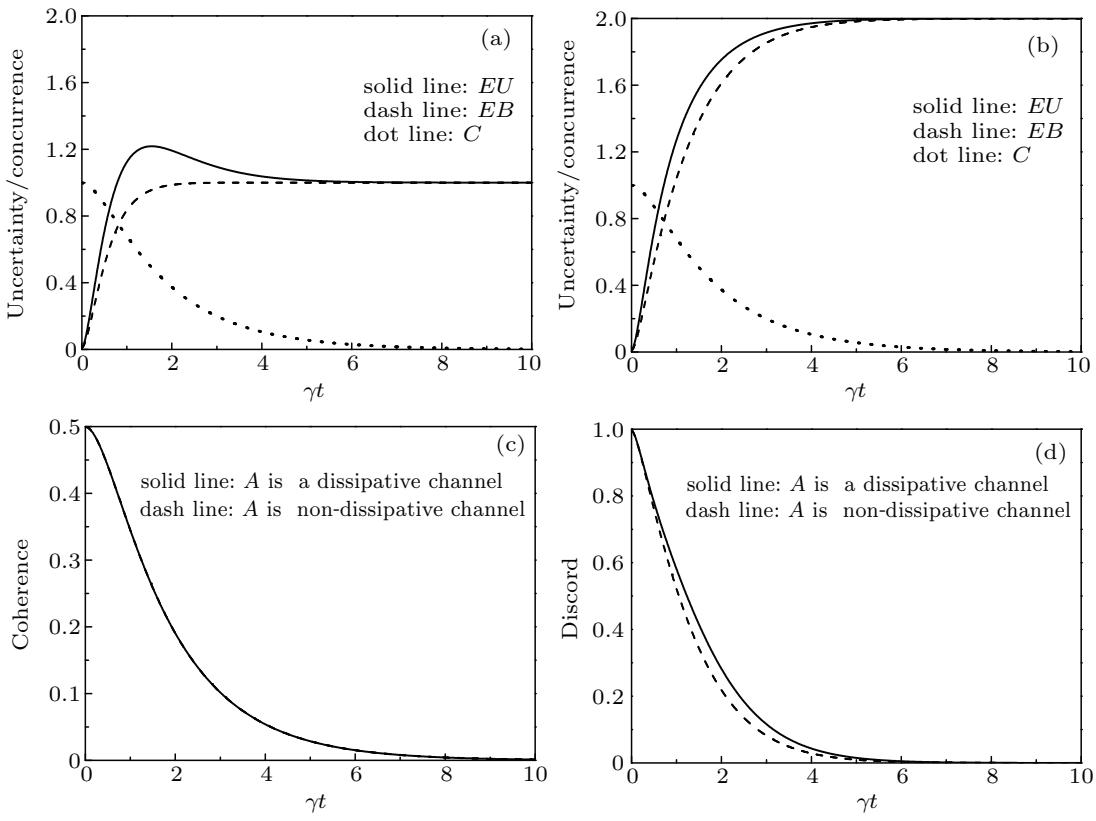


Fig. 1. Dynamics of correlations as a function of the dimensionless quantity γt in a single-sided Markovian channel with $r = 1.0$, $\theta = \pi/4$, and $\delta_A = \delta_B = 0.0$. (a) The channel A is a dissipative channel with $\lambda_A = 3.0\gamma$, and the channel B is an ideal channel. (b) The channel A is an ideal channel, and the channel B is a dissipative channel with $\lambda_B = 3.0\gamma$. Panels (c) and (d) denote respectively the difference between the coherence and quantum discord under panels (a) and (b).

The dynamical behavior of the quantum system in the single-sided amplitude damping channel as a function of dimensionless time γt is plotted in Figs. 1–4. Obviously, we can find that when the qubits used as quantum registers are placed in an ideal channel without dissipation, one can reduce the entropic uncertainty and its lower bound of the system. On the contrary, the entropic uncertainty and its lower bound increase obviously. The greater the entropic uncertainty is, the greater the noise of the measurement output is, which will affect its application in quantum communication. In addition, figure 1 shows that the entropic uncertainty and its lower bound firstly increases and then turns down to a steady state value in the Markovian channel. In contrast, with the evolution of the entropic uncertainty, the quantum coherence, quantum entanglement, and quantum discord of the system all monotonically decrease in the Markovian channel. The difference of the dynamical features indicates that there should be other factors affecting the entropic uncertainty of the system. In the non-Markovian channel, from Fig. 2, the entropic uncertainty, quantum coherence, quantum entanglement, and quantum discord oscillate with time.

