ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理
Chinese Physics B

Photoelectron longitudinal momentum distributions containing nondipole effects

Yi-Ning Huo(霍一宁), Jian Li(李健), Feng-Cai Ma(马凤才)
Citation: Chin. Phys. B . 2018, 27(1): 013203. **doi:** 10.1088/1674-1056/27/1/013203

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Isolated attosecond pulse generation with few-cycle two-color counter-rotating circularly polarized laser pulses

Jin-Song Wu(吴劲松), Zheng-Mao Jia(贾正茂), Zhi-Nan Zeng(曾志男)
Chin. Phys. B . 2017, 26(9): 093201. **doi:** 10.1088/1674-1056/26/9/093201

Equivalent electron correlations in nonsequential double ionization of noble atoms

Shansi Dong(董善思), Qiujing Han(韩秋静), Jingtao Zhang(张敬涛)
Chin. Phys. B . 2017, 26(2): 023202. **doi:** 10.1088/1674-1056/26/2/023202

Ionization in an intense field considering Coulomb correction

Jian Li(李健), Yi-Ning Huo(霍一宁), Zeng-Hua Tang(唐增华), Feng-Cai Ma(马凤才)
Chin. Phys. B . 2017, 26(2): 023203. **doi:** 10.1088/1674-1056/26/2/023203

Theoretical study on non-sequential double ionization of carbon disulfide with different bond lengths in linearly polarized laser fields

Kai-Li Song(宋凯莉), Wei-Wei Yu(于伟威), Shuai Ben(贲帅), Tong-Tong Xu(徐彤彤), Hong-Dan Zhang(张宏丹), Pei-Ying Guo(郭培莹), Jing Guo(郭静)
Chin. Phys. B . 2017, 26(2): 023204. **doi:** 10.1088/1674-1056/26/2/023204

Electron localization of H_2^+ in a dc electric field

Z M Jia(贾正茂), Z N Zeng(曾志男), W T Tang(唐文涛), R X Li(李儒新)
Chin. Phys. B . 2017, 26(1): 013201. **doi:** 10.1088/1674-1056/26/1/013201

Photoelectron longitudinal momentum distributions containing nondipole effects*

Yi-Ning Huo(霍一宁)[1,†], Jian Li(李健)[2,3], and Feng-Cai Ma(马凤才)[1,3,‡]

[1] School of Physics, Liaoning University, Shengyang 110036, China
[2] Department of Physics, Shenyang Aerospace University, Shengyang 110003, China
[3] School of Physics Science and Technology, Inner Mongolia University, Hohhot 010021, China

(Received 11 August 2017; revised manuscript received 30 September 2017; published online 28 November 2017)

This paper proposes a modified strong field approximation model for evaluating nondipole effects on the ionization of an atom in an intense laser field. The photoelectron longitudinal momentum distributions (PLMD) of a hydrogen-like atom exposed to a mid-infrared laser field is calculated. The theoretical results indicate an obvious asymmetry in the PLMD, and an offset of the PLMD peak appears in the opposite direction of the beam propagation due to nondipole effects. The peak offsets of the PLMD increased with the laser intensity, imposed by the initial state of the hydrogen-like atom.

Keywords: strong field approximation, photoelectron, longitudinal momentum distributions, nondipole effects

PACS: 32.80.Rm, 33.80.Rv **DOI:** 10.1088/1674-1056/27/1/013203

1. Introduction

The photoionization of atoms or molecules in an electromagnetic field is the foundation for many physical phenomena. A considerable amount of attention has been given to above threshold ionization (ATI),[1,2] double ionization (DI),[3–5] and high-order harmonic generation (HHG).[6,7] The so-called strong field approximation (SFA) or Keldysh–Faisal–Reiss (KFR) theory[8–10] provides an insightful method for investigating ionization. The electric dipole approximation employed in SFA assumes that any spatial dependence of the field and the homogeneous magnetic field component can be neglected when the radiation wavelength is much larger than the size of the atom, and dipole transitions are not forbidden by selection rules. This approximation usually holds well for the most commonly used near-infrared laser sources and intensities.[1] The dipole approximation can be expected to break down towards short wavelengths, where the wavelength is comparable to the target size. However, recent theoretical works have shown that nondipole effects can manifest themselves even for small-frequency infrared (IR) laser fields.[11–13]

