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Some studies of the interaction between two two-level atoms and $SU(1,1)$ quantum systems

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Herein, we present an approach to look for the best phenomenon to measure quantum correlation. The system of two isolated qubits each interacting with a single-mode cavity was theoretically created to study the quantum correlation. Some of the phenomena, such as the quantum discord and concurrence, were generated through such a system. The influences of initial state purity, qubit motion, and detuning parameters were discussed for the phenomena. These parameters for a specific value show that the behavior of phenomena are analogous. It is interesting to mention that some values of detuning undergo a sudden death of phenomena, and the quantum discord still captures the qubits quantum correlation. We predict that the quantum discord may be a better measure of quantum correlation than concurrence.

Keywords: an isolated atom with a cavity $SU(1,1)$, quantum discord concurrence

PACS: 02.20.Bb **DOI:** 10.1088/1674-1056/27/10/100201

1. Introduction

In recent years, the main resource for tasks of quantum information processing was quantum correlations (QCs). One of the important kinds of QCs was the quantum entanglement. Entanglement played a main role in quantum information and computation,[1–7] quantum state teleportation,[8] and quantum cryptography.[9] Entanglement was observed for some theoretical works[10] and experimental researches.[11] Some of these works were mixed separable states and allowed us to show the quantum algorithms advantages over their classical analogies. The classical algorithms were observed for some systems without entanglement.[12,13]

Besides, another important type of QCs was firstly defined by Ollivir[14] called quantum discord (QD). QD was used to quantify all nonclassical correlation in a system. Relative to the definition of mutual information, QD explains that two classical equivalent ways turn out to be inequivalent in the quantum domain. It has been previously mentioned that QD is an independent one and a more general measure of QCs that may include entanglement. Quantum states with nonclassical correlations were provided by a larger region through QD, and a nonzero QD but not entanglement may be responsible for the efficiency of quantum computing.[15–17] Such observations were suggested that QD could be another asset for quantum information processing.

Recently, some works were studied for QD[18–20] but the analytical calculation of QD was comparatively difficult. On the other hand, the authors in Ref. [21] obtained the analytical formulas of QD for a family of two-qubit states. For more general quantum states, the authors in Refs. [22], [23], [24], and [25] derived some straightforward expressions to obtain QD, and concluded that QD, classic correlation, and entanglement were independent measures of correlations.

This paper is organized as follows. Section 2 gives the model and its solution. Section 3 is devoted to studying two types of measures to analyze quantum correlation. Section 4 gives the numerical results and discussion. Finally, we give our conclusions in Section 5.

2. Main system and its density operator

In the present work, we employ the interaction system of two identical moving qubits (A and B) or atoms with two single-mode $SU(1,1)$ cavities (a and b). We assume that the system of each qubit–cavity is isolated. This means that the atom "A" at a constant velocity only interacted with a field "a", and the atom "B" is only interacted with a field "b", see Fig. 1.

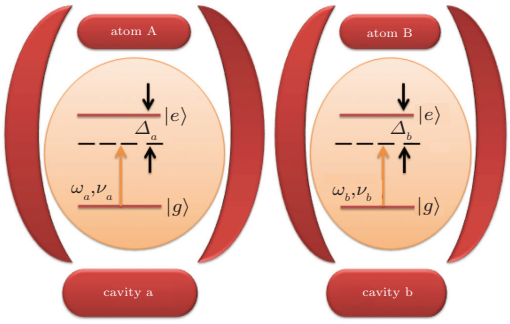


Fig. 1. (color online) Schematic structure for the interaction of each isolated atom with a cavity.

The time-depended Hamiltonian for such a system is given by

$$\hat{H}(t) = \nu_j \hat{K}_j^z + \frac{\omega_j}{2}\hat{\sigma}_j^z + \lambda g(\nu_j t)(\hat{K}_j^- \hat{\sigma}_j^+ + \hat{K}_j^+ \hat{\sigma}_j^-) \qquad (1)$$

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with $\hbar = 1$. The system (1) contains some abbreviations: (i) $\hat{K}_j^z = (\hat{n}_j + 1/2)$, $\hat{n}_j = a_j^\dagger a_j$ $\hat{K}_j^- = a_j\sqrt{\hat{n}_j}$, and $\hat{K}_j^+ = \sqrt{\hat{n}_j}a_j^\dagger$ refer to the symmetric Lie group generators of $SU(1,1)$; (ii) a_j ($\hat{\sigma}_-^j$) is the annihilation operator of photon (atom), $\hat{\sigma}_z^j$ is the Pauli spin matrix, ω_j is the atomic transition frequency, ν_j is the field frequency, λ refers to the coupling constant, $j = a, b$ for cavity operators and $j = A, B$ for atoms operators; (iii) g refers to the cavity shape function. Now, we rewrite system (1) as the following formula.

