ISSN 1674-1056
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Citation: Chin. Phys. B . 2018, 27(1): 010504. **doi:** 10.1088/1674-1056/27/1/010504

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Controllable optical superregular breathers in the femtosecond regime*

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(Received 30 August 2017; revised manuscript received 30 September 2017; published online 17 November 2017)

We investigate optical superregular breathers in the femtosecond regime under dispersion management in an inhomogeneous fiber governed by the nonautonomous higher-order nonlinear Schrödinger equation (NLSE). Exact solutions describing the dynamics of superregular breathers are obtained. Furthermore, we discuss the propagation behaviors of controllable superregular breathers, including stabilization and recurrence in an exponential dispersion fiber and a periodic distributed fiber system. Particularly, the nonlinear dynamics of superregular modes evolved from an identical initial small-amplitude modulation is analyzed in detail.

Keywords: nonlinear wave, superregular breather, controllable dynamics, modulation instability

PACS: 05.45.Yv, 02.30.Ik, 42.81.Dp **DOI:** 10.1088/1674-1056/27/1/010504

1. Introduction

Localized wave structures on a plane-wave background have become a subject of intense research in nonlinear physics, since their nonlinear dynamics is strongly associated with the ubiquitous modulation instability (MI).[1–7] In particular, a new type of breather mode, superregular (SR) breather, has been recently proposed by Zakharov and Gelash.[8] In fact, the SR breather is one type of higher-order breather mode, which is formed by the interaction of pairs of quasi-Akhmediev breathers with specific phase shift parameters. Physically, the SR breather describes a nontrivial quasi-annihilation of breather collisions and a nonlinear stage of MI from a small amplitude localized modulation,[8–10] which is absent in the classical soliton interaction theory.[11,12] Specifically, the MI scenario of SR breathers is different from the known higher-order MI formed by multiple Akhmediev breathers that requires an initial periodic modulation in the whole space.[13] Interestingly, the SR breather has been verified experimentally in the picosecond regime of an ideal optical fiber[10] based on the autonomous standard nonlinear Schrödinger equation (NLSE). However, in view of the complexity of practical nonlinear systems, it is important to study the characteristics of SR breathers in a real optical fiber system.

For long distance transmission, a real optical fiber possesses several disadvantages such as fiber loss and periodic lumped amplification.[14] Nevertheless, these two disadvantages can be rectified by employing dispersion management.[14,15] Indeed, the study of pulse propagation under dispersion management has been paid a great deal of attention.[16–18] On the other hand, ultrashort pulses in the femtosecond regime are tempting and desirable to improve the capacity of high-bit-rate transmission systems. It is demonstrated that the phenomenon of optical localized waves in the femtosecond regime is significantly more complicated than that in the picosecond regime,[19–33] since the higher-order terms, such as the third-order dispersion, self-steepening, and the delayed nonlinear response effect must be taken into account. Therefore, it is important to study optical localized waves in the femtosecond regime under dispersion management.

Recently, the inhomogeneous fibers have aroused extensive attention to reflect the inhomogeneities of media, nonuniformities of boundaries, and external forces.[34–40] Furthermore, different types of femtosecond localized waves in the nonautonomous higher-order NLSE have been investigated, including solitons,[41–44] rogue waves,[45] breathers,[46] and so on.[47,48] In particular, the bright solitons in an exponential dispersion fiber and a periodic distributed fiber system have been observed experimentally.[49] It is reported that management opens a venue into experimental manipulation of localized waves, which needs more exploration. However, to the best of our knowledge, optical SR breathers in the femtosecond regime under dispersion management have not been reported so far.

In this paper, we investigate the characteristics of optical femtosecond SR breathers in an optical fiber by the nonautonomous higher-order NLSE. Our presentation will be structured as follows. In Section 2, we present the theoretical model

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11547302, 11705145, and 11434013).
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and the SR breather solution by using the Darboux transformation and similarity transformation. In Section 3, we first exhibit the characteristics of three types of SR modes. Then we discuss the exponential dispersion management SR modes in an exponential dispersion fiber and the periodical dispersion management SR modes in a periodic distributed fiber system. Section 4 gives our conclusion.

