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Super-sensitive phase estimation with coherent boosted light using parity measurements*

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We consider a passive and active hybrid interferometer for phase estimation, which can reach the sub-shot-noise limit in phase sensitivity with only the cheapest coherent sources. This scheme is formed by adding an optical parametric amplifier before a Mach–Zehnder interferometer. It is shown that our hybrid protocol can obtain a better quantum Cramer–Rao bound than the pure active (e.g., SU(1,1)) interferometer, and this precision can be reached by implementing the parity measurements. Furthermore, we also draw a detailed comparison between our scheme and the scheme suggested by Caves [*Phys. Rev. D* **23** 1693 (1981)], and it is found that the optimal phase sensitivity gain obtained in our scheme is always larger than that in Caves' scheme.

Keywords: phase estimation, quantum metrology

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1. Introduction

Interferometers, as an extremely useful and flexible precise measuring tool, play a key role in the field of quantum metrology.[1–21] There are mainly two classes of interferometers:[2,3] passive (e.g., Mach–Zehnder interferometer (MZI)) and active (e.g., SU(1,1)) ones. The ultimate goal of both setups is to obtain the sub-shot-noise limit for phase estimation. The sensitivity of these setups crucially depends on the input states as well as the detection scheme.

It is well known that, for the MZI to beat the shot-noise limit (SNL), i.e., $1/\sqrt{N}$ with N the total photon number, nonclassicality of the input states, such as the single mode squeezed vacuum states[1,22,23] and the two-mode squeezed vacuum states,[2,24,25] is necessary. In contrast to the MZI, the potential advantage of the SU(1,1) interferometer is that even for bright classical coherent sources of the input states, the SNL can also be surpassed.[4,5] In Refs. [4] and [5], the authors have shown that when inputting coherent states into this interferometer the SNL can be beaten through a photon intensity measurement. However, in general, the photon intensity measurement is not optimal, so the best phase estimation precision given by the quantum Cramer–Rao bound (CRB),[26–29] that is $\Delta\phi_{\min} = 1/\sqrt{F}$ with F the quantum Fisher information (QFI), is usually hard to be reached in this case.

Recently, parity detection[24,25,30–36] has been considered to be a very useful measurement method in MZI with practically many input states, which can achieve the CRB for phase sensitivity. References [24] and [25] have shown that the extreme sensitivity of parity detection is equivalent to measure all the phase-bearing terms, including the off-diagonal terms, in the density matrix of the two-mode light field before the second beam splitter.[24] However, this detection method cannot be used on the SU(1,1) interferometer directly when the beam splitters in MZI are replaced by the nonlinear optical elements. To the best of our knowledge, the optimal measurement scheme for the SU(1,1) interferometer remains absent.

In this work, we combine the advantages of passive and active interferometers by adding an optical parametric amplifier (OPA) before an MZI. We show that our protocol can obtain a better CRB than the pure active interferometer; and this precision can be achieved by a parity measurement. Comparing with other quantum metrology schemes utilizing the parity detection, the main advantage of our scheme is that it has the potential to reach the phase uncertainties far below the SNL, and only needs the cheapest coherent sources. To demonstrate the advantage of our scheme, we also compare the optimal phase sensitivity gains (OPSG) with respect to the SNL between these two schemes. It is found that the gain obtained in our scheme is always higher than that obtained in Ref. [1] in the whole range of r.

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2. Phase estimation in a hybrid interferometer

We consider a passive and active hybrid interferometer, which is formed by an OPA, two 50:50 beam splitters, and two phase shifters, as shown in Fig. 1(a). This setup requires a strong pump beam in addition to two coherent light beams labeled by $|\alpha_0\rangle$ and $|\beta_0\rangle$. The OPA performs the unitary transformation $S(r)=\exp[r(a^\dagger b^\dagger - ab)]$, where r is the two-mode squeezing parameter, which is relevant to the strength of the nonlinearity and the intensity of the pump beam. The beam splitter is described by $B=\exp\left[\mathrm{i}\frac{\pi}{4}(a^\dagger b + ab^\dagger)\right]$; and the phase-shift unitary operator is $U=\exp[\mathrm{i}(\varphi_a a^\dagger a+\varphi_b b^\dagger b)]=\exp(\mathrm{i}N_\mathrm{d}\phi_\mathrm{d}/2)\exp(\mathrm{i}N_\mathrm{s}\phi_\mathrm{s}/2)$, where we have introduced the difference and sum phase shifts $\phi_\mathrm{d}=\varphi_a-\varphi_b$ and $\phi_\mathrm{s}=\varphi_a+\varphi_b$, which can be precisely estimated by the setups in Figs.1(a) and 1(b), respectively. The corresponding difference and sum number operators are given by $N_\mathrm{d}=a^\dagger a-b^\dagger b$ and $N_\mathrm{s}=a^\dagger a+b^\dagger b$, respectively.