$$\hat{H}(t) = \nu_j\hat{K}_j^z + \frac{\nu_j}{2}\hat{\sigma}_j^z + \frac{\Delta_j}{2}\hat{\sigma}_j^z + \lambda g(v_j t)(\hat{K}_j^-\hat{\sigma}_j^+ + \hat{K}_j^+\hat{\sigma}_j^-), \tag{2}$$

where $\Delta_j = \omega_j - \nu_j$ is the detuning parameter of qubit–field interaction. Applying the transformation $\hat{T} = \exp(\mathrm{i}\nu_j t(\hat{K}_j^z + \hat{\sigma}_j^z/2))$ to the system (2), then the Hamiltonian in the new transformation becomes

$$\begin{aligned}\hat{H}_j(t) &= \hat{T}\hat{H}(t)\hat{T}^\dagger(t) - \mathrm{i}\hat{T}\frac{\partial\hat{T}^\dagger}{\partial t}\\ &= \frac{\Delta_j}{2}\hat{\sigma}_j^z + \lambda g(v_j t)(\hat{K}_j^-\hat{\sigma}_j^+ + \hat{K}_j^+\hat{\sigma}_j^-),\end{aligned} \tag{3}$$

where $[\hat{K}_j^z, \hat{K}_j^\pm] = \pm\hat{K}_j^\pm$ and $[\hat{K}_j^-, \hat{K}_j^+] = 2\hat{K}_j^z$. Now, we define another form of shape function as follows:

$$\begin{aligned}\theta = \theta(t) &= \int_0^\tau g(v_j\tau)\,\mathrm{d}\tau\\ &= \int_0^\tau \{p_1 + p_2\cos(p_j v_j \pi\tau/L_j)\}\,\mathrm{d}\tau\\ &= p_1 t + \{p_2 L_j \sin(p_j v_j \pi t/L_j)\}/(p_j\pi v_j),\end{aligned} \tag{4}$$

where v_j is the atomic velocity, (p_1 and p_2) are the atomic motion parameters, and p_j represents the cavity half-wave number with the length L_j. Here, assuming that $\Delta_\mathrm{A} = \Delta_\mathrm{B} = \Delta$, $L_\mathrm{a} = L_\mathrm{b} = L$, $p_\mathrm{a} = p_\mathrm{b} = p$, and $v_\mathrm{A} = v_\mathrm{B} = v = \lambda L/\pi$, then the new shape function takes the form: $\theta(t) = tp_1 + p_2\sin(p\lambda t)/\lambda p$. The effect of qubit motion exists when $p_1 = 0$ and $p_2 = 1$, and it is absent when $p_1 = 1$ and $p_2 = 0$.

To compute the density operator analytically, which will employ searching for the best measure of quantum correlation, we start by calculating the time evolution of system (3) as follows:

$$\hat{U}_j(t) = \exp\Big(-\mathrm{i}\int_0^t \hat{H}_j(\tau)\,\mathrm{d}\tau\Big) = \begin{pmatrix}\hat{u}_{11j} & -\mathrm{i}\hat{u}_{12j}\\ -\mathrm{i}\hat{u}_{21j} & \hat{u}_{22j}\end{pmatrix}, \tag{5}$$

where

$$\begin{aligned}&\left[\int_0^t \hat{H}_j(\tau)\,\mathrm{d}\tau,\ \hat{H}_j(t)\right] = 0,\\ &\hat{u}_{11j} = \cos(\theta\hat{\mu}_{1j}) - \mathrm{i}\frac{\delta_j}{2}\frac{\sin(\theta\hat{\mu}_{1j})}{\hat{\mu}_{1j}},\\ &\hat{u}_{12j} = \frac{\sin(\theta\hat{\mu}_{1j})}{\hat{\mu}_{1j}}\hat{K}_j^-,\\ &\hat{u}_{21j} = \frac{\sin(\theta\hat{\mu}_{2j})}{\hat{\mu}_{2j}}\hat{K}_j^+,\\ &\hat{u}_{22j} = \cos(\theta\hat{\mu}_{2j}) + \mathrm{i}\frac{\delta_j}{2}\frac{\sin(\theta\hat{\mu}_{2j})}{\hat{\mu}_{2j}},\\ &\hat{\mu}_{1j} = \sqrt{\Big(\frac{\delta_j}{2}\Big)^2 + \hat{K}_j^-\hat{K}_j^+},\\ &\hat{\mu}_{2j} = \sqrt{\Big(\frac{\delta_j}{2}\Big)^2 + \hat{K}_j^+\hat{K}_j^-}.\end{aligned}$$

Based on the previous assumption, the two qubits do not interact, the time evolution for each qubit can be expressed as follows:

$$\begin{aligned}\hat{U}_{AB}(t) &= \hat{U}_A(t)\otimes\hat{U}_B(t)\\ &= \begin{pmatrix}\hat{u}_{11A} & -\mathrm{i}\hat{u}_{12A}\\ -\mathrm{i}\hat{u}_{21A} & \hat{u}_{22A}\end{pmatrix}\otimes\begin{pmatrix}\hat{u}_{11B} & -\mathrm{i}\hat{u}_{12B}\\ -\mathrm{i}\hat{u}_{21B} & \hat{u}_{22B}\end{pmatrix}.\end{aligned} \tag{6}$$