2. The model and exact solution

The dynamics of SR breathers in the femtosecond regime for an optical fiber under dispersion management is described by the nonautonomous higher-order NLSE[41–48]

$$\begin{aligned} &\mathrm{i}E_z + D_2(z)E_{tt} + R(z)|E|^2E + \mathrm{i}D_3(z)E_{ttt} \\ &+ \mathrm{i}\alpha_0(z)(|E|^2E)_t + \mathrm{i}f(z)E(|E|^2)_t = \mathrm{i}\Gamma(z)E, \end{aligned} \tag{1}$$

where z and t represent the propagation distance and retarded time, respectively. $E(z,t)$ is the slowly varying envelope field. $D_2(z)$, $D_3(z)$, and $R(z)$ denote the group-velocity dispersion (GVD), the third-order dispersion, and the nonlinearity, respectively. $\alpha_0(z)$ and $f(z)$ are related to the self-steepening and the delayed nonlinear response effect. $\Gamma(z)$ is the gain or loss parameter.

By means of the Darboux transformation and similarity transformation,[45,50] we can obtain the exact SR breather solution of Eq. (1). Its exact expression can be written as

$$E = AE_0\left[1 - 4\rho\rho_1\frac{(\mathrm{i}\rho_1-\rho)\Xi_1 + (\mathrm{i}\rho_1+\rho)\Xi_2}{a(\rho^2\Xi_3+\rho_1^2\Xi_4)}\right]\mathrm{e}^{\mathrm{i}\Phi}, \tag{2}$$

where

$$\begin{aligned} &\Xi_1 = \varphi_{21}\phi_{11} + \varphi_{22}\phi_{21},\ \ \Xi_2 = \varphi_{11}\phi_{21} + \varphi_{21}\phi_{22}, \\ &\Xi_3 = \varphi_{11}\phi_{22} - \varphi_{21}\phi_{12} - \varphi_{12}\phi_{21} + \varphi_{22}\phi_{11}, \\ &\Xi_4 = (\varphi_{11}+\varphi_{22})(\phi_{11}+\phi_{22}), \\ &\phi_{jj} = \cosh(\Theta_2 \mp \mathrm{i}\psi) - \cos(\Phi_2 \mp \alpha), \\ &\varphi_{jj} = \cosh(\Theta_1 \mp \mathrm{i}\psi) - \cos(\Phi_1 \pm \alpha), \\ &\phi_{j3-j} = \pm\mathrm{i}\cosh(\Theta_2 \mp \mathrm{i}\alpha) \mp \mathrm{i}\cos(\Phi_2 \mp \psi), \\ &\varphi_{j3-j} = \pm\mathrm{i}\cosh(\Theta_1 \pm \mathrm{i}\alpha) \mp \mathrm{i}\cos(\Phi_1 \mp \psi), \end{aligned}$$

with

$$\begin{aligned} &\Theta_j = 2\eta_{\mathrm{r}j}(T - V_{\mathrm{gr}j}Z) + \mu_j, \\ &\Phi_j = 2\eta_{\mathrm{i}j}(T - V_{\mathrm{ph}j}Z) - \theta_j, \\ &V_{\mathrm{gr}j} = v_{1j} + v_{2j}\frac{\eta_{\mathrm{i}j}}{\eta_{\mathrm{r}j}},\ \ V_{\mathrm{ph}j} = v_{1j} - v_{2j}\frac{\eta_{\mathrm{r}j}}{\eta_{\mathrm{i}j}}, \\ &v_{1j} = q + \rho_j - \beta(2a^2 - 3q^2 + 4\rho^2 - 6q\rho_j - 4\rho_j^2), \\ &v_{2j} = -6\beta q_{\mathrm{s}j}\rho\left(1 - \frac{q}{q_{\mathrm{s}j}}\right),\ \ q_{\mathrm{s}j} = -\frac{1}{6\beta}(1 + 8\rho_j\beta), \\ &\rho_2 = -\rho_1,\ \eta_{\mathrm{r}1} = \eta_{\mathrm{r}2} = \eta_{\mathrm{r}},\ \eta_{\mathrm{i}1} = -\eta_{\mathrm{i}2} = \eta_{\mathrm{i}}, \\ &\eta_{\mathrm{r}} = \frac{a}{2}(R - 1/R)\cos\alpha,\ \ \eta_{\mathrm{i}} = \frac{a}{2}(R + 1/R)\sin\alpha, \\ &\rho_1 = \frac{a}{2}(R - 1/R)\sin\alpha,\ \ \rho = \frac{a}{2}(R + 1/R)\cos\alpha, \\ &\psi = \arctan[\mathrm{i}(1-R)/(1+R)], \end{aligned}$$

$$\begin{aligned} &T = k\left[p\left(\frac{k}{\beta} - 3p\right)\int_0^z D_3\,\mathrm{d}z + t\right] + T_0, \\ &Z = \frac{-k^3}{\beta}\int_0^z D_3\,\mathrm{d}z + Z_0,\ A = A_0\exp\left(\int_0^z \Gamma\,\mathrm{d}z\right), \\ &\Phi = p\left[\left(\frac{kp}{2\beta} - 2p^2\right)\int_0^z D_3\,\mathrm{d}z + t\right] + \Phi_0. \end{aligned}$$

