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Current transport and mass separation for an asymmetric fluctuation system with correlated noises*

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We discuss the transport of an underdamped particle driven by an external fluctuation force in a spatially periodic asymmetric potential with correlated noises. The corresponding mathematical model is established. The movement of the steady current of an underdamped particle is presented by the method of the numerical simulation. It is indicated that the value of the current may be negative, zero, or positive. The external fluctuation force and correlated noises can effect the current direction. Under the appropriate parameters, the correlated noises intensity may even raise a reversal of the current. Besides, we have noticed a phenomenon that particles with different weight have different directions during movement by the impact of the correlated noises and external fluctuation force. Therefore, the Brownian particles can be effectively separated according to their masses.

Keywords: current transport, correlated noises, fluctuation force, Brownian motion

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1. Introduction

Transport phenomena is ubiquitous in nature and can be observed in a number of physical and biological systems. In recent years, the research has focused on noise-induced transport, especially the correlated noises. In certain situations, noises may not be independent, but correlated, because noises may have a common origin and be correlated to each other.[1,2] In fact, in many physical as well as biological systems, many studies indicate that the noises or correlated noises widely exist.[3–7] For instance, in the physical system, the transport has been addressed by using the noise-driven Langevin equation. The motion of the current in a periodic structure with a white-noise was stressed in Ref. [8]. We also note a recent interesting essay in Refs. [9] and [10] discussing the transport of particles in a spatially periodic potential subjected to the correlated noises. In the ecosystem, the noise induce frequent catastrophic regime shifts and even more accelerate it.[4] The effects of time delay and correlation intensity on the signal-to-noise ratio between the internal noise and external noise are discussed.[5,11] It can be concluded that the correlated noises play a great role in the nonequilibrium motion process. In addition, there were also some studies on rocking ratchets,[12,13] the inertia ratchets,[14] feedback-control ratchet,[15] uncoupled neuron system,[16] the static bias,[17–19] mass separation,[20] entropy barrier,[21] the motion of fiber suspension,[22] non-markovian effect,[23] etc.

While the researches had so far mainly concentrated on the transport of overdamped particles in dynamical systems, the mass of the particle is neglected. Recently, however, some researchers are increasingly aimed at examining the movement of inertia Brownian in periodic potential. The processes have been studied analytically and numerically in a few papers.[24–27] In Ref. [27], Sancho discussed the transport of Brownian particles in an inertial system. They found that different masses of particles can be separated by controlling the system. The study of nonequilibrium dynamics of underdamped Brownian particles in an asymmetric potential ratchet was addressed in Ref. [28], and they demonstrated that the current reversal phenomenon exists for less weight particles. It should be pointed out that works aforementioned did not consider the external force or only mentioned the constant force, but they did not take into account the external force which is changeable and inconsistent. That is to say, the system is always disturbed by fluctuation force. In Ref. [29], they stressed the transport behaviors of two harmonically coupled Brownian particles in an asymmetric saw-tooth potential. The competition between noises parameters and external forces is a necessary condition for the current reversal.[30] However, the role of the fluctuation force in inertial system has not been fully explored. Therefore, the question is raised in this thesis which examines the influence of external fluctuation force with the correlated noises in an asymmetric periodic potential on inertial Brownian motors.

In this essay, the motion of underdamped particles by an external fluctuation force and correlated noises in an asymmetric potential is discussed. The rest of the paper is organized as follows. In Section 2, we introduce the Langevin equation of the model. In Section 3, current transport induced by noises and external fluctuation force are discussed. The conclusion and some final remarks are concluded in Section 4.

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2. Model

Let us consider the underdamped movement of a Brownian particle under the influence of a time-dependent spatially potential $V(x,t)$ and subjects to the external fluctuation force $F(t)$. The corresponding equation of motion is described by the following Langevin equation in the dimensionless form:

$$m\ddot{x}+\dot{x}=-V'(x,t)+F(t)+\xi(t), \tag{1}$$

where m is the mass of the particle, $V'(x,t)$ points to the potential function $V(x,t)$ derivative of x and the dot represents the derivative of t; here $\xi(t)$ is Gaussian white noise with $\langle\xi(t)\rangle=0$ and $\langle\xi(t)\xi(s)\rangle=2\alpha\delta(t-s)$, in which α is the intensity of the noise $\xi(t)$ and relates to the temperature. $F(t)$ is an external fluctuation force and it is displayed in Fig. 1(a). It satisfies $F(t+\tau)=F(t)$; $\int_0^\tau F(t)\mathrm{d}t=0$ and has the following property:[31]