Notably, from Figs. 1 and 2, whether the quantum register is located in a dissipative channel or in a non-dissipative channel, the time evolutions of quantum coherence and quantum entanglement are identical. However, it is interesting to notice that when the quantum register is located in a non-dissipative channel, the amplitude of quantum discord is larger than that in a dissipative channel, which indicates that the system presents more quantum correlation when the quantum register is located in a dissipative channel.

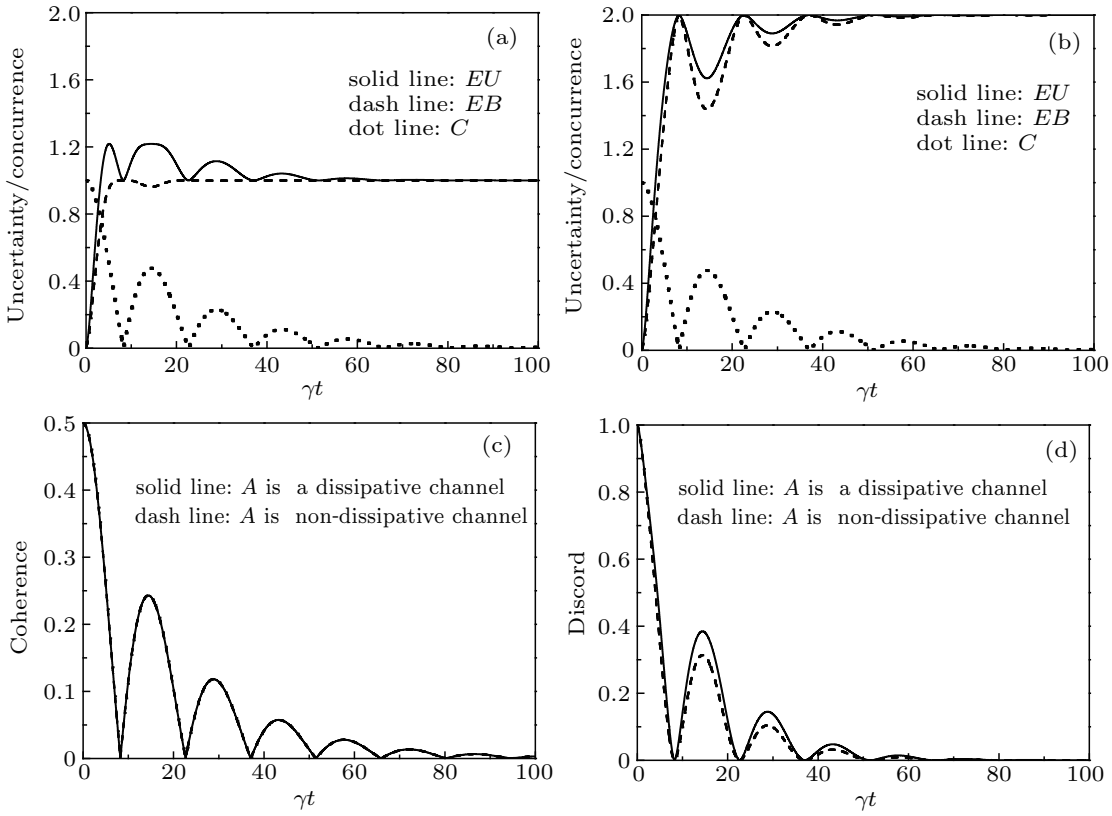


Fig. 2. Dynamics of correlations as a function of the dimensionless quantity γt in a single-sided non-Markovian channel with $r = 1.0$, $\theta = \pi/4$, and $\delta_A = \delta_B = 0.0$. (a) Channel A is a dissipative channel with $\lambda_A = 0.1\gamma$, and channel B is an ideal channel. (b) Channel A is an ideal channel, and channel B is a dissipative channel with $\lambda_B = 0.1\gamma$. Panels (c) and (d) denote respectively the difference between the coherence and quantum discord under panels (a) and (b).

