ISSN 1674-1056
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Citation: Chin. Phys. B . 2018, 27(1): 014501. **doi:** 10.1088/1674-1056/27/1/014501

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Envelope solitary waves and their reflection and transmission due to impurities in a granular material*

Wen-Qing Du(杜文青)[1], Jian-An Sun(孙建安)[1], Yang-Yang Yang(杨阳阳)[2], and Wen-Shan Duan(段文山)[1,†]

[1] College of Physics and Electronic Engineering and Joint Laboratory of Atomic and Molecular Physics of NWNU & IMP CAS, Northwest Normal University, Lanzhou 730070, China

[2] Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

(Received 15 May 2017; revised manuscript received 9 September 2017; published online 28 November 2017)

A nonlinear Schrödinger equation in one-dimensional bead chain is first obtained and an envelope solitary wave of the system is verified numerically in this system. The reflection and the transmission of an incident envelope solitary wave due to impurities has also been investigated. It is found that the magnitudes of both the reflection and the transmission not only depend on the characters of impurity materials, the wave number, the incident wave amplitude, but also on the impurity number. This can be used to detect the character and the number of the impurity materials in the bead chain by measuring the reflection and the transmission of an incident pulse.

Keywords: impurities, envelope solitary wave, numerical simulation

PACS: 45.05.+x, 05.45.–a, 45.70.–n **DOI:** 10.1088/1674-1056/27/1/014501

1. Introduction

The nonlinear Schrödinger equation (NLSE) serves as a central model in nonlinear science, especially in the nonlinear wave.[1] The one-dimensional (1D) NLSE can show some very exciting and fascinating nonlinear phenomena, such as the modulation instability and envelope solitons.[2,3] The NLSE belongs to the remarkable class of integrable systems[4] and can be solved by using the integration techniques. The phenomenon of the envelope solitary waves and the modulation instability (MI) have been extensively studied in the past years.[5–11] Solitary wave solution of NLSE has been extensively studied in many branches of the science such as the deep water wave,[4] the dynamics of the Duffing oscillator[4] in optics,[12] plasmas,[13] etc.

Granular chains have been an interesting topic in many branches of science.[14] The force displacement law between two grain beads in contact is governed by Hertzian interaction.[14] Granular crystals provide a versatile type of metamaterial for both fundamental physical phenomena and applications.[15,16]

Granular chains have been used to investigate numerous coherent structures such as traveling waves, breathers, and dispersive shock waves.[17–22] It is found that the granular chains support a new type of solitary wave which is qualitatively different from the well-known weakly nonlinear Korteweg–de Vries (KdV) solitary waves when it is without initial pre-stress.[14] These kinds of waves do not support sound waves. Different groups investigated numerically and experimentally the properties of these waves.[17–22] On the other hand, the KdV solitary wave with the initial pre-stress is also found in the bead chain.[23] The reflection and the transmission of these solitary waves at the interface of two different bead chains, from imperfections or a wall, have been thoroughly studied.[23,27]

However, envelope solitary wave described by the NLSE has not been reported until now. This work presents an NLSE in 1D bead chain and then finds the envelope solitary waves in this system. Furthermore, the reflection and the transmission of an incident envelope solitary wave due to impurities are also studied. Interesting phenomena are observed.

2. Model

The simplest granular systems are 1D chains of elastic spheres. The dynamics of the 1D bead chain is described by N coupled nonlinear equations[14]

$$m_i \frac{\mathrm{d}^2 u_i}{\mathrm{d}t^2} = nK[(2R+u_{i-1}-u_i)^{n-1} - (2R+u_i-u_{i+1})^{n-1}], \quad i=1,2,\ldots,N, \qquad (1)$$

where m_i is the bead mass, $u_i=(2i-1)R+s_i$ is the absolute position of the bead i, s_i is the displacement from its equilibrium position, $n=5/2$, $K=\frac{2}{5}\frac{(R/2)^{1/2}}{D}$, $D=\frac{3}{2}\frac{1-\sigma^2}{Y}$, R is the bead radius, Y is Young's modulus, and σ is the Poisson ratio of the bead material.