$$F(t)=\begin{cases}\dfrac{1+\varepsilon}{1-\varepsilon}F_0, & n\tau\le t<n\tau+\dfrac{1}{2}\tau(1-\varepsilon),\\ -F_0, & n\tau+\dfrac{1}{2}\tau(1-\varepsilon)<t\le(n+1)\tau,\end{cases} \tag{2}$$

where τ is its period and ε is the temporal asymmetric parameter of the external fluctuation force. The time-dependent potential $V(x,t)$ consists of a ratchet part $V(x)$ (spatially asymmetric periodic potential) and time modulated by a stochastic process in the following from:[32,33]

$$V(x,t)=V(x)(1+\eta(t)). \tag{3}$$

The asymmetric periodic potential $V(x)$ is given by

$$V(x)=-\frac{1}{2\pi}(\sin(2\pi x)+\frac{1}{4}\sin(4\pi x)), \tag{4}$$

and figure 1(b) depicts the potential $V(x)$. The multiplicative noise $\eta(t)$ represents another Gaussian white noise with zero mean and its autocorrelation is $\langle\eta(t)\eta(s)\rangle=2D\delta(t-s)$, D is the intensity of the noise $\eta(t)$. Most of previous articles treat them as independent noises, while in certain situations noises may be correlated as well, so here we assume that the two noises are correlated with[34,35]

$$\langle\xi(t)\eta(s)\rangle=\langle\eta(t)\xi(s)\rangle=2\lambda\sqrt{D\alpha}\delta(t-s), \tag{5}$$

where λ refers to the intensity of the correlation between $\xi(t)$ and $\eta(t)$. Thus, equation (1) turns into

$$m\ddot{x}+\dot{x}=-V'(x)-V'(x)\eta(t)+F(t)+\xi(t). \tag{6}$$

To decouple the two cross-correlation noises, the following transformation is adopted:[36]

$$\xi'(t)=\xi(t)-\lambda\sqrt{\alpha/D}\eta(t). \tag{7}$$

Here the noises $\xi'(t)$ and $\eta(t)$ are no longer correlated. Then equation (6) can be rewritten as

$$m\ddot{x}+\dot{x}=-V'(x)+F(t)+(-V'(x)+\lambda\sqrt{\alpha/D})\eta(t)+\xi'(t). \tag{8}$$

For the convenience of analysis and discussion, we can write Eq. (8) as

$$\dot{x}=y, \tag{9}$$

$$\dot{y}=\frac{1}{m}(f(x,t)+g(x)\eta(t)+\xi'(t)-y), \tag{10}$$

where $f(x,t)=-V'(x)+F(t)$ and $g(x)=-V'(x)+\lambda\sqrt{\alpha/D}$. The Stratonovich interpretation of Eqs. (9) and (10) yields the Fokker–Planck equation[37]

$$\frac{\partial P(x,y,t)}{\partial t}=-\frac{\partial J(x,y,t)}{\partial x}, \tag{11}$$

and the probability density $P(x,y,t)$ satisfies

$$\frac{\partial P(x,y,t)}{\partial t}=-y\frac{\partial P(x,y,t)}{\partial x}-\frac{1}{m}\frac{\partial(y+f(x,t))P(x,y,t)}{\partial y}+\frac{1}{m}(D(g(x))^2+\alpha(1-\lambda^2))\frac{\partial^2P(x,y,t)}{\partial y^2}. \tag{12}$$

Equation (12) cannot be analytically solved even for the stationary case since detailed balance is broken, but it can be solved by applying numerical methods.