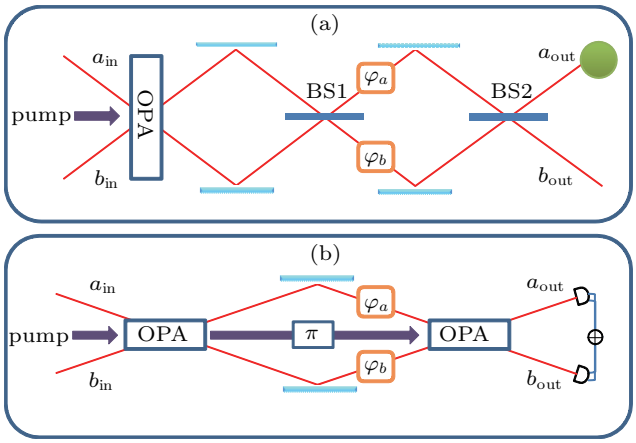


Fig. 1. (color online) (a) Schematic diagram of the passive and active hybrid interferometry, which is formed by adding an OPA before an MZI. This setup is sensitive to the differential phase shift $\phi_\mathrm{d}=\varphi_a-\varphi_b$, and the information is extracted by a parity detection on one of the second beam splitter's outputs. (b) Schematic drawing of an ideal SU(1,1) interferometer. The beam splitters of the MZI have been replaced by OPAs. There is a π phase shift between the two OPAs. Different from the setup in panel (a), the SU(1,1) interferometer is sensitive to the sum phase shift $\phi_\mathrm{s}=\varphi_a+\varphi_b$ and the total photon intensity measurement is taken on the second OPA's outputs.

Assuming there is no loss in the configuration, then for the setup shown in Fig. 1(a), we can obtain the QFI $F=N_\mathrm{c}\cosh(4r)+2|\alpha_0\beta_0|\sinh(4r)+\sinh^2(2r)$, where $N_\mathrm{c}=|\alpha_0|^2+|\beta_0|^2$ is the total amount of coherent light with $\alpha_0=|\alpha_0|\mathrm{e}^{\mathrm{i}\theta_1}$ and $\beta_0=|\beta_0|\mathrm{e}^{\mathrm{i}\theta_2}$. To obtain the optimal QFI, here we consider the phase-matching condition and choose the phases $\theta_1=\theta_2=0$. Therefore, the best phase sensitivity given by the CRB, i.e., $\Delta\phi_{\mathrm{d}\min}=1/\sqrt{F}$, for our scheme is

$$\Delta\phi_{\mathrm{d}\min}=\frac{1}{\sqrt{N_\mathrm{c}\cosh(4r)+2|\alpha_0\beta_0|\sinh(4r)+\sinh^2(2r)}}, \quad (1)$$

which is far below the SNL, i.e., $\Delta\phi_{\mathrm{dSNL}}=1/\sqrt{N_\mathrm{T}}$. Here $N_\mathrm{T}=N_\mathrm{c}\cosh(2r)+n_\mathrm{s}$ is the total photon number inside the interferometer, where the first term corresponds to contributions from the stimulated parametric process, and the second term results from the spontaneous process with $n_\mathrm{s}=2\sinh^2 r$. In the limit of $|\alpha_0|=|\beta_0|\to 0$, equation (1) reduces to the case of two-mode squeezed vacuum states[2,24,25] and the CRB is $\Delta\phi_{\min}=\sqrt{1/(n_\mathrm{s}+2)n_\mathrm{s}}$, which realizes the Heisenberg limit.