It should be noted that solution 2 is based on the system constrained parameters given as

$$\begin{aligned} &D_3(z) : D_2(z) : R(z) : f(z) : \alpha_0(z) \\ &= 1 : \left(3p - \frac{k}{2\beta}\right) : \frac{6k^2\beta p - k^3}{\beta A_0^2\exp(2\int_0^z \Gamma\,\mathrm{d}z)} \\ &\quad : \frac{-6k^2}{A_0^2\exp(2\int_0^z \Gamma\,\mathrm{d}z)} : \frac{6k^2}{A_0^2\exp(2\int_0^z \Gamma\,\mathrm{d}z)}. \end{aligned} \tag{3}$$

The above expressions depend on the background amplitude a, frequency q, the phase parameters θ_j, μ_j, and the real parameters β, p, k, and A_0. Here R (>1) and α are the radius and angle of the polar coordinates, which are introduced by the Jukowsky transform of the spectral parameter. $V_{\mathrm{gr}j}$ and $V_{\mathrm{ph}j}$ represent the group velocity and phase velocity of the waves propagating on the plane wave background, respectively. Parameters θ_j and μ_j $(j = 1,2)$ are phase shift parameters. More specifically, μ_j determines the collision location in the z–t plane. Thus, without loss of generality, we can set $\mu_{1,2} = 0$, implying that the collision emerges at $(z,t) = (0,0)$. Here the remaining parameters determine the degree of complexity of the wave profile around $(z,t) = (0,0)$.

Firstly, we exhibit the effect that θ_j will bring about on the complexity of the wave profile around $(z,t) = (0,0)$ without the management (corresponding to $D_3(z) = D_{30}$ and $\Gamma(z) = 0$). In this case, equation (1) becomes the autonomous Hirota equation. Figure 1 shows some particular arrangements of one-pair quasi-Akhmediev breathers with the following values of phase shifts: $\theta_{1,2} = 0$, π, and $\pi/2$ ($\mu_{1,2} = 0$ for all cases).

Figure 1(a) depicts the synchronous interaction structure obtained for $\theta_{1,2} = 0$. It is shown that one-pair quasi-Akhmediev breathers collide with each other and lead to a high amplitude peak structure at the origin $(z,t) = (0,0)$.[51–53] A similar synchronized collision of Akhmediev breathers has been reported in Ref. [54] by controlling the phase and velocity differences between the breathers.

Another particular interaction is obtained for $\theta_{1,2} = \pi$ (see Fig. 1(b)). The collision point is just near another maximum of either breather solution. Each breather then appears seemingly without much influence of the collision process.[10]

When $\theta_{1,2} = \pi/2$, we can see a novel quasi-annihilation phenomenon of two quasi-Akhmediev breathers, which is called the standard SR modes. Figure 1(c) describes the quasi-annihilation of breathers collision at the line $z = 0$. It also can be regarded as an initially small localized perturbation that generates a pair of quasi-Akhmediev breathers propagating with very fast group velocities in opposite directions.