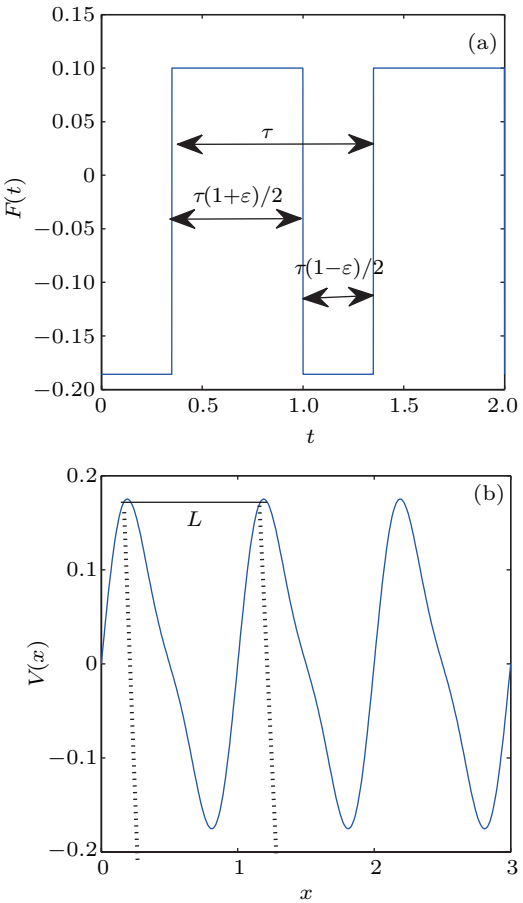


Fig. 1. (color online) Fluctuation force and potential: (a) fluctuation force $F(t)$, $\langle F(t)\rangle=0$; $F(t+\tau)=F(t)$; where the temporal asymmetry is given by the parameter ε. (b) Potential $V(x)=V(x+L)$, L is the period of the potential $V(x)$.

3. Transitions and discussion

In the following, we adopt our numerical simulation directly by using the Langevin Eq. (8). According to the Euler method for numerical simulation algorithm,[37–39] the results of the current versus other parameters are plotted in Figs. 2–7, respectively. Every point in the figures is simulated 500 different trajectories and the 10^5 different discrete times. Here the time step is taken as $\Delta t = 0.01$. In order to guarantee that the system is in the stationary state, we take the average after $t = 1000$. There is a quasi-steady state if $F(t)$ turns very slowly with the time t. In other words, its period is longer than any other time scale of the system. Therefore, we choose a period that is long enough for $F(t)$ so that quasi-steady state can occur.

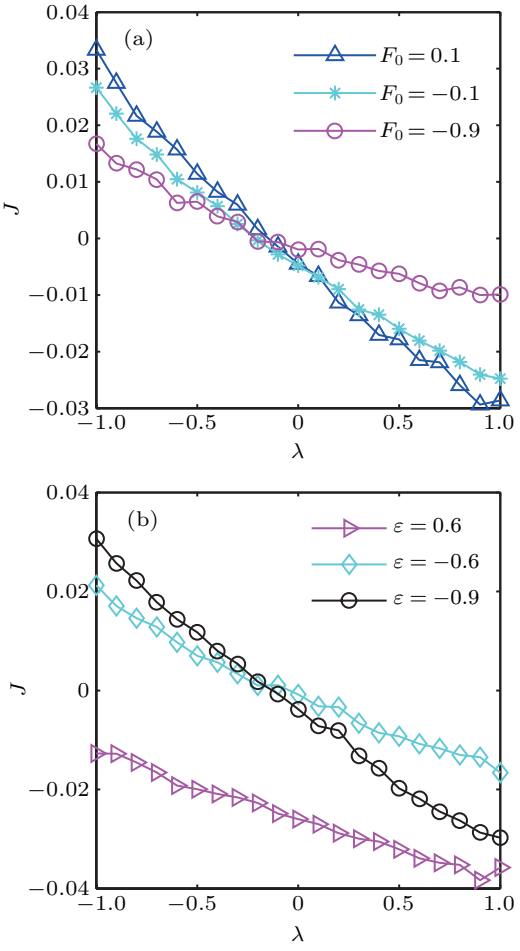


Fig. 2. (color online) Current J versus the correlated noises intensity λ with different F_0 and ε. The values of the parameter are $m = 1, D = 0.1, \alpha = 2$ with (a) $\varepsilon = 0$, $F_0 = 0.1, -0.1, -0.9$; (b) $F_0 = -0.9$, $\varepsilon = 0.6, -0.6, -0.9$.