If we give an initial pre-stress p_0 that leads to an initial average bead overlap $\delta_0 \sim p_0^{3/2}$, the average distance between two nearest neighbors becomes $R'=R-\frac{\delta_0}{2}$, where δ_0 is the

*Project supported by the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDA01020304) and the National Natural Science Foundation of China (Grant Nos. 91026005, 11547304, 11275156, 11047010, and 61162017).

†Corresponding author. E-mail: duanws@nwnu.edu.cn

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pre-stress. Under the conditions of long wavelength approximation and the small amplitude assumption, $s_i - s_{i+1} \ll \delta_0$, equation (1) becomes[23]

$$\frac{\mathrm{d}^2 s_i}{\mathrm{d}t^2} = A\left(\frac{\mathrm{d}^2 s_i}{\mathrm{d}i^2} + \frac{1}{12}\frac{\mathrm{d}^4 s_i}{\mathrm{d}i^4}\right) + B\frac{\mathrm{d}s_i}{\mathrm{d}i}\frac{\mathrm{d}^2 s_i}{\mathrm{d}i^2}, \tag{2}$$

where $A = \frac{15K\sqrt{\delta_0}}{4m_i}$ and $B = \frac{-15K}{8m_i\sqrt{\delta_0}}$. By introducing the following stretched coordinates: $\xi = \varepsilon(x - v_s t)$, $\tau = \varepsilon^2 t$, where $x = i$.[23–25] $s_i = \sum_{m=1}^{\infty} \varepsilon^m \sum_{l=-\infty}^{+\infty} s_l^{(m)}(\xi, \tau)\, \mathrm{e}^{\mathrm{i}l(kx-\omega t)}$, we obtain the well-known NLSE

$$\mathrm{i}\frac{\partial s_1^{(1)}}{\partial \tau} + P\frac{\partial^2 s_1^{(1)}}{\partial \xi^2} + Q \mid s_1^{(1)} \mid^2 s_1^{(1)} = 0, \tag{3}$$

where $P = \frac{2A - Ak^2 - 2v_s^2}{4\omega}$, $Q = \frac{3B^2k^6}{12Ak^2\omega - 4Ak^4\omega - 12\omega^3}$, the group velocity $v_s = A(\frac{k}{\omega} - \frac{k^3}{6\omega})$, and the dispersion relation $\omega = k\sqrt{A(1 - \frac{k^2}{12})}$. It is found that $PQ > 0$. The envelope solitary wave solution of Eq. (3) is as follows: $s_1^{(1)} = \sqrt{\frac{2(Pk^2 - \omega)}{Q}}\,\mathrm{sech}(\sqrt{\frac{Pk^2 - \omega}{P}}\xi)$. In the experimental coordinates, the displacements of the beads of the envelope solitary wave can be expressed as follows:

$$s_i = \varepsilon\sqrt{\frac{2(Pk^2 - \omega)}{Q}} \times \mathrm{sech}\left[\sqrt{\frac{Pk^2 - \omega}{P}}\varepsilon(x - v_s t)\right]\mathrm{e}^{[\mathrm{i}(kx - \omega t)]}. \tag{4}$$

3. Numerical results of the envelope solitary wave

We let the initial displacements of all the beads satisfy Eq. (4) and their corresponding velocities be $v = \frac{\mathrm{d}s}{\mathrm{d}t}$, set the values of the parameters as $\varepsilon = 0.005$ and $k = 0.1$. In our simulation, the periodic boundary condition is applied. We choose the steel beads: the radius of the bead is 0.001 m, Young's modulus $Y = 2 \times 10^{11}$ Pa, Poisson's ratio $\sigma = 0.3$, the density of the bead $\rho = 7.78 \times 10^3$ kg/m^3, the number of the bead $N = 3001$, and the international system of unit is used. The bead motion is obtained by numerical integration of Eq. (1).