Figures 3 and 4 show that, in the case of the single dissipative channel, the increase of the detuning can improve the quantum coherence, quantum entanglement, and quantum discord, and reduce the entropic uncertainty and its lower bound. Its physical mechanism is that the detuning inhibits the attenuation rate of the system, maintains the coherence and correlations, and reduces the entropic uncertainty of the system. However, figures 3 and 4 also show that the effects of the detuning on quantum coherence, quantum entanglement, and quantum discord are closely related to the dissipative characteristics of the environment. In the Markovian channel, the effect is not obvious; however, in the non-Markovian channel, the effect is very obvious since the non-Markovian effect makes the lost quantum information return from the environment to the quantum system. On the other hand, the detuning accelerates the exchange between the qubit and reservoir, and reduces the coupling strength between the quantum system and the thermal reservoir, which is beneficial for maintaining and improving the quantum coherence and quantum correlation.

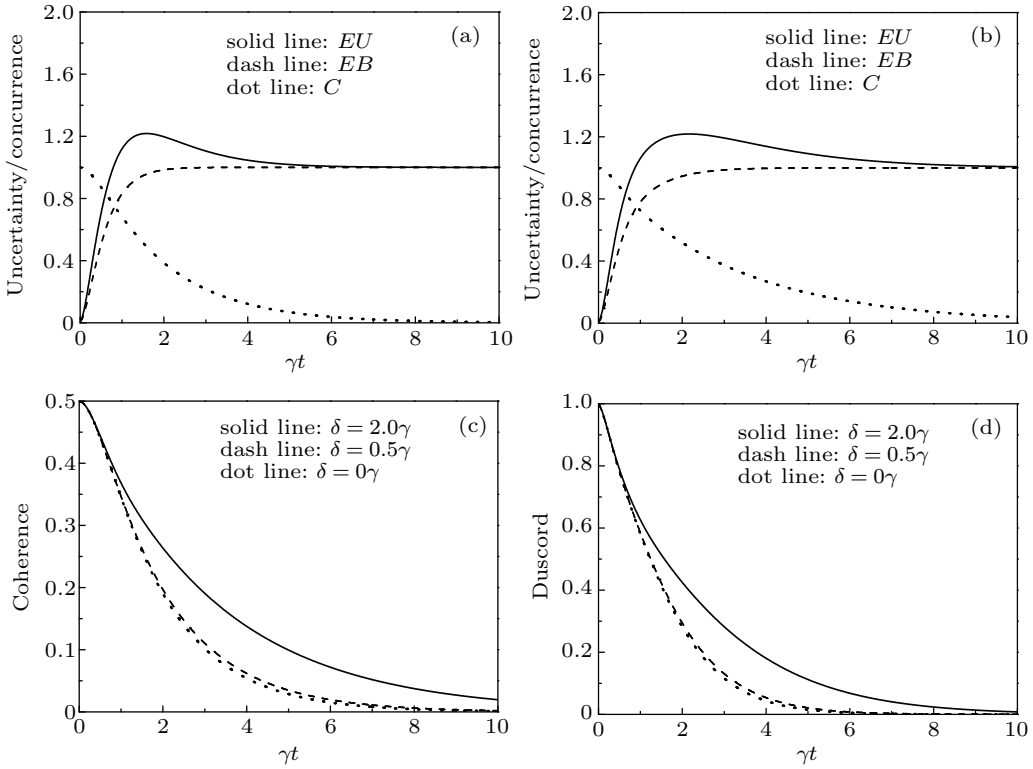


Fig. 3. Dynamics of correlations as a function of the dimensionless quantity γt in $\rho_{\psi}(r,\theta)$ state with $r = 1.0$, $\theta = \pi/4$, and $\lambda_A = 3.0\gamma$. (a) $\delta_A = \delta_B = 0.5$. (b) $\delta_A = \delta_B = 2.0$. Panels (c) and (d) denote respectively the difference between the coherence and quantum discord under panels (a) and (b).