In this work, it is assumed that the two qubits are initially in the Werner state

$$\rho_{AB}(0) = \frac{r}{2}\big(|eg\rangle\langle eg| + |eg\rangle\langle ge| + |ge\rangle\langle eg| + |ge\rangle\langle ge|\big) + \frac{(1-r)}{4}I_4,$$

where I_4 is the identity matrix, $\rho_{AB}(0)$ is called the atom–atom density operator for zero time, and r is the purity of the initial state varying from 0 to 1. Also, the two cavity fields "a" and "b" are initially in the states $\rho_a(0) = |m_a, k_a\rangle\langle k_a, m_a|$ and $\rho_b(0) = |m_b, k_b\rangle\langle k_b, m_b|$. For $r = 0$, the state becomes a maximal state, while for $r = 1$ it becomes a Bell states. Consequently, the density evolution operator of the system can be expressed as follows:

$$\begin{aligned}\hat{\rho}(t) &= \hat{U}_{AB}(t)\big[\rho_{AB}(0)\otimes\rho_a(0)\otimes\rho_b(0)\big]\hat{U}_{AB}^\dagger(t)\\ &= \hat{\rho}_{AB}(t)\otimes\hat{\rho}_{ab}(t).\end{aligned} \tag{7}$$

We taking the trace over the field variables to obtain the reduced density matrix $\hat{\rho}_{AB}(t)$ of two qubits. Relative to the computational basis of states $|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle$, the matrix $\rho_{AB}(t)$ can be written as follows:

$$\hat{\rho}_{AB}(t) = \begin{pmatrix}\rho_{11}(t) & 0 & 0 & 0\\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0\\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0\\ 0 & 0 & 0 & \rho_{44}(t)\end{pmatrix}. \tag{8}$$

The elements of the matrix are given by

$$\begin{aligned}\rho_{11}(t) &= \frac{1}{4}[(1-r)(|x_{11}|^2 + |x_{14}|^2) + (1+r)(|x_{12}|^2 + |x_{13}|^2)],\\ \rho_{22}(t) &= \frac{1}{4}[(1-r)(|x_{21}|^2 + |x_{24}|^2) + (1+r)(|x_{22}|^2 + |x_{23}|^2)],\\ \rho_{33}(t) &= \frac{1}{4}[(1-r)(|x_{31}|^2 + |x_{34}|^2) + (1+r)(|x_{32}|^2 + |x_{33}|^2)],\\ \rho_{44}(t) &= \frac{1}{4}[(1-r)(|x_{41}|^2 + |x_{44}|^2) + (1+r)(|x_{42}|^2 + |x_{43}|^2)],\\ \rho_{23}(t) &= \frac{r}{2}x_{22}x_{33}^\dagger,\\ \rho_{32}(t) &= \rho_{23}^\dagger(t).\end{aligned}$$

With

$$
\begin{aligned}
x_{11} &= u_{11a}(m_a,k_a,t)u_{11b}(m_b,k_b,t),\\
x_{12} &= u_{11a}(m_a,k_a,t)u_{12b}(m_b,k_b,t),\\
x_{13} &= u_{12a}(m_a,k_a,t)u_{11b}(m_b,k_b,t),\\
x_{14} &= u_{12a}(m_a,k_a,t)u_{12b}(m_b,k_b,t),\\
x_{21} &= u_{11a}(m_a,k_a,t)u_{21b}(m_b,k_b,t),\\
x_{22} &= u_{11a}(m_a,k_a,t)u_{22b}(m_b,k_b,t),\\
x_{23} &= u_{12a}(m_a,k_a,t)u_{21b}(m_b,k_b,t),\\
x_{24} &= u_{12a}(m_a,k_a,t)u_{22b}(m_b,k_b,t),\\
x_{31} &= u_{21a}(m_a,k_a,t)u_{11b}(m_b,k_b,t),\\
x_{32} &= u_{21a}(m_a,k_a,t)u_{12b}(m_b,k_b,t),\\
x_{33} &= u_{22a}(m_a,k_a,t)u_{11b}(m_b,k_b,t),\\
x_{34} &= u_{22a}(m_a,k_a,t)u_{12b}(m_b,k_b,t),\\
x_{41} &= u_{21a}(m_a,k_a,t)u_{21b}(m_b,k_b,t),\\
x_{42} &= u_{21a}(m_a,k_a,t)u_{22b}(m_b,k_b,t),\\
x_{43} &= u_{22a}(m_a,k_a,t)u_{21b}(m_b,k_b,t),\\
x_{44} &= u_{22a}(m_a,k_a,t)u_{22b}(m_b,k_b,t),\\
u_{11i} &= \cos[\theta(t)\mu(m_i,k_i,t)]\\
&\quad - \mathrm{i}\delta\sin[\theta(t)\mu(m_i,k_i,t)]/2\mu(m_i,k_i,t),\\
u_{12i} &= \sin[\theta(t)\mu(m_i-1,k_i,t)]\\
&\quad \cdot\sqrt{m_i(m_i+2k_i-1)}/\mu(m_i-1,k_i,t),\\
u_{21i} &= \sin[\theta(t)\mu(m_i,k_i,t)]\\
&\quad \cdot\sqrt{(m_i+1)(m_i+2k_i)}/\mu(m_i,k_i,t),\\
u_{22i} &= \cos[\theta(t)\mu(m_i-1,k_i,t)]\\
&\quad + \mathrm{i}\delta\sin[\theta(t)\mu(m_i-1,k_i,t)]\\
&\quad \cdot\left[2\mu(m_i-1,k_i,t)\right]^{-1}\\
\mu(m_i,k_i,t) &= \sqrt{(\delta/2)^2+(m_i+1)(m_i+2k_i)},\\
&\quad (i=a,b).
\end{aligned}
$$