The profiles of the envelope solitary waves of both the analytical results and the numerical ones are shown in Fig. 1, in which $\varepsilon = 0.005$ and $k = 0.1$. It is observed that the envelope solitary wave propagates, as predicted by the analytical result. Both are in good agreement. We then conclude that the envelope solitary wave does exist in the bead chain. However, it is well known that the analytical solution of envelope solitary wave of Eq. (4) is obtained by the approximate reductive perturbation method. Can it correctly express the real envelope solitary wave, or what is the application scope of the envelope solitary wave expressed by Eq. (4). In order to answer these questions, the comparisons between the analytical results of Eq. (4) and the numerical ones for different parameters of ε and k are presented. It is noted that the differences between two are observed when either ε or k becomes large.

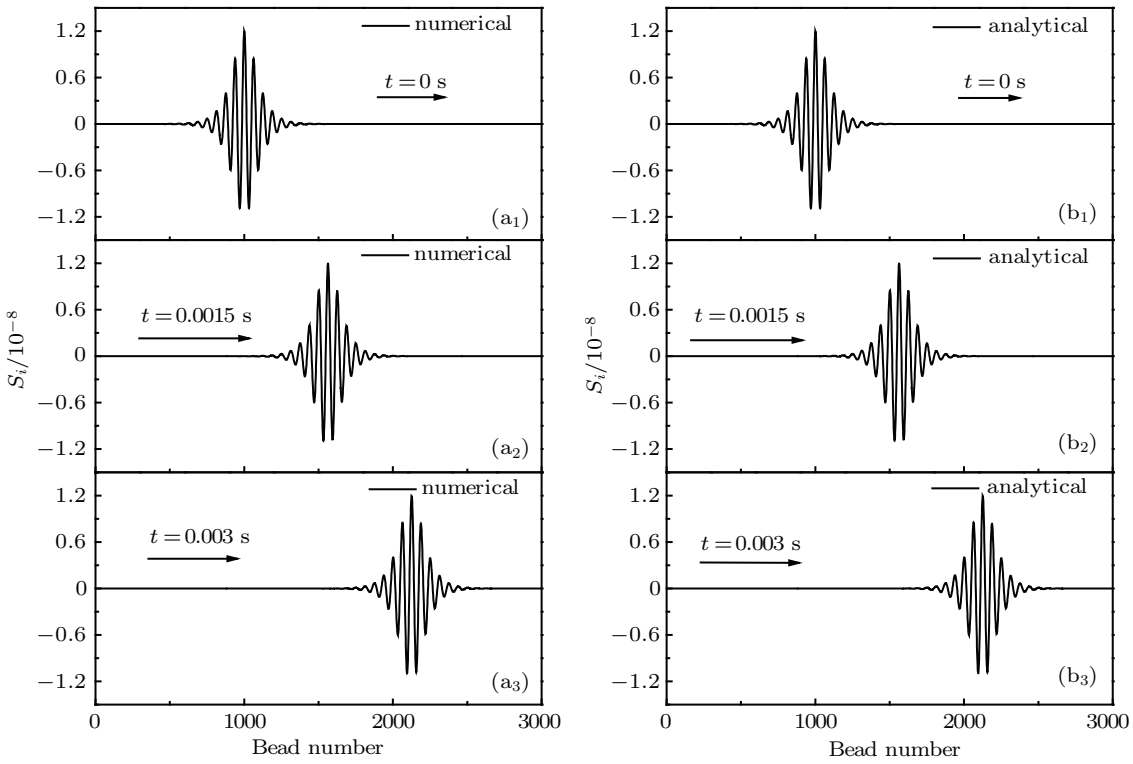


Fig. 1. Comparisons between the numerical results on the left-hand side and the analytical one on the right-hand side from Eq. (4) of the evolution of the envelope solitary wave with the different time, where the values of the bead parameters are as follows: $Y = 2.03 \times 10^{11}$ Pa, $\sigma = 0.3$, $\rho = 7.78 \times 10^3$ kg/m^3, $R = 0.001$ m, $\delta_0 = 8.46154 \times 10^{-7}$ m, $\varepsilon = 0.005$, and $k = 0.1$.

