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Plasma-screening effects on positronium formation*

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Plasma-screening effects on positronium (Ps) formation for positron–hydrogen collisions in a Debye plasma environment is further investigated using the screening approximation model with the inclusion of the modified structure of Ps. More accurate Ps formation cross sections ($n = 1,2$) are obtained for various Debye lengths from the Ps formation thresholds to 50 eV. The influence of considering modified bound-state wave functions and eigenenergies for the Ps is found to result in the reduction of the Ps formation cross sections at low energies, whereas it cannot counteract the enhancement of the Ps formation by the Debye screening.

Keywords: plasma-screening effect, positron, scattering, positronium formation

PACS: 34.80.Uv, 52.20.Hv, 34.80.Lx **DOI:** 10.1088/1674-1056/27/1/013401

1. Introduction

Coulomb interaction screening in classical hot, dense plasmas has been the subject of extensive study for several decades.[1,2] This collective and many-particle effect can be described by the Debye–Hückel potential,[1,2] which is characterized by the Debye screening length D, in a pair correlation approximation. A particular value of D corresponds to a range of plasma conditions. Smaller values of D correspond to stronger screening conditions. A recent review[3] on atomic collisions in plasmas provided a comprehensive overview of the recent studies in atomic physics in Debye plasmas, including the work on atomic structure, photon collisions, electron collisions, and heavy particle collisions in a wide range of plasma screening conditions.

In recent years, collision processes involving positrons in hot and dense plasmas have attracted growing interest not only because of the conclusive proof of the existence of positrons in the interstellar medium,[4] but also because of the wide applications in diverse fields of physics and astrophysics.[5–8] A number of works[9–20] have been carried out to study the positron–atom elastic, excitation, rearrangement processes, and resonances in plasma-embedded positron–atom systems. It has been noted that Debye screening profoundly affects the dynamics of the collision processes.

Positronium (Ps) formation, which is a special rearrangement process in positron–atom collision, has been found to significantly affect the scattering process for low projectile energies in a vacuum.[21–30] As the screening effect increases in Debye plasmas, the Ps-formation threshold decreases, which results in a more important Ps process at low scattering energies. Moreover, the rate of Ps plays a decisive role in the rate of annihilation and measurement of the spectrum and distribution of the annihilation radiation can provide us with fruitful information to develop our understanding regarding the physical properties of the plasma medium.[10] It is worth investigating the Ps formation process in Debye plasmas at low projectile energies and obtain a more accurate Ps formation cross section.

Recently, we reported the screening approximation model results[18] of positron scattering on the hydrogen atom in Debye plasmas, where interesting features were observed with the reduction of Debye length D, such as the enhancement of the Ps formation cross sections at low energies and the appearance of the local minima in the total Ps formation cross sections ($n = 1+2$). The influence of the plasma environment on the eigenenergies and wave functions of a hydrogen atom and the Coulomb potentials was taken into account in that work, while the variation of the bound states of Ps formation in Debye plasmas was omitted. It should be noted that screened Coulomb potential also affects the structures of Ps states. As the Debye length D decreases, the binding energy of the Ps decreases and the wave function peaks decrease, broaden, and become more diffuse. The importance of considering modified bound-state wave functions and eigenenergies for the Ps was pointed out by Sen *et al.*[10] when they studied Ps formation in positron–helium collisions with a screened Coulomb interaction. In this work, we have taken account of the modified structure of the Ps in order to obtain a more accurate Ps cross section.

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In the screening approximation model, a complex polarization potential has been derived to describe Ps formation in positron–atom systems. The motion of the positronium center of mass is represented by the plane wave, while the bound state of Ps is chosen to be orthogonalized to the initial state of a hydrogen atom, from which the electron is captured. The Ps formation cross sections ($n = 1, 2$) in positron–hydrogen collisions for various Debye lengths from the Ps formation thresholds to 50 eV are calculated using the screening approximation model, taking account of the influence of the plasmas screening on the Coulomb potentials and the structures of target states and Ps states.

