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Detection of meso-micro scale surface features based on microcanonical multifractal formalism*

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Synthetic aperture radar (SAR) is an effective tool to analyze the features of the ocean. In this paper, the microcanonical multifractal formalism is used to analyze SAR images to obtain meso-micro scale surface features. We use the Sobel operator to calculate the gradient of each point in the image, then operate continuous variable scale wavelet transform on this gradient matrix. The relationship between the wavelet coefficient and scale is built by linear regression. This relationship decides the singular exponents of every point in the image which contain local and global features. The manifolds in the ocean can be revealed by analyzing these exponents. The most singular manifold, which is related to the streamlines in the SAR images, can be obtained with a suitable threshold of the singular exponents. Experiments verify that application of the microcanonical multifractal formalism to SAR image analysis is effective for detecting the meso-micro scale surface information.

Keywords: multifractal microcanonical singularity analysis

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1. Introduction

Rich information is contained in the ocean surface. Observations and deep understanding of the ocean surface flow system can provide great benefits to the study of the ocean and the environment. The main challenge is how to obtain global ocean information at rates of high frequency. Satellite remote sensing technology makes this possible. At present, the relatively mature sensors include scatterometer and synthetic aperture radar (SAR). The resolution of scatterometer data can reach 25 km, while SAR can reach 50 m. Moreover, with its all-weather and all-time capabilities, SAR can be considered to be more suitable for studying the ocean surface than the scatterometer.

In order to obtain and study information data sets for the ocean surface, the turbulent processes usually need to be observed and analyzed. For now, the main method of analyzing the turbulent processes is statistical analysis, which needs abundant data to build numerical models. The microcanonical multifractal formalism allows us to analyze the observations directly. This approach is a purely geometrical approach which is able to describe and exploit the geography properties of the tracer variable. In this paper, the microcanonical multifractal formalism is studied. Based on the analysis of the characteristics of ocean SAR images, a method for singularity analysis is presented and used to reveal the multifractal structure of ocean SAR images. The most singular manifold method is used to analyze the maximum singularity streamfunction and to obtain the main streamlines in the ocean SAR image.

2. General theory

In the 1970s, Mandelbrot firstly introduced the concept of fractal to the natural science field. In simple terms, fractal means that in a given system, there is some self-similarity between the whole and the part.[1] This self-similarity is found widely in the natural world. By studying the application of fractal thought, many mathematical difficulties can be solved in the analysis of physical and economic phenomena.

Suppose that an n-dimensional space with volume 1 is divided into n equal parts which are D-dimensional geometries with the side length of r. We can obtain

$$nr^D = 1. \tag{1}$$

The dimension of the space can be obtained as

$$D = -\frac{\ln n}{\ln r}. \tag{2}$$

Mandelbrot calculated that the dimension of the British coastline is 1.25. Then the length of the coastline can be determined. So in practice, we can use the fractal dimension to describe the complex geometries.[2]

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Mathematically, multifractals are characterized by many or infinitely many power-law relations.[3] These relations are built by exponents. To describe the complex systems, we need a set of continuous exponents, which is a singular spectrum. Every exponent, which is a singular exponent, reveals the singularity of every point in the system. This method is recognized as multifractal.

There are two ways to analyze the multifractal system and its energy property. The traditional method is the canonical formalism (CF), which only deals with the statistical property. The singular exponents are calculated by the box-counting method[4] or its improved method.[5] The other method is the microcanonical formalism (μCF), which assigns a geometrical property to every point in the system. Thus, we can analyze the fractal structure, obtain its energy characteristics, and process the accurate geometric classification on fractal manifolds. In addition, by the application of μCF, it has been shown that the multifractal signal can be reconstructed from the most singular points.

Coherent vortices in a turbulent flow strongly interact and fold small-scale filaments ejected from the vertex cores and generate small-scale tracer gradients between eddies.[6] So the spatial structure of a tracer inherits some properties of the underlying flow, according to what has been theoretically and experimentally shown in some papers.[7] A new scheme for extracting the structural information from the ocean scalar variable has arisen, that of singularity analysis in the microcanonical multifractal formalism.

With more researching on the microcanonical multifractal formalism, its application range becomes wider. It can be used in analysis of topography and geomorphology,[8] ecosystem, digital image processing,[9,10] power systems, theoretical physics,[11] urban traffic,[12] signal and information processing,[13] the processing of radar and sonar signals,[14] and so on.

