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Decoherence of macroscopic objects from relativistic effect*

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We study how the decoherence of macroscopic objects originates intrinsically from the relativistic effect. With the degree of freedom of the center of mass (CM) characterizing the collective quantum state of a macroscopic object (MO), it is found that an MO consisting of N particles can decohere with a time scale of no more than $\sqrt{N}^{-1}$. Here, the special relativity can induce the coupling of the collective motion mode and the relative motion modes in an order of $1/c^2$, which intrinsically results in the above minimum decoherence.

Keywords: decoherence, macroscopic object, special relativistic effect

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1. Introduction

Quantum superposition lies in the heart of both quantum mechanics[1,2] and the current quantum technologies such as quantum communication and computation.[3,4] Any superposition of different states whose evolution is governed by Schrödinger's equation[5] still satisfies the same evolution equation and remains valid in the quantum world. Upon one measurement, Born's rule determines the probability of one definite outcome.[6] Another feature of interest in quantum mechanics is the quantum coherence, which is depicted by superposition. For instance, the fringes in the interference experiment of electrons show the coherence of the electron state in different paths. However, interaction between a physical system with its environment may ruin this coherence. This environment-induced process, known as decoherence or dissipation,[7] destroys the coherence of the system, i.e., suppresses strongly the interference of states of the system or dissipates its energy, and singles out a set of states which behave like classical states.[8,9] In the quantum theory of measurement where the system, apparatus, and environment are all treated as quantum objects (governed by Schrodinger's equation), decoherence is proposed to interpret the outcome of measurements without the collapse of wave packets.[9,10] A number of models of environments which do not dissipate energy of the system but contribute to its decoherence have been studied in the past few decades. For example, the environment can be chosen as a ring of spin $1/2$,[11,12] a reservoir of harmonic oscillators,[7,13] and many other external environments.[14]

It seems that quantum theory, while tested thoroughly at the microscopic level, is somehow counter-intuitive in the macroscopic domain. In Schrödinger's well-known gedanken experiment,[15] a cat, which is described as a macroscopic object (MO), is in a superposition state of alive and dead, which has never been observed in the classical world. In our daily life, the cat is either alive or dead with the same chance but not in the superposition state. In fact, decoherence plays an important role in this transition from the quantum world to the classical world.[8,9] In the universe, isolated systems barely exist, especially the MOs, which must interact with environments (with a large scale of degrees of freedom). Generally speaking, the larger the scale of a system, the faster it decoheres.[16]

As we stated above, various interactions will lead to this quantum-classical transition phenomena. In this paper, we focus on its intrinsic origin, which results in the minimum decoherence even in the absence of any usual environments. It has been found that the collective mode of MOs can be coupled with its inner motion modes.[17–19] Carazza has studied the decoherence effect of the collective variable for free quasi-relativistic particles.[19] Nevertheless, for macroscopic objects, a more reasonable scenario should take the interaction between particles into account, since it is the coupling between particles that bound them into a macroscopic object. Recently, Igor Pikovski *et al.* has also studied decoherence due to gravitational time dilation in 2015.[20] They phenomenologically thought that the internal movement energy of the system can contribute to its total mass, and the center of mass (CM) was coupled to the internal movement due to the general relativistic effect.

In this paper, we revisit the effect of the special relativity on the decoherence of the collective mode of macroscopic ob-

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jects. We study a ring of relativistic particles with the nearest–neighbouring interaction, and under some transformation we find there exist interactions between the CM and the internal degrees of freedom. Then we look into the decoherence of CM motion after time evolution and obtain the decoherence time $\tau \sim \left(3\sqrt{N}\left|\Delta E_{1,2}\right|\omega/2Mc^2\right)^{-1}$ where N is the particle number, $\Delta E_{1,2}$ is the energy difference of two initial states, ω is the coupling strength, and M is the total mass of CM.

The remainder of this paper is organized as follows. In Section 2, we describe the relativistic macroscopic system consisting of N particles and obtain its effective Hamiltonian with the lowest relativistic effect to depict the decoherence of CM motion. In the decoherence dynamics, we choose the initial state as a product state of the superposition states of CM momentum (coherent states) and ground states of simple oscillators, and figure out the time evolution of this state. Then we obtain the reduced density matrix and analyze its decoherence. In Section 3, we study the decoherence of free particles with the same initial state and compare it with the outcomes in Section 2. The conclusions and discussions are in Section 4.

2. The quasi-relativistic macroscopic object

2.1. The Hamiltonian of the macroscopic particle ring

In this section, we start from a relativistic MO composed of N particles, each of which obeys the Dirac equation, with the nearest-neighbouring interaction being considered. In special relativistic quantum theory, the Dirac Hamiltonian of a free fermion reads [?]

$$H_0 = \beta mc^2 + c\boldsymbol{\alpha}\cdot\boldsymbol{p},$$

where p and m are the momentum operator and mass of the particle respectively, and

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad (i = 1,2,3)$$

are the Dirac matrices and c denotes the light velocity. It is well known that there are four eigenvectors where two correspond to positive energy $\sqrt{p^2c^2+m^2c^4}$ and the others correspond to negative energy $-\sqrt{p^2c^2+m^2c^4}$. In other words, one can diagonalize this Hamiltonian in a block with a unitary transformation $U = \exp\left[\beta\boldsymbol{\alpha}\cdot\boldsymbol{p}\arctan\left(p/mc\right)/2p\right]$ (Ref. [22]) as $H_0' = UH_0U^\dagger$, i.e.,

$$H_0' = \beta\sqrt{p^2c^2+m^2c^4}.$$

The positive and negative energy spaces are separated. In the following we will focus on the positive energy part and consider the lowest-order relativistic correction of non-relativistic particles. Actually, the above argument is carried out for Dirac particles. For scalar particles, one can also obtain the mass-energy relation with the Klein–Gordon equation.

Thus the total Hamiltonian of the relativistic MO is

$$H = \sum_i \sqrt{p_i^2c^2+m^2c^4} + \frac{1}{2}m\omega^2\left(x_i - x_{i+1}\right)^2, \tag{1}$$

where m is the mass of a single particle, p_i and x_i are the momentum and position operator at site i respectively as we set the lattice distance $a = 1$. The Hamiltonian to the second-order approximation, which contains the lowest-order term with the relativistic effect, becomes

$$H \simeq Nmc^2 + \sum_i^N \left[\frac{{p_i}^2}{2m} - \frac{{p_i}^4}{8m^3c^2} + m\omega^2\left(x_i^2 - x_i x_{i+1}\right)\right]. \tag{2}$$

For simplicity, we choose N as an odd number without loss of generality and take the Fourier transformation

$$\begin{aligned} p_k &= \sqrt{\frac{1}{N}}\sum_{j=1}^N p_j \exp\left(-\frac{\mathrm{i}2\pi}{N}kj\right), \\ x_k &= \sqrt{\frac{1}{N}}\sum_{j=1}^N x_j \exp\left(\frac{\mathrm{i}2\pi}{N}kj\right), \end{aligned} \tag{3}$$

to get the normal modes with p_k and x_k, the momentum and position operators at momentum space respectively. It is worth mentioning that $p_{k=N} = \sum_{j=1}^N p_j \exp\left(-\mathrm{i}2\pi j\right)/\sqrt{N} = P/\sqrt{N}$, and $x_{k=N} = \sum_{j=1}^N x_j \exp\left(\mathrm{i}2\pi j\right)/\sqrt{N} = \sqrt{N}X$ are the momentum and position operators of the CM. That is to say, the N-th mode describes the motion of CM while the other $N-1$ modes describe the internal relative motion. Therefore, we introduce the CM and relative coordinates in the relativistic Hamiltonian by this Fourier transformation. In this center of mass reference frame, the Hamiltonian in the low-energy limit becomes,

$$\begin{aligned} H \simeq\ & \frac{P^2}{2M} - \frac{P^4}{8M^3c^2} + \sum_{k=1}^{N-1}\frac{p_k p_{-k}}{2m} \\ & + 2m\omega^2\sum_{k=1}^{N-1} x_k x_{-k}\sin^2\left(\frac{\pi}{N}k\right) - \frac{3P^2}{2M^2c^2}\sum_{k=1}^{N-1}\frac{p_k p_{-k}}{2m} \\ & - \frac{P}{2m^2Mc^2\sqrt{N}}\sum_{k_1,k_2,k_3=1}^{N-1} p_{k_1}p_{k_2}p_{k_3}\delta_{k_1+k_2+k_3,qN} \\ & - \frac{1}{8m^2Mc^2}\sum_{k_1,k_2,k_3,k_4=1}^{N-1} p_{k_1}p_{k_2}p_{k_3}p_{k_4}\delta_{k_1+k_2+k_3,qN}, \end{aligned} \tag{4}$$

where $\delta_{k_1+k_2+k_3,qN}$ for q being one of nonzero integers, is the Kronecker Delta function. It can be seen in Eq. (4) that the first two terms describe the Hamiltonian of CM while the third and fourth terms describe the relative motions. Obviously the last three terms characterize the interaction between CM and relative motion and the higher order correction.