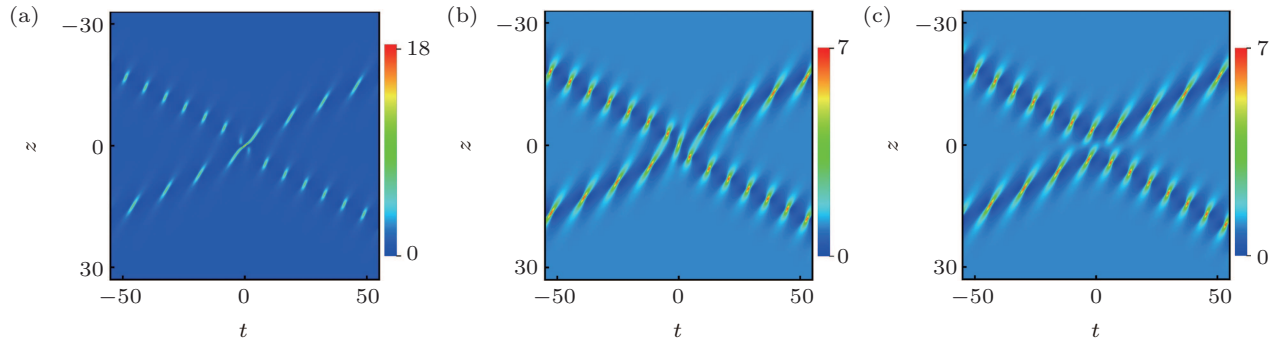


Fig. 1. (color online) Special cases of the one-pair breathers solution ($R = 1.2$, $\alpha = \pi/5$) compared to the single breather solution. One-pair breathers solution with the following phase shifts: (a) $\theta_{1,2} = 0$, (b) $\theta_{1,2} = \pi$, and (c) $\theta_{1,2} = \pi/2$ (superregular case). $\mu_{1,2} = 0$ for all cases. Other parameters are $a = 1$, $q = -1.56754$, $\beta = 0.2$, $k = 1/2$, $A_0 = 1$, and $p = 1/2$.

Besides these special cases, it should be noted that equation (2) only describes the ordinary interaction of a pair of quasi-Akhmediev breathers.

3. Dynamics of the nonautonomous superregular modes

3.1. Three types of superregular modes

In this part, we discuss the basic characteristics of SR modes. We find that the characteristics of SR modes are influenced by $V_{\mathrm{gr}j}$ and $V_{\mathrm{ph}j}$, which are closely related to the background frequency q. Particularly, q is an adjustable and important experiment parameter. Thus, we shall reveal the characteristics of SR modes by the properties of $V_{\mathrm{gr}j}$ and $V_{\mathrm{ph}j}$ with q in the following. The corresponding results are shown in Fig. 2.

Figure 2(a) shows the evolution characteristics of $V_{\mathrm{gr}i}$ and $V_{\mathrm{ph}i}$ with q. Specifically, the unique properties of SR modes have been found, which are reflected on the intersections at $q = q_{\mathrm{f}}$ and $q_{\mathrm{s}j}$ (include $q_{\mathrm{s}1}$, $q_{\mathrm{s}2}$). These results are absent in the standard NLSE. Here we will pay our attention to $q = q_{\mathrm{f}}$ and $q_{\mathrm{s}1}$. We omit the results in the case of $q = q_{\mathrm{s}2}$, since the structure is similar to the one in the case of $q = q_{\mathrm{s}1}$ with the same initial physical parameters.