2. Model and method

The Hamiltonian for the positron–hydrogen system in Debye plasmas is given by

$$\begin{aligned} H &= K_1 + K_2 + V_1 + V_2 + V_{12} \\ &= -\frac{1}{2}\nabla_1^2 - \frac{1}{2}\nabla_2^2 - \frac{\exp\left(-r_1 D^{-1}\right)}{r_1} \\ &\quad + \frac{\exp\left(-r_2 D^{-1}\right)}{r_2} - \frac{\exp\left(-r_{12} D^{-1}\right)}{r_{12}}, \end{aligned} \tag{1}$$

where K_1 and K_2 stand for the kinetic energy operators of the electron and the positron, V_1 and V_2 are the electron–nucleus and positron–nucleus potential operators, V_{12} is the positron–electron interaction potential operator, r_1 and r_2 are the coordinates of electron and positron relative to the nucleus respectively, and r_{12} is their relative distance. The Debye screening length D is a function of electron density n_e and electron temperature T_e, as

$$D = (k_\mathrm{B} T_\mathrm{e} / 4\pi^2 n_\mathrm{e})^{1/2}, \tag{2}$$

where k_B is the Boltzmann constant.

A polarization potential $W_\mathrm{Ps}^{(Q)}$ is set up to describe the positronium formation. The form used for the matrix element of $W_\mathrm{Ps}^{(Q)}$, which has been described in the work of Zhou *et al.*[31] in detail, is given as

$$\begin{aligned} \langle \boldsymbol{k}' i | W_\mathrm{Ps}^{(Q)} | j\boldsymbol{k} \rangle &= \sum_{n \in Q} \langle \boldsymbol{k}' i | V_2 + V_{12} | X_n^{(-)} \rangle \\ &\quad \times \frac{1}{E^{(+)} - \epsilon_n - K_\mathrm{Ps}^2} \langle X_n^{(-)} | V_2 + V_{12} | j\boldsymbol{k} \rangle, \end{aligned} \tag{3}$$

where $X_n^{(-)}$ represents the time reverse of the exact state vector for a reaction starting in channel n. The notation n is a discrete notation for the two-body positronium formation channels. $\boldsymbol{K}_\mathrm{Ps}$ is the momentum of the positronium center of mass.

The matrix element $\langle X_n^{(-)} | V_2 + V_{12} | j\boldsymbol{k} \rangle$ (j is the ground state of hydrogen target) under Debye screening is written as

$$\begin{aligned} \langle X_n^{(-)} | V_2 + V_{12} | j\boldsymbol{k} \rangle &= \int \mathrm{d}r_1 \int \mathrm{d}r_2 \varphi_\mu^*(\boldsymbol{r}_{12}) \exp\left(-\mathrm{i}\boldsymbol{K}_\mathrm{Ps}\right) \\ &\quad \times \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2) \left[-\frac{\exp(-r_{12} D^{-1})}{r_{12}} \right. \\ &\quad \left. + \int \mathrm{d}^3 r_2 \frac{|\phi(\boldsymbol{r}_2)|^2 \exp(-r_{12} D^{-1})}{r_{12}} \right] \\ &\quad \times \phi_{1s}(\boldsymbol{r}_2) \exp(\mathrm{i}\boldsymbol{k}_2 \cdot \boldsymbol{r}_2). \end{aligned} \tag{4}$$

The positron–electron interaction term is expanded as

$$\begin{aligned} \frac{\exp\left(-r_{12} D^{-1}\right)}{r_{12}} &= -D^{-1} \sum_{l=0}^{\infty} (2l+1) \mathrm{j}_l(\mathrm{i} D^{-1} r_<) \\ &\quad \times \mathrm{h}_l^{(1)}(\mathrm{i} D^{-1} r_>) \mathrm{P}_l(\cos\theta), \end{aligned} \tag{5}$$

where $r_> = \max(r_1, r_2), r_< = \min(r_1, r_2)$, and P_l, j_l, and $\mathrm{h}_l^{(1)}$ denote the Legendre polynomials, the spherical Bessel functions, and the spherical Hankel functions of the first kind, respectively.