We then treat the CM system as the "system" and the relative motion system as the "internal environment". Therefore, this division of the Hamiltonian in Eq. (4) implies that the motion of the system will be influenced by the environment. As a consequence

$$H \simeq H_{\mathrm{S}} + H_{\mathrm{E}} + H_{\mathrm{SE}}, \tag{5}$$

where

$$H_{\rm S} = \frac{P^2}{2M},$$
$$H_{\rm E} = \sum_{k=1}^{N-1} \frac{p_k p_{-k}}{2m} + 2m\omega^2 x_k x_{-k} \sin^2\left(\frac{\pi}{N}k\right),$$
$$H_{\rm SE} = -\frac{3P^2}{2M^2c^2}\sum_{k=1}^{N-1}\frac{p_k p_{-k}}{2m},$$

where the high order collective terms are neglected. It can be checked immediately that $[H_{\rm S}, H_{\rm SE}] = 0$ and $[H_{\rm E}, H_{\rm SE}] \neq 0$, which means that evolution governed by the Hamiltonian in Eq. (5) may cause entanglement between the system and the environment. According to the decoherence theory, this kind of interaction will induce a transition from the quantum superposition state to the classical statistical mixture in the system without energy dissipation.[7,8]

We have obtained the decoherence model, but there remains another problem: different modes of relative motions are not independent but coupled in pairs. We then introduce the Bogoliubov transformation. This transformation is given by $\left(\begin{array}{cccc} Q_{k=1} & Q_{k=2} & \cdots & Q_{k=N-1}\end{array}\right)^{\rm T} = W_{(N-1)\times(N-1)}\left(\begin{array}{cccc} q_{k=1} & q_{k=2} & \cdots & q_{k=N-1}\end{array}\right)^{\rm T}$, where Q stands for P,X, which are the momentums and the displacements of the $N-1$ independent relative motions, q for p,x respectively. The transformation W is

$$W = \sqrt{\frac{1}{2}}\begin{pmatrix} 1 & 0 & \ldots & 0 & 0 & 1 \\ 0 & 1 & \ldots & 0 & 1 & 0 \\ & & \cdots & & & \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -i & i & 0 & 0 \\ 0 & -i & \ldots & 0 & i & 0 \\ -i & 0 & \ldots & 0 & 0 & i \end{pmatrix}. \tag{6}$$

For $j \in [1, N-1]$, only two elements are non-vanishing in every row and column of W. Especially, $W_{j,j} = W_{j,N-j} = 1/\sqrt{2}$ when $j \in [1, (N-1)/2]$ and $W_{j,j} = W^*_{j,N-j} = {\rm i}/\sqrt{2}$ when $j \in [(N+1)/2, N-1]$. We can check that $W \cdot W^\dagger = I_{(N-1)\times(N-1)}$, and

$$\sum_{k=1}^{N-1} q_k q_{-k} = \sum_{k=1}^{N-1} Q_k^2. \tag{7}$$

Finally, the diagonalized Hamiltonian becomes ($\omega_k = 2\omega\sin(\pi k/N)$)

$$\begin{aligned} H &= \frac{P^2}{2M} + \sum_{k=1}^{N-1}\left(\frac{P_k^2}{2m} + \frac{m}{2}\omega_k^2 X_k^2\right) - \frac{3P^2}{2M^2c^2}\sum_{k=1}^{N-1}\frac{P_k^2}{2m} \\ &= \frac{P^2}{2M} + \sum_{k=1}^{N-1} H_{{\rm E},k} + H_{I,k}(P). \end{aligned} \tag{8}$$

Now we know that there is only the kinetic term in the system Hamiltonian, the environment contains $N-1$ modes of a simple harmonic oscillator with $N-1$ eigenfrequencies and the motion of CM couples with all the relative motion modes.

2.2. Decoherence dynamics

In the previous subsection, we obtained the Hamiltonian of the decoherence shown in Eq. (8). With the interaction between system and environment, the evolution equation of the reduced density matrix of the system is the quantum master equation, which is not unitary due to the interaction with its environment.[7] According to Born's rule, the diagonal terms of the reduced density matrix describe the probabilities of getting some outcome in one measurement, while the off-diagonal ones characterize the interference of different quantum states and show the coherence properties of this system. When the coherence of the system decays with time while the probability terms remain stable, the system undergoes a transition from quantum to classical, i.e., decoherence.[9,16]

We can see that, in the Hamiltonian (8), the relative motions are $N-1$ simple harmonic oscillations and the minimal energy difference between two neighbouring levels in the large N limit is $2\hbar\omega\pi/N$. Assuming the temperature of the environment is too low to excite the relative mode, i.e., $2\hbar\omega\pi/N \gg k_{\rm B}T$, all the relative motions stay in the ground state. Then we choose the initial state as $|\varphi(0)\rangle = (|P_1\rangle + |P_2\rangle)/\sqrt{2} \otimes \prod_{k=1}^{N-1}|0\rangle_k$ where $|P_i\rangle$ $(i = 1,2)$ is the eigenstate of P with eigenvalue P_i and $|0\rangle_k$ is the ground state of the k-th mode of the relative motion Hamiltonian. The total density matrix at time t evolves as

$$\rho(t) = {\rm e}^{-{\rm i}Ht/\hbar}|\varphi(0)\rangle\langle\varphi(0)|\,{\rm e}^{{\rm i}Ht/\hbar}. \tag{9}$$

The reduced density matrix of the motion of CM is

$$\rho_{\rm c.m.}(t) = {\rm Tr}_{\rm E}\,\rho(t).$$

As $|P_1\rangle$ and $|P_2\rangle$ are the eigenstates of $H_{\rm S}$, the diagonal terms of the local (reduced) density matrix in basis $|P_i\rangle$ are independent of time, i.e., $\rho_{\rm S}^{1,1}(t) = \rho_{\rm S}^{1,1}(0) = \rho_{\rm S}^{2,2}(t) = \rho_{\rm S}^{2,2}(0) = 1/2$. This feature indicates that we are dealing with a pure decoherence process without dissipation. The off-diagonal term reads

$$\begin{aligned} \left|\rho^{12}_{\rm c.m.}(t)\right| &= \frac{1}{2}\prod_{k=1}^{N-1}\left|\langle 0|\,{\rm e}^{{\rm i}H_k(P_1)t/\hbar}\,{\rm e}^{-{\rm i}H_k(P_2)t/\hbar}\,|0\rangle_k\right| \\ &= \frac{1}{2}\prod_{k=1}^{N-1}\left|f_k(P_1,P_2,t)\right|, \end{aligned} \tag{10}$$

where

$$\begin{aligned} &H_k(P_i) = H_{{\rm E},k} + H_{I,k}(P_i), \\ &f_k(P_1,P_2,t) = \langle 0|\,S_k(\xi_1\,{\rm e}^{{\rm i}\pi})S_k(r_1)S_k(r_2\,{\rm e}^{{\rm i}\pi})S_k(\xi_2)\,|0\rangle_k, \end{aligned}$$

where $S(r)$ is a squeeze operator. Thus the coherence of this system is related with the expectation value of four squeeze operators over the vacuum state (for more details see Appendixes A and B). After some calculation, we obtain

$$\lim_{N\gg 1}\left|\rho^{12}_{\rm c.m.}(t)\right| = \frac{1}{2}\exp\left[-\frac{N}{N_0}\left(1 - {\rm J}_0(4\omega t)\right)\right], \tag{11}$$

where $N_0 = \left(32M^2c^4/9\left(\Delta E_{1,2}\right)^2\right)$, $\mathrm{J}_n(x)$ is the first kind of Bessel function and $\Delta E_{1,2} = \left(P_1^2 - P_2^2\right)/2M$. One can conclude from Eq. (11) that in the large N limit the decoherence function depends on the scale of the system, the energy difference of initial states, the coupling strength of real particles, and time t. One may also find that as $\omega t \ll 1$, $\mathrm{J}_0(4\omega t) \simeq 1 - 4\omega^2 t^2$, thus the decoherence function becomes

$$\left|\rho_{\mathrm{c.m.}}^{12}(t)\right| \simeq \frac{1}{2}\exp\left[-4N\omega^2 t^2/N_0\right]. \tag{12}$$