Following the same procedure, we can also obtain the phase sensitivity of the pure SU(1,1) interferometer (Fig. 1(b)) for the same input states. When setting $\theta_1=\theta_2=0$, the CRB turns out to be

$$\Delta\phi_{\mathrm{s}\min}=\frac{1}{\sqrt{N_\mathrm{c}\sinh^2(2r)+2|\alpha_0\beta_0|\sinh(4r)+\sinh^2(2r)}}. \quad (2)$$

Comparing Eq. (1) with Eq. (2), we can easily find that the quantum CRB obtained in our hybrid interferometer is better than that obtained in the pure SU(1,1) interferometer, since $\cosh(4r)>\sinh^2(2r)$. Moreover, as we know that the CRB can very well quantify the potential advantage of a certain scheme in quantum metrology, but not every detection method is able to achieve this advantage. For example, the precision given in Eq. (2) cannot be realized through the current measurement scheme, such as photon intensity detection and parity detection. In Refs. [4] and [5], the authors obtained the phase sensitivity $\Delta\phi_\mathrm{s}=1/[\sinh(2r)\sqrt{N_\mathrm{c}}]$ in some certain cases by detecting the total photon intensity on the second OPA's outputs of the SU(1,1) interferometry. It is clear to see that this precision does not reach the CRB. Moreover, we can check that for the SU(1,1) interferometry, the parity detection scheme is even worse than the photon intensity detection when $|\alpha_0|,|\beta_0|>0$.

3. Achieve the quantum CRB by parity detection

Next, we will demonstrate that, in our hybrid scheme (Fig. 1(a)), the sensitivity in Eq. (1) can be obtained by just implementing the parity detection, which was first introduced into optical interferometry by Gerry,[31] and has been applied to precise phase estimation. When specifying a as the mode to measure, the signal of the parity detection is $\langle\Pi_a\rangle=\langle(-1)^{a^\dagger a}\rangle$.

In our scheme, the expectation value of the parity operator $\langle\Pi_a\rangle$ in mode a can be obtained as

$$\langle\Pi_a\rangle=\exp(-2N_\mathrm{c}+\Lambda)/\sqrt{1+\sinh^2(2r)\cos^2\phi_\mathrm{d}}, \quad (3)$$

where

$$\begin{aligned}\Lambda &= 2X^2/\cosh(2r)+\sin\phi_\mathrm{d}\sinh(2r)\\ X &= (|\beta_0|\cosh r-|\alpha_0|\sinh r)\cos(\phi_\mathrm{d}/2)\\ &\quad +(|\beta_0|\sinh r-|\alpha_0|\cosh r)\sin(\phi_\mathrm{d}/2).\end{aligned}$$

Note that, in Eq. (3) the parity signal $\langle\Pi_a\rangle$ does not depend on the sum phase shift $\phi_\mathrm{s}=\varphi_a+\varphi_b$.

The phase sensitivity $\Delta\phi_{\rm d}$ of an interferometer is characterized by using the error propagation formula $\Delta\phi_{\rm d} = \Delta\Pi_a/|\partial\langle\Pi_a\rangle/\partial\phi|$, where $\Delta\Pi_a = \sqrt{\langle\Pi_a^2\rangle - \langle\Pi_a\rangle^2}$. When $|\alpha_0|^2 = |\beta_0|^2 = N_{\rm c}/2$, the explicit form of the optimal phase sensitivity with parity detection reads

$$\Delta\phi_{\rm d\,min} = 1/\sqrt{N_{\rm c}\,{\rm e}^{4r} + \sinh^2(2r)} \tag{4}$$

in the limit $\phi_{\rm d} \to -\pi/2$. Notably, equation (4) saturates the quantum CRB given in Eq. (1) since $2|\alpha_0\beta_0|\sinh(4r) = N_{\rm c}\sinh(4r)$ in this case. This implies that the parity detection is the optimal measurement scheme for our hybrid interferometer.