The photoelectron momentum is a worthwhile and meaningful issue for researching and understanding the nature of photoelectrons in a strong laser field in association with other critical phenomena.[14,15] The PLMD refers to the distributions that are parallel to the pulse propagation direction and has gradually been recognized for its importance because it provides direct information about the ionization process. In the past few decades, the PLMD was initially believed to exhibit a Gaussian-like structure.[4,5,16–18] However, recent studies have presented different viewpoints.[12,19–26] Arissian *et al.*[19] and Smeenk *et al.*[20] experimentally measured the momentum spectra of photoelectron ionized by circularly polarized pulses. A small positive shift of the centre of the PLMD from $p_z = 0$ was observed when argon and neon atoms are ionized by circularly polarized lasers at wavelengths of 800 and 1400 nm with an intensity of 10^{14}–10^{15} W/cm^2.[12,21] Ludwig *et al.* measured the PLMD for the ionization of noble gases with few-cycle mid-IR linearly polarized pulses at a moderate intensity range of 2×10^{13}–8×10^{13} W/cm^2.[22] An asymmetric PLMD was observed, and the peak of the PLMD was found to shift in the opposite direction of the beam propagation. Tao *et al.* investigated the PLMD by using a semi-classical model to include the magnetic Lorentz force by considering the rescattering process.[25,26] An anti-intuitive negative shift of the PLMD peak was obtained, which agreed with the experimental results.[22] Analysis revealed that the experimental results presented above can be interpreted through the breakdown of the dipole approximation at low light intensities (i.e., $I = 10^{14}$ W/cm^2), where the electron energies are far from the relativistic regime[11,13,27] and the laser wavelength λ is longer than the characteristic atomic size $2a_0 = 0.106$ nm. These unusual experimental phenomena motivated us to consider how the nondipole effects affect the photoelectron momentum distribution.

In this work, to obtain the leading-order nondipole corrections of the PLMD, an approximate form is derived in the SFA framework to calculate the PLMD while considering the

*Project supported by the National Natural Science Foundation of China (Grant No. 11274149) and the Program of Shenyang Key Laboratory of Optoelectronic Materials and Technology, China (Grant No. F12-254-1-00).

†Corresponding author. E-mail: huoyining_jiangnan@126.com
‡Corresponding author. E-mail: fcma@lnu.edu.cn

© 2018 Chinese Physical Society and IOP Publishing Ltd

nondipole effects. In the model, the hydrogen-like atom is exposed in a mid-IR laser field, the polarization of the pulses is in the x direction, and the laser propagated along the z direction. The atom–field interaction is described in the velocity gauge,[9,10,28] which retained a spatial variable in the vector potential $\boldsymbol{A}(\boldsymbol{r},t)$. The vector potential $\boldsymbol{A}(\boldsymbol{r},t)$ is expected to depend on space and time in the interaction Hamiltonian. The nondipole effect has a significant influence on the photoelectron spectra and is adopted to explain the characteristic asymmetry. The atomic units $m=\hbar=e=a=1$ are used throughout this paper unless specially indicated otherwise. The paper is organized as follows. Section 2 introduces the model for calculating the PLMD in the SFA framework while considering the leading-order nondipole corrections. Section 3 presents the numerical results and discussion. Section 4 concludes the paper.

2. Theory

This paper proposes a theoretical model for calculating the PLMD with the lowest-order nondipole correction, where the nondipole effects are introduced to modify the SFA. When a hydrogen-like atom is exposed in a linearly polarized laser field, the interaction between the photoelectron and laser can be described by the interaction Hamiltonian $\hat{H}_{\rm I}=\frac{\boldsymbol{A}\cdot\boldsymbol{p}}{c}+\frac{(\boldsymbol{A})^2}{2c^2}$. The components of the laser vector potential are $\boldsymbol{A}(\boldsymbol{r},t)=a\boldsymbol{\varepsilon}\cos(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})$ for considering the breakdown of the electric dipole approximation. By expanding the vector potential around $\boldsymbol{r}=0$ up to the linear order, the first and second terms in the expanded expression can be obtained

$$\boldsymbol{A}(\boldsymbol{r},t)=a\boldsymbol{\varepsilon}\cos(\omega t)+a\boldsymbol{\varepsilon}\sin(\omega t)\cdot\boldsymbol{k}\cdot\boldsymbol{r}. \tag{1}$$

In the Coulomb gauge $[\boldsymbol{p},\boldsymbol{A}]=0$, $\boldsymbol{k}\cdot\boldsymbol{A}=0$, and the Hamiltonian is given by

$$\begin{aligned}\hat{H}_{\rm I} &= \hat{H}_{\rm I}^0+\hat{H}_{\rm I}^1\\ &= \frac{a}{c}\boldsymbol{\varepsilon}\cdot\boldsymbol{p}\cos(\omega t)+\frac{a^2|\boldsymbol{\varepsilon}|^2}{2c^2}\cos^2(\omega t)\\ &\quad+\frac{a}{c}\boldsymbol{\varepsilon}\cdot\boldsymbol{p}\sin(\omega t)\boldsymbol{k}\cdot\boldsymbol{r}+\frac{a^2}{2c^2}\sin(2\omega t)\boldsymbol{k}\cdot\boldsymbol{r},\end{aligned} \tag{2}$$

where ω is the frequency of the laser field, $\boldsymbol{\varepsilon}$ is the unit polarization vector, and a is the amplitude of the potential vector. $\hat{H}_{\rm I}^0$ is a Hamiltonian involving the dipole approximation and can be expressed as[9]

$$\hat{H}_{\rm I}^0=\frac{\boldsymbol{p}\cdot\boldsymbol{\varepsilon}}{c}a\cos(\omega t)+\frac{a^2\cos^2(\omega t)}{2c^2}. \tag{3}$$