Then the decoherence time (assuming $N \gg N_0$) is

$$\tau \sim \frac{2\sqrt{2}Mc^2}{3\sqrt{N}\left|\Delta E_{1,2}\right|\omega}. \tag{13}$$

In the long time limit, $\omega t \gg 1$, $\mathrm{J}_0(4\omega t) \simeq 0$ and $\left|\rho_{\mathrm{c.m.}}^{12}(t)\right| \simeq \exp\left[-N/N_0\right]/2$ which indicates that only when $N \gg N_0$, $\left|\rho_{\mathrm{S}}^{12}(t)\right| \overset{\omega t \gg 1}{\longrightarrow} 0$, i.e., there is a restriction on the scale of the whole system.

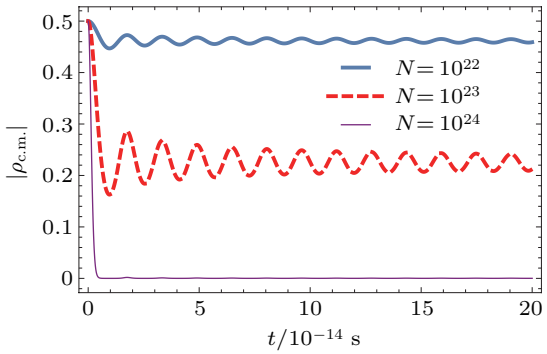


Fig. 1. (color online) The off-diagonal matrix element of $\left|\rho_{\mathrm{c.m.}}^{12}(t)\right|$ as a function of t with different N: $N = 10^{22} < N_0$ (blue, solid), $N = 10^{23} \simeq N_0$ (red, dashed), $N = 10^{24} \gg N_0$ (purple, thin). All the parameters are chosen as $\omega = 10^{14}$ Hz, and the two superposed initial velocities $V_1 = 200$ m/s, $V_2 = 1000$ m/s.

Equations (12) and (13) are the main results of this paper. The decoherence process of CM depends on the scale of the system, the interaction strength of real particles and the difference of the initial kinetic energy. The larger the system is, the faster it decoheres. Table 1 shows a list of decoherence results with respect to systems of different magnitudes (e.g., the universe, the Earth, a person, and C_{60}). For simplicity, we have assumed that all these systems are composed of only carbon atoms. As an example, we consider C_{60} molecules with coupling strength $\omega \simeq 10^{14}$ Hz. With the two superposed initial velocities of this carbon ring as 200 m/s and 1000 m/s,[23] the lower bound of the particle number is $N_0 = 1.3 \times 10^{23}$ which has the magnitude of Avogadro constant $N_A = 6.02 \times 10^{23}$. Thus no decoherence occurs in a single C_{60} molecule. Figure 1 shows $\left|\rho_{\mathrm{c.m.}}^{12}(t)\right|$ as a function of t with different N. First, when $N \ll N_0$, the off-diagonal element $\left|\rho_{\mathrm{c.m.}}^{12}(t)\right|$ oscillates around 0.48, which means the system barely decoheres; then, as $N \simeq N_0$, $\left|\rho_{\mathrm{c.m.}}^{12}(t)\right|$ decreases in the beginning and oscillates around 0.25 later, indicating that a portion of coherence is retained in the system; at last, for $N \gg N_0$, compared with the above two cases, $\left|\rho_{\mathrm{c.m.}}^{12}(t)\right|$ decreases to zero immediately thus the state of the CM in this system decoheres. And the decoherence time of this MO is about 10^{-14} s.

It is shown in Eq. (11) that the decoherence process depends on $P_1^2 - P_2^2$. It seems obscure that when $P_1 = -P_2$ the coherence of CM remains unchanged. In fact, this can be seen in Eqs. (4) and (5) that we drop the term $\sim P\sum_{k_1,k_2,k_3=1}^{N-1} p_{k_1}p_{k_2}p_{k_3}\delta_{k_1+k_2+k_3,N}$ as a higher order term and only keep the one $\sim P^2\sum_{k=1}^{N-1} p_k p_{-k}$. This reduction may be related with our results $\propto \left(P_1^2 - P_2^2\right)^2$. Now it is time to find out the decoherence time as $P_1 = -P_2$. In this case, the off-diagonal term of the reduced density matrix is calculated as

$$\left|\rho_{\mathrm{c.m.}}^{12}(t)\right| = \frac{1}{2}\left|\langle 0|\, \mathrm{e}^{\mathrm{i}H(P_1)t/\hbar}\,\mathrm{e}^{-\mathrm{i}H(P_2)t/\hbar}\,|0\rangle\right|,$$

where

$$H(P_1) = \frac{P_1^2}{2M} + \sum_{k=1}^{N-1}\frac{P_k^2}{2m} + \frac{m}{2}\sum_{k=1}^{N-1}X_k^2\omega_k^2 - \frac{3P_1^2}{2M^2c^2}\sum_{k=1}^{N-1}\frac{P_k^2}{2m} - \frac{P_1}{2m^2Mc^2\sqrt{N}}\sum_{k_1,k_2,k_3=1}^{N-1} W^{\dagger}\left(P_{k_1}P_{k_2}P_{k_3}\right).$$

Here $\sum_{k_1,k_2,k_3=1}^{N-1} W^{\dagger}\left(P_{k_1}P_{k_2}P_{k_3}\right)$ denotes the term $\sum_{k_1,k_2,k_3=1}^{N-1} p_{k_1}p_{k_2}p_{k_3}\delta_{k_1+k_2+k_3,N}$ transformed by W given in Eq. (6). Keeping terms up to t^2, one finds

$$\begin{aligned}\left|\rho_{\mathrm{c.m.}}^{12}(t)\right| &\simeq \frac{1}{2}\left|1 + \mathrm{i}\frac{t}{\hbar}\langle 0|H(P_1) - H(P_2)|0\rangle - \frac{t^2}{2\hbar^2}\langle 0|\left(H(P_1) - H(P_2)\right)^2|0\rangle\right| \\ &= \frac{1}{2}\exp\left(-\frac{t^2}{\tau^2} - \frac{t^2}{\tau'^2}\right).\end{aligned} \tag{14}$$

The decoherence times τ and τ' are obtained, which are caused by the P^2 and P terms of interaction respectively. What is more important,

$$\begin{aligned}\tau^{-1} &= \frac{9}{32}\frac{\left(V_1^2 - V_2^2\right)^2}{c^4}\omega^2, \\ \tau'^{-1} &\simeq \frac{\left(V_1 - V_2\right)^2\omega^2}{32Mc^4}\hbar\omega, \\ \frac{\tau'}{\tau} &\simeq Nm\frac{\left(V_1 + V_2\right)^2}{\hbar\omega},\end{aligned} \tag{15}$$

where $V_1(V_2) = P_1(P_2)/M$. Here we know that when $P_1 = -P_2$, the decoherence effect originates from the interaction term P with time scale $\sim \left(V_1 - V_2\right)^2\hbar\omega^3/32Mc^4$. Actually, when $P_1^2 \neq P_2^2$ and in the large system limit, we also find that the influence of the P^2 term dominates, i.e., $\tau \ll \tau'$. In other words, the decoherence process caused by the interaction term P^2 is much faster than the process caused by term P. This is why we only keep the interaction term P^2 in the very beginning of our paper.

Table 1. Decoherence time and particle number bound of systems of different scale with coupling strength $\omega = 10^{14}$ Hz.