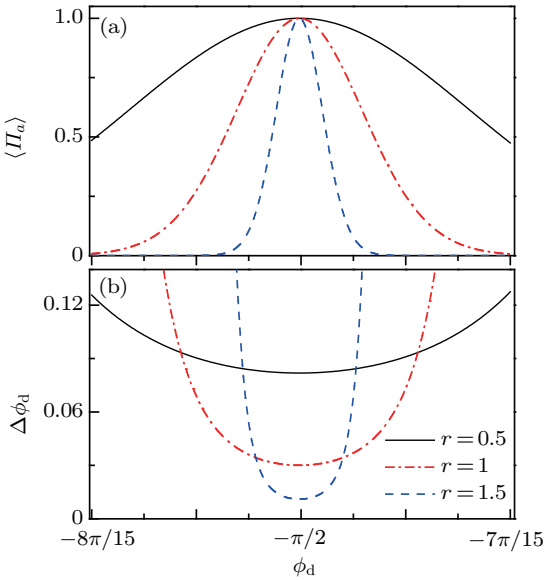


Fig. 2. (color online) (a) The parity signal $\langle\Pi_a\rangle$ and (b) the phase sensitivity $\Delta\phi_{\rm d}$ as a function of differential phase $\phi_{\rm d} = \varphi_a - \varphi_b$ when r takes different values, where $|\alpha_0|^2 = |\beta_0|^2 = N_{\rm c}/2 = 10$.

In Fig. 2, we plot the parity signal $\langle\Pi_a\rangle$ and the phase sensitivity $\Delta\phi_{\rm d}$ as a function of differential phase $\phi_{\rm d} = \varphi_a - \varphi_b$. There is a central peak at $\phi_{\rm d} \to -\pi/2$ for the parity signal, and the width of the peak becomes narrower and narrower when increasing the value of r (see Fig. 2(a)). This leads to an increased magnitude of slope (i.e., increased $|\partial\langle\Pi_a\rangle/\partial\phi|$) around $\phi_{\rm d} \to -\pi/2$, and hence the phase sensitivity is improved. From Fig. 2(b), we can clearly see that the best phase sensitivity given by CRB is found at $\phi_{\rm d} \to -\pi/2$; and for fixed photon number $N_{\rm c}$, when increasing r the phase uncertainty decreases gradually. This point can be seen from Eq. (4).

We now use the numerical techniques to investigate the optimal phase sensitivity $\Delta\phi_{\rm dmin}$ in the case of $|\alpha_0| \neq |\beta_0|$, namely, $k = |\alpha_0|^2/N_{\rm c}$, by setting $N_{\rm c} = 500$. From Fig. 3, we find that the optimal sensitivity is reached around $k = 0.5$, but it is also shown that, if $r > 1.5$ the values of the phase sensitivity hardly have changes when $N_{\rm c} = 500$. In other words, it is insensitive to the value of k. Hence, we can safely say that as long as r is not too small, for a fixed total amount of coherent light $N_{\rm c}$, the phase sensitivity given in Eq. (4) can still be acquired.

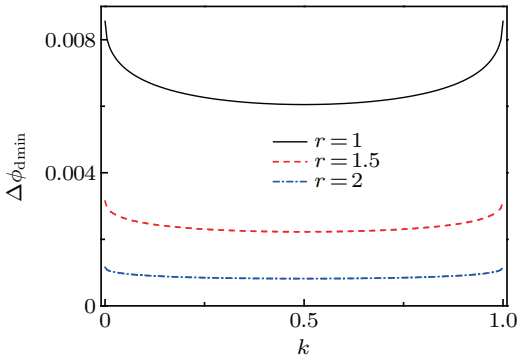


Fig. 3. (color online) The quantum CRB $\Delta\phi_{\rm dmin}$ as a function of the fraction $k \equiv |\alpha_0|^2/N_{\rm c}$ when r takes different values. Here the total coherent photon number is $N_{\rm c} = 500$.

It is worthwhile noting that in the high power regime (i.e., for large $N_{\rm c}$ or large r), under our consideration, the accurate number-resolving may be a difficult task. However, there are some proposals[25,30] which can implement the parity detection while they do not need to have photon number counting. These efforts confirm that the high-intensity input states scheme can be constructed with current technology, and then reach the sub-SNL of phase sensitivity. Below, we shall compare our method with another high-intensity quantum-enhanced method presented by Caves.