The model used for the positronium formation is

$$\left| X_n^{(-)} \right\rangle = \left(1 - \sum_j |\phi_j\rangle \langle \phi_j| \right) \left| x_n^{(-)} \right\rangle. \tag{6}$$

$|X_n^{(-)}\rangle$ satisfies the orthogonalization to the orbital ϕ_i, from which the electron is captured, and

$$\left| x_n^{(-)} \right\rangle = \left| \varphi_\mu \boldsymbol{K}_\mathrm{Ps} \right\rangle, \tag{7}$$

where φ_μ is the bound state of the Ps. The plane wave represents the motion of positronium, since only short-range terms in the positron–ion potential survive.

In the Debye plasma environment, the eigenenergies of hydrogen target ϵ_n and target functions ϕ_j are calculated by using piecewise exact power series expansions of the radial functions.[32] Highly correlated bound-state wave functions and eigenenergies of the Ps are calculated by Sabyasachi Kar.[33–35] In Table 1, we present the binding energies of Ps for unscreened ($D = \infty$) and screened cases with $D = 100$, 50, 40, 30, and 25 a.u. Radial probability distribution function of the 1s, 2s, and 2p states are presented in Figs. 1–3 for various Debye lengths. The excited states are effected by the Debye screening much more than the ground state.

Table 1. Energy levels of nl states for positronium formation under different Debye lengths in Rydberg units.

D	1s	2s	2p
∞	−0.250	−0.0625	−0.0625
100	−0.240	−0.0531	−0.0530
50	−0.231	−0.0474	−0.0471
40	−0.226	−0.0409	−0.0404
30	−0.281	−0.0350	−0.0342
25	−0.212	−0.0307	−0.0296

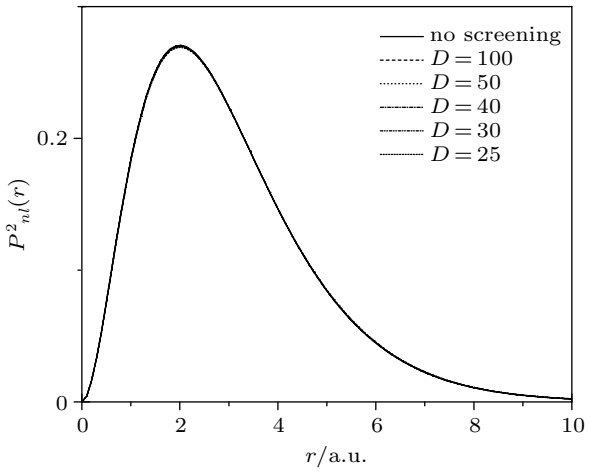


Fig. 1. Radial probability distribution function of the 1s positronium formation under different Debye lengths.

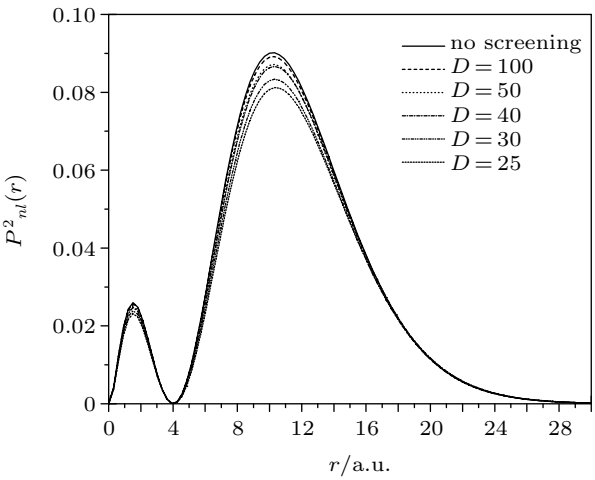


Fig. 2. Radial probability distribution function of the 2s positronium formation under different Debye lengths.