3. Multifractal structure of SAR image

3.1. Singularity analysis

In singularity analysis, the key is assigning the local scale exponent to every point in the image. The local singular exponent $h(x)$ of a given field $s(x)$ at point x can be obtained, for any scalar r small enough, from the following scaling:

$$\frac{1}{r}\left|s(x+r)-s(x)\right| \sim r^{h(x)}. \tag{3}$$

The results from wind tunnels and other laboratory experiments reveal that such scaling relations are generally observed.[15,16] The local singular exponent can reveal the continuity degree around a given point. This method can be used in analyzing natural images,[17] econometric time series,[18] meteorological images,[19] bacterial genes,[20] speckle fields,[21] random signals,[22,23] nanofluids,[24] target detection studies,[25] and oceanographic satellite images.[26]

However, in most practical cases, it does not hold, because the existence of long-range correlations can mask the softest singular exponents.[27] To eliminate this effect, we can use the wavelet transform to filter $s(x)$ without destroying the scale invariance properties. The counterpart to Eq. (3) is the wavelet transformed result in the way

$$\mathrm{T}_{\psi} s(x,r) \sim r^{h(x)}, \tag{4}$$

where $\mathrm{T}_{\psi} s(x,r)$ is the wavelet projection of s over the wavelet Ψ at the point x and the scale r, defined as

$$\mathrm{T}_{\psi} s(x,r) \equiv \frac{1}{r^2}\int_{-\infty}^{\infty} s(x)\Psi\left(\frac{x-x_0}{r}\right)\mathrm{d}x. \tag{5}$$

The wavelet Ψ used here must have some zero-crossings. These points may reduce the resolution in the determination of singular exponents. To solve this problem, we can process the wavelet transform on the modulus of its gradient instead of the signal itself, as

$$\mathrm{T}_{\psi}\left|\nabla s\right|(x,r) \equiv \frac{1}{r^2}\int_{-\infty}^{\infty} \left|\nabla s\right|(x)\Psi\left(\frac{x-x_0}{r}\right)\mathrm{d}x. \tag{6}$$

In this relation, Ψ needs not to have zero-crossing. We can choose positive wavelets to process the wavelet transform. If the signal is multifractal, the relationship between the wavelet coefficients and the singular exponents can be obtained as

$$\mathrm{T}_{\psi}\left|\nabla s\right|(x,r) \sim r^{h(x)}, \tag{7}$$

where $h(x)$ can be obtained by

$$h(x) = \frac{\ln[\mathrm{T}_{\psi}\left|\nabla s\right|(x,r)]}{\ln r}. \tag{8}$$

In ocean SAR images, redundancy and high resolution reduce the difference between the neighbor points. The traditional method of singularity analysis is unsuited for this analysis. To solve these problems, the singularity analysis method is improved as shown in Fig. 1.

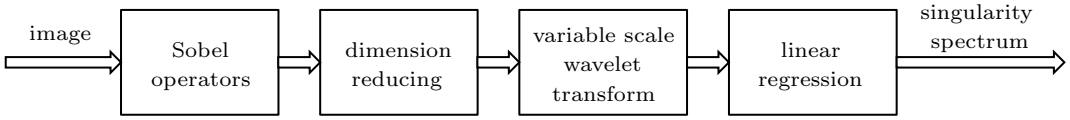


Fig. 1. Improved singularity analysis process.

The difference between most neighbor points is so small that simply calculating the gray level gradient cannot produce enough features. So we use the Sobel operators instead of the traditional gradient calculation to highlight the surface features. This calculation is processed in the x and y directions separately, which are shown as

$$\boldsymbol{Sobel}_x=\begin{bmatrix}-1 & 0 & +1\\ -2 & 0 & +2\\ -1 & 0 & +1\end{bmatrix}*\boldsymbol{S},\quad \boldsymbol{Sobel}_y=\begin{bmatrix}+1 & +2 & +1\\ 0 & 0 & 0\\ -1 & -2 & -1\end{bmatrix}*\boldsymbol{S}, \tag{9}$$

where $\boldsymbol{S}$ is the gray-level matrix of the SAR image. Thus, we obtain two matrixes, $\boldsymbol{Sobel}_x$ and $\boldsymbol{Sobel}_y$. In other words, there are two values, assumed to be s_x and s_y, on every point. To perform wavelet transform on the matrix, there must be one value on every point. Moreover, this value must contain the so-called feature on the x direction and the y direction all together. So we must reduce the dimension to get the gradient for every point as