$\hat{H}_{\rm I}^1$ is the modification related to the nondipole effects

$$\hat{H}_{\rm I}^1=\frac{\boldsymbol{p}\cdot\boldsymbol{\varepsilon}}{c}a(\boldsymbol{k}\cdot\boldsymbol{r})\sin(\omega t)+\frac{a^2}{2c^2}\sin(2\omega t)(\boldsymbol{k}\cdot\boldsymbol{r}). \tag{4}$$

Based on the SFA,[9] the interaction Hamiltonian is introduced into the expression of the transition amplitude

$$(S-1)_{\rm fi}=-{\rm i}\int{\rm d}t\left(\psi_{\rm f},\hat{H}_{\rm I}\Phi_{\rm i}\right). \tag{5}$$

The initial state $\Phi_{\rm i}$ is the bound state of the hydrogen-like atom, including the ground and excited states. The final state

$$\psi_{\rm f}=\frac{1}{(2\pi)^{3/2}}\exp\left\{{\rm i}\boldsymbol{p}\cdot\boldsymbol{r}-{\rm i}\frac{p^2}{2}t-\int_{-\infty}^{t}{\rm d}t'H\left(t'\right)\right\}$$

is a long-wavelength approximation of the Gordon–Volkov solution in the Coulomb gauge.[9,29] By inserting the initial and final states as well as the Hamiltonian into Eq. (5), the transition amplitude can be divided into two parts

$$S_{\rm fi}=S_{\rm fi}^{(0)}+S_{\rm fi}^{(1)}. \tag{6}$$

The first term describes the transition amplitude. It is obtained by using a method similar to that described by Reiss[9]

$$\begin{aligned}(S-1)_{\rm fi}^{(0)} &= 2\pi{\rm i}\tilde{\phi}_{\rm i}^{(0)}(p)\sum_n J_n(\zeta_l,-\frac{z}{2})\left(\frac{p^2}{2}+E_{\rm B}\right)\\ &\quad\times\delta\left(\frac{p^2}{2}-E_{\rm i}-n\omega+z\omega\right),\end{aligned} \tag{7}$$

where $\tilde{\phi}_{\rm i}^{(0)}(p)=\frac{1}{(2\pi)^{3/2}}\int{\rm d}^3\boldsymbol{r}\exp\{-{\rm i}\boldsymbol{p}\cdot\boldsymbol{r}\}\phi_{\rm i}(\boldsymbol{r})$. Here, $\phi_{\rm i}(\boldsymbol{r})$ is the initial bound state of the electron in an atom, and $J_n(\zeta_l,-\frac{z}{2})$ is a general Bessel function.[9] The intensity-dependent amplitude ζ_l and intensity parameter z are defined as

$$\zeta_l=\frac{-a\boldsymbol{\varepsilon}\cdot\boldsymbol{p}}{\omega c}, \tag{8a}$$

$$z=\frac{a^2}{4\omega c^2}. \tag{8b}$$

The second term of Eq. (6) contributes to the transition amplitude associated with the nondipole effects, which causes the asymmetry of the PLMD considered here. Similar to the method derived by Reiss,[9] it can be written as

$$\begin{aligned}(S-1)_{\rm fi}^{(1)} &= \tilde{\phi}_{\rm i}^{(1)}(\boldsymbol{p})\sum_n J_n\left(\zeta_l,-\frac{z}{2}\right)\int{\rm d}t\left[\zeta_l\sin(\omega t)+2z\sin(2\omega t)\right]\\ &\quad\times\exp\left[{\rm i}\left(\frac{p^2}{2}+E_{\rm B}+U_{\rm p}-n\omega\right)t\right].\end{aligned} \tag{9}$$

By employing the expression $\sin x=\frac{1}{2}\left({\rm e}^{{\rm i}x}-{\rm e}^{-{\rm i}x}\right)$ and the property of the δ function, the integration is performed over t

$$\begin{aligned}(S-1)_{\rm fi}^{(1)} &= \tilde{\phi}_{\rm i}^{(1)}(\boldsymbol{p})\pi\left\{\zeta_l\left(A^{(1)}-A^{(-1)}\right)\right.\\ &\quad\left.-2z\left(A^{(2)}-A^{(-2)}\right)\right\},\end{aligned} \tag{10}$$

where $\tilde{\phi}_{\rm i}^{(1)}(p)=\frac{1}{(2\pi)^{3/2}}\int{\rm d}\boldsymbol{r}^3(\boldsymbol{k}\cdot\boldsymbol{r})\exp\left\{-\frac{\rm i}{\hbar}\boldsymbol{p}\cdot\boldsymbol{r}\right\}\phi_{\rm i}(\boldsymbol{r})$ and $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, and $A^{(4)}$ can be expressed as

$$A^{(1)}=\omega\sum_{n\geq n_1}J_n\left(\zeta_l,-\frac{z}{2}\right)\delta\left(\frac{p^2}{2m}-E_{\rm i}+U_{\rm p}-(n-1)\omega\right),$$

$$n_1 = n_0 + 1, \tag{11a}$$

$$A^{(-1)} = \omega \sum_{n\geq n_{-1}} J_n\left(\zeta_l, -\frac{z}{2}\right)\delta\left(\frac{p^2}{2m} - E_\mathrm{i} + U_\mathrm{p} - (n+1)\omega\right),$$