4. Comparison with Caves' scheme

The scheme presented by Caves employs a high-intensity coherent state and a low-intensity squeezed vacuum state as the input states of an MZI,[1] and the quantum CRB turns out to be[22,35] $\Delta\phi'_{\rm d\,min} = 1/\sqrt{N_{\rm c}\,{\rm e}^{2r} + \sinh^2 r}$. Such a phase sensitivity can be realized either by implementing parity detection[25] or using Bayesian parameter estimation.[22] Now, we will demonstrate the advantages of our scheme when comparing with Caves' scheme. First, we consider an interesting case of $N_{\rm c} \gg \sinh^2 r$, from Eq. (4) we immediately have $\Delta\phi_{\rm d\,min} \simeq {\rm e}^{-2r}/\sqrt{N_{\rm c}}$, while for Caves' scheme we have $\Delta\phi'_{\rm d\,min} \simeq {\rm e}^{-r}/\sqrt{N_{\rm c}}$. This means that our scheme is exponentially better than Caves'. However, this is not to say that our scheme is directly more efficient, since the total photon numbers are different between these two schemes. To make a fair comparison, below we introduce the OPSG with respect to the SNL, which is defined as[12]

$$g_{\rm min} \equiv -10\log_{10}(\Delta\phi_{\rm min}/\Delta\phi_{\rm SNL}). \tag{5}$$

For our scheme, we have

$$g_{\rm min} = -5\log_{10}\left[(N_{\rm c}\,{\rm e}^{4r} + \sinh^2 2r)/(N_{\rm c}\cosh(2r) + 2\sinh^2 r)\right].$$

Whereas for Caves' scheme, the OPSG is given by

$$g'_{\min} = -5\log_{10}\left[(N_c + \sinh^2 r)/(N_c\, e^{2r} + \sinh^2 r)\right],$$

since the total photon number in this scheme is $N'_T = N_c + \sinh^2 r$. If $g_{\min} > g'_{\min}$, it indicates that our scheme is more efficient than Caves'.

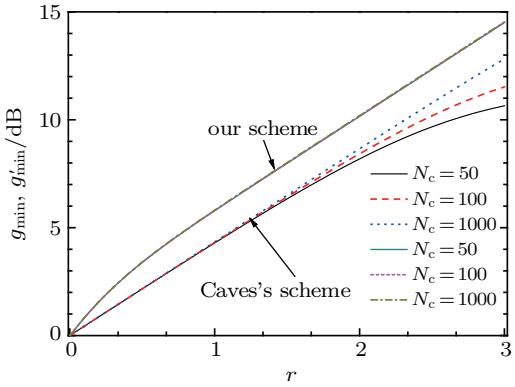


Fig. 4. (color online) A comparison of the optimal phase sensitivity gains between our scheme $g_{\min}$ and Caves' scheme $g'_{\min}$ as a function of squeezing parameter r for various coherent photon numbers N_c. Note that in our scheme the values of OPSG are independent of N_c.

In Fig. 4, we compare the OPSG between our scheme ($g_{\min}$) and Caves' scheme ($g'_{\min}$) as a function of squeezing parameter r for various coherent photon numbers N_c. As can be seen, the phase sensitivity gains increase when increasing the squeezing parameter r, and we always have $g_{\min} > g'_{\min}$ in the whole range of r. For our scheme, when $r = 1$ it is possible to overcome the SNL by 5.8 dB, while reaching 14.6 dB when $r = 3$ (this two-mode squeezed parameter is available in current experience[37]); and these results are independent of the coherent photon numbers N_c. In contrast to these, for Caves' scheme we have $g'_{\min} \simeq 4.2$ dB with $r = 1$, and for larger r there are bifurcations when increasing coherent photon numbers N_c. It can be seen that the larger N_c is, the better OPSG can be obtained. For example, when $r = 3$ we have $g'_{\min} \simeq 10.5$ dB with $N_c = 50$, while $g'_{\min} \simeq 13$ dB with $N_c = 1000$.

5. Conclusion

In summary, we have investigated a hybrid scheme for optical interferometry, which assembles the advantages of both passive and active interferometers. This scheme can reach a scaling far below the SNL for phase estimation with bright sources. Comparing with the pure active interferometer schemes with coherent light input, our protocol can obtain a better QCR, and precisely this can be achieved when a parity measurement is utilized. Furthermore, we have compared our scheme with Caves' scheme; it is found that our scheme is not only more sensitive, but also more efficient when the same total photon number is input.