$$\nabla s=\sqrt[2]{(s_x)^2+(s_y)^2}. \tag{10}$$

We use this ∇s instead of the traditional gradient to highlight the difference among the adjacent points. Every SAR image has a finite energy and fits the square integrable condition. So we use Eq. (6), at every point x, with different scales $r_1, r_2, \ldots, r_n$, and obtain relevant wavelet coefficients as $\mathrm{T}_\psi|\nabla s|(x,r_1)$, $\mathrm{T}_\psi|\nabla s|(x,r_2),\ldots,\mathrm{T}_\psi|\nabla s|(x,r_n)$. At a fixed scale, $h(x)$ can be obtained by Eq. (8). At multiscale, $h(x)$ can be obtained by linear regression. Set $b_i=\ln[\mathrm{T}_\psi|\nabla s|(x,r_i)]$, $a_i=\ln r_i$, then we obtain

$$h(x)=\frac{\sum_{i=1}^{n}(a_i-\bar{a})(b_i-\bar{b})}{\sum_{i=1}^{n}(b_i-\bar{b})^2}=\frac{\sum_{i=1}^{n}a_ib_i-n\bar{a}\bar{b}}{\sum_{i=1}^{n}b_i^2-n\bar{b}}. \tag{11}$$

Equation (11) calculates the regression coefficient by the least-squares fitting. Indeed, according to the scale invariance property in fractal analysis, this coefficient is just the singularity exponent.

3.2. Result

Using the microcanonical multifractal formalism in analyzing SAR ocean images can get properties and detect the surface currents. Some researchers have studied its application in analyzing scatterometer images.[6] But the scatterometer cannot provide ocean information at any time and in all weather. Besides, the resolution of the scatterometer data is too low to detect the meso-micro scale vortex. Normalized radar cross section (NRCS) presents the electromagnetic backscattering intensity obtained by SAR. The results of electromagnetic scattering from a fractal sea surface can be used in the remote sensing of ocean variables, for example, retrievals of wave spectra and marine winds, ship detection, oil slick recognition, pollution analyses, and related problems.[28–30] Some studies have shown that NRCS of the ocean surface has a fractal structure.[31–33] So NRCS is selected to be studied for the ocean SAR images.

Figure 2 is the SST image in 22 May 2009 of part of the Northeast American coast. There is a black area in the lower-right corner, which is caused by clouds or other phenomena. Analysis of this SST image by application of μCF leads to Fig. 3. Thus we can see the main manifold in this area (except the black area). Moreover, the manifold around the black area is also deteriorated. When the SST image has large or multiple black areas, we perhaps cannot see the manifold at all. Because of the lower resolution of the SST image, we can only see some large scale currents in Fig. 3. At the bottom of Fig. 3, there are many dark lines which constitute simply a mess. Some meso-micro scale vortex phenomena may be located there, but we cannot clearly see the manifold.

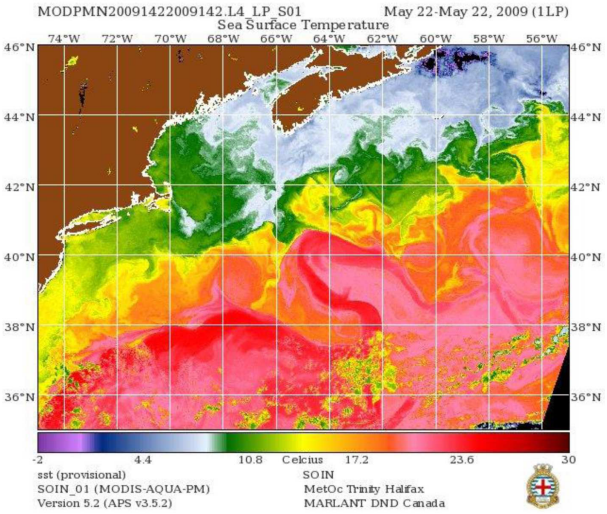


Fig. 2. SST image.

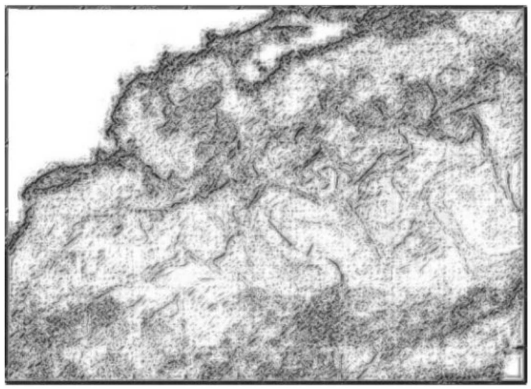


Fig. 3. Singular exponents detected from the SST image.