3. Quantum correlation

Now, to find the better measure of quantum correlation between the two moving and non-moving qubits of the present system, we study some of the phenomena, such as quantum discord and entanglement.

3.1. Entanglement phenomenon

Here, to investigate the entanglement of two qubits, we study the concurrence (C) measure,[26] which is considered as an effective approach to quantify entanglement and is defined as follows:

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{9}$$

where λ_1, λ_2, λ_3, and λ_4 are the eigenvalues of the matrix, $\rho_{AB}(\hat{\sigma}_y \otimes \hat{\sigma}_y)\rho^{\dagger}_{AB}(\hat{\sigma}_y \otimes \hat{\sigma}_y)$, and $C \in [0,1]$. The unentangled states referring to the concurrence for theirs are zero while maximally entangled states referring to it are one. Now, for the density matrix ρ_{AB}, we rewrite the definition (9) in the following formula

$$C = 2\max(0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}). \tag{10}$$

3.2. Quantum discord phenomenon

Quantum discord is considered to be one of the important measures of quantum correlation, which is defined as a difference between the total correlation measured by quantum mutual information and the classical correlation[27,28] as follows:

$$
\begin{aligned}
\mathrm{QD}(\rho_{AB}) &= \{S(\rho_A) + S(\rho_B) - S(\rho_{AB})\}\\
&\quad - \max_{B_k}\{S(\rho_A) - S(\rho_{AB}|B_k)\}\\
&= S(\rho_B) - S(\rho_{AB}) + \min_{B_k}\Big(\sum_k p_k S(\rho_k)\Big).
\end{aligned} \tag{11}
$$

The symbol $S(\rho_j)$ of Eq. (11) represents the Von Neumann entropy (j refers to either the total system or the subsystem) and $\sum_k p_k S(\rho_k)$ is the quantum conditional entropy with $\rho_k = \mathrm{Tr}_B[(I\otimes B_k)\rho_{AB}(I\otimes B_k)]/p_k$, $p_k = \mathrm{Tr}_{AB}[(I\otimes B_k)\rho_{AB}(I\otimes B_k)]$ and B_k is to a projectors complete set for measuring the subsystem B. Based on the previous work[27,28] and the definition of Shannon entropy $\Gamma(x) = -x\log_2 x - (1-x)\log_2(1-x)$, the definition (11) can be rewritten in the following form

$$\mathrm{QD} = \min(\Omega_1, \Omega_2), \tag{12}$$

where

$$\Omega_1 = \sum_{i=1}^{4}(\lambda_i \log_2 \lambda_i - \rho_{ii}\log_2\rho_{ii})$$

with the eigenvalues

$$
\begin{aligned}
\lambda_1 &= \rho_{11},\\
\lambda_2 &= (\rho_{22}+\rho_{33}+\sqrt{[(\rho_{22}-\rho_{33})^2+4|\rho_{23}|^2]})/2,\\
\lambda_3 &= \rho_{44},\\
\lambda_4 &= (\rho_{22}+\rho_{33}-\sqrt{[(\rho_{22}-\rho_{33})^2+4|\rho_{23}|^2]})/2,
\end{aligned}
$$

and

$$
\begin{aligned}
\Omega_2 &= \Gamma(\rho_{11}+\rho_{22}) + \sum_{i=1}^{4}\lambda_i\log_2\lambda_i\\
&\quad + \Gamma\left(1+\sqrt{[(\rho_{11}+\rho_{22}-\rho_{33}-\rho_{44})^2+4|\rho_{23}|^2]}/2\right).
\end{aligned}
$$

In the following, we employ Eqs. (10) and (12) to discuss the dynamics behavior of these phenomena.