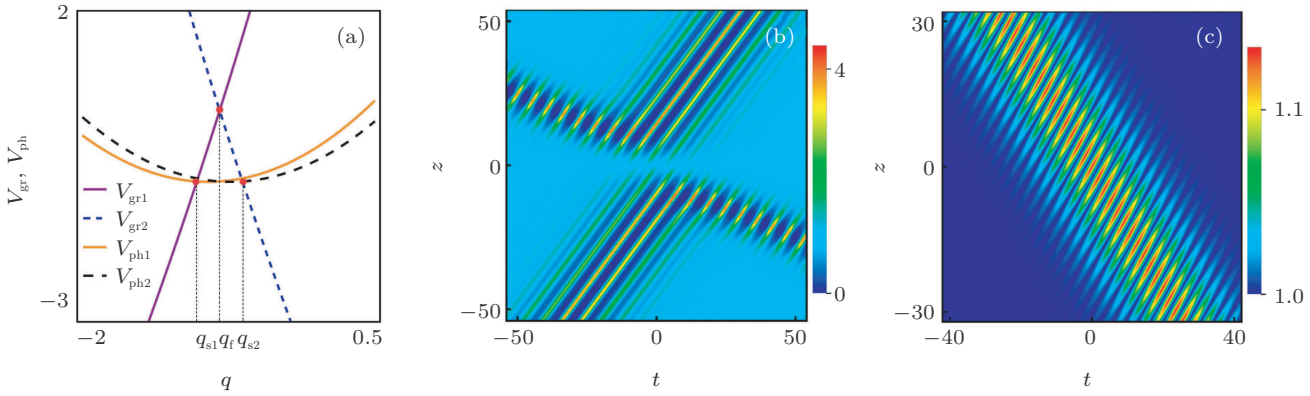


Fig. 2. (color online) (a) Evolution of $V_{\mathrm{gr1,2}}$ and $V_{\mathrm{ph1,2}}$ with q. The intersections at $q = q_{\mathrm{s}j}, q_{\mathrm{f}}$ reflect the nontrivial properties of the SR modes. (b) The half-transition SR mode when $V_{\mathrm{gr1}} = V_{\mathrm{ph1}}$ and $V_{\mathrm{gr2}} \neq V_{\mathrm{ph2}}$ (thus $q = q_{\mathrm{s}1}$). (c) The full-suppression SR mode when $V_{\mathrm{gr1}} = V_{\mathrm{gr2}}$ (thus $q = q_{\mathrm{f}}$). Other parameters are $R = 1.2$, $\alpha = \pi/3$, $\beta = 0.2$, $k = 1/2$, $A_0 = 1$, and $p = 1/2$.

Compared with the standard SR breather formed by a pair of quasi-Akhmediev breathers, a half-transition mode[55,56] is shown in Fig. 2(b). The latter describes a mix of a quasi-Akhmediev breather and a quasi-periodic wave when $V_{\mathrm{gr1}} = V_{\mathrm{ph1}}$ and $V_{\mathrm{gr2}} \neq V_{\mathrm{ph2}}$ (thus $q = q_{\mathrm{s}1} = -\beta(1+8\rho_j\beta)/6$). Namely, a quasi-Akhmediev breather and a quasi-periodic wave collide and form a small-amplitude wave structure around $(z,t) = (0,0)$ (i.e., quasi-annihilation). Then this small-amplitude perturbation is amplified exponentially and eventually becomes a mix of quasi-Akhmediev breather and quasi-periodic waves. Whereas figure 2(c) shows the full-suppression mode when $V_{\mathrm{gr1}} = V_{\mathrm{gr2}}$ (thus $q = q_{\mathrm{f}} = -1/6\beta$).[55,56] Surprisingly, the full-suppression mode describes a crucial non-amplifying nonlinear dynamics of localized small perturbations along z. Interestingly, both of these two localized small perturbations at $z = 0$ are exactly the same as the one of the standard SR breather. Note that the standard SR breather exists when $V_{\mathrm{gr1}} \neq V_{\mathrm{gr2}}$ and $V_{\mathrm{gr}j} \neq V_{\mathrm{ph}j}$ (thus $q \neq q_{\mathrm{f}}, q_{\mathrm{s}j}$). We note that similar half-transition SR structures and full-suppression structures have been demonstrated in recent studies.[55–57]

3.2. Exponential dispersion management superregular modes

We present several cases to illustrate the characteristics of nonautonomous SR breathers under the dispersion management. As shown above, $D_3(z)$ is the third-order dispersion coefficient, and $\Gamma(z)$ is the gain or loss parameter. Without loss of generality, in this work, we choose $\Gamma_0 = 0$, it just denotes a constant background net gain or loss. We consider an exponential dispersion decreasing fiber system with[41,42,47]

$$D_3(z) = D_{30}\exp(-\sigma z),\ \Gamma(z) = \Gamma_0, \tag{4}$$

where D_{30} and σ are the parameters related to the third-order dispersion.

The previous subsection illustrates the essential characteristics of SR modes in the non-management case. In this part, the dynamical processes of SR modes under the exponential dispersion management are investigated, as shown in Fig. 3. It exhibits that the localized small perturbations on the plane wave background evolve to stable quasi-periodic waves. In order to demonstrate the dynamical processes of the exponential dispersion management SR modes more clearly, we depict them from $z = 0$.