References

[1] Caves C M 1981 *Phys. Rev. D* **23** 1693
[2] Yurke B, McCall S L and Klauder J R 1986 *Phys. Rev. A* **33** 4033
[3] Sanders B C, Milburn G J and Zhang Z 1997 *J. Mod. Opt.* **44** 1309
[4] Plick W N, Dowling J P and Agarwal G S 2010 *New J. Phys.* **12** 083014
[5] Marino A M, Corzo Trejo N V and Lett P D 2012 *Phys. Rev. A* **86** 023844
[6] Holland M J and Burnett K 1993 *Phys. Rev. Lett.* **71** 1355
[7] Dorner U, Demkowicz-Dobrzanski R, Smith B J, Lundeen J S, Wasilewski W, Banaszek K and Walmsley I A 2009 *Phys. Rev. Lett.* **102** 040403
[8] Giovannetti V, Lloyd S and Maccone L 2011 *Nat. Photon.* **5** 222
[9] Sanders B C and Milburn G J 1995 *Phys. Rev. Lett.* **75** 2944
[10] Ma J, Wang X, Sun C P and Nori F 2011 *Phys. Rep.* **509** 89
[11] Humphreys P C, Barbieri M, Datta A and Walmsley I A 2013 *Phys. Rev. Lett.* **111** 070403
[12] Pezzé L and Smerzi A 2013 *Phys. Rev. Lett.* **110** 163604
[13] Dowling J P 2008 *Contemp. Phys.* **49** 125
[14] Lvovsky A I, Sanders B C and Tittel W 2009 *Nat. Photon.* **3** 706
[15] Escher B M, de Matos Filho R L and Davidovich L 2011 *Nat. Phys.* **7** 406
[16] Hudelist F, Kong J, Liu C, Jing J, Ou Z Y and Zhang W P 2014 *Nat. Commun.* **5** 3049
[17] Yue J D, Zhang Y R and Fan H 2014 *Sci. Rep.* **4** 5933
[18] Tan Q S, Liao J Q, Wang X and N Franco 2014 *Phys. Rev. A* **89** 053822
[19] Liu G Q, Zhang Y R, Chang Y C, Yue J Do, Fan H and Pan X Y 2015 *Nat. Commun.* **6** 6726
[20] Wang Y T, Tang J S, Hu G, Wang J, Yu S, Zhou Z Q, Cheng Z D, Xu J S, Fang S Z, Wu Q L, Li C F and Guo G C 2016 *Phys. Rev. Lett.* **117** 230801
[21] Wei C P, Hu X Y, Yu Y F and Zhang Z M 2016 *Chin. Phys. B* **25** 040601
[22] Pezzé L and Smerzi A 2008 *Phys. Rev. Lett.* **100** 073601
[23] Jarzyna M and Demkowicz-Dobrzański R 2012 *Phys. Rev. A* **85** 011801
[24] Anisimov P M, Chiruvelli A, Plick W N, Huver S D, Lee H and Dowling J P 2010 *Phys. Rev. Lett.* **104** 103602
[25] Plick W N, Anisimov P M, Dowling J P, Lee H and Agarwal G S 2010 *New J. Phys.* **12** 113025
[26] Cramer H 1946 *Mathematical Methods of Statistics* (Princeton: Princeton University) p. 500
[27] Helstrom C W 1976 *Quantum Detection and Estimation Theory* (New York: Academic Press) p. 20
[28] Holevo A S 1982 *Probabilistic and Statistical Aspects of Quantum Theory* (Amsterdam: North-Holland) p. 110
[29] Braunstein S L and Caves C M 1994 *Phys. Rev. Lett.* **72** 3439
[30] Royer A 1977 *Phys. Rev. A* **15** 449
[31] Gerry C C 2000 *Phys. Rev. A* **61** 043811
[32] Gerry C C and Mimih J 2010 *Contemp. Phys.* **51** 497
[33] Gerry C C and Mimih J 2010 *Phys. Rev. A* **82** 013831
[34] Chiruvelli A and Lee H 2011 *J. Mod. Opt.* **58** 945
[35] Seshadreesan K P, Anisimov P M, Lee H and Dowling J P 2011 *New J. Phys.* **13** 083026
[36] Seshadreesan K P, Kim S, Dowling J P and Lee H 2013 *Phys. Rev. A* **87** 043833
[37] Martini F D, Sciarrino F and Vitelli C 2008 *Phys. Rev. Lett.* **100** 253601