To get more insight into how the analytical results present the real envelope solitary wave, the comparisons between the analytical results and the numerical ones for different parameters of ε and k are shown in Fig. 2. Dependence of amplitude, group velocity, and the width of the envelope solitary wave on the parameter k are shown on the left-hand side of Fig. 2, where $\varepsilon = 0.01$ and $0.05 \leq k \leq 0.95$. Dependence of the amplitude, group velocity, and the width of the envelope solitary wave on the parameter ε are shown on the right-hand side of Fig. 2, where $k = 0.1$ and $0.05 \leq \varepsilon \leq 0.6$. It is noted that both the amplitude of the envelope solitary waves are in good agreement between the numerical results and the analytical ones if both the wavenumber k and the parameter ε are small enough (see Fig. 2(a)), while the differences between the two are observed when either the wave number k or the parameter ε is large enough. Furthermore, it is observed that both the group velocity and the width of the envelope solitary waves are in good agreement between the numerical results and the analytical ones.

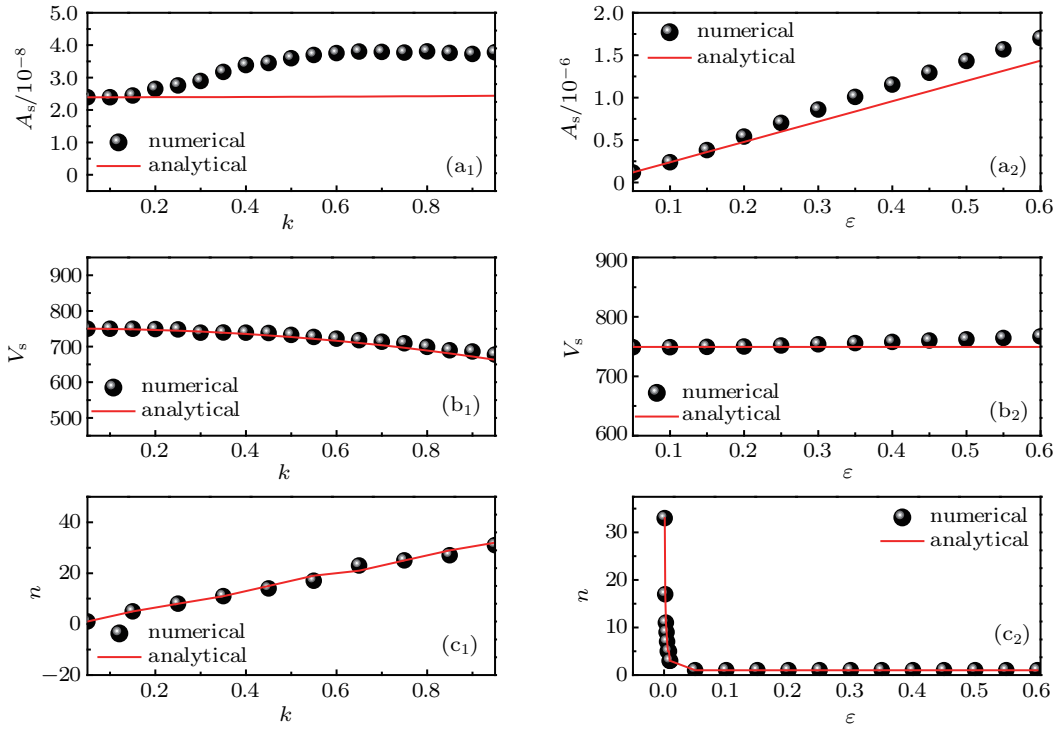


Fig. 2. (color online) Comparison between the analytical results of Eq. (4) and the numerical ones of the envelope solitary wave. The dependence of amplitude, group velocity, and the width of the envelope solitary wave on the parameter k are shown in (a_1), (b_1), and (c_1), respectively, where $\varepsilon = 0.01$ and $0.05 \leq k \leq 0.95$. The dependence of amplitude, group velocity, and the width of the envelope solitary wave on the parameter ε are shown in (a_2), (b_2), and (c_2), respectively, where $k = 0.1$ and $0.05 \leq \varepsilon \leq 0.6$.