3.3. Phenomena behavior and their discussion

The effect of parameter r on the time evolution of concurrence is shown in Fig. 2, where concurrence is plotted for r and scaled time λt in the absence of the qubit motion effect. If $0 \leq r \leq 0.32$, one can note that C is zero while if $0.32 < r \leq 1$ and $\Delta/\lambda = 0$, it is observed that C evolves periodically and the sudden death of C appears, see Fig. 2(a). We observed that C is randomly evolved and the entanglement sudden death phenomenon disappears for $0.32 < r \leq 1$ and $\Delta/\lambda = 10$, see Fig. 2(b). For the off-resonance case the value of r is increased, the local maximum value of the concurrence is increased, and it tends to be 1 when $r = 1$.

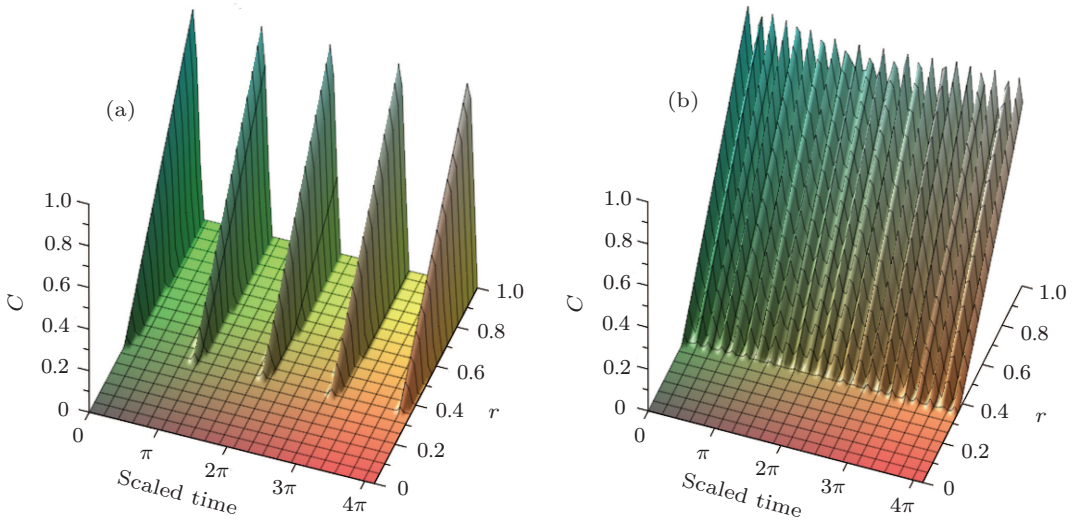


Fig. 2. (color online) Concurrence against scaled time λt and parameter r in the absence of the qubit motion effect for the fixed value of the Bargmann index $k_a = k_b = 1/2$ and the nonnegative integer $m_a = m_b = 1$. Panel (a) for $\Delta/\lambda = 0$ and panel (b) for $\Delta/\lambda = 10$.

On the other hand, to explore the effects of r on the quantum discord, we plot in Fig. 3, the QD against λt and r. In the resonance case, one observes that the time behavior of QD is periodical with period $n\pi$, $n = 0, 1, 2, 3\ldots$, see Fig. 3(a). Also, for the off-resonance case, in Fig. 3(b), the quantum discord is developed randomly and the periods of QD are early increased by increasing the purity of the initial state.

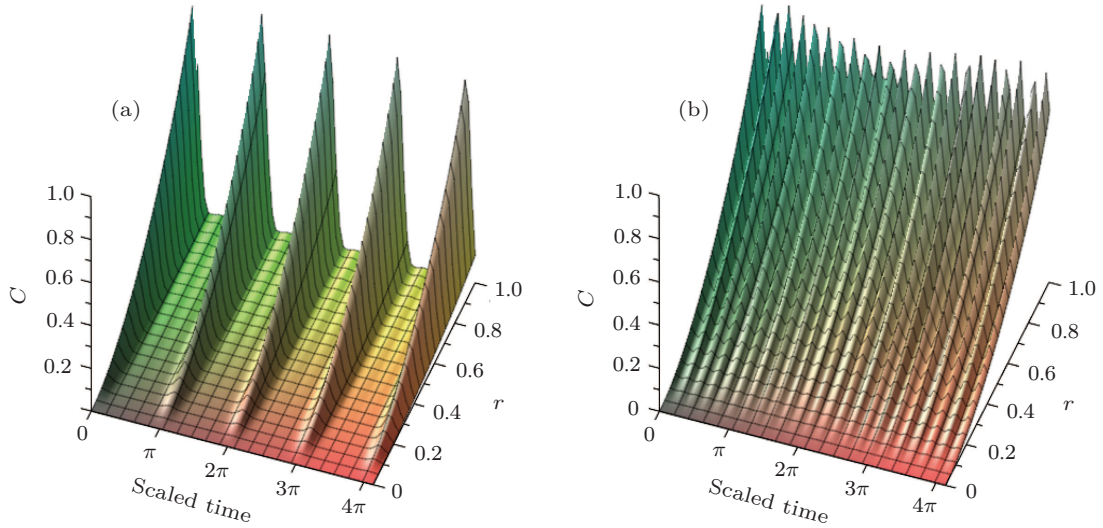


Fig. 3. (color online) The time evolution of quantum discord in the absence of the qubit motion effect with different values of $r \in [0, 1]$ for $k_a = k_b = 1/2$ and $m_a = m_b = 1$. Panel (a) for $\Delta/\lambda = 0$ and panel (b) for $\Delta/\lambda = 10$.