$$n_{-1} = n_0 - 1, \tag{11b}$$

$$A^{(2)} = \omega \sum_{n\geq n_2} J_n\left(\zeta_l, -\frac{z}{2}\right)\delta\left(\frac{p^2}{2m} - E_\mathrm{i} + U_\mathrm{p} - (n-2)\omega\right),$$

$$n_2 = n_0 + 2, \tag{11c}$$

$$A^{(-2)} = \omega \sum_{n\geq n_{-2}} J_n\left(\zeta_l, -\frac{z}{2}\right)\delta\left(\frac{p^2}{2m} - E_\mathrm{i} + U_\mathrm{p} - (n+2)\omega\right),$$

$$n_{-2} = n_0 - 2. \tag{11d}$$

Here, $n \geq n_0 = [z\omega - E_\mathrm{i}]/\omega + 1 = [U_\mathrm{p} + E_\mathrm{B}]/\omega + 1$, and $[\,]_\mathrm{int}$ represents the integer for the quantity in the bracket. $E_\mathrm{B} = -E_\mathrm{i}$ is the binding energy of the electron and $U_\mathrm{p} = \frac{I}{4\omega^2} = z\omega$ is the ponderomotive energy. The ionization rate per unit time can be written as

$$w = \lim_{t\to\infty}\frac{1}{t}\left|(S-1)^{(0)}_\mathrm{fi} + (S-1)^{(1)}_\mathrm{fi}\right|^2. \tag{12}$$

The total transition probability is

$$w = w_l^{(0)} + w_l^{(1)}, \tag{13}$$

where

$$w_l^{(0)} = 4\pi^2\left|\tilde{\phi}_\mathrm{i}^{(0)}(p)\right|^2(n-z)^2\omega^2 \times \sum_{n\geq n_0}\left|J_n\left(\zeta_l, -\frac{z}{2}\right)\right|^2\delta\left(\frac{p^2}{2m} - E_\mathrm{i} - n\omega + z\omega\right) \tag{14}$$

is the contribution of the total transition probability with the dipole approximation. Then, the contribution caused by the correction of the nondipole approximation is given by

$$w_l^{(1)} = 2\pi^2\omega^2\zeta_l\left(\mathrm{i}\tilde{\phi}_\mathrm{i}^{(0)^*}(p)\cdot\tilde{\phi}_\mathrm{i}^{(1)}(p) - \mathrm{i}\tilde{\phi}_\mathrm{i}^{(0)}(p)\cdot\tilde{\phi}_\mathrm{i}^{(1)^*}(p)\right) \times \left\{(\zeta_l - 2z)(n_0 - z)\left|J_{n0}\left(\zeta_l, -\frac{z}{2}\right)\right|^2 \times \delta\left(\frac{p^2}{2m} - E_\mathrm{i} - n_0\omega + z\omega\right) - 2z(n_1 - z)\left|J_{n1}\left(\zeta_l, -\frac{z}{2}\right)\right|^2 \times \delta\left(\frac{p^2}{2m} - E_\mathrm{i} - n_1\omega + z\omega\right)\right\}. \tag{15}$$

The terms proportional to k^2 have been neglected. In order to clearly analyze the experimental phenomena,[22] the PLMD can be written in cylindrical coordinates

$$\frac{\mathrm{d}W_l}{\mathrm{d}p_z} = \int w p_\perp \mathrm{d}p_\perp \mathrm{d}\varphi. \tag{16}$$

where $p_\perp = \sqrt{p^2 - p_z^2}$ represents the momentum component on the polarization plane, and φ is the azimuthal angle between the direction of the polarization vector and the momentum projection on the polarization plane. At this coordinate, the propagation direction of the electromagnetic field corresponds to the z axis, and the direction of the polarization vector is along the x axis.

For the ground state of the hydrogen-like atom, the PLMD (i.e. the distribution of the momentum in the propagation direction of the laser) can be written as

$$\frac{\mathrm{d}W_l}{\mathrm{d}p_z} = \int \mathrm{d}\varphi\left\{\sum_{n\geq n_0}\frac{32Z^5\sqrt{2(n\omega - E_\mathrm{B} - z\omega) - p_z^2}}{(Z^2 + 2(n\omega - E_\mathrm{B} - z\omega))^4} \times \left|J_n\left(\xi, -\frac{z}{2}\right)\right|^2\left(\frac{p^2}{2} + E_\mathrm{B}\right)^2 + \frac{128\omega^2 k p_z\sqrt{2(n_0\omega - E_\mathrm{B} - z\omega) - p_z^2}}{[(Z)^2 + 2(n_0\omega - E_\mathrm{B} - z\omega)]^5} \times (Z)^5(\zeta_l - 2z)(n_0 - z)\left|J_{n_0}\left(\zeta_l, -\frac{z}{2}\right)\right| - \frac{128\omega^2 k p_z\sqrt{2(n_1\omega - E_\mathrm{B} - z\omega) - p_z^2}}{[(Z)^2 + 2(n_1\omega - E_\mathrm{B} - z\omega)]^5} \times (Z)^5 2z(n_1 - z)\left|J_{n_1}\left(\zeta_l, -\frac{z}{2}\right)\right|\right\}, \tag{17}$$