4. Reflection and transmission of the incident envelope solitary wave and general acoustic wave

Nesterenko proposed that the propagation of a perturbation in a chain of beads in the Hertzian contact can evolve into soliton-like pulse.[28] Later, several investigations confirmed the existence of such soliton-like pulses.[17,29–31] Moreover, the interaction between a solitary wave and the boundary has also been studied.[21,26,27,32–34]

We now study the scattering of an envelope solitary wave at an interface, as shown in Fig. 3. We place a chain of N identical beads from $i = -N$ to $i = -1$ and from $i = 1$ to $i = N$, while there is a different bead at $i = 0$. Let an envelope solitary wave propagate from the region $i < 0$, arrive at the interface $i = 0$, and split into transmitted waves in the region $i > 0$, and waves are reflected in the region $i < 0$, as already observed previously.[23] The numerical results of the reflection and the transmission of an envelope solitary wave are shown in Fig. 4 in which the reflected wave and the transmitted wave are observed. It is found that the amplitudes of the reflected waves are different for different impurity materials.

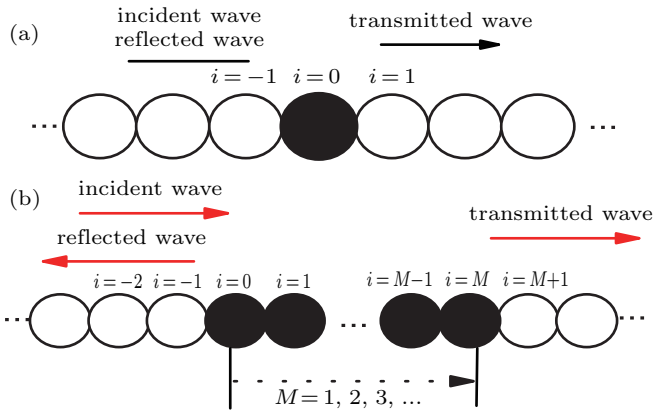


Fig. 3. (color online) Granular chain composed of several impurities.