Figure 2 shows the current J as a function of the correlated noises intensity λ with different F_0 and ε. No matter what the value of the F_0 and ε, the current J decreases monotonically with λ. From Fig. 2(a) we can see that the current J occurs a reversal with the increase in λ. The reversal point is near zero, that is, when the correlation between the two noises is very small, the current disappears. Moreover, with the change of λ, the large F_0 corresponds to the large current value. However, after the inversion occurs, large F_0 values correspond to the small current values. Figure 2(b) depicts that the current reverses with $\varepsilon < 0$ ($\varepsilon = -0.6, -0.9$). Nevertheless, when $\varepsilon > 0$ ($\varepsilon = 0.6$), there is no current reversal. Combined with Figs. 2(a) and 2(b), the current reversal is not only affected by correlated noises intensity but also by other parameters (i.e., F_0, ε). Therefore, correlated noises intensity λ may be a factor in causing current reversal.

We display the current J versus the force F_0 and asymmetric parameter ε for different correlated noise intensity λ in Figs. 3 and 4. In Fig. 3, the current J is positive for $\lambda < 0$ ($\lambda = -0.9, -0.5$), and negative for $\lambda > 0$ ($\lambda = 0.9$) or $\lambda = 0$. The curve is smooth when the two noises are uncorrelated ($\lambda = 0$), and when $\lambda \neq 0$, there is a value of $F_0 \approx 0$ that makes the current reach an extreme value. That is, when the external force nearly disappears, the current can reach a maximum or a minimum value. In addition, with the increase of F_0, if $F_0 < 0$, $\lambda > 0$ or $F_0 > 0$, $\lambda < 0$, the current decreases; if $F_0 < 0$, $\lambda < 0$ or $F_0 > 0$, $\lambda > 0$, the current increases, namely, the current increases with the same sign of F_0 and λ, and the current decreases with the contrary sign of F_0 and λ. In Fig. 4 one can obtain the similar case with the current being positive for $\lambda < 0$ ($\lambda = -0.9$), and negative for $\lambda > 0$ ($\lambda = 0.5, 0.9$). However, the situation is the opposite and the current is positive when $\lambda = 0$. The force F_0 exhibits high variability when the two noises are correlated especially when the correlation of two noises is strong, while the sample of the current is more smoother for asymmetric parameter ε. In order to obtain a significant change of the current, it is better to adjust the value of force F_0.

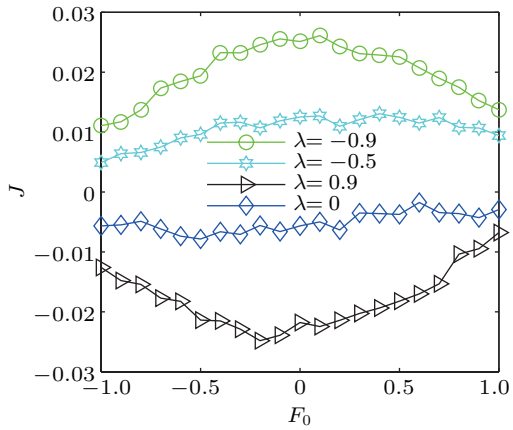


Fig. 3. (color online) The current J versus F_0 for different values of the parameter λ. The values of the parameter are $D = 0.1$, $m = 1$, $\alpha = 2$, $\varepsilon = 0$, $\lambda = -0.9, -0.5, 0, 0.9$.

Another interesting property is exhibited in Fig. 5, here the steady state current J as a function of additive noise intensity α is shown for different λ. The curve is observed to be bell shaped. There are no transitions when $\alpha \to 0$, namely, the particle cannot pass the barrier and there is no current. When

$\alpha \to \infty$, that is, α is very big, the effect of the ratchet disappears and the current tends to be zero, also. Furthermore, in the case of $\lambda < 0$ ($\lambda = -0.5, -0.9$), the current is positive, while the current is negative in the case of $\lambda > 0$ ($\lambda = 0.5, 0.9$). In both cases, there exists an optimal value of α at which the current takes its extremal value, and this optimal value of α is obtained when α is small. Besides, when the additive noise intensity α is weak, the curve has a noticeable change, which shows that this small range has a great impact on the current.