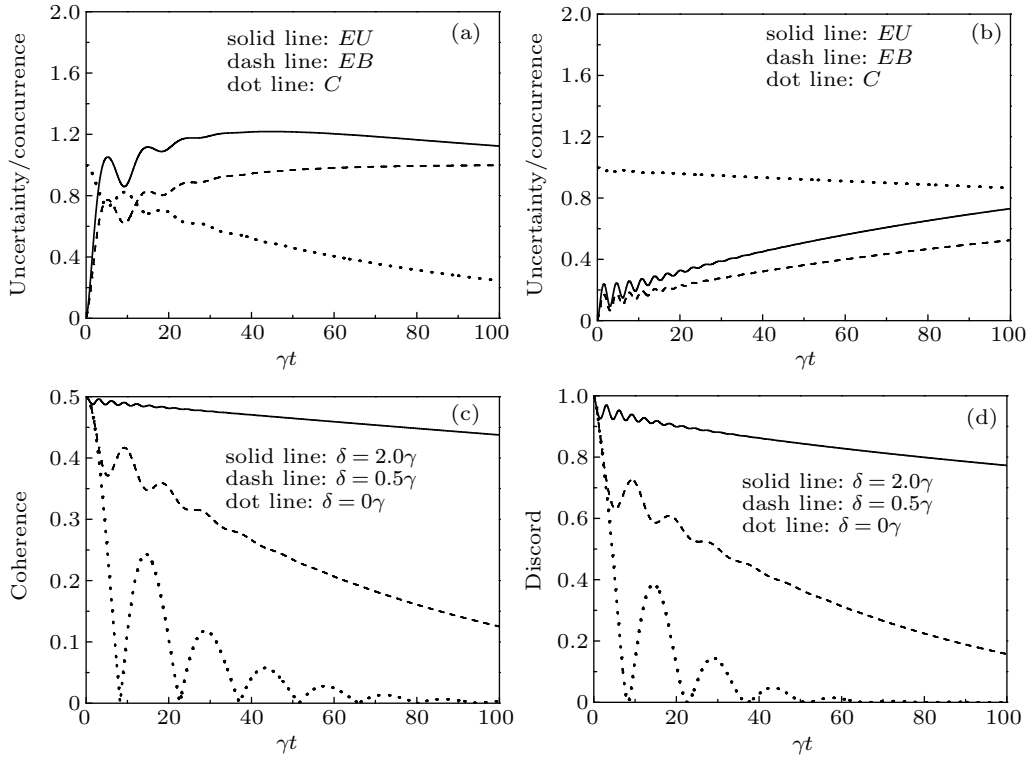


Fig. 4. Dynamics of correlations as a function of the dimensionless quantity γt in $\rho_{\psi}(r,\theta)$ state with $r = 1.0$, $\theta = \pi/4$, and $\lambda_A = 0.1\gamma$. (a) $\delta_A = \delta_B = 0.5$. (b) $\delta_A = \delta_B = 2.0$. Panels (c) and (d) denote respectively the difference between the coherence and quantum discord under panels (a) and (b).

4.2. The identical local dissipative channel

Figures 5 and 6 present the dynamical behavior of the quantum system as a function of the dimensionless time γt in the Markovian channel and non-Markovian channel, respectively. Consistent with the expected results, under the same condition, the dynamical behaviors of the entropic uncertainty and quantum entanglement are similar to those in the single dissipative channel (see Figs. 1–4). The same results can be

obtained for quantum coherence and quantum discord (not presented). However, the entropic uncertainty of the system increases with the growth of decoherence because the quantum register is also affected by the decoherence effect. Besides, due to the quantum entanglement between qubits, the original independent dissipative channel leads to the composite effect of the Markovian process and increases the uncertainty of the system. The composite effect not only causes each qubit to be affected by the dissipation effect in all channels, but also causes the backflow effect in the non-Markovian channel. Therefore, it is not only harmful to the quantum entanglement, but also leads to the increase of the system uncertainty.

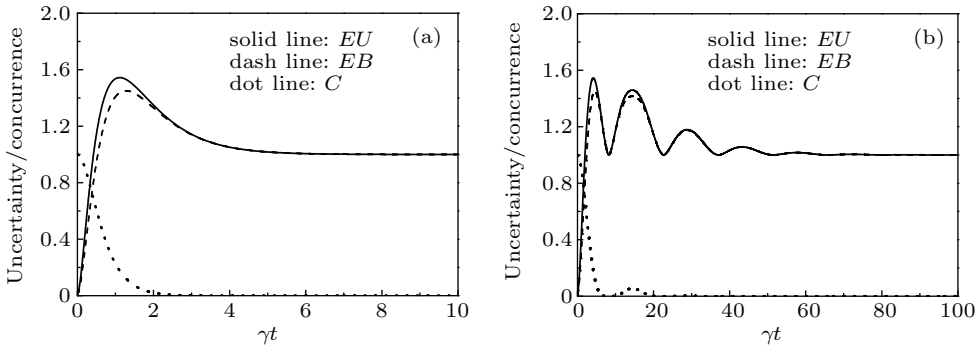


Fig. 5. Dynamics of quantum correlations as a function of the dimensionless quantity γt in $\rho_\psi(r,\theta)$ state with $r = 1.0$, $\theta = \pi/4$, and $\delta = 0.0$. (a) $\lambda_A = \lambda_B = 3.0\gamma$. (b) $\lambda_A = \lambda_B = 0.1\gamma$.