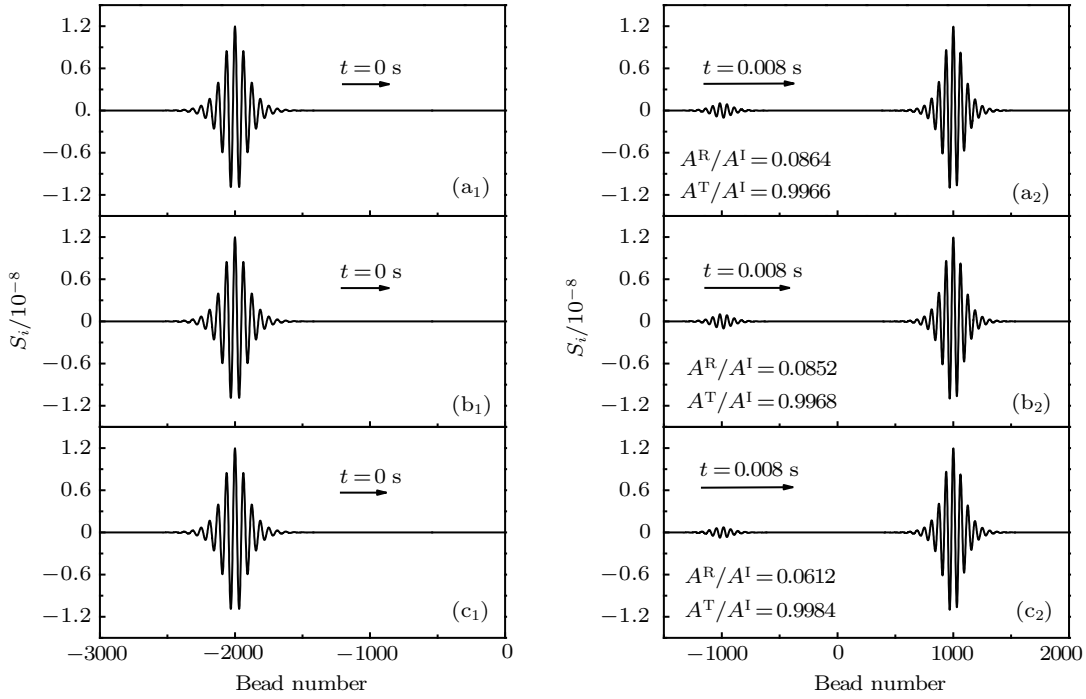


Fig. 4. Reflection and transmission due to an incident wave where there is an impurity bead of plexiglass in (a), aluminium alloy in (b), and tungsten carbide in (c).

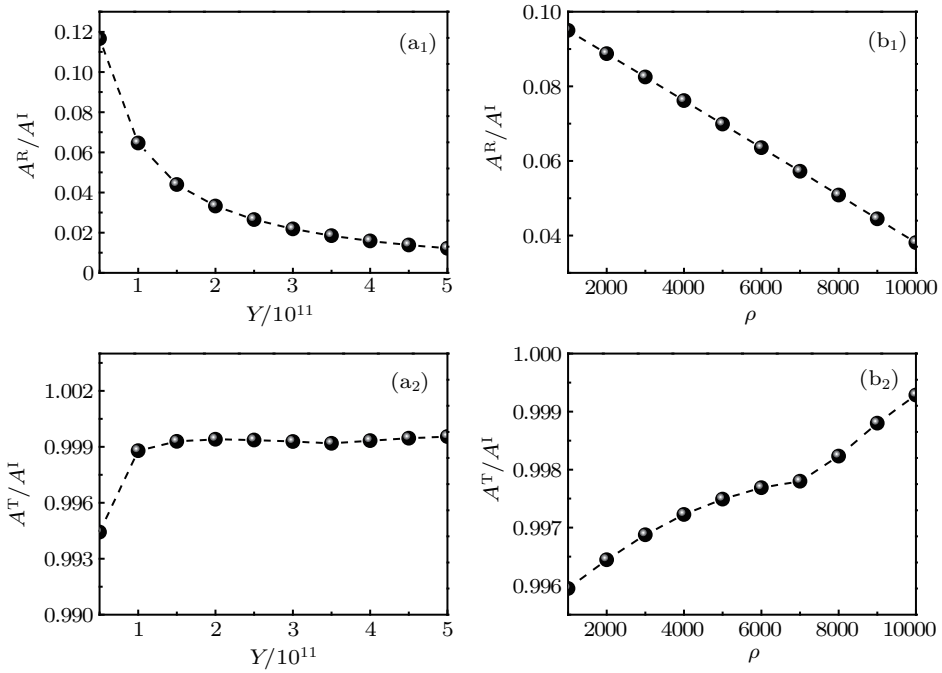


Fig. 5. Dependence of the amplitude ratios of the transmitted wave to the incident wave, $\frac{A^{\rm T}}{A^{\rm I}}$, the amplitude ratios of the reflected wave to the incident wave, $\frac{A^{\rm R}}{A^{\rm I}}$, on the Young's modulus and the density of the impurity materials.

To further study the dependence of the reflection and the transmission on the impurity material the numerical results of the dependence of the amplitude ratios of both the reflected wave to the incident wave ($\frac{A^{\rm R}}{A^{\rm I}}$) and the transmitted wave to the incident wave ($\frac{A^{\rm T}}{A^{\rm I}}$) on Young's modulus and the density of the impurity material are shown in Fig. 5. It indicates that the reflected wave amplitude decreases, while the transmitted wave amplitude increases as Young's modulus of the impurity material increases. In addition, the reflected wave amplitude decreases, but the transmitted wave amplitude increases as the density of the impurity material increases.