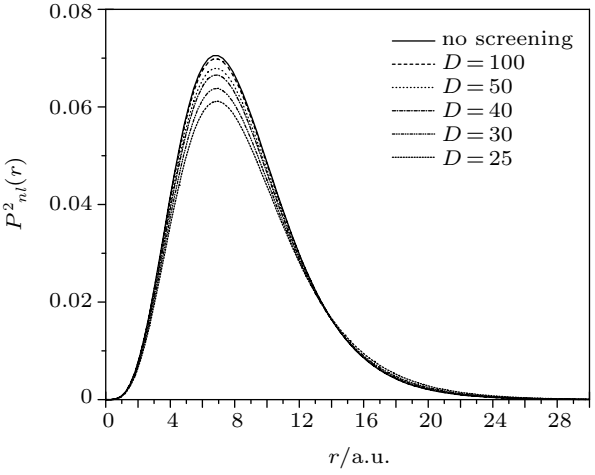


Fig. 3. Radial probability distribution function of the 2p positronium formation under different Debye lengths.

A simple estimate of the total cross section of Ps formation is[36,37]

$$\sigma_{\rm Ps} = \frac{2}{k}(2\pi)^3 {\rm Im}\,\langle \boldsymbol{k}0|\, W^{(Q)}_{\rm Ps}\,|0\boldsymbol{k}\rangle\,. \tag{8}$$

3. Results and discussion

The Ps ($n=1$), Ps ($n=2$), and total Ps ($n=1+2$) formation cross sections in positron–hydrogen collisions for various Debye lengths, along with that for a free hydrogen atom, are presented in Tables 2–4. These results are reasonably accurate, because the quality of the present description for Ps formation cross sections has been checked.[18]

Table 2. Positronium ($n=1$) formation cross section (in units of πa_0^2) as a function of incident positron energy (in units of eV) in positron–hydrogen collisions for various Debye lengths D. The notation $a[b]$ stands for $a\times 10^b$.

E	$D=\infty$	$D=100$	$D=50$	$D=40$	$D=30$	$D=25$
6.8			3.300[−3]	4.800[−3]	5.510[−3]	1.306[−2]
6.9	4.700[−2]	5.044[−2]	5.575[−2]	5.847[−2]	5.985[−2]	7.299[−2]
7	1.170[−1]	1.240[−1]	1.310[−1]	1.350[−1]	1.360[−1]	1.530[−1]
8	1.133	1.161	1.172	1.178	1.181	1.207
9	2.013	2.060	2.069	2.074	2.078	2.101
10	2.604	2.664	2.670	2.675	2.677	2.694
11	2.954	3.021	3.025	3.028	3.030	3.042
12	3.130	3.120	3.201	3.203	3.204	3.211
13	3.183	3.255	3.253	3.254	3.254	3.257
14	3.155	3.224	3.222	3,223	3.222	3.222
15	3.075	3.141	3.138	3.138	3.137	3.135
16	2.961	3.024	3.020	3.020	3.018	3.015
17	2.830	2.887	2.883	2.882	2.881	2.876
18	2.687	2.741	2.736	2.735	2.734	2.728
19	2.541	2.590	2.586	2.585	2.583	2.577
20	2.396	2.441	2.436	2.435	2.433	2.427
25	1.745	1.773	1.769	1.768	1.766	1.760
30	1.262	1.278	1.275	1.274	1.272	1.268
35	9.058[−1]	9.297[−1]	9.276[−1]	9.266[−1]	9.252[−1]	9.217[−1]
40	6.790[−1]	6.852[−1]	6.837[−1]	6.829[−1]	6.817[−1]	6.791[−1]
50	3.860[−1]	3.903[−1]	3.895[−1]	3.890[−1]	3.883[−1]	3.868[−1]

Table 3. Positronium ($n=2$) formation cross section (in units of πa_0^2) as a function of incident positron energy (in units of eV) in positron–hydrogen collisions for various Debye lengths D. The notation $a[b]$ stands for $a\times 10^b$.