where Z is the nuclear charge. Here, $Z = 1$ for the hydrogen-like atom. For the excited state with $(n,l,m) = (2,1,1)$ and $(n,l,m) = (2,1,-1)$, the PLMD results are as follows:

$$\frac{\mathrm{d}W_l}{\mathrm{d}p_z} = \int \mathrm{d}\varphi\left\{2\left(\frac{Z}{2}\right)^7\sum_{n\geq n_0}\left|J_n\left(\zeta_l, -\frac{z}{2}\right)\right|^2\left(\frac{p^2}{2} + E_\mathrm{B}\right)^2 \times \frac{\left(2(n\omega + E_\mathrm{i} - z\omega) - p_z^2\right)^{3/2}}{\left[(Z/2)^2 + 2(n\omega + E_\mathrm{i} - z\omega)\right]^6} + \omega^2\left(12k\left(\frac{Z}{2}\right)^7\frac{\left(2(n_0\omega + E_\mathrm{i} - z\omega) - p_z^2\right)^{3/2}p_z}{\left[(Z/2)^2 + 2(n_0\omega + E_\mathrm{i} - z\omega)\right]^7}\right) \times (\zeta_l - 2z)(n_0 - z)\left|J_{n_0}\left(\zeta_l, -\frac{z}{2}\right)\right|^2 - \omega^2\left(12k\left(\frac{Z}{2}\right)^7\frac{\left(2(n_1\omega + E_\mathrm{i} - z\omega) - p_z^2\right)^{3/2}p_z}{\left[(Z/2)^2 + 2(n_1\omega + E_\mathrm{i} - z\omega)\right]^7}\right) \times 2z(n_1 - z)\left|J_{n_1}\left(\zeta_l, -\frac{z}{2}\right)\right|^2\right\}. \tag{18}$$

For the case where the initial state is $(n,l,m) = (2,1,0)$, the numerical calculation shows that the transition probability is very small. This agrees well with the phenomenon observed by Barth and Smirnova.[30–33] A reasonable explanation is that the destructive interference largely suppresses the ionization of the p orbital when the initial state is $(n,l,m) = (2,1,0)$. Therefore, we mainly focused on the ground and excited states with $(n,l,m) = (2,1,1)$ and $(n,l,m) = (2,1,-1)$, respectively, in this work.

3. Numerical results and discussion

We used the modified SFA model to evaluate the PLMD of a released electron from a hydrogen-like atom exposed in a mid-IR laser field. The initial states of the atom considered here are the ground and first excited states. The wavelength

and intensity of the laser field are selected to be similar to those used in the experiment.[22] For the initial ground state of the hydrogen-like atom, the laser wavelength is 3.4 μm with an intensity of 6.0×10^{13} W/cm^2. The calculated results are shown in Fig. 1. Clearly, the momentum distribution is dramatically affected by the correction of the nondipole effect, and the peak of the PLMD is clearly shifted from $p_z=0$, implying an obvious asymmetry (see Fig. 1(a)). The peak of the PLMD shifts in the opposite direction of the laser propagation. When the PLMD is produced by a shorter-wavelength laser, a similar shift in the direction for the peak of the momentum distribution is obtained. However, the offset is not evident in the case of the PLMD produced by a shorter-wavelength laser for the ground state, as shown in Fig. 1(b), and is clearly shown in Fig. 4.

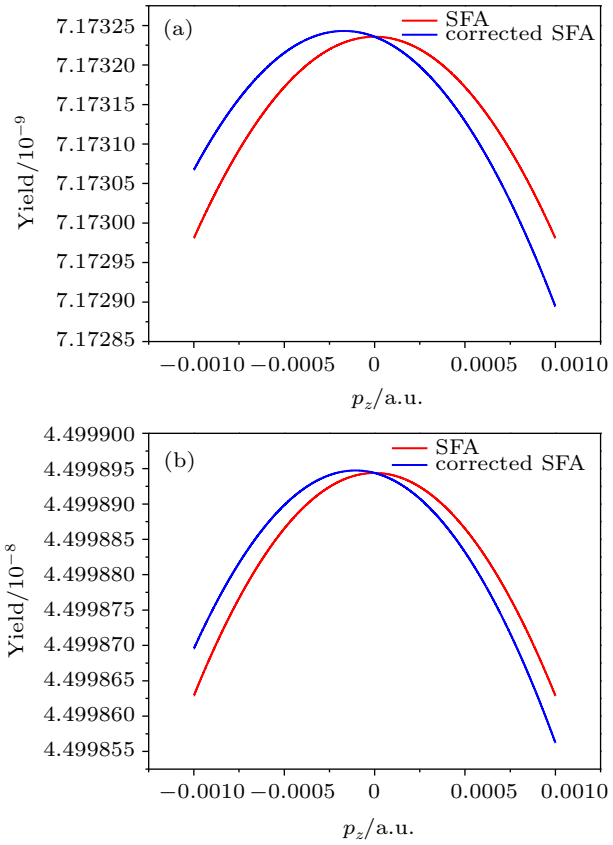


Fig. 1. (color online) (a) The PLMD for the ground state of a hydrogen-like atom produced by a linearly polarized laser at an intensity of 6.0×10^{13} W/cm^2 and wavelength of 3.4 μm. The red line represents the spectra by SFA without the calculated correction of the nondipole effect, and the blue line shows the spectra with the correction of the nondipole approximation. (b) The PLMD for the ground state of hydrogen-like atoms in the linearly polarized laser at an intensity of 6.0×10^{13} W/cm^2 and wavelength of 800 nm. The red line represents the spectra by SFA without correction of the calculated nondipole effect, and the blue line shows the spectra with the correction of the nondipole approximation.