Dependence of the reflection and the transmission on the impurity number are shown in Fig. 6. It is observed that both ($\frac{A^{\rm R}}{A^{\rm I}}$) and ($\frac{A^{\rm T}}{A^{\rm I}}$) vibrate with the impurity number until it reaches a critical value of N_0, as shown in Fig. 6. It is also noted that the amplitude of the vibration decreases as the impurity number increases. The reflection amplitude will reach its maximum value when the transmission amplitude reaches its mini-

mum value. Both tend to be constants when the impurity number is larger than the critical number N_0.

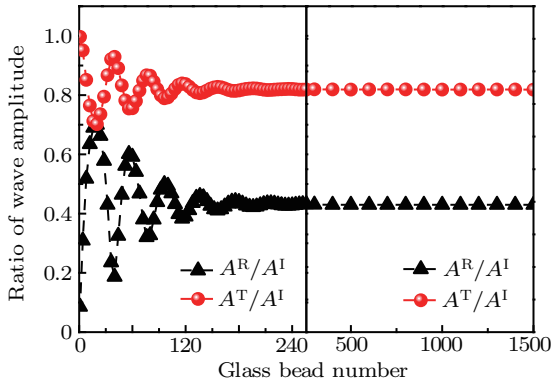


Fig. 6. (color online) Dependence of the amplitude ratios of both $\frac{A^T}{A^I}$, and $\frac{A^R}{A^I}$ on the number of the impurity materials.

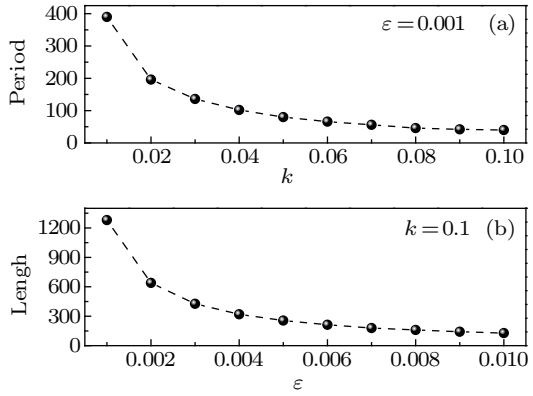


Fig. 7. Dependence of the period and the length of the vibration of the amplitude ratios, $\frac{A^T}{A^I}$, or $\frac{A^R}{A^I}$ on the wave number k and the parameter ε.

The dependence of the length and the period of the vibration observed in Fig. 6 on the wavenumber k and the parameter ε are shown in Fig. 7. It is found that the period of the vibration only depends on the wavenumber k, while the length of the vibration only depends on the parameter ε. The period of vibration decreases as the wavenumber k increases and the length of the vibration decreases as ε increases.

For more general case, we calculate the ratios of the kinetic energy of all beads of both transmitted wave to the incident wave and the reflected wave to the incident wave. We give the incident wave as follows:

$$\mathbf{s}_i = \begin{cases} A\cos[k(x-x_0)-\omega t]\,\mathrm{e}^{\alpha(x-x_1)}, & 0 < x < x_1, \\ A\cos[k(x-x_0)-\omega t], & x_1 < x < x_2, \\ A\cos[k(x-x_0)-\omega t]\,\mathrm{e}^{-\alpha(x-x_2)}, & x_2 < x, \end{cases} \quad (5)$$

We choose $x_0 = 1000$. ω satisfies the dispersion relation. The results are given by changing the parameters of A, α, x_1, and x_2. Dependence of the kinetic energy ratios of both reflected waves, $\frac{E^R}{E^I}$, and transmitted wave, $\frac{E^T}{E^I}$, on the number of the impurity materials are shown in Fig. 8. Similar results to that of Fig. 6 are observed. Both $\frac{E^R}{E^I}$ and $\frac{E^T}{E^I}$ vibrate with the impurity number until it reaches a critical point. It is also noted that the period of the vibration only depends on the wave number k. The length of vibration depends on both parameters of α and $\Delta L = x_2 - x_1$. However, both the ratios are independent of A. It is interesting to know that the magnitude of the reflection and the transmission not only depend on the impurity materials, the wave number, and the incident wave amplitude, but also especially on the impurity number.