E	$D=\infty$	$D=100$	$D=50$	$D=40$	$D=30$	$D=25$
11.8					5.560[−3]	3.511[−2]
11.9		1.129[−2]	2.575[−2]	3.501[−2]	4.567[−2]	5.782[−2]
12	3.940[−2]	4.481[−2]	5.175[−2]	5.638[−2]	6.403[−2]	7.368[−2]
13	1.250[−1]	1.361[−1]	1.412[−1]	1.446[−1]	1.506[−1]	1.580[−1]
14	1.620[−1]	1.775[−1]	1.844[−1]	1.887[−1]	1.963[−1]	2.048[−1]
15	1.812[−1]	2.042[−1]	2.130[−1]	2.182[−1]	2.272[−1]	2.369[−1]
16	1.980[−1]	2.225[−1]	2.329[−1]	2.389[−1]	2.490[−1]	2.595[−1]
17	2.080[−1]	2.350[−1]	2.465[−1]	2.531[−1]	2.640[−1]	2.750[−1]
18	2.150[−1]	2.431[−1]	2.553[−1]	2.622[−1]	2.735[−1]	2.847[−1]
19	2.160[−1]	2.475[−1]	2.602[−1]	2.671[−1]	2.787[−1]	2.897[−1]
20	2.170[−1]	2.490[−1]	2.618[−1]	2.687[−1]	2.802[−1]	2.909[−1]
25	1.970[−1]	2.283[−1]	2.393[−1]	2.450[−1]	2.543[−1]	2.623[−1]
30	1.630[−1]	1.884[−1]	1.966[−1]	2.007[−1]	2.072[−1]	2.126[−1]
35	1.361[−1]	1.486[−1]	1.543[−1]	1.572[−1]	1.616[−1]	1.652[−1]
40	1.010[−1]	1.150[−1]	1.190[−1]	1.210[−1]	1.240[−1]	1.263[−1]
50	6.000[−2]	6.866[−2]	7.061[−2]	7.154[−2]	7.296[−2]	7.399[−2]

Table 4. Positronium ($n = 1+2$) formation cross section (in units of πa_0^2) as a function of incident positron energy (in units of eV) in positron–hydrogen collisions for various Debye lengths D. The notation $a[b]$ stands for $a \times 10^b$.

E	$D=\infty$	$D=100$	$D=50$	$D=40$	$D=30$	$D=25$
6.8			3.300[−3]	4.800[−3]	5.510[−3]	1.306[−2]
6.9	4.700[−2]	5.044[−2]	5.575[−2]	5.847[−2]	5.985[−2]	7.299[−2]
7	1.170[−1]	1.240[−1]	1.310[−1]	1.350[−1]	1.360[−1]	1.530[−1]
8	1.133	1.161	1.172	1.178	1.181	1.207
9	2.013	2.060	2.069	2.074	2.078	2.101
10	2.604	2.664	2.670	2.675	2.677	2.694
11	2.954	3.022	3.025	3.028	3.030	3.042
12	3.169	3.245	3.252	3.259	3.268	3.284
13	3.308	3.390	3.394	3.398	3.405	3.415
14	3.316	3.402	3.407	3,411	3.418	3.427
15	3.258	3.345	3.351	3.356	3.364	3.372
16	3.159	3.247	3.253	3.259	3.268	3.274
17	3.036	3.123	3.130	3.135	3.145	3.151
18	2.900	2.984	2.992	2.997	3.007	3.013
19	2.758	2.838	2.846	2.852	2.861	2.867
20	2.612	2.690	2.698	2.704	2.713	2.718
25	1.942	2.001	2.008	2.013	2.020	2.022
30	1.424	1.467	1.472	1.475	1.480	1.480
35	9.058[−1]	1.078	1.082	1.084	1.087	1.087
40	7.800[−1]	8.003[−1]	8.027[−1]	8.038[−1]	8.057[−1]	8.054[−1]
50	4.460[−1]	4.590[−1]	4.601[−1]	4.605[−1]	4.612[−1]	4.608[−1]

From the tables, we notice that the peak of the positronium formation is enhanced with the decreasing of Debye length at low energies, which is consistent with the previous result[18] that the effect of screening causes an enhancement in the values of the Ps formation cross sections. However, the increase is not substantial as the previous results in which the modified structure of the Ps in Debye plasmas was not taken into consideration. For example, as shown in Table 3, the value of the Ps ($n = 2$) formation cross section for $D = 50$ a.u. at 12 eV is 0.05175 πa_0^2, while in the previous calculations the value is 0.12061 πa_0^2.[18] It means that the influence of Debye screening is reduced with the inclusion of the modified structure of the Ps.