To solve these problems, we use this method on the SAR NRCS. Figures 4 and 5 show two areas in this region, which are within the broken lines, imaged as NRCS by Radarset-2 at the same time.

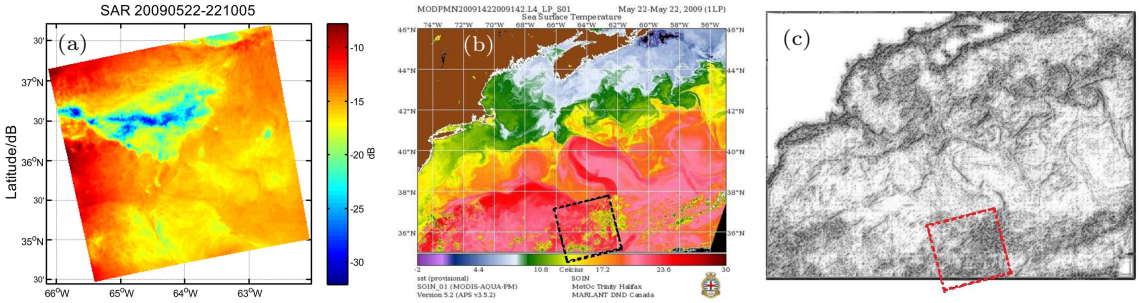


Fig. 4. (a) SAR image obtained at 22:10:05, (b) location in SST image, (c) location in the singular exponents image (RADARSAT-2 data and products from MacDonald, Dettwiler, and Associates Ltd. All Rights Reserved).

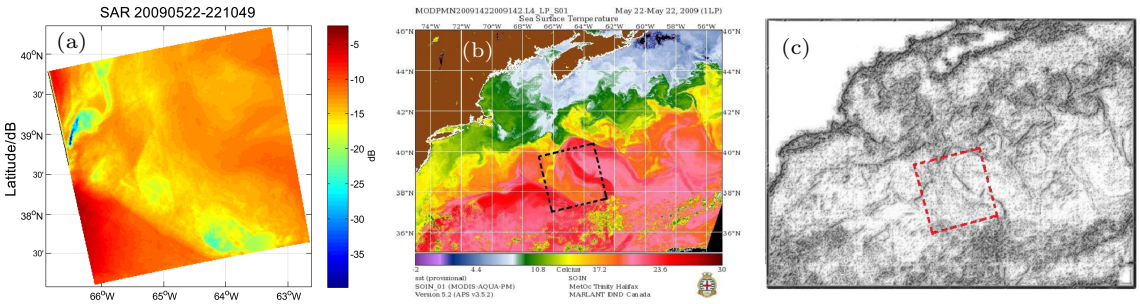


Fig. 5. (a) SAR image obtained at 22:10:49, (b) location in SST image, (c) location in the singular exponents image (RADARSAT-2 data and products from MacDonald, Dettwiler, and Associates Ltd. All Rights Reserved).

From the SAR images, we can see some vortexes, but in the same area in the SST image and singular exponents image, we cannot see these vortex features clearly. In the SAR image obtained at 22:10:05, we can see an almost complete vortex located near 30°N and 64.5°W. We can use the microcanonical multifractal formalism in analyzing this SAR image if it has a fractal structure indeed. In Fig. 6, we can see the relationship between $\ln[T_\psi|\nabla s|(x,r_i)]$ and $\ln r_i$ at any point in the SAR image. The values of $\ln[T_\psi|\nabla s|(x,r_i)]/\ln r_i$ at every r_i are similar to the regression coefficient. This is a prevalent phenomenon in the whole image, which fits the scale invariance property of fractal. Then we use the microcanonical multifractal formalism in analyzing this SAR image and obtain Fig. 7.