Now, we give fixed values for the parameter r to visualize the difference of QD from C. The time evolution of QD and C is shown in Fig. 4 for the fixed values of $k_a = k_b = 1/2$, $m_a = m_b = 1$, and different values of Δ/λ. One can observe that the detuning has a strong effect on these phenomena. It is noted that the amplitude of QD decreases during the time evolution when Δ/λ takes a suitable value, and a high amount of QD is obtained, see Fig. 4(c2). In addition, for Figs. 4(a1), 4(a2), 4(c1), and 4(c2), the evolutions of C in the resonance and off-resonance cases are always zero while those of QD are still nonzero for the region $0 \leq r \leq 0.32$. Besides, for $0.32 < r \leq 1$ and $\Delta = 0$, the concurrence phenomenon completely disappears and the sudden

death of entanglement happens while the QD phenomenon remains nonzero with its corresponding fluctuations. Here, it is found that the amplitude of QD is smaller than the amplitude of C. Also, we find that the dynamics of QD is distinguished from C, where the behavior of C suddenly disappears during the evaluation while the behavior of QD still does not disappear. As a result, the absence of the entanglement does not indicate the absence of QD, see Figs. 4(b1) and 4(b2). So, the quantum discord still can capture the quantum correlation between the two qubits when C is zero. Here, for a suitable value of the detuning parameter, the amplitude of these phenomena decreases through the time evolution and obtains a high oscillation amount of QD and C, see Figs. 4(d1) and 4(d2). By increasing the values of r and detuning parameters more and more, we reach an important prediction: the amplitudes of these phenomena completely disappeared during the time evolution.