In order to have a clear insight into the effect of the inclusion of the modified structure of the Ps, a comparison is displayed between the previous results[18] and the present results, namely OA and OB, respectively. The OA and OB results of Ps ($n = 1$), Ps ($n = 2$), and total Ps ($n = 1+2$) formation cross sections for Debye lengths $D = 100$, 50, and 25 a.u. are displayed in Figs. 4(a)–4(i). As shown in Figs. 4(a)–4(i), the Ps formation threshold energy increases once considering the modified structure of the Ps in Debye plasmas, which occurs as a result of the decreasing binding energy of the Ps. From the comparison of the OA and OB results in Fig. 4, we observe that the inclusion of the modified structure of Ps causes a shift of the maximum position of the Ps formation cross section to higher energies and a reduction in its value.

Moreover, the value of the Ps formation cross section decreases increasingly as the screening length decreases. That is, values of the Ps formation cross sections are reduced by considering the modified structure of Ps by the screened Coulomb potential, and this influence is enhanced with the increasing screening effect. Thus, slight differences exist among the values of the Ps formation cross sections under different Debye lengths at the same scattering energy, as can be seen in Tables 2 and 4. However, Ps formation cross sections in the presence of the plasmas are still larger than that of the free hydrogen atom case at low energies, which means that the influence of the inclusion of the modified structure of Ps cannot counteract the enhancement of the Ps formation by the Debye screening. The screened interaction, described by the Debye–Hückel potential, always results in the enhancement of the Ps formation cross sections at low energies.

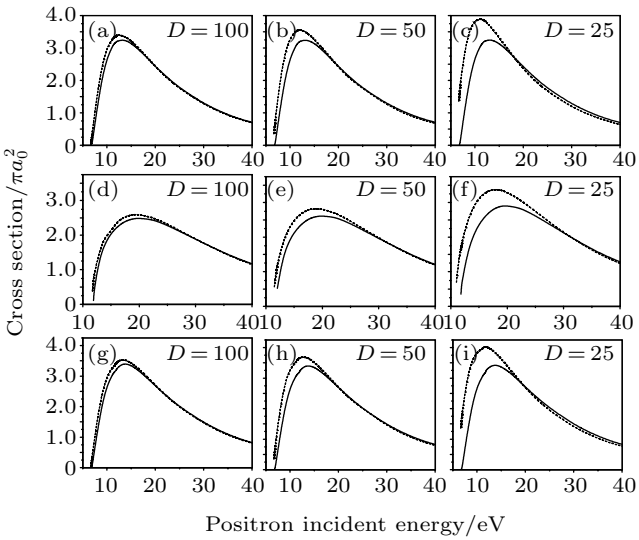


Fig. 4. A comparison of positronium formation cross sections under different Debye lengths between the previous calculations in Ref. [18] (dashed lines) and the present calculations (solid lines) with the inclusion of the modified structure of the Ps. (a)–(c): Ps ($n = 1$) formation cross sections, (d)–(f): Ps ($n = 2$) formation cross sections, and (g)–(i): Ps ($n = 1+2$) formation cross sections.

In Figs. 5 and 6, we compare our results of Ps ($n = 1$) formation cross sections for Debye lengths $D = 50$ and 100 with the corresponding results obtained by using different theoretical approaches.[9,11] The previous results[18] are also shown. From Fig. 5, the present results are in closer agreement with the theoretical results of Sen *et al.*[9] than the previous results.[18] From Fig. 6, the agreement between the present results and the results of Ghoshal *et al.*[11] is excellent above 20 eV. At lower impact energies, the previous results[18] are closer to the results of Ghoshal *et al.*[11] It is probably attributed to the present inclusion of the modified structure of Ps.