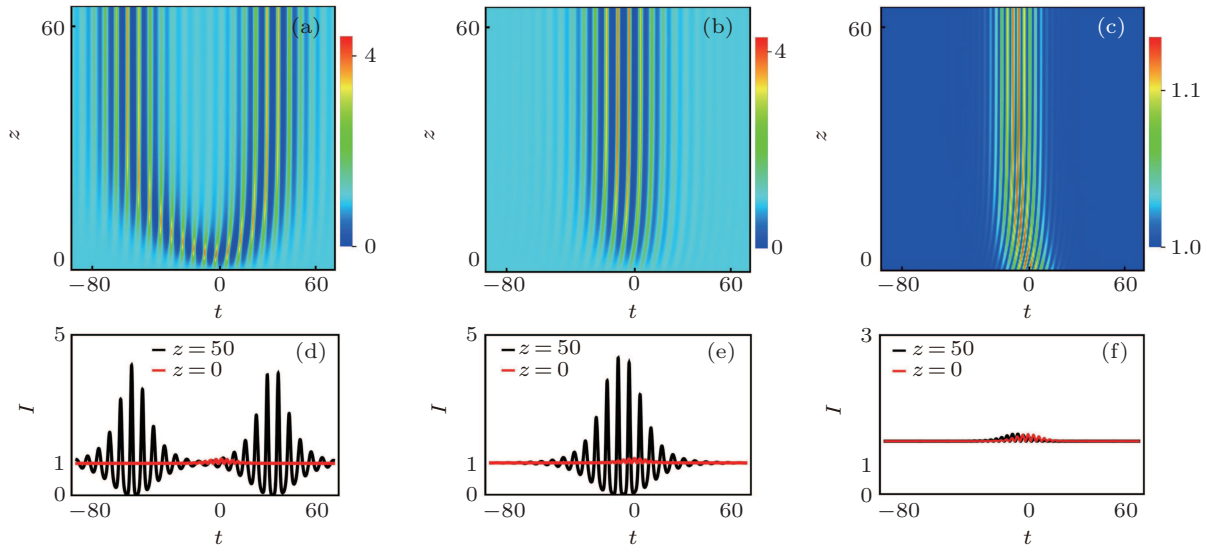


Fig. 3. (color online) (a) Exponential dispersion management of the standard SR breather ($q = 1.5q_{s1}$). (b) Exponential dispersion management of the half-transition SR mode ($q = q_{s1}$). (c) Exponential dispersion management of the full-suppression SR mode ($q = q_f$). Panels (d)–(f) are the profiles of the modes in panels (a)–(c) at $z = 0$ and $z = 50$, respectively. Other parameters are $R = 1.2$, $\alpha = \pi/3$, $\sigma = 1/10$, $\beta = 0.2$, $k = 1/2$, $A_0 = 1$, and $p = 1/2$.

In the case of $V_{gr1} \neq V_{gr2}$ and $V_{grj} \neq V_{phj}$, which imply $q \neq q_f, q_{sj}$, where $q = 1.5q_{s1}$, the localized small perturbation evolves to one-pair quasi-Akhmediev breathers firstly and eventually becomes one-pair quasi-periodic waves along the propagating direction z, as shown in Fig. 3(a). It should be noted that this result is different from the localized small perturbation evolving to a pair of quasi-Akhmediev breathers propagating with fast group velocities in opposite directions in the non-management case. Specifically, comparing the peak intensities of $z = 0$ and $z = 50$, the small localized perturbation (red circles) increases to two high amplitude multi-peaks (black circles) (see Fig. 3(d)), namely, this small localized perturbation on the plane wave background displays the feature of MI.

When $V_{gr1} = V_{ph1}$ and $V_{gr2} \neq V_{ph2}$ (thus $q = q_{s1}$), the localized small perturbation just immediately evolves to a quasi-periodic wave along the propagating direction z (see Fig. 3(b)). This result is distinctly different from the localized small perturbation evolving to a quasi-Akhmediev breather and a quasi-periodic wave in Fig. 2(b). The corresponding profiles at $z = 0$ and $z = 50$ are shown in Fig. 3(e). As we all know, the classical SR breather scenario of MI exhibits a complex evolution of multiple quasi-Akhmediev breathers, which evolves from a localized small perturbation at the position of breather interaction.[8–10] In contrast, the exponential dispersion management SR modes can be seen as a stabilization of the classical SR modes, which describe the stabilization of the nonlinear stage of the MI.

When $V_{gr1} = V_{gr2}$, thus $q = q_f$, we can see the exponential dispersion management full-suppression SR mode. Namely, the growth of the initial perturbation amplitude vanishes (Fig. 3(c)). The small localized perturbation evolves to a small amplitude quasi-periodic wave with a small phase shift (Fig. 3(f)). It can be inferred that this phenomenon arises from the regime where the MI is suppressed completely when $q = q_f$.