Figure 2 illustrates the calculated PLMD for the excited state of $(n,l,m)=(2,1,1)$ with the same laser field parameters. As indicated in Fig. 2(a), when the laser wavelength is 3.4 μm, the PLMD shows clear asymmetry compared to the ground state. The PLMD produced by the laser with a wavelength of 800 nm also reveals asymmetry, as shown in Fig. 2(b). By comparing Figs. 2(a) and 2(b), it can be seen that the shorter laser wavelength does not reduce the offset of the PLMD peak for the excited state, compared with the longer-wavelength laser that is divergent from the condition for the ground state. Our calculated results indicate that the excited state with $(n,l,m)=(2,1,-1)$ exhibits the same distribution as the excited state with $(n,l,m)=(2,1,1)$ when the same laser field is employed.

According to Serebryannikov and Zheltikov's recent work,[34] ultrafast photoionization can occur through quantum pathways that cannot be categorized as multiphoton ionization or ground-state tunneling in an intense laser field. In this regime, tunneling from electronic excited states dominates the ground-state tunnelling, drastically enhancing photoionization and modifying its dynamics. It is meaningful to investigate the ionization rate and PLMD of the excited state.

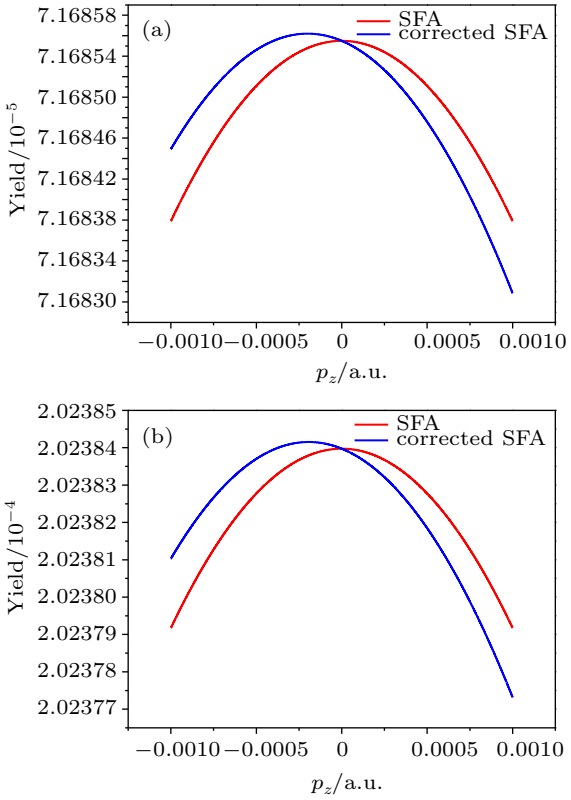


Fig. 2. (color online) (a) The PLMD for the excited state of a hydrogen-like atom with $(n,l,m)=(2,1,1)$ produced by a linearly polarized laser at an intensity of 6.0×10^{13} W/cm^2 and wavelength of 3.4 μm. The red line represents the spectra without the calculated correction of the nondipole effect by SFA, and the blue line shows the PLMD with the correction of the nondipole approximation. (b) PLMD for the excited state of hydrogen-like atoms with $(n,l,m)=(2,1,1)$ produced by a linearly polarized laser at an intensity of 6.0×10^{13} W/cm^2 and wavelength of 800 nm. The red line indicates the spectra without the calculated correction of the nondipole effect by SFA, and the blue line shows the PLDM with the correction of the nondipole approximation.

The peak shift of PLMD versus the laser intensity is shown in Fig. 3. The peak shift is more obvious when the laser

wavelength is 3.4 μm than the laser wavelength of 800 nm for the ground state. Moreover, the peak offset increased with the laser intensity. Conversely, it is found that the peak offset varies when the laser wavelength is 800 nm revealed by the blue dotted line in Fig. 3 for the excited state. Therefore, the initial state of the atom is a key factor for PLMD.