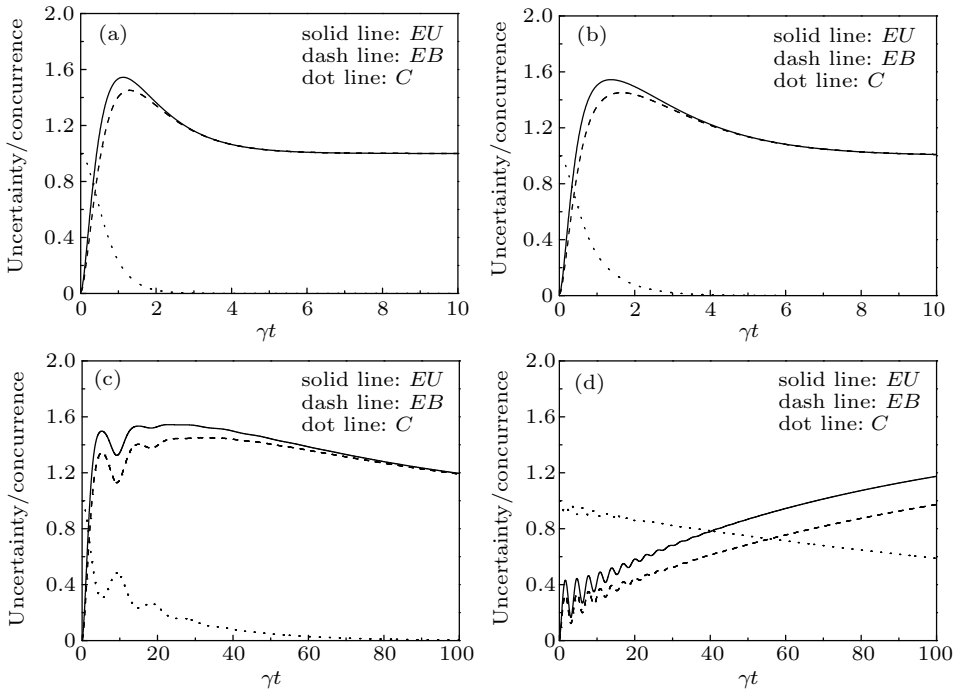


Fig. 6. Dynamics of correlations as a function of the dimensionless quantity γt in the two-sided channel case with $r = 1.0$ and $\theta = \pi/4$. (a) and (c) $\delta = 0.5\gamma$; (b) and (d) $\delta = 2.0\gamma$. (a) and (b) Markovian, $\lambda = 3\gamma$; (b) and (d) non-Markovian, $\lambda = 0.1\gamma$.

4.3. Different local dissipative channels

To further study the influence of two-sided amplitude damping channels with different dissipative characteristics on the uncertainty, we assume that one is a non-Markovian channel and the other is a Markovian channel. The coupling strengths between system and environment for two cases are the same $\gamma_A = \gamma_B = \gamma$, but the spectral widths of the noise channel are different, namely, $\lambda_A \neq \lambda_B$.

We perform numerical simulations on the selection of a channel for the quantum register. Comparing Fig. 7(a) with Fig. 7(b), we find that the quantum register should be placed in the non- Markovian channel since it not only reduces the uncertainty of the system but also significantly reduces the lower bound of uncertainty. If the quantum register is placed in the Markovian channel, the entropic uncertainty and its lower bound will obviously increase. From Fig.7, we also find that whether the non-Markovian channel or Markovian channel will change the evolution of the quantum coherence and quantum entanglement, except for the dynamical process of quantum discord. The stronger the non-Markovian effect is, the greater the quantum discord is, the smaller the uncertainty is, and the lower the noise of the measurement is.