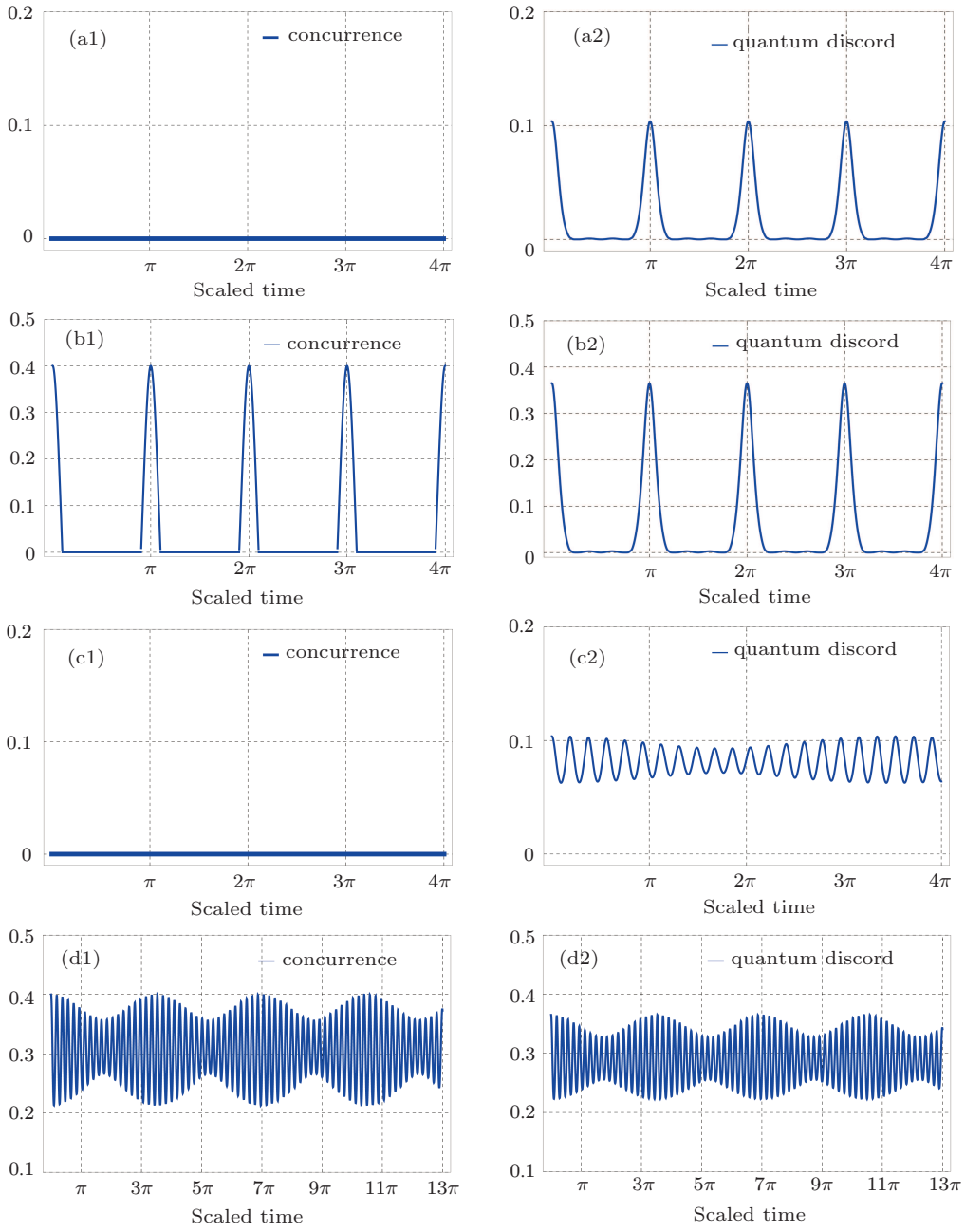


Fig. 4. (color online) The quantum discord and C versus λt in the absence of qubit motion influences for the resonance case in panels (a1), (a2), (b1), and (b2) and the off-resonance case in panels (c1), (c2), (d1), and (d2). Panels (a1) and (a2) for $\Delta/\lambda = 0$, $r = 0.3$; Panels (b1) and (b2) for $\Delta/\lambda = 0$, $r = 0.6$; Panels (c1) and (c2) for $\Delta/\lambda = 10$, $r = 0.3$; Panels (d1) and (d2) for $\Delta/\lambda = 10$, $r = 0.6$.

To visualize the influences of atomic motion and field-mode structure on quantum discord and concurrence C, the behavior of the phenomena with $r = 1$, $k_a = k_b = 1/2$, and $m_a = m_b = 1$ are displayed in Fig. 5. We observed that from Figs. 5(a1) and 5(a2), the time evolution of the phenomena behavior is similar to the above discussion and it exhibits the periodic dynamics with the same period as shown in the previous figures. So, the simulations of Figs. 5(a1) and 5(a2) are the same as those discussed

above. In addition, by increasing the value of p the evolution periods of phenomena are decreased, as can be seen in Figs. 5(b1) and 5(b2).

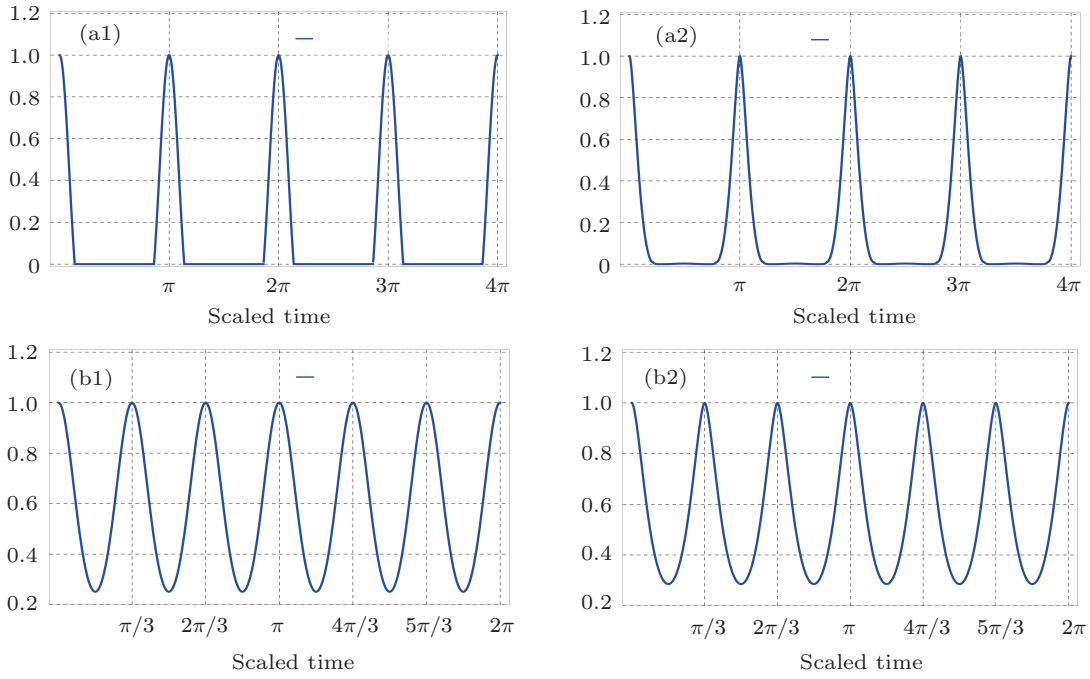


Fig. 5. (color online) The behavior in the resonance case through the influences of atomic motion (p_2) and field-mode structure (p). Panels (a1) and (a2) for $\Delta/\lambda = 0$, $r = 1$, and $p = 1$; Panels (b1) and (b2) for $\Delta/\lambda = 0$, $r = 1$, and $p = 3$.