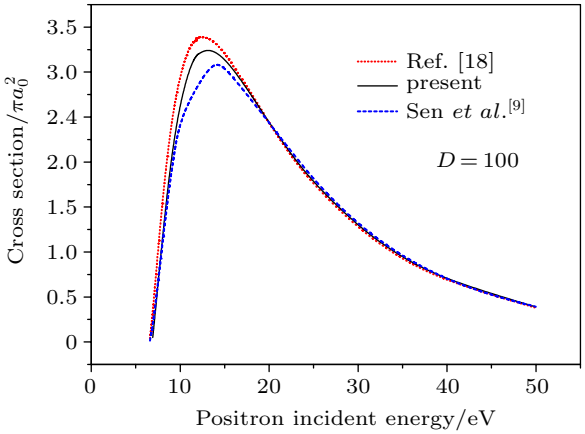


Fig. 5. (color online) A comparison of positronium ($n = 1$) formation cross section (in units of πa_0^2) among present results (solid lines), previous results[18] (dot lines), and the results of Sen *et al.*[9] (dash lines) in positron–hydrogen collisions for $D = 100$.

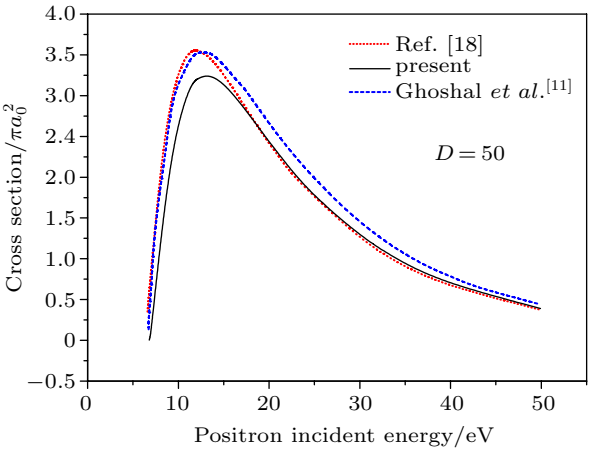


Fig. 6. (color online) A comparison of positronium ($n = 1$) formation cross section (in units of πa_0^2) among present results (solid lines), previous results[18] (dot lines), and the results of Ghoshal *et al.*[11] (dash lines) in positron–hydrogen collisions for $D = 50$.

4. Conclusion

A screening approximation model has been employed to calculate the Ps formation cross sections ($n = 1, 2$) in positron–hydrogen collisions for various Debye lengths from the Ps formation thresholds to 50 eV, taking account of the influence of the plasmas screening on the Coulomb potentials and the structures of target states and Ps states. The influence of considering modified bound-state wave functions and eigenenergies for the Ps by the screened Coulomb potential on the Ps formation process is investigated for the first time. Our present investigation reveals that it reduces the values of the Ps formation cross section at low energies and this influence is enhanced as the screening effect increases, whereas it cannot counteract the enhancement of the Ps formation by the Debye screening. The screened Coulomb potential always increases the values of the Ps formation cross sections at low energies. With the inclusion of the modified structure of Ps, we obtain a more accurate Ps formation cross section in this charge transfer process. We hope that our present investigate will provide useful information to plasma physics, astrophysics, and positron physics.