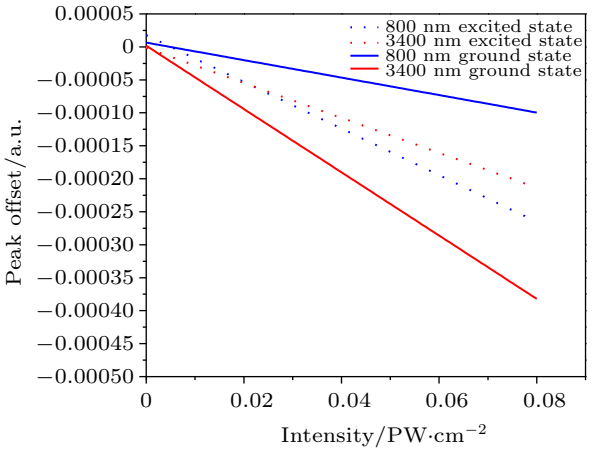


Fig. 3. (color online) The peak offsets along the laser propagation direction as an approximately linear function primarily depends on the laser intensity and frequency for different initial states as shown in Fig. 3. An increasingly negative tendency is clearly revealed. The red and blue solid lines represent the peak offsets when the lasers adopted to produce photoelectrons with wavelengths of 3.4 μm and 800 nm are used for the ground state, respectively. The tendency is different when the excited state is considered, as the red and blue dotted lines clearly show. The red dotted line illustrates the peak offset when the laser wavelength is 3.4 μm. It is noteworthy that an obvious peak shift is observed for the excited state when the 800 nm laser is considered.

A normalized asymmetry $A(p_z)$ is cited to describe the tendency of the offsets[26] in PLMD, defined as

$$A(p_z) = \frac{f(p_z) - f(-p_z)}{f(p_z) + f(-p_z)}. \tag{19}$$

The offset is directly visualized in PLMD and is represented by $f(p_z)$. From Fig. 4 it is concluded that the offset is more apparent when $\lambda = 3.4$ μm, than for the $\lambda = 800$ nm ground state. Observing the asymmetry coefficient $A(p_z)$ is a simple method to analyze the PLMD. The slope of the $A(p_z)$ can be used to analyze the asymmetry in PLMD. Under no conditions, when considering the correction of the dipole approximation, $A(p_z) = 0$.

According to Fig. 5, the asymmetry structure is produced when the laser intensity is large enough, demonstrating that shorter pulses are capable of causing the asymmetry in PLMD.[26] Based on Fig. 3 the asymmetry is exceedingly apparent for the excited state. Figure 5 shows the PLMD of the excited state $(n,l,m) = (2,1,1)$ produced by a linearly polarized laser with a wavelength of 800 nm and intensity of 0.70×10^{16} W/cm^2. This indicates that the dipole approximation breaks down in such an intense laser field according to the long wavelength limit $I_{\text{limit}} = 8c\omega^3$.[11,13,27]

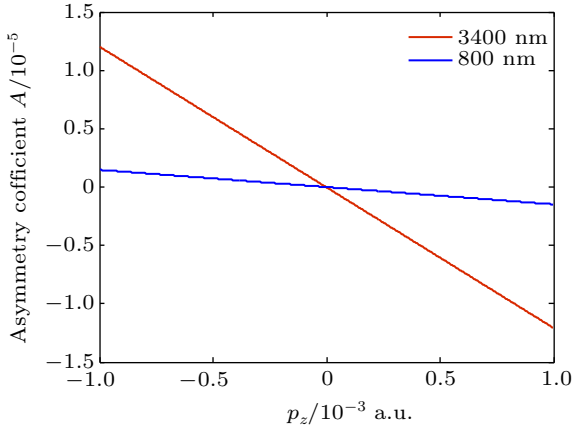


Fig. 4. (color online) The asymmetry coefficient A corresponding to the ionization caused by the laser with a wavelength of 3.4 μm and at an intensity of 6.0×10^{13} W/cm^2 (red line), and a wavelength of 800 nm and at an intensity of 6.0×10^{13} W/cm^2 (blue line). The more manifest offset is indicated through the asymmetry coefficient A when the laser with a wavelength of 3.4 μm is adopted for ground state.

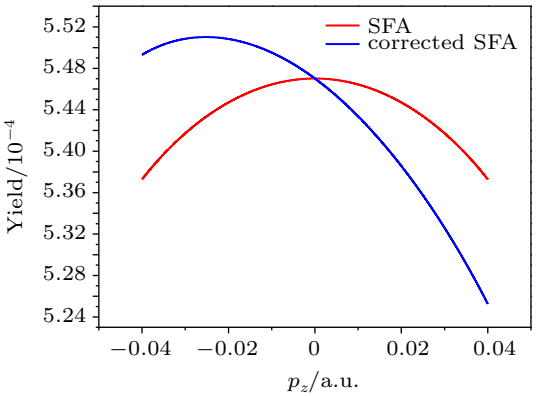


Fig. 5. (color online) The PLMD for the excited state $(n,l,m) = (2,1,1)$ produced by a linearly polarized laser at a wavelength of 800 nm and intensity of 0.70×10^{16} W/cm^2. The red line represents the spectra without the calculated correction of the nondipole effects by SFA, and the blue line shows the photoelectron momentum spectra with the correction of the nondipole approximation.