3.3. Periodical dispersion management superregular modes

In real experiments, periodic management of parameters of nonlinear systems has been shown to be an effective technique for generating several types of breathers.[58] Gener-

ally, the periodic management of parameters depends on the trigonometric functions which are physically relevant. The research on this case is more conducive to the characteristics of nonlinear waves because they provide alternating regions of positive and negative dispersion and nonlinearity.[59] In this section, we analyze the periodical dispersion management SR modes in the periodic distributed fiber system with[43,44]

$$D_3(z) = D_{30}\cos(\kappa z), \quad \Gamma(z) = \Gamma_0, \tag{5}$$

where D_{30} and κ are the parameters related to the third-order dispersion management.

When $V_{\mathrm{gr1}} \neq V_{\mathrm{gr2}}$ and $V_{\mathrm{gr}j} \neq V_{\mathrm{ph}j}$, which imply $q \neq q_{\mathrm{f}}, q_{\mathrm{s}j}$, in the periodic distributed system, the standard SR breather recurs periodically with the period $2\pi/\kappa$ (see Fig. 4(a)). Especially, in contrast to the non-management standard SR breather propagating to $\pm\infty$, here the periodical dispersion management standard SR breather is localized in a certain scope in the t-axis direction. It looks like one-pair quasi-Akhmediev breathers propagating with periodic oscillation along z. Interestingly, the one-pair quasi-Akhmediev breathers go through the amplification and quasi-annihilation process in each half-period.

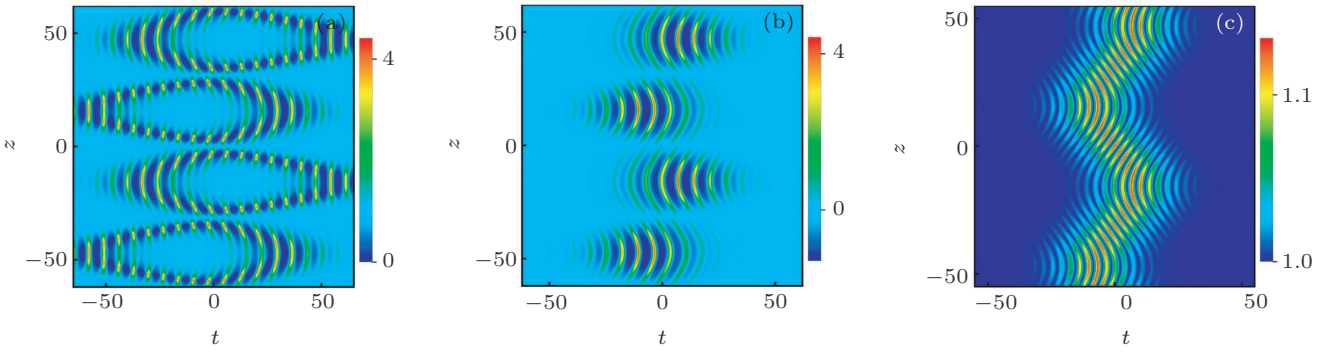


Fig. 4. (color online) (a) Periodical dispersion management of the standard SR breather ($q = 1.5q_{\mathrm{s1}}$). (b) periodical dispersion management of the half-transition SR mode ($q = q_{\mathrm{s1}}$). (c) periodical dispersion management of the full-suppression SR mode ($q = q_{\mathrm{f}}$). Other parameters are $R = 1.2$, $\alpha = \pi/3$, $\kappa = 1/10$, $\beta = 0.2$, $k = 1/2$, $A_0 = 1$ and $p = 1/2$.

When $V_{\mathrm{gr1}} = V_{\mathrm{ph1}}$ and $V_{\mathrm{gr2}} \neq V_{\mathrm{ph2}}$ (thus $q = q_{\mathrm{s1}}$), the half-transition SR breather recurs periodically with the period $2\pi/\kappa$, as shown in Fig. 4(b). Compared to the non-management case, the structure of the periodical dispersion management half-transition SR breather is totally changed. It is clear that a multi-peak structure goes through the amplification and quasi-annihilation process from a small perturbation in each direction at $z = n\pi/\kappa$ (n is an integral number). Specifically, compare Figs. 4(a) and 4(b), the growth rate of the perturbation decreases gradually as $q \to q_{\mathrm{s1}}$. As a result, these periodic motions of SR modes from an initially small localized perturbation possess more complex growth-evolution processes than the standard one, which exhibits only one growth-evolution cycle.