References

[1] Salzman D 1998 *Atomic Physics in Hot Plasmas* (Oxford: Oxford University Press)
[2] Murillo M S and Weisheit J C 1998 *Phys. Rep.* **302** 1
[3] Janev R K, Zhang S B and Wang J G 2016 *Matter and Radiation at Extremes* **1** 237
[4] Weidenspointner G, Skinner G, Jean P, Knodlseder J, von Ballmoons P, Bignami G, Diehl R, Strong A W, Cordier B, Schanne S and Winkler C 2008 *Nature* **451** 159
[5] Frey A R and Reid N B 2013 *Phys. Rev. D* **87** 103508
[6] Bell A R and Kirk J G 2008 *Phys. Rev. Lett.* **101** 200403
[7] Chupp E L, Forrest D J, Higbie P R, Suri A N, Tsai C and Dunphy P P 1973 *Nature* **241** 333
[8] Amoretti G M, Amsler C, Bonomi G M, Bouchta A, Bowe P D, Carraro C, Cesar C L, Charlton M, Doser M, Filippini V, Fontana A, Fujiwara M C, Funakoshi R, Genova P, Hangst J S, Hayano R S, Jurgensen L V, Lagomarsino V, Landua R, Lindelof D, Lodi Rizzini E, Macri M, Madsen N, Manuzio G, Montagna P, Pruys H, Regenfus C, Rotondi A, Testera G, Variola A, and van der Werf D P 2003 *Phys. Rev. Lett.* **91** 055001
[9] Sen S, Mandal P and Mukherjee 2011 *Euro. Phys. J. D* **62** 379
[10] Sen S, Mandal P and Mukherjee 2012 *Euro. Phys. J. D* **66** 230
[11] Ghoshal A, Kamali M Z M and Ratnavelu 2013 *Phys. Plasmas* **20** 013506
[12] Rej P and Ghoshal A 2014 *Phys. Plasmas* **21** 093507
[13] Nayek S and Ghoshal A 2012 *Phys. Plasmas* **19** 113501
[14] Nayek S and Ghoshal A 2012 *Phys. Scr.* **85** 035301
[15] Nayek S and Ghoshal A 2011 *Euro. Phys. J. D* **64** 257
[16] Zhang S B, Qi Y Y, Qu Y Z, Chen X J and Wang J G 2010 *Chin. Phys. Lett.* **27** 013401
[17] Ghoshal A, Nayek S, Kamali M Z M and Ratnavelu K 2014 *AIP Conf. Proc.* **1588** 94
[18] Ma J, Cheng Y, Wang Y C and Zhou Y 2012 *Phys. Plasmas* **19** 063303
[19] Rej P and Ghoshal A 2016 *J. Phys. B-At. Mol. Opt. Phys.* **49** 125203
[20] Pandey M K, Lin Y C and Ho Y K 2016 *J. Phys. B-At. Mol. Opt. Phys.* **49** 034007
[21] Ma J, Cheng Y, Wang Y C and Zhou Y 2011 *J. Phys. B-At. Mol. Opt. Phys.* **44** 175203
[22] Utamuratov R, Kadyrov A S, Fursa D V, Bray I and Stelbovics A T 2010 *Phys. Rev. A* **82** 042705
[23] Liu F, Cheng Y J and Zhou Y J 2012 *Chin. Phys. B* **21** 053403
[24] Yu R M, Cheng Y J, Wang Y and Zhou Y J 2012 *Chin. Phys. B* **21** 053402
[25] Ma J, Zhou Y J and Wang Y C 2012 *Chin. Phys. B* **21** 123403
[26] Cheng Y J, Zhou Y J and Jiao L G 2012 *Chin. Phys. B* **21** 013405
[27] Cheng Y J and Zhou Y J 2010 *Chin. Phys. B* **19** 063405
[28] Lin L, Wang H N and Jiao L G 2017 *Chin. Phys. B* **26** 033401
[29] Wu X G, Cheng Y J, Liu F and Zhou Y J 2017 *Chin. Phys. B* **26** 023401
[30] Yu R M, Pu C Y, Huang X Y, Ying F R, Liu X Y, Jiao L G and Zhou Y J 2016 *Chin. Phys. B* **25** 073401
[31] Zhou Y, Ratnavelu K and McCarthy I E 2005 *Phys. Rev. A* **71** 042703
[32] Salvat F, Fernandez-Varea J M and Williamson W 1995 *Comput. Phys. Commun.* **90** 151
[33] Kar S and Ho Y K 2006 *Chem. Phys. Lett.* **424** 403
[34] Kar S and Ho Y K 2006 *Phys. Rev. A* **73** 032502
[35] Kar S and Ho Y K 2009 *Few-Body Syst.* **46** 173
[36] McCarthy I E and Zhou Y 1994 *Phys. Rev. A* **49** 4597
[37] McCarthy I E and Stelbovics A T 1980 *Phys. Rev. A* **22** 502