In addition, in Fig. 6, these phenomena have been plotted as a contour versus time and r with the same conditions as given in Fig. 5. Also, it is concluded that the quantum discord still is obviously nonzero and the long-living quantum discord can be obtained.

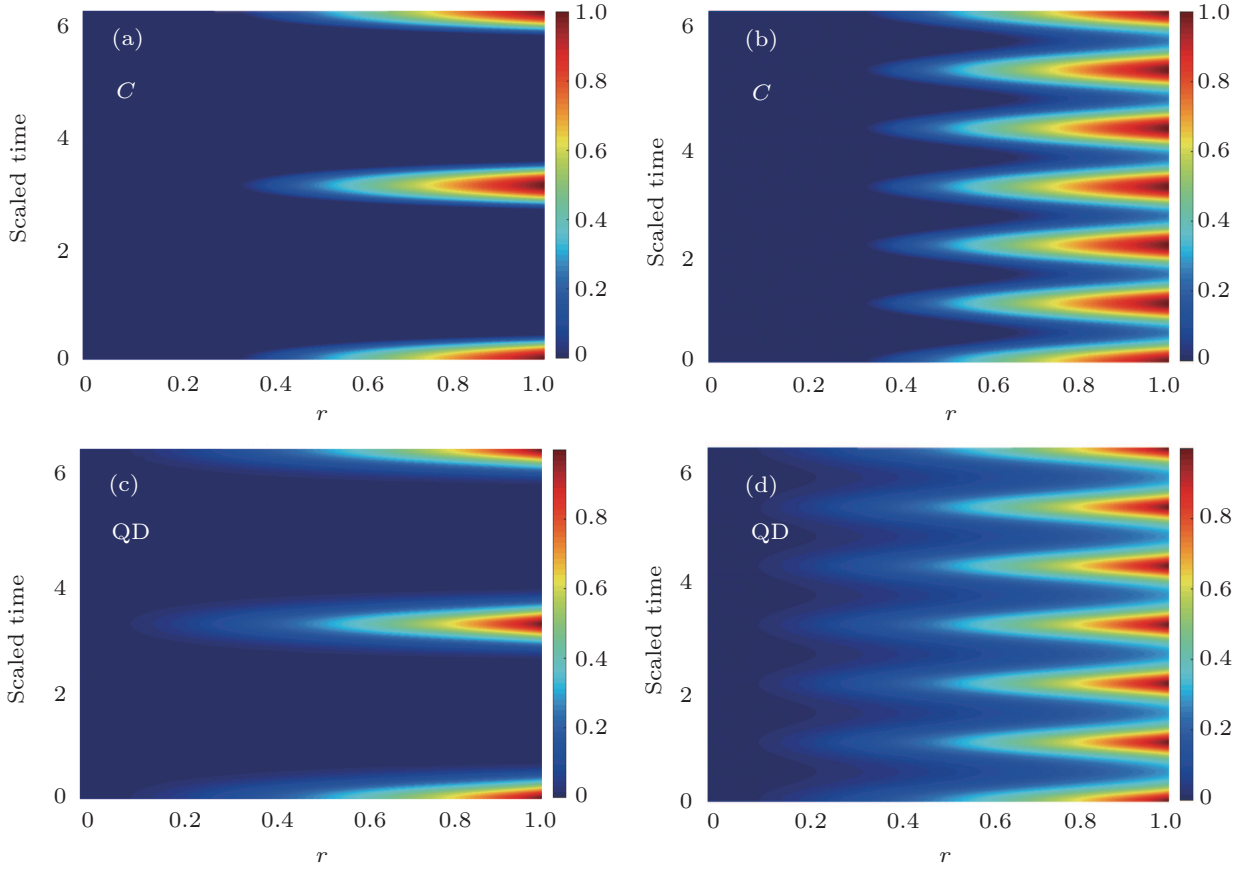


Fig. 6. (color online) Contour plots of phenomena behavior in the resonance case. Panels (a) and (c) for $\Delta/\lambda = 0$, $p = 1$; Panels (b) and (d) for $\Delta/\lambda = 0$, $p = 3$.

4. Summary

In summary, some of the phenomena, such as the quantum discord and concurrence, have been demonstrated between two isolated moving qubits each separately interacting with a single-mode $SU(1,1)$ cavity. The effects of some parameters, such as the field-mode structure, purity of the initial state, atomic motion, and detuning have been studied for the current phenomena. These phenomena are strongly influenced due to these parameters. It is observed that in the absence of qubit motion and a specific r for the resonance and off-resonance cases the entanglement completely disappears while the quantum discord remains unchanged. Also, a sudden death via the detuning effect for these phenomena appears. Besides, in the presence of qubit motion and field-mode structure for $r = 1$, the phenomena exhibit the same behavior for the resonance and off-resonance cases. It is concluded that a minimum value of quantum discord is always greater than that of concurrence, and then when concurrence is zero the quantum discord can still capture the quantum correlation between the two qubits. Hence, the quantum discord is considered as a best measure of quantum correlation.

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