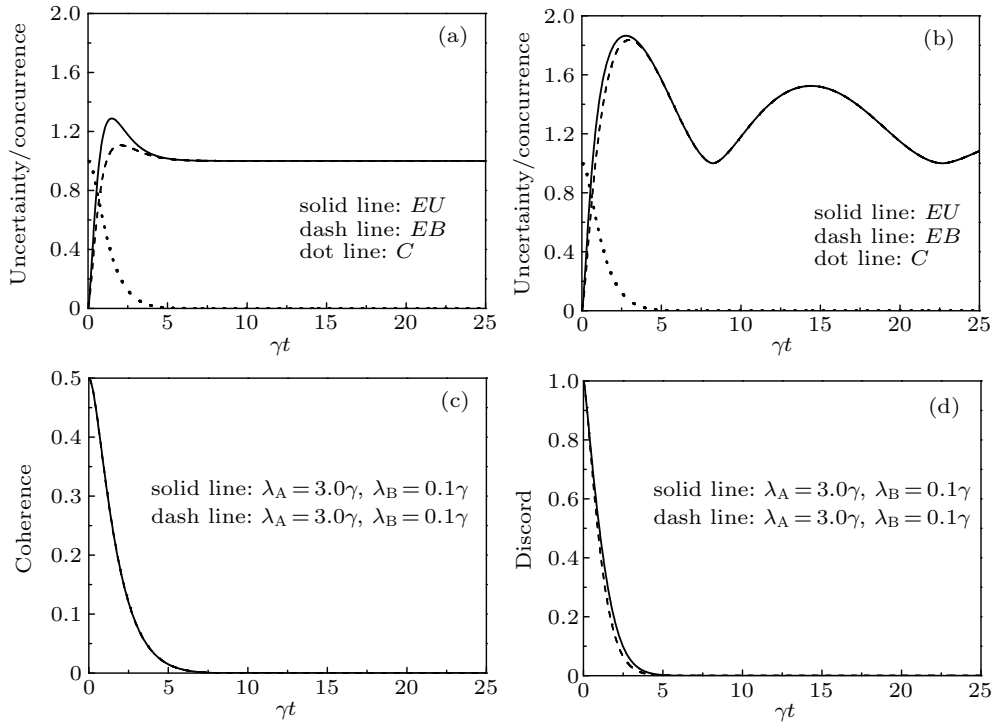


Fig. 7. Dynamics of correlations as a function of the dimensionless quantity γt in $\rho_{\psi}(r,\theta)$ state with $r = 1.0$, $\theta = \pi/4$, and $\delta_A = \delta_B = 0$. (a) $\lambda_A = 3.0\gamma$, $\lambda_B = 0.1\gamma$. (b) $\lambda_A = 0.1\gamma$, $\lambda_B = 3.0\gamma$. Panels (c) and (d) denote respectively the difference between the coherence and quantum discord under panels (a) and (b).

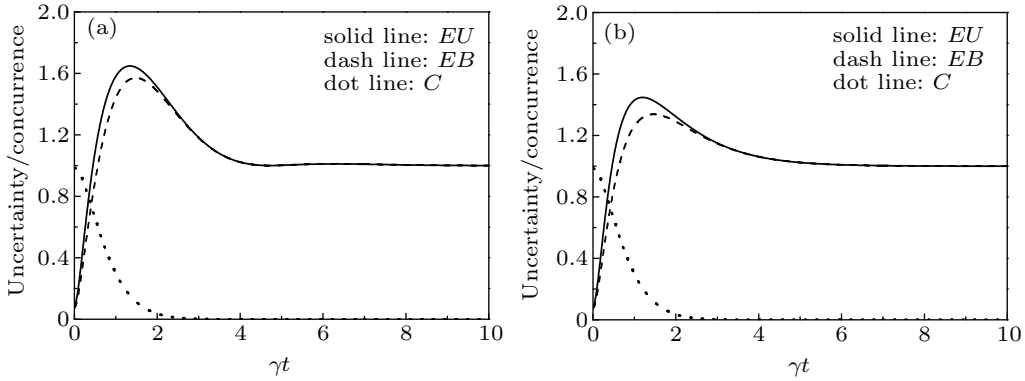


Fig. 8. Dynamics of correlations as a function of the dimensionless quantity γt in $\rho_{\psi}(r,\theta)$ state with $r = 1.0$, $\theta = \pi/4$, and $\delta_A = \delta_B = 0.0$. (a) $\lambda_A = 1.0\gamma$, $\lambda_B = 5.0\gamma$. (b) $\lambda_A = 5.0\gamma$, $\lambda_B = 1.0\gamma$.