Figure 7 shows the manifold of this meso-micro scale vortex clearly. The manifold indicates the streamlines in the ocean. The SAR image has a higher resolution, so the echo intensities of adjacent points have little difference to one another, which will reduce the gradients. When we calculate the local singular exponents of every point, the results are centered around 0. This conclusion can be comfirmed by Fig. 8, which is the probability density function (PDF) for the singular exponents obtained from the SAR image. To show the manifold and analyze the streamline of the ocean, we disperse the results by nonlinear stretching, which allows us to see the manifold clearly but breaks the streamlines at some points. This breakage can be removed by some specific algorithms when

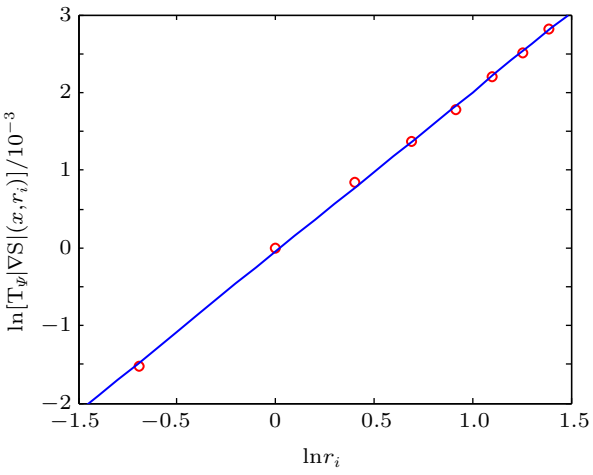


Fig. 6. Regression curve of a point in the SAR image.

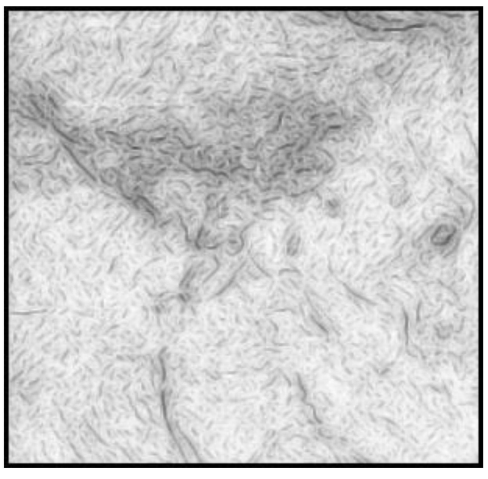


Fig. 7. Singular exponents.

some specific features are needed, such as the vortex's edge, area size, and so on. From the latitude and longitude coordinates, we can see that the maximum width of this vortex is about 200 km. The ocean surface feature of this scale cannot be detected by SST fields determined by scatterometer images, so using the microcanonical multifractal formalism on SAR images to analyze the meso-micro scale feature is reasonable and efficient.

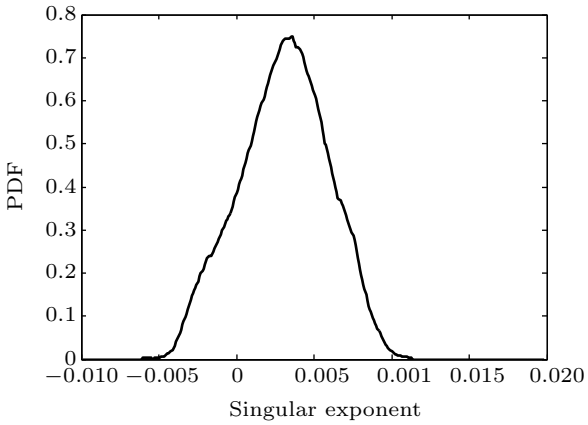


Fig. 8. PDF for the singular exponents.

4. Most singular manifold analysis

4.1. Maximum singularity streamfunction method

Once every point is assigned a singular exponent, it is possible to decompose the signal into different patterns (the fractal components) classified from the most singular (usually associated to transitions) to the less singular (associated to smooth, continuous areas). This "multiple-fractal" character shows up when the image is split in different singular components F_h. They are formed by the points sharing the same singular exponent h

$$F_h = \{h(x) = h\}. \tag{12}$$

The components F_h can be predicted from the statistical properties of the image.[29] In the multifractal structure, we are mainly interested in the most singular points, because they have many dynamical and statistical properties. The most singular manifold (MSM) is denoted as $\hat{F}_\infty$, which is given by

$$\hat{F}_\infty \equiv \{x : h(x) = h_\infty\}, \tag{13}$$

where h_∞ is the value of the most singular exponent, that is, the minimum observed value of singularity. From a statistical standpoint, the MSM is related with the vertex of the energy cascade. So we can analyze the image by its MSM, which is called the maximum singularity streamfunction method.