	Universe	Earth	Person	C_{60}
Initial velocity of CM/(m/s)	2.0×10^{6}, 1.0×10^{6}	3.0×10^{4}, 2.0×10^{4}	10, 5.0	1.0×10^{3}, 2.0×10^{2}
Particle number	5.0×10^{78}	2.5×10^{50}	5.0×10^{27}	60
Particle number bound	1.3×10^{10}	4.0×10^{17}	2.0×10^{31}	1.3×10^{23}
Decoherence time/s	2.0×10^{-49}	2.0×10^{-31}	3.0×10^{-13}	2.4×10^{-5}

2.3. Decoherence of cat: Superposition of coherent states

It is well known that the eigenstates of momentum operators are ideal quantum states, and that they are difficult to prepare in experiments. More "classical" states in quantum mechanics are coherent states, like $|\alpha\rangle$, which are generated by applying the displacement operators on the vacuum states. One important property of the coherent state is that it satisfies the minimum uncertainty relation. Moreover, coherent states behave as Gaussian wave packets in both momentum and position space. In this subsection, we study the decoherence of a more "classical" quantum state, cat state, i.e., the superposition of coherent states. Here the Wigner function in phase space is a useful tool in exploring the non-classicality of quantum states.[24]

Here we consider that the CM is prepared in a macroscopic superposition state $\sim|\alpha\rangle+|\beta\rangle$ initially, and the relative modes are still in ground states, describing a non-excited internal environment,

$$|\Psi(0)\rangle=\frac{1}{\Xi}\left(|\alpha\rangle+|\beta\rangle\right)\bigotimes\prod_{k=1}^{N-1}|0\rangle_k, \tag{16}$$

where Ξ is the normalization factor and $\alpha\,(\beta)$ is a complex number. The reduced density matrix of the motion of CM is

$$\rho_{\mathrm{c.m.}}(t)=\frac{1}{|\Xi|^2}\int\int \mathrm{d}P_1\,\mathrm{d}P_2\Pi\left(\alpha,\beta,P_1,P_2,t\right)|P_1\rangle\langle P_2|, \tag{17}$$

where

$$\begin{aligned}&\Pi\left(\alpha,\beta,P_1,P_2,t\right)\\&=\prod_{k=1}^{N-1}f_k(P_1,P_2,t)\left(\langle P_1\mid\alpha\rangle+\langle P_1\mid\beta\rangle\right)\left(\langle\alpha\mid P_2\rangle+\langle\beta\mid P_2\rangle\right).\end{aligned}$$

As illustrated in Subsection 2.2, the decoherence function relies on the overlap of two quantum states, $\prod_{k=1}^{N-1}f_k(P_1,P_2,t)$. For simplicity, we replace this overlap function with its modulus,

$$\prod_{k=1}^{N-1}f_k(P_1,P_2,t)\simeq\prod_{k=1}^{N-1}\left|f_k(P_1,P_2,t)\right|.$$

In momentum representation, the coherent state behaves as a Gaussian wave packet with packet width $\hbar/2\sigma$ and the mean momentum $\hbar\Im(\alpha)/\sigma$,

$$\begin{aligned}\langle P\mid\alpha\rangle\;=\;&\left(\frac{2\sigma^2}{\pi\hbar^2}\right)^{1/4}\exp\left[-\frac{\sigma^2}{\hbar^2}\left(P-\frac{\hbar}{\sigma}\Im(\alpha)\right)^2\right]\\&\times\exp\left[-2\mathrm{i}\frac{\sigma}{\hbar}\Re(\alpha)P\right].\end{aligned} \tag{18}$$

At first glance, it seems difficult to deal with the reduced density matrix given in Eq. (17) as its dimension is infinite. In order to quantify this decoherence process of CM, we introduce the quasi-probability distribution, Wigner function, which is defined as

$$W(p,q)=\frac{1}{\pi\hbar}\int_{-\infty}^{\infty}\mathrm{d}y\,\mathrm{e}^{\mathrm{i}2yp/\hbar}\langle q-y|\,\rho\,|q+y\rangle. \tag{19}$$

Although the Wigner function is a real function, it cannot be interpreted as a probability distribution function since it can be negative. Nevertheless, if integrating it over p (q), one will get the probability distribution function of q (p).

Inserting Eqs. (17) and (18) into Eq. (19), we calculate the Wigner function of the center of mass

$$W_{\mathrm{c.m.}}(p,q,t)=W^{\alpha}(p,q,t)+W^{\beta}(p,q,t)+W^{I}(p,q,t), \tag{20}$$

where

$$\begin{aligned}W^{\alpha}(p,q,t)\;=\;&\frac{1}{|\Xi|^2\pi\hbar}\exp\left[-G_q^2\left(-\Re(\alpha)\right)/2\sigma^2\right]\exp\left[-2\left(\sigma p-\hbar\Im(\alpha)\right)^2/\hbar^2\right]\\&\times\exp\left[-16\gamma\left(1-\mathrm{J}_0(4\omega t)\right)p^2\frac{\sigma^2}{4N^4\hbar^2}\left(1-\frac{G_q^2\left(-\Re(\alpha)\right)}{\sigma^2}\right)\right],\\W^{\beta}(p,q,t)\;=\;&\frac{1}{|\Xi|^2\pi\hbar}\exp\left[-G_q^2\left(-\Re(\beta)\right)/2\sigma^2\right]\exp\left[-2\left(\sigma p-\hbar\Im(\beta)\right)^2/\hbar^2\right]\\&\times\exp\left[-16\gamma\left(1-\mathrm{J}_0(4\omega t)\right)p^2\frac{\sigma^2}{4N^4\hbar^2}\left(1-\frac{G_q^2\left(-\Re(\beta)\right)}{\sigma^2}\right)\right],\end{aligned} \tag{21}$$

which are the direct Wigner functions contributed by quantum state $|\alpha\rangle$ and $|\beta\rangle$ respectively, and

$$W^I(p,q,t) = \frac{1}{|\Xi|^2\pi\hbar}\exp\left[-2\frac{\sigma^2}{\hbar^2}\left(p+\frac{\mathrm{i}\hbar}{2\sigma}(\alpha-\beta^*)\right)^2\right] \times \exp\left[-(\alpha-\beta^*)^2/2-\Im(\alpha)^2-\Im(\beta)^2\right]\exp\left[-G_q^2\left(-\frac{\alpha+\beta^*}{2}\right)/2\sigma^2\right] \times \exp\left[-16\gamma(1-\mathrm{J}_0(4\omega t))\,p^2\frac{\sigma^2}{4N^4\hbar^2}\left(1-\frac{G_q^2\left(-\frac{\alpha+\beta^*}{2}\right)}{\sigma^2}\right)\right]+\mathrm{h.c.}$$

is the one caused by the interference term $|\alpha\rangle\langle\beta|+|\beta\rangle\langle\alpha|$, and we have set $G_q(x)\equiv q+2\sigma x$ and $\gamma=9N\hbar^4/128\sigma^4m^4c^4$.

In the beginning, the two direct Wigner functions in phase space are centered at $(q=2\sigma\Re(\alpha), p=\hbar\Im(\alpha)/\sigma)$ and $(q=2\sigma\Re(\beta), p=\hbar\Im(\beta)/\sigma)$ respectively, which are exactly the mean position and momentum of two coherent states, while the interference one is approximately centered at $(q=\sigma(\Re(\alpha)+\Re(\beta)), p=\hbar(\Im(\alpha)+\Im(\beta))/2\sigma)$ (the midpoint of the centers of the two direct terms). Figure 2(a) shows the Wigner function in phase space at $t=0$, where two packets correspond to the two coherent states and the oscillations correspond to the interference term. Since p stands for the momentum of the whole system (center of mass), p/N indicates the mean momentum of a single particle. In this case, the two packets are centered at $(p/N=0.3, q=10)$ and $(p/N=0.7, q=6)$ respectively. This raises the following question: How does the total Wigner function evolve with time?