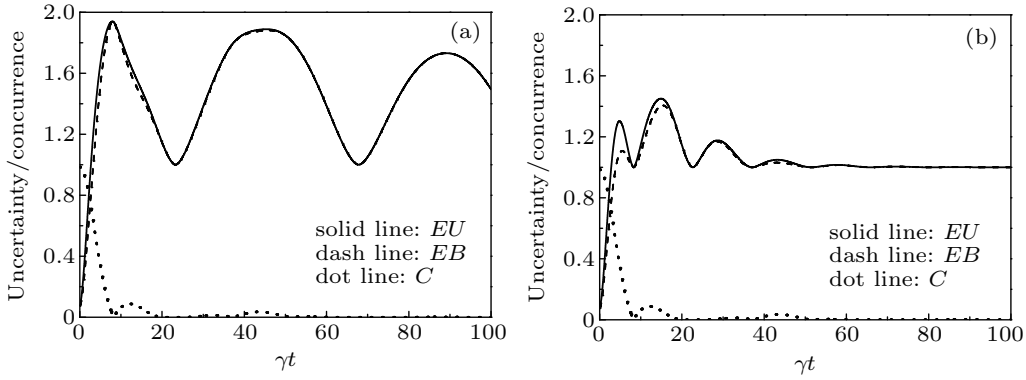


Fig. 9. Dynamics of correlations as a function of the dimensionless quantity γt in $\rho_{\psi}(r,\theta)$ state with $r = 1.0$, $\theta = \pi/4$, and $\delta_A = \delta_B = 0.0$. (a) $\lambda_A = 0.01\gamma$, $\lambda_B = 0.1\gamma$. (b) $\lambda_A = 0.1\gamma$, $\lambda_B = 0.01\gamma$.

Figures 8 and 9 also indicate that for the amplitude-damping channel, the entropic uncertainty and its lower bound of the system can be effectively reduced by placing the quantum register in the larger non-Markovian effect channel, no matter which channel the system is in, such as a Markovian channel or non-Markovian channel.

4.4. Discussions

The above investigations show that in order to reduce the uncertainty, the quantum register should be placed in the larger non-Markovian channel. In this case, the dynamical process of quantum discord will be changed, but the evolutions of quantum coherence and quantum entanglement will not. These numerical results show that: (i) the uncertainty of the system is closely related to quantum discord, therefore quantum discord plays an important role in reducing the uncertainty of the system; (ii) the quantum register has been placed in the larger degree of the non-Markovianity channel, which is helpful to use the memory effect of the non-Markovian environment, and promotes the loss of quantum information to return to the quantum system from the environment. That is to say, the quantum register in the non-Markovian channel is more likely to play the role of quantum discord resources; (iii) quantum discord is indeed a quantum resource that is different from both quantum coherence and quantum entanglement. The method of maintaining quantum coherence or quantum entanglement is not the same as protecting quantum discord. In some cases, the method of maintaining quantum coherence or quantum entanglement cannot effectively protect quantum discord.

In order to take advantage of the non-Markovian effect, we need to emphasize that in the actual quantum experiment, the non-Markovian dynamics evolution of a qubit (such as qubit B) can be realized by coupling the qubit B with a non-Markovian bath, for example, placing qubit B in photonic band gap materials, or coupling qubit B with an optical cavity of a high quality factor. Furthermore, the control of an environment characterized by non-Markovianity can be achieved by controlling the coupling strength between the qubit and the environment, or by designing a non-Markovian bath (such as a quality factor designed for a good optical cavity).

5. Conclusion

We investigate how to reduce the uncertainty of the system by optimizing the configuration of the quantum register in the Markovian and non-Markovian channels. The results show that, for the amplitude-damping channel, the uncertainty of the system can be effectively reduced by placing the quantum register in the stronger non-Markovian channel. Moreover, the entropic uncertainty and its lower bound can be reduced by increasing the detuning. The entropic uncertainty is closely related with the quantum discord, which can play a key role in reducing the uncertainty of the system. The increases of detuning and non-Markovian effect are beneficial to protecting the quantum discord and reduce the uncertainty of the system.

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