But in practice, this definition is too restrictive. Because the whole image to be analyzed is composed of different areas. They have different mutifractal parameters. So the h_∞ of the areas are not exactly the same. To solve this problem, we can use a sufficiently small h_0 instead of h_∞. The coarser version of MSM is obtained as

$$F_\infty \equiv \{x : h(x) < h_0\}. \tag{14}$$

MSM is the main result of studying the ocean surface by SAR images. It relates to the vertex of the energy cascade in turbulence. So a lot of information regarding the ocean can be obtained by analyzing it, such as the stream functions, ocean surface velocities, and so on. This method also can be used to analyze the ocean surface salinity[26] and study the surface ocean partial pressure of CO_2, pCO_2, and air–sea CO_2 fluxes.[30]

4.2. Result

According to the above analysis, the choice of h_0 decides the MSM that we get. Figure 9 shows different MSM obtained by different h_0.

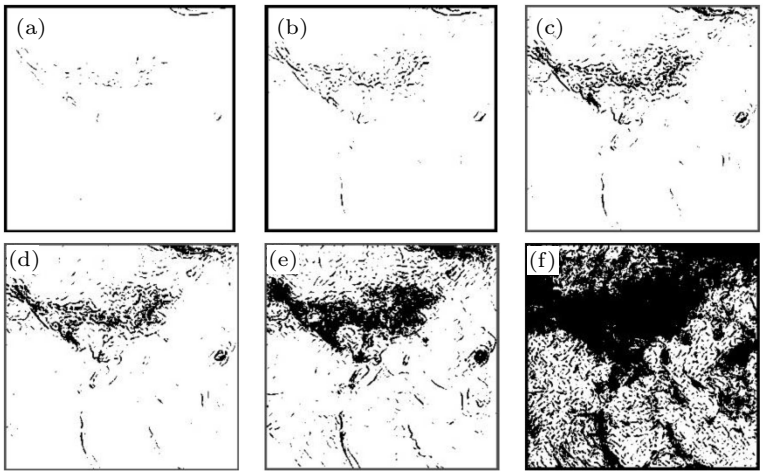


Fig. 9. MSM obtained using different thresholds: (a) $h_0 = -0.002$, (b) $h_0 = 0$, (c) $h_0 = 0.002$, (d) $h_0 = 0.005$, (e) $h_0 = 0.007$, (f) $h_0 = 0.01$.

In these images, we can see that too small h_0 results in excessive filtering of some coherent streamlines. On the contrary, when h_0 increases, many points that have less singularity are also reserved. In this case, we cannot capture as many patterns as possible. Considering these two aspects, a reasonable threshold is necessary to obtain the clear and complete set of streamlines. Thus, we set the threshold as t, which can separate the spectrum image into two parts: the foreground and the background. Set the percentage of the points which are larger than t (according to the foreground) as w_0, that of those which are smaller than t (according to the background) as w_1. Set the average of the foreground as u_0, that of the background as u_1. Then we traverse t from $h(x)_{\min}$ to $h(x)_{\max}$ with step length of 0.0001 and find t to make the following value maximal:

$$g = w_0 w_1 (u_0 - u_1)^2. \tag{15}$$

This g represents the class variance. By the method of maximum class variance, we can obtain the optimal effect of binarization. Then we determine that the threshold is 0.0022. For easier determination of observations and comparisons, we set $h_0 = 0.002$. From Fig. 9, we can see that by this threshold, the main streamline is retained. The meso-micro scale vortex on the upper left corner in this area has a clear and complete structure. Although the MSM should represent streamlines, numerical limitations and data resolution may force some lines to end within the flow or intersect between them. But the main structure of the streamline is obtained. This information has significant value in analyzing the ocean features.

5. Conclusion

In this paper, the microcanonical multifractal formalism is used to analyze a SAR image to obtain meso-micro scale ocean surface features. According to the characteristics of the SAR image, calculation of the gray level gradients is replaced by the Sobel operator to distinguish the small differences between adjacent points. By the continuous variable scale wavelet transform, the long-range correlations are filtered and the accurate relationship between the wavelet coefficient and scale is built. According to the scale invariance property, linear regression can be used to obtain the local singular exponents. The image of the singular exponents can reveal the ocean surface features. Using the maximum singularity streamfunction method with a suitable threshold, which is obtained by the method of maximum class variance, we can obtain MSM and streamlines on the ocean surface. Experiments verify that the application of the microcanonical multifractal formalism in SAR image analyses is effective for detecting the meso-micro scale surface information.

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