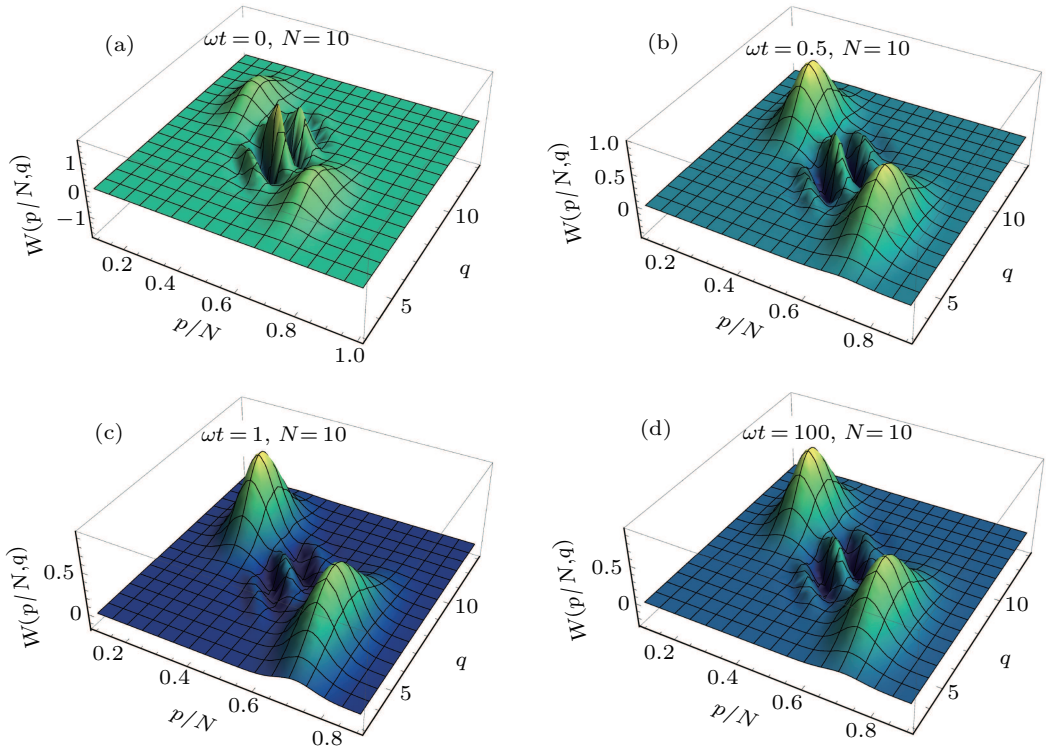


Fig. 2. (color online) The Wigner function in phase space $(p/N,q)$ at different time where we have chosed the natural units $\hbar=c=1$ and assumed $\sigma=1$, $N=10$, $\gamma=10$, $\alpha=5+3\mathrm{i}$, and $\beta=3+7\mathrm{i}$: (a) $t=0$; (b) $\omega t=0.5$; (c) $\omega t=1$; (d) $\omega t=100$.

First, let us turn to a mathematical function $\Omega(x,y)=\exp\left[-x^2-2(ay-b)^2-dy^2(1-2x^2)\right]$ where $a>0, d\geq 0$. One can prove that if $2a^2\gg d$ and $a^2>2b^2d$, the position of its peak is $x=0, y=b/a$, $\Omega(x,y)\,|_{\mathrm{peak}}=\Omega(x=0, y=b/a)=\exp\left[-db^2/a^2\right]$. Then in our occasion, if $2\gamma(1-\mathrm{J}_0(4\omega t))\ll N^4$, $8\gamma(1-\mathrm{J}_0(4\omega t))\Im(\alpha)^2<N^4$, and $8\gamma(1-\mathrm{J}_0(4\omega t))\Im(\beta)^2<N^4$ (our choice mentioned above meets all these conditions), the positions of the peaks of three Wigner functions terms remain unchanged with time (as shown in Fig. 2)

$$W^\alpha(p,q,t)\,|_{\mathrm{peak}} = \frac{1}{|\Xi|^2\pi\hbar}\mathrm{e}^{-4\gamma(1-\mathrm{J}_0(4\omega t))\Im(\alpha)^2/N^4},$$

$$W^\beta(p,q,t)\,|_{\mathrm{peak}} = \frac{1}{|\Xi|^2\pi\hbar}\mathrm{e}^{-4\gamma(1-\mathrm{J}_0(4\omega t))\Im(\beta)^2/N^4},$$

$$W^I(p,q,t)\,|_{\mathrm{peak}} \simeq W^I\left(\frac{\hbar}{2\sigma}\Im(\alpha+\beta), \sigma\Re(\alpha+\beta), t\right). \quad (22)$$

It is depicted by the above equation that the peak value of the three Wigner function terms is at time t. Since the two direct terms describe the probability distributions of two coherent states respectively and the oscillation term describes the interference effect, it sounds reasonable to quantify the decoherence of the superposition of coherent states with the peak values of the Wigner function. A useful quantity introduced by Zurek[24] is the fringe visibility function

$$F(\alpha,\beta,t) \simeq \frac{1}{2}\frac{W^I(p,q,t)\mid_{\text{peak}}}{\left(W^\alpha(p,q,t)\mid_{\text{peak}} W^\beta(p,q,t)\mid_{\text{peak}}\right)^{1/2}}. \quad (23)$$

Obviously, the fringe visibility function describes the decay of the peak value of the interference term. In other words, it shows the decoherence of the system, which originates from its interaction with the environment. Here, the fringe visibility function gives

$$F(\alpha,\beta,t) \propto \exp\left[-\gamma(1-\mathrm{J}_0(4\omega t))\left(\Im(\alpha)^2-\Im(\beta)^2\right)^2\right].$$

When expanded at the small time and large time limit, the above equation becomes

$$\begin{aligned}\lim_{\omega t\ll 1} F(\alpha,\beta,t) &\propto \exp\left[-\frac{4N}{N_0'}\omega^2 t^2\right],\\ \lim_{\omega t\gg 1} F(\alpha,\beta,t) &\propto \exp\left[-\frac{N}{N_0'}\right],\end{aligned} \quad (24)$$

where $N_0' = 32M^2c^4/9\left(\triangle E_{\alpha,\beta}\right)^2$ is the new particle number restriction, $\Delta E_{\alpha,\beta} = \hbar^2\left(\Im(\alpha)^2-\Im(\beta)^2\right)/2M\sigma^2$ is the difference of mean kinetic energy of the two coherent states. Figure 2 shows the time evolution of the Wigner function. When $t<1/\omega$ the interference term is damped over time while when $\omega t = 100 \gg 1$, a portion of coherence still remains. Figure 3 shows the Wigner function at the large time limit with different N while the centers of the two direct terms remain unchanged. It is shown that the larger the particle number N is, the less coherence it keeps in the large time limit. All these features coincide with the analysis results given in Eq. (24). Now we obtain the decoherence function of a macroscopic superposition quantum state, $\sim|\alpha\rangle+|\beta\rangle$, which depends on the mean momentum of the two superposed coherent states $\hbar\Im(\alpha)/\sigma$ and $\hbar\Im(\beta)/\sigma$. What is more, this function $F(\alpha,\beta,t)$ is highly similar to the result we get in Subsection 2.2,

$$\begin{aligned}\lim_{\omega t\ll 1}\left|\rho^{12}_{\text{c.m.}}(t)\right| &\simeq \frac{1}{2}\exp\left[-\frac{9N}{8}\omega^2t^2\left(\frac{\triangle E_{1,2}}{Mc^2}\right)^2\right],\\ \lim_{\omega t\gg 1}\left|\rho^{12}_{\text{c.m.}}(t)\right| &\simeq \frac{1}{2}\exp\left[-\frac{9N}{32}\left(\frac{\triangle E_{1,2}}{Mc^2}\right)^2\right],\end{aligned} \quad (25)$$

where $\triangle E_{1,2}$ is the difference of the kinetic energy of the two superposed states. Similar to the outcome in Subsection 2.2, in the superposition of the coherent states case, the decoherence time depends on the scale of the total system, the interaction strength of real particles, and the difference of the initial kinetic energy.