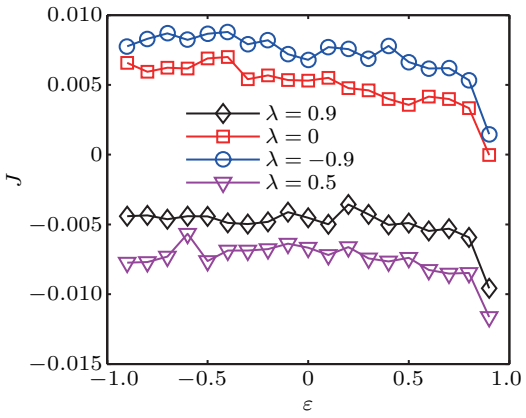


Fig. 4. (color online) Current J versus the ε for different values of the parameter λ. The values of the parameter are $D = 0.01, F_0 = -0.1, m = 1, \alpha = 2, \lambda = -0.9, 0, 0.5, 0.9$.

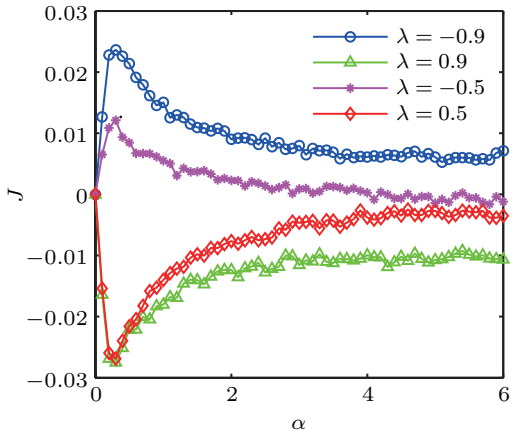


Fig. 5. (color online) The current J versus the α for different values of the parameter λ. The values of the parameters are $D = 0.01$, $\varepsilon = 0$, $m = 1$, $F_0 = -0.1$, $\lambda = -0.5, -0.9, 0.9, 0.5$.

Figures 6(a) and 6(b) show the current J versus the multiplicative noise intensity D of the different parameters λ and ε. In Fig. 6(a), the current J is negative when $\lambda > 0$ ($\lambda = 0.7, 0.9$), and positive when $\lambda < 0$ ($\lambda = -0.9$). With the increase of noise intensity D, the current decreases monotonically when $\lambda > 0$. However, when $\lambda < 0$, the current increases monotonically. Figure 6(b) is analogous to Fig. 6(a). When $\varepsilon > 0$ ($\varepsilon = 0.4$), the current J is positive and increases monotonically with the increase of the D, while the current J is negative and decreases with the D under the circumstance of $\varepsilon < 0$ ($\varepsilon = -0.4, -0.6$). Namely, we can decide the direction of the current according to the positive and negative of the correlated noises intensity and asymmetric parameters. We compare the additive noise in Fig. 5 with the multiplicative noise in Fig. 6(a). The effect of additive and multiplicative noise on the current is different. The curves show nonmonotonic behavior with the additive noise intensity, yet the current is monotonically changing with the multiplicative noise intensity. The effect of additive noise on current is more significant than that of multiplicative noise.