On the other hand, when $V_{\mathrm{gr1}} = V_{\mathrm{gr2}}$ (thus $q = q_{\mathrm{f}}$), we can obtain the periodical dispersion management full-suppression SR mode (see Fig. 4(c)). In this case, the small perturbation cannot be amplified, but propagates with small oscillations along z. It also can be inferred that this phenomenon arises from the MI being suppressed completely when $q = q_{\mathrm{f}}$. It is consistent with the result in the exponential dispersion management case. Namely, the suppression of MI is related with the background frequency and cannot be changed by the dispersion management.

Very recently, the nonlinear Talbot effect of the rogue waves of the NLSE[60] and the nonlinear Talbot-like property of the Kuznetsov–Ma breathers of the variable-coefficient derivative NLSE[61] have been focused on widely. Interestingly, the periodical dispersion management half-transition SR mode shown in Fig. 4(b) has many characteristics that are the same as the nonlinear Talbot effect.[60–62] In Fig. 4(b), the localized nonlinear Talbot-like property, where the periodic incident wave exactly reappears at certain distances, can be clearly seen.

Figure 5(a) shows the intensities of the localized wave in Fig. 4(b) at $z = 10$, $10 + z_{\mathrm{T}}/2$, $10 + z_{\mathrm{T}}$. Here $z_{\mathrm{T}} = 2\pi/\kappa$ is the Talbot effect length. We find that, if the initial state is at the non-zero place ($z = 10$), the localized structure recurs after the full Talbot effect length but has a phase shift after half Talbot effect length. Interestingly, the quasi-annihilation phenomenon around $z = 0$ shows a special characteristic. As shown in Fig. 5(b), we find that the localized structures at z_{T} and $z_{\mathrm{T}}/2$ are the same as the one at the initial place $z = 0$. This probably means that the phase shift has been suppressed by the quasi-annihilation phenomenon. Hereby, we demonstrate the localized nonlinear Talbot-like property from the half-transition SR mode based on the nonautonomous higher-order NLSE.

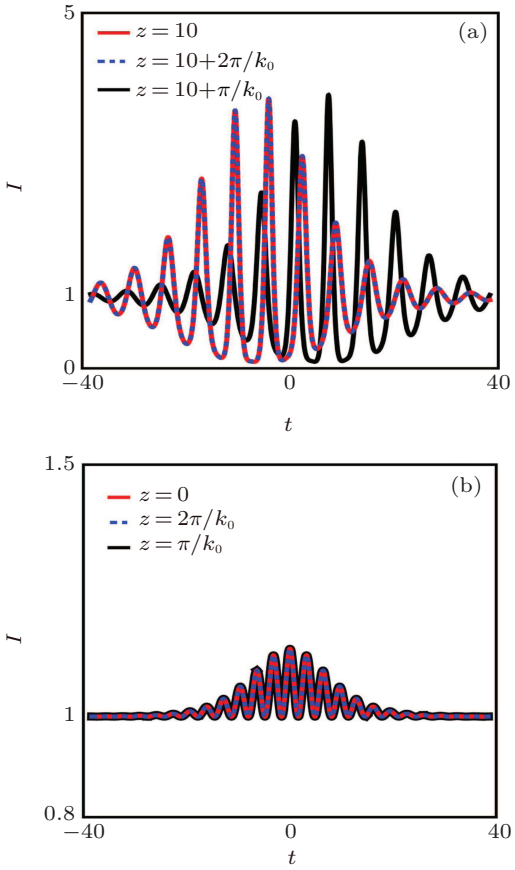


Fig. 5. (color online) (a) Profiles of Fig. 4(b) at $z = 10$, $10 + 2\pi/k_0$, and $10 + \pi/k_0$; (b) profiles of Fig. 4(b) at $z = 0$, $2\pi/k_0$, and π/k_0.

4. Conclusion

We have presented the exact solutions describing the dynamics of superregular breathers for the nonautonomous higher-order NLSE. For a real optical fiber system, the nonautonomous model provides us with a more realistic model than its constant coefficient counterpart. The exact superregular breathers solutions of the nonautonomous higher-order NLSE can be obtained via a transformation connected with the autonomous Hirota equation. Based on the procedure, we find that the dispersion management superregular modes exhibit stabilization and recurrence in an exponential dispersion fiber and a periodic distributed fiber system. Particularly, the nonlinear dynamics of superregular modes evolved from an identical initial small-amplitude modulation is analyzed in detailed. Our results could provide certain theoretical assistance to the experimental control and manipulation of superregular modes dynamics in an inhomogeneous fiber.

Acknowledgment

We are grateful to Jian-Wen Qi and Liang Duan for our valuable discussion.

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