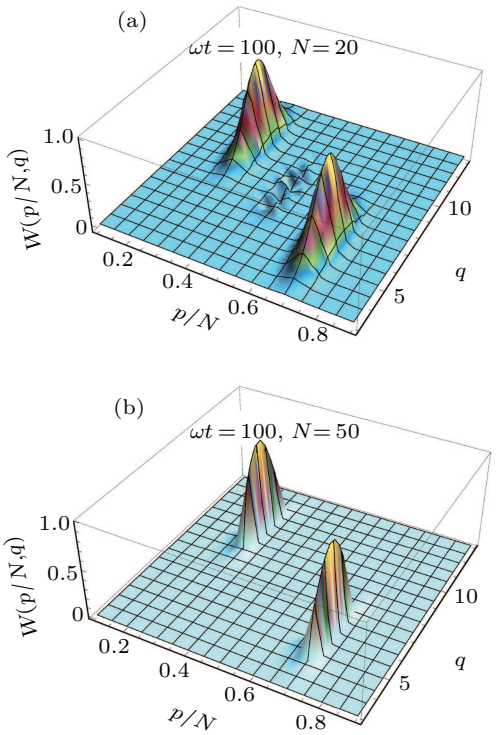


Fig. 3. (color online) The Wigner function in phase space $(p/N,q)$ with different N.

3. Free-particles evolution

In Subsection 2, we studied the decoherence of the CM in a ring with N relativistic particles with the nearest-neighbouring interaction and find that a restriction on particle number is necessary for the decoherence of CM. Next, we will explore whether or not this particle number restriction is induced by the nearest-neighbouring interaction. To this end, in this section, we investigate the system of N free relativistic particles

$$H = \sum_i \sqrt{p_i^2c^2+m^2c^4}.$$

We point out that the results about the cases without intercoupling could not be simply achieved from the above consequence by asumming the couplings vanish.

Making the Fourier and Bogoliubov transformations mentioned above in Eqs. (3) and (6), one finds this Hamiltonian, to the second order approximation, becomes

$$H_f \simeq \frac{P^2}{2M}+\sum_{k=1}^{N-1}\frac{P_k^2}{2m}-\frac{3P^2}{2M^2c^2}\sum_{k=1}^{N-1}\frac{P_k^2}{2m}, \quad (26)$$

where P describes the momentum of CM and P_k the momentum of the k-th mode of relative motion.

As there are only momentum terms in Eq. (26), the three terms commute pairwise. Therefore, for an initial state such as the product state of the collective and relative momentum operators, no decoherence will occur. What is more, in Section 2 the initial state of the kth mode relative motion is chosen to

be the ground state of the simple harmonic oscillator, which in position representation behaves as the Gaussian wave packet with width $\hbar/(4m\omega\sin(\pi/N)k)$. To make sure that the two models start from the same condition, we set the initial state of the free-particle model as

$$|\psi(0)\rangle_{\rm f}=\frac{|P_1\rangle+|P_2\rangle}{\sqrt{2}}\bigotimes\prod_k|\phi\rangle_k,$$

where $\langle x\,|\,\phi\rangle_k=\left(2\pi\sigma_k^2\right)^{-1/4}\exp\left[-x^2/4\sigma_k^2\right]$ is a Gaussian wave packet with packet width $\sigma_k^2=\hbar/\left(4m\omega\sin\frac{\pi}{N}k\right)$. As the evolution of the state is governed by Eq. (26), the state at time t reads

$$\begin{aligned}|\psi(t)\rangle_{\rm f}=&\frac{1}{\sqrt{2}}\exp\left[-{\rm i}\frac{P_1^2}{2\hbar M}t\right]|P_1\rangle\bigotimes\prod_k\int{\rm d}pg_k(p)\\&\times\exp\left[-\frac{\rm i}{\hbar}\left(1-\frac{3P_1^2}{2M^2c^2}\right)\frac{p^2}{2m}t\right]|p\rangle_k\\&+\frac{1}{\sqrt{2}}\exp\left[-{\rm i}\frac{P_2^2}{2\hbar M}t\right]|P_2\rangle\bigotimes\prod_k\int{\rm d}pg_k(p)\\&\times\exp\left[-\frac{\rm i}{\hbar}\left(1-\frac{3P_2^2}{2M^2c^2}\right)\frac{p^2}{2m}t\right]|p\rangle_k,\end{aligned}\tag{27}$$

where $g_k(p)=\left(2\sigma_k^2/\pi\hbar^2\right)^{1/4}{\rm e}^{-p^2\sigma_k^2/\hbar^2}$. Then the off-diagonal element of the reduced density matrix becomes

$$\left|\rho_{\rm f}^{12}(t)\right|=\frac{1}{2}\exp\left[-\frac{1}{4}\sum_{k=1}^{N-1}\ln\left(1+\frac{9\hbar^2}{16\sigma_k^4m^2}\left(\frac{\triangle E}{Mc^2}\right)^2t^2\right)\right].$$

For small t, we obtain

$$\left|\rho_{\rm f}^{12}(t)\right|\simeq\frac{1}{2}\exp\left[-\frac{9}{8}N\omega^2t^2\left(\frac{\triangle E}{Mc^2}\right)^2\right],\tag{28}$$

with decoherence time

$$\tau_{\rm f}=\frac{2\sqrt{2}Mc^2}{3\sqrt{N}\left|\Delta E_{1,2}\right|\omega}.\tag{29}$$

Comparing these two model, we find that with the same initial state the CM decoheres at the same rate as the outcome we obtained in Section 2, while there is no restriction on the particle number (N_0) in the free-particle model. This finding concludes that the restriction is introduced by the nearest-neighboring interaction between relativistic particles in the ring. In fact, this product state in the collective and relative movement reference frame corresponds to an entanglement state in the real-particle movements frame. Moreover, this entanglement state is difficult to prepare in experiments since particles are all free. By the way, the transformation between the two frames considered in this paper is

$$P_k=\begin{cases}\displaystyle\sum_{j=1}^N\sqrt{\frac{2}{N}}p_j\cos\left(\frac{2\pi}{N}kj\right), & k\in\left[1,\frac{N-1}{2}\right]\\ \displaystyle\sum_{j=1}^N\sqrt{\frac{2}{N}}p_j\sin\left(\frac{2\pi}{N}kj\right), & k\in\left[\frac{N-1}{2},N-1\right]\\ \displaystyle\sum_{j=1}^N p_j, & k=N.\end{cases}$$

4. Conclusions and discussions

We have considered the relativistic modification in the Hamiltonian of the N-particle-ring system with the nearest-neighbouring interaction. It is found that interactions exist between the CM motion and the relative motion that originated from the relativistic effect. As the part of relative motions behaves as a harmonic oscillator bath environment, this interaction causes the decoherence of the CM without dissipation.

Under the particle number condition, $N\gg N_0$, the decoherence time of the system depends on the particle number of the system, the coupling constant and the initial kinetic energy difference, i.e., $\tau\sim\left(3\sqrt{N}\left(\Delta E_{1,2}\right)\omega/2Mc^2\right)^{-1}$. One can conclude that macroscopic objects decoherence faster than microscopic ones. With a more classical state as the initial state, the superposition of coherent states, the CM decoheres in a similar way to the former case where the decoherence time depends on the two expectation values of the CM momentum. Through a further study, we find that in the free-particle system (without the particle interaction), for the same initial state, the system decoheres with the same rate as the former case while the particle number restriction disappears. Therefore, the interaction between particles is responsible for the particle number restriction and keeps the small system from decoherence. In the example we take above, only in macroscopic systems with a particle number $N\sim N_A$, does the CM decohere.

We finally remark that we only studied the minimum decoherence mechanism for the decoherence of MOs, which is dipicted by its CM coupled with relative movements due to the relativistic effect. In the real world, as we said before, a physical system, especially a macroscopic system (with lots of degrees of freedom) must interact with its external environment.[9,14] The decoherence effect caused by this external environment may dominate and the intrinsic decoherence effect we considered here can be ignored. In other words, in practice the MO will already be in a statistical mixture long before reaching at the decoherence time we obtained in this paper.