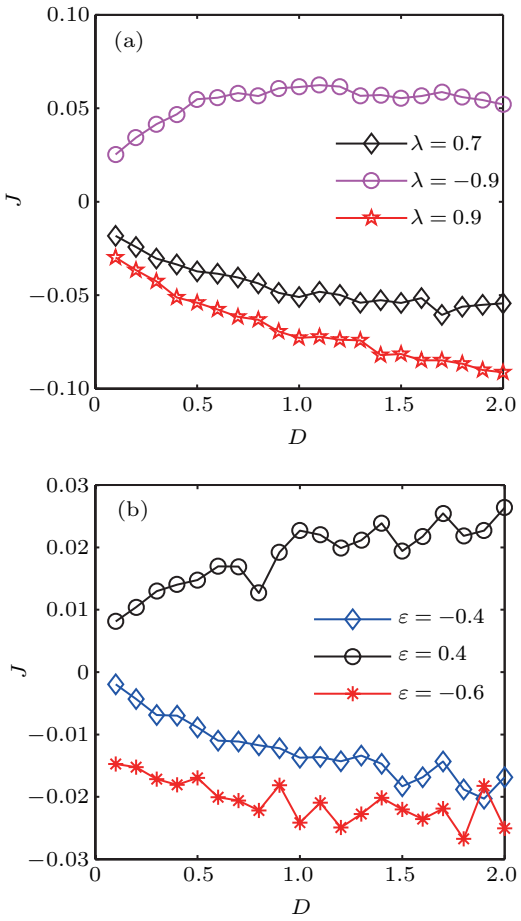


Fig. 6. (color online) Current J versus D for different values of the parameters λ and ε. The values of the parameters are $m = 1$, $\alpha = 2$, $F_0 = 0.5$. (a) $\varepsilon = 0$, $\lambda = 0.7, 0.9, -0.9$. (b) $\lambda = 0.9$, $\varepsilon = 0.4, -0.6, -0.4$.

We next plot the current J as a function of the mass m for different F_0 and λ in Fig. 7. With the increase of m, the current increases for $\lambda > 0$ ($\lambda = 0.5, 0.9$), and the current decreases when the $\lambda < 0$ ($\lambda = -0.5, -0.9$). On the other hand, when $F_0 = 0.5$, $\lambda = 0.9$ or $F_0 = -0.5$, $\lambda = -0.9$, the current reverses with the increase of m. However, the current does not reverse when $F_0 = 0.5$, $\lambda = -0.9$ or $F_0 = -0.5$, $\lambda = 0.9$. In other words, the current may reverse when F_0 and λ have the same sign. That is, different masses of the particles have different directions. Therefore, we can separate the Brownian particles with different masses according to the parameters of noises and forces. The method seems to be of possible interest in biology. For example, the kinesins and dyneins move along the microtubules. We know that current reversal should

cause them to move simultaneously in opposite directions by controlling the system, so that they can be separated.

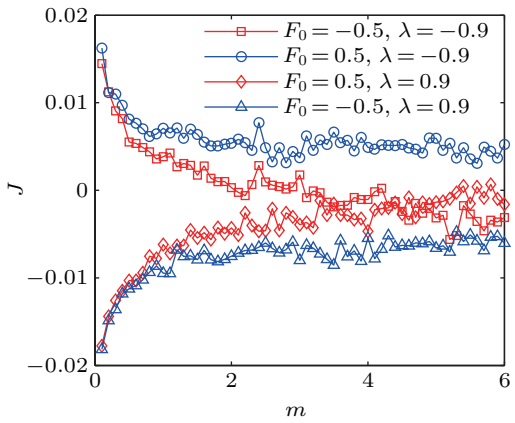


Fig. 7. (color online) The current J versus m for different values of the parameters λ and F_0. The values of the parameters are $D = 0.01$, $\varepsilon = -0.4$, $\alpha = 2$, $F_0 = 0.5$, $\lambda = 0.9$; $F_0 = 0.5$, $\lambda = -0.9$; $F_0 = -0.5$, $\lambda = 0.9$; $F_0 = -0.5$, $\lambda = -0.9$..

For Figs. 2–7, the system is controlled by fluctuation force and noises (Figs. 3–6), the current is positive when $\lambda < 0$, and when $\lambda > 0$, the current is negative. The current may be reversed by correlated noises intensity (Figs. 2 and 7).

4. Conclusion

In this article, the transport of an underdamped particle in a nonequilibrium system which consists of correlated noises and an external fluctuation force is investigated. We simulate the probability current in the quasi-steady state limit. A simple and imaginable ratchet system is discussed, it found that the asymmetric parameter ε of the external fluctuation force, and correlated noise parameter λ may affect the net current direction, even lead to the reversal of the current. On the other hand, the different masses of the particles possess different directions of the transport. Consequently, particles can be separated according to their masses.

Our results may be useful for separation devices, especially in biological systems. The motion of macro-molecules is just a case in point. We know that macromolecular movement may be the cause of eukaryotic cell vesicle transport. Typically, proteins move along a microtubule, we can separat the kinesins and dyneins, because it is well known that they move along tubulin filaments in opposite directions.[40] Similarly, Kinesin and non-claret disjunctional also move in opposite directions along microtubules,[41