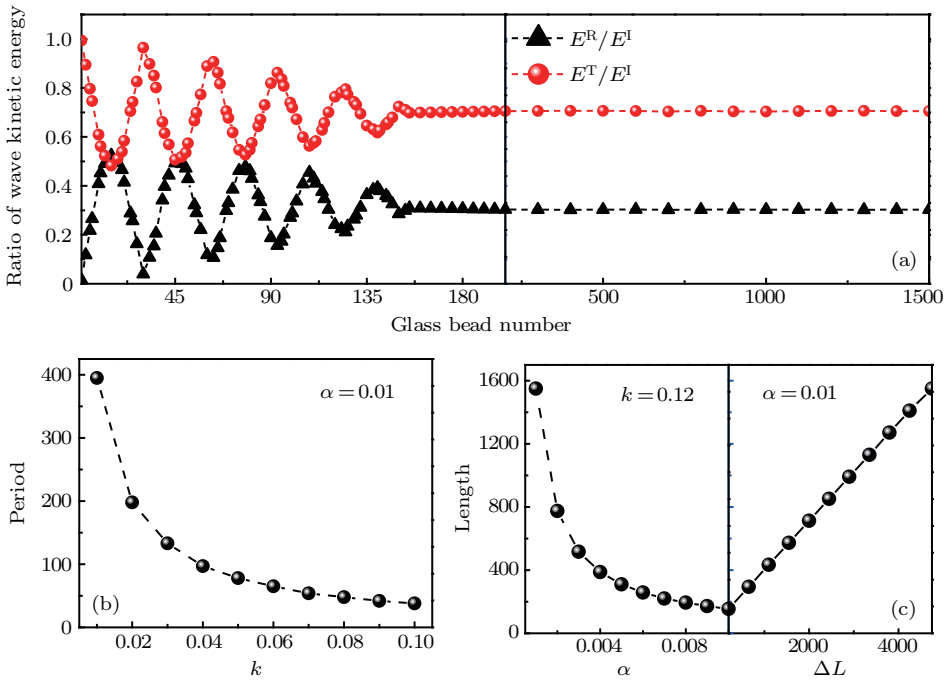


Fig. 8. (color online) (a) Dependences of the kinetic energy ratios of both transmitted wave $\frac{E^T}{E^I}$ and reflected wave $\frac{E^R}{E^I}$ on the number of the impurity materials. Dependences of the period and the length of the vibration of the kinetic energy ratios on the wavenumber k and the parameter α and $\Delta L = x_2 - x_1$.

5. Conclusion

An NLSE in 1D bead chain has been first given in the present work and an envelope solitary wave solution of the NLSE is verified numerically in this system. Furthermore, the reflection and the transmission of an incident envelope solitary wave due to impurities are also studied. It is found that the magnitudes of both the reflection and the transmission not only depend on the impurity materials, the wave number, the incident wave amplitude, but also on the impurity number. Similar results are obtained for more general incident waves.

The obtained results can be used to detect the impurities in the bead chain by measuring the reflection of a given pulse. The reflection due to an incident pulse contains the information of the material and the number of the impurity. Our results may help to devise a bead protection system. Disintegrating impulse or shock wave into many weak impulses is one of the possible ways. We can then protect something important from different kinds of disastrous external impacts, such as an earthquake, bomb explosion, automobile collision, etc. It seems that it is very important to further investigate the reflection and the transmission by one or several impurities from a given incident wave.

This phenomenon is actually the acoustic diode effect. We can control the reflection or the transmission from an arbitrary incident acoustic waves by changing the numbers of the impurities and adding several identical pieces of impurities in the bead chains. It can be employed to design tunable information transportation lines with the unique possibility to manipulate the signal delay and scrambling of security-related information.

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