Acknowledgment

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Appendix A: Squeeze operator in simple harmonic oscillator

The Hamiltonian

$$H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2-\delta\frac{p^2}{2m}$$

$$= \frac{p'^2}{2m'} + \frac{1}{2}m'\omega'^2 x'^2, \tag{A1}$$

where $m' = m/(1-\delta)$, $\omega' = \omega\sqrt{1-\delta}$. There are two kinds of definition of operators

$$\begin{aligned} x &= \sqrt{\frac{\hbar}{2m\omega}}(a+a^\dagger), \\ p &= -\mathrm{i}\sqrt{\frac{\hbar m\omega}{2}}(a-a^\dagger), \\ x' &= \sqrt{\frac{\hbar}{2m'\omega'}}(b+b^\dagger), \\ p' &= -\mathrm{i}\sqrt{\frac{\hbar m'\omega'}{2}}(b-b^\dagger). \end{aligned} \tag{A2}$$

Then the Hamiltonian becomes

$$\begin{aligned} H &= \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \frac{\delta}{2}\frac{\hbar\omega}{2}(a-a^\dagger)^2 \\ &= \hbar\omega'\left(b^\dagger b + \frac{1}{2}\right). \end{aligned} \tag{A3}$$

The eigenvalue of the system is $(n+1/2)\hbar\omega'$. And

$$\begin{aligned} b &= \sqrt{\frac{m'\omega'}{2\hbar}}\left(x' + \mathrm{i}\frac{p'}{m'\omega'}\right) \\ &= \frac{1}{2}\left(\sqrt{\frac{m'\omega'}{m\omega}}(a+a^\dagger) + \sqrt{\frac{m\omega}{m'\omega'}}(a-a^\dagger)\right) \\ &= S^\dagger(r)aS(r), \end{aligned} \tag{A4}$$

where $r = |r|\exp(\mathrm{i}\theta)$, $|r| = -\ln(1-\delta)/4$, $\theta = \pi$, and $S(r) = \exp(r^* a^2/2 - r a^{\dagger 2}/2)$. Thus

$$H = \hbar\omega'\left(S^\dagger(r)a^\dagger a S(r) + \frac{1}{2}\right). \tag{A5}$$

Appendix B: Squeeze operator in simple harmonic oscillator

$$\left|\rho^{12}_{\mathrm{c.m.}}(t)\right| = \frac{1}{2}\prod_{k=1}^{N-1}\left|\langle 0|\,\mathrm{e}^{\mathrm{i}(H_k+H_{I,k}(P_1))t/\hbar}\,\mathrm{e}^{-\mathrm{i}(H_k+H_{I,k}(P_2))t/\hbar}\,|0\rangle_k\right|$$

$$= \frac{1}{2}\prod_{k=1}^{N-1}\left|f_k(P_1,P_2,t)\right|.$$

where

$$\begin{aligned} f_k(P_1,P_2,t) &= \langle 0|\,S_k^\dagger(r_1)\,\mathrm{e}^{\mathrm{i}\omega_k(P_1)a^\dagger at}S_k(r_1)S_k^\dagger(r_2) \\ &\quad\times \mathrm{e}^{-\mathrm{i}\omega_k(P_2)a^\dagger at}S_k(r_2)\,|0\rangle_k, \end{aligned}$$

and

$$\begin{aligned} \omega_k(P) &= \omega_k\sqrt{1-3P^2/2M^2c^2}, \\ S_k(r_i) &= \exp(-|r_i|\,a^2/2 + |r_i|\,a^{\dagger 2}/2), \\ |r_i| &= -\frac{1}{4}\ln(1-3P_i^2/2M^2c^2). \end{aligned}$$

Then we obtain

$$f_k(P_1,P_2,t) = \langle 0|\,S_k(\xi_1\,\mathrm{e}^{\mathrm{i}\pi})S_k(r_1)S_k(r_2\,\mathrm{e}^{\mathrm{i}\pi})S_k(\xi_2)\,|0\rangle_k.$$

where

$$\begin{aligned} \xi_1 &= |r_1|\exp\left[\mathrm{i}(\pi - 2\omega_k(P_1)t)\right], \\ \xi_2 &= |r_2|\exp\left[\mathrm{i}(\pi - 2\omega_k(P_2)t)\right]. \end{aligned}$$

The squeeze operator can be transformed to[25]

$$\begin{aligned} S(|z|\,\mathrm{e}^{\mathrm{i}\phi}) &= \exp(|z|\,\mathrm{e}^{-\mathrm{i}\phi}a^2/2 - |z|\,\mathrm{e}^{\mathrm{i}\phi}a^{\dagger 2}/2) \\ &= \exp\left[-\mathrm{e}^{\mathrm{i}\phi}\tanh|z|\,L_+\right]\exp\left[-2\log(\cosh|z|)\,L_3\right] \\ &\quad\times\exp\left[\mathrm{e}^{-\mathrm{i}\phi}\tanh|z|\,L_-\right], \end{aligned}$$

where $L_+ = a^{\dagger 2}/2$, $L_- = a^2/2$, and $L_3 = \left(a^\dagger a + 1/2\right)/2$ form a realization of the $SU(l,l)$ Lie algebra.

$$\begin{aligned} f_k(P_1,P_2,t) &= \langle 0|\,S_k(|r_1|\exp\left[-2\mathrm{i}\omega_k(P_1)t\right])S_k(|r_1|\exp\left[-\mathrm{i}\pi\right])S_k(|r_2|)S_k(|r_2|\exp\left[i(\pi-2\omega_k(P_2)t)\right])\,|0\rangle_k \\ &= \frac{1}{\sqrt{\cosh|r_1|\cosh|r_2|}}\,\langle 0|\exp\left[g_1^k L_-\right]\exp\left[\tanh|r_1|\,L_+\right]\exp\left[-2\log(\cosh|r_1|)\,L_3\right]\exp\left[-\tanh|r_1|\,L_-\right] \\ &\quad\times\exp\left[-\tanh|r_2|\,L_+\right]\exp\left[-2\log(\cosh|r_2|)\,L_3\right]\exp\left[\tanh|r_2|\,L_-\right]\exp\left[g_2^k L_+\right]\,|0\rangle_k, \end{aligned}$$

where $g_1^k = \exp(2\mathrm{i}\omega_k(P_1)t)\tanh|r_1|$, $g_2^k = \exp(-2\mathrm{i}\omega_k(P_2)t)\tanh|r_2|$. Following from inserting the identity operator $\int \mathrm{d}^2\alpha/\pi|\alpha\rangle\langle\alpha| = I$, we obtain

$$\begin{aligned} f_k(P_1,P_2,t) = \frac{1}{\sqrt{\cosh|r_1|\cosh|r_2|}}\frac{1}{\pi^3}\int \mathrm{d}^2\alpha_1\ldots\mathrm{d}^2\alpha_3\,\langle 0|\exp\left[g_1^k L_-\right]|\alpha_1\rangle\,\langle\alpha_1|\exp\left[\tanh|r_1|\,L_+\right] \\ \times\exp\left[-2\log(\cosh|r_1|)\,L_3\right]\exp\left[-\tanh|r_1|\,L_-\right]|\alpha_2\rangle\,\langle\alpha_2|\exp\left[-\tanh|r_2|\,L_+\right] \\ \times\exp\left[-2\log(\cosh|r_2|)\,L_3\right]\exp\left[\tanh|r_2|\,L_-\right]|\alpha_3\rangle\,\langle\alpha_3|\exp\left[g_2^k L_+\right]\,|0\rangle_k \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\cosh|r_1|\cosh|r_2|}\frac{1}{\pi^3}\int \mathrm{d}^2\alpha_1\ldots\mathrm{d}^2\alpha_3\,\langle 0|\exp\left[\frac{g_1^k}{2}\alpha_1^2\right]|\alpha_1\rangle\langle\alpha_1|\exp\left[\frac{\tanh|r_1|}{2}\alpha_1^{*2}\right]\\
&\quad\times\exp\left[-\log\left(\cosh|r_1|\right)a^\dagger a\right]\exp\left[-\frac{\tanh|r_1|}{2}\alpha_2^2\right]|\alpha_2\rangle\langle\alpha_2|\exp\left[-\frac{\tanh|r_2|}{2}\alpha_2^{2*}\right]\\
&\quad\times\exp\left[-\log\left(\cosh|r_2|\right)a^\dagger a\right]\exp\left[\frac{\tanh|r_2|}{2}\alpha_3^2\right]|\alpha_3\rangle\langle\alpha_3|\exp\left[g_2^k L_+\right]|0\rangle_k\,. \qquad \text{(B1)}
\end{aligned}
$$