4. Remarks and conclusions

To investigate the nondipole effect on the photoionization of an atom, a modified SFA model has been developed. Only the leading-order nondipole corrections are introduced into the interaction Hamiltonian in the SFA framework. The initial and final states are similar to those used in SFA theory including the bound state of an atom and the Gordon–Volkov plane wave. The PLMD of a hydrogen-like atom exposed to a mid-IR laser is calculated with the modified SFA theory. Our numerical results reveal asymmetry in the PLMD, and an offset of the PLMD peak appears in the opposite direction of the beam propagation. The asymmetric structure of the PLMD is more obvious when the excited state is the initial state of the atom than the ground state for short-wavelength pulses. The modified SFA model is a useful method to obtain a better understanding of the relevant experiments, such as the main characteristics of the experiment reported by Ludwig *et al.*:[22] (i) the symmetry of the PLMD is destroyed, and

an offset of the peak of the PLMD appears; (ii) the magnitude of the offset of the PLMD peak increases with the wavelength of the laser for the ground state, agreeing well with Reiss's prediction;[11,12] (iii) for the same wavelength, a higher intensity of the laser field promotes a more obvious offset of the PLMD peak for the ground state; and (iv) the initial state of the hydrogen-like atom is a crucial element to decide the peak shift in PLMD. More obvious asymmetry is found for the excited states compared to the ground state. The nondipole effects are convincingly found to play an important role in the excited states of an atom, providing a hint for further investigation in future theoretical and experimental works.

References

[1] Milošević D B, Paulus G G, Bauer D and Becker W 2006 *J. Phys. B* **39** R203
[2] Wu C Y, Yang Y D, Liu Y Q, Gong Q H, Wu M, Liu X, Hao X L, Li W D, He X T and Chen J 2012 *Phys. Rev. Lett.* **109** 043001
[3] I'Huillier A, Lompre L A, Mainfray G and Manus C 1983 *Phys. Rev. A* **27** 2503
[4] Weber Th, Weckenbrock M, Staudte A, Spielberger L, Jagutzki O, Mergel V and Afanech F 2000 *Phys. Rev. Lett* **84** 443
[5] Chen J and Nam C H 2002 *Phys. Rev. A* **66** 053415
[6] van de Sand Gerd and Rost Jan M 1999 *Phys. Rev. Lett* **83** 524
[7] Catoire F, Ferre A, Hort O, Dubrouoil A Quintard L, Descamps D, Petit S, Burgy F, Mevel E, Mairesse Y and Constant E 2016 *Phys. Rev. A* **94** 063401
[8] Keldysh L V 1965 *Sov. Phys. JETP* **20** 1307
[9] Reiss H R 1980 *Phys. Rev. A* **22** 1786
[10] Faisal F H M 1973 *J. Phys. B* **6** L89
[11] Reiss H R 2008 *Phys. Rev. Lett* **101** 043002
[12] Reiss H R 2013 *Phys. Rev. A* **87** 033421
[13] Reiss H R 2014 *J. Phys. B* **47** 204006
[14] Wu M Y, Wang Y L, Liu X J, Li W D, Hao X L and Chen J 2013 *Chin. Phys. Lett.* **30** 073202
[15] Hao X L, Li W D, Liu J and Chen J 2012 *Chin. Phys. B* **21** 083304
[16] Moshammer R Ullrich J, Feuerstein B Fischer D, Dorn A, Schroter C D, Crespo Lopez-Urrutia J R and Hoehr C 2003 *Phys. Rev. Lett.* **91** 113002
[17] Chen Z J, Morishita T Le Anh-Thu, Wickenhauser M Tong X M and Lin C D 2006 *Phys. Rev. A* **74** 053405
[18] Ivanov I A 2014 *Phys. Rev. A* **90** 013418
[19] Arissian L, Smeenk C, Turner F Trallero C, Sokolov A V, Villeneuve D M, Staudte A and Corkum P B 2010 *Phys. Rev. Lett.* **105** 133002
[20] Smeenk C T L, Arissian L, Zhou B, Mysyrowicz A, Villeneuve D M, Staudte A and Corkum P B 2011 *Phys. Rev. Lett.* **106** 193002
[21] Titi A S and Drake G W 2012 *Phys. Rev. A* **85** 041404
[22] Ludwig A, Maurer J, Mayer B W, Phillips C R, Gallmann L and Keller U 2014 *Phys. Rev. Lett.* **113** 243001
[23] Yakaboylu E Klaiber M, Bauke H, Hatsagortsyan K Z and Keitel C H 2013 *Phys. Rev. A* **88** 063421
[24] Bauer J H 2013 *J. Phys. B* **46** 045601
[25] Tao J F Xia Q Z, Cai J Fu L B and Liu J 2017 *Phys. Rev. A* **95** 011402
[26] Chelkowski S, André D B and Corkum P B 2017 *Phys. Rev. A* **95** 053402
[27] Reiss H R 2000 *Phys. Rev. A* **63** 013409
[28] Tang Z H, Li Y Q and Ma F C 2013 *Chin. Phys. B* **22** 084203
[29] Gordon W 1926 *Z. Phys.* **40** 117
[30] Barth I and Smirnova O 2011 *Phys. Rev. A* **84** 063415
[31] Barth I and Smirnova O 2012 *Phys. Rev. A* **85** 029906
[32] Barth I and Smirnova O 2012 *Phys. Rev. A* **85** 039903
[33] Barth I and Smirnova O 2013 *Phys. Rev. A* **87** 013433
[34] Serebryannikov E E and Zheltikov A M 2016 *Phys. Rev. Lett.* **116** 123901