As the coherent state is over complete, the overlap of two different coherent states is nonzero, i.e., $\langle\alpha\mid\beta\rangle = \exp[-(|\alpha|^2+|\beta|^2-2\alpha^*\beta)/2]$. We also notice that the coherent state is not the eigenstate of particle number operator $a^\dagger a$, then

$$
\exp\left[-\lambda a^\dagger a\right]|\alpha\rangle = \mathrm{e}^{-\lambda a^\dagger a}\exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\mathrm{e}^{|\alpha\mathrm{e}^{-\lambda}|^2/2}\left|\alpha\,\mathrm{e}^{-\lambda}\right\rangle.
$$

In the following, we denote $\alpha_i = x_{2i-1}+\mathrm{i}x_{2i}$, $\int \mathrm{d}^2\alpha_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}x_{2i-1}\,\mathrm{d}x_{2i}$, $(i=1,2,3)$.

$$
\begin{aligned}
f_k(P_1,P_2,t) &= \frac{1}{\cosh|r_1|\cosh|r_2|}\frac{1}{\pi^3}\int \mathrm{d}^2\alpha_1\ldots\mathrm{d}^2\alpha_3\exp\left[\frac{g_1^k}{2}\alpha_1^2\right]\exp\left[-\frac{|\alpha_1|^2}{2}\right]\exp\left[\frac{\tanh|r_1|}{2}\alpha_1^{*2}\right]\\
&\quad\times\exp\left[-\frac{|\alpha_2|^2}{2}-\frac{|\alpha_1|^2}{2}+\alpha_1^*\alpha_2\,\mathrm{e}^{-\log(\cosh|r_1|)}\right]\exp\left[-\frac{\tanh|r_1|}{2}\alpha_2^2\right]\exp\left[-\frac{\tanh|r_2|}{2}\alpha_2^{2*}\right]\\
&\quad\times\exp\left[-\frac{|\alpha_2|^2}{2}-\frac{|\alpha_3|^2}{2}+\alpha_2^*\alpha_3\,\mathrm{e}^{-\log(\cosh|r_2|)}\right]\exp\left[\frac{\tanh|r_2|}{2}\alpha_3^2\right]\exp\left[-\frac{|\alpha_3|^2}{2}\right]\exp\left[\frac{g_2^k}{2}\alpha_3^{*2}\right]\\
&\quad\times\exp\left[-x_1^2-x_2^2-x_3^2-x_4^2-x_5^2-x_6^2\right]\\
&= \frac{1}{\cosh|r_1|\cosh|r_2|}\frac{1}{\pi^3}\int \mathrm{d}x_1\ldots\mathrm{d}x_6\exp\left[-\frac{1}{2}\sum_{i,j=1}^{6}A_{ij}^k x_i x_j\right], \qquad \text{(B2)}
\end{aligned}
$$

where

$$
A^k(t) = \begin{pmatrix} \Theta_1 & \Omega_1 & 0\\ \Omega_1^{\mathrm{T}} & \Lambda_{1,2} & \Omega_2\\ 0 & \Omega_2^T & \Theta_2\end{pmatrix},
$$

$$
\Theta_1 = \begin{pmatrix} 2-g_1^k-\tanh|r_1| & \mathrm{i}\left(\tanh|r_1|-g_1^k\right)\\ \mathrm{i}\left(\tanh|r_1|-g_i^k\right) & 2+g_1^k+\tanh|r_1|\end{pmatrix},
$$

$$
\Theta_2 = \begin{pmatrix} 2-g_2^k-\tanh|r_2| & -\mathrm{i}\left(\tanh|r_2|-g_2^k\right)\\ -\mathrm{i}\left(\tanh|r_2|-g_2^k\right) & 2+g_2^k+\tanh|r_2|\end{pmatrix},
$$

$$
\Omega_i = \begin{pmatrix} -\cosh^{-1}|r_i| & -\mathrm{i}\cosh^{-1}|r_i|\\ \mathrm{i}\cosh^{-1}|r_i| & -\cosh^{-1}|r_i|\end{pmatrix},
$$

$$
\Lambda_{1,2} = \begin{pmatrix} 2+\tanh|r_1|+\tanh|r_2| & \mathrm{i}\left(\tanh|r_1|-\tanh|r_2|\right)\\ \mathrm{i}\left(\tanh|r_1|-\tanh|r_2|\right) & 2-\tanh|r_1|-\tanh|r_2|\end{pmatrix}.
$$

With the help of Gaussian integral,[26] we obtain

$$
\begin{aligned}
f_k(P_1,P_2,t) &= \frac{2^3}{\cosh|r_1|\cosh|r_2|}\frac{1}{\sqrt{\det\left[A^k(t)\right]}}\\
&= \frac{2^3}{\cosh|r_1|\cosh|r_2|}\frac{1}{\sqrt{\det\left[A^k(t)\right]}}. \qquad \text{(B3)}
\end{aligned}
$$

In Eq. (4), the terms higher than $(p^2)^2$ are neglected. Therefore, the higher terms in $\left|\det\left[A^k(t)\right]\right|$ should also be ignored, i.e., $|r_i|\sim 3P_i^2/8M^2c^2$, $\cosh|r_i|\sim 1+|r_i|^2/2$ and $\tanh|r_1|\sim|r_i|$. After further calculation,

$$
\begin{aligned}
&\left|\cosh^2|r_1|\cosh^2|r_2|\det\left[A^k(t)\right]\right|\\
&\simeq\left|2^6(1+|r_1|^2+|r_2|^2)\,\mathrm{e}^{-2\mathrm{i}\omega_k(P_2)t}\right|\\
&\times\Big|\left[|r_1||r_2|(1-\mathrm{e}^{2\mathrm{i}\omega_k(P_1)t}-\mathrm{e}^{2\mathrm{i}\omega_k(P_2)t}+\mathrm{e}^{2\mathrm{i}\omega_k(P_2)t}\mathrm{e}^{2\mathrm{i}\omega_k(P_1)t}\right)\\
&\quad-|r_1|^2\,\mathrm{e}^{2\mathrm{i}\omega_k(P_1)t}\mathrm{e}^{2\mathrm{i}\omega_k(P_2)t}-|r_2|^2+\mathrm{e}^{2\mathrm{i}\omega_k(P_2)t}\Big]\Big|\\
&\simeq 2^6\Big|\left[1+|r_1|^2+|r_2|^2-|r_1|^2\,\mathrm{e}^{2\mathrm{i}\omega_k(P_1)t}-|r_2|^2\,\mathrm{e}^{-2\mathrm{i}\omega_k(P_2)t}\right.\\
&\quad+|r_1||r_2|(\mathrm{e}^{2\mathrm{i}\omega_k(P_1)t}+\mathrm{e}^{-2\mathrm{i}\omega_k(P_2)t}\\
&\quad\left.-\mathrm{e}^{-2\mathrm{i}\omega_k(P_2)t}\mathrm{e}^{2\mathrm{i}\omega_k(P_1)t}-1)\right]\Big|\\
&= 2^6\left[1+(|r_1|-|r_2|)^2(1-\cos(2\omega_k t))\right], \qquad \text{(B4)}
\end{aligned}
$$

where we have assumed $\omega_k(P_i)\simeq\omega_k$.

$$
\begin{aligned}
\left|\rho_{\mathrm{c.m.}}^{12}(t)\right| &= \frac{1}{2}\prod_{k=1}^{N-1}|f_k(P_1,P_2,t)|\\
&= \frac{1}{2}\left(\prod_{k=1}^{N-1}2^{-6}\left|\cosh|r_1|\cosh|r_2|\det\left[A^k(t)\right]\right|\right)^{-1/2}\\
&\simeq \frac{1}{2}\prod_{k=1}^{N-1}\left[1-\frac{9}{32}\frac{(\Delta E_{1,2})^2}{M^2c^4}(1-\cos(2\omega_k t))\right]. \qquad \text{(B5)}
\end{aligned}
$$

There is a product of $(N-1)$ terms in Eq. (B5) and we can take its logarithm,

$$
\ln 2\left|\rho_{\mathrm{c.m.}}^{12}(t)\right| = \sum_{k=1}^{N-1}\ln\left[1-\frac{9}{32}\frac{(\Delta E_{1,2})^2}{M^2c^4}(1-\cos(2\omega_k t))\right]
$$

$$\simeq -\frac{9}{32}\frac{(\Delta E_{1,2})^2}{M^2c^4}\sum_{k=1}^{N-1}(1-\cos(2\omega_k t)). \quad \text{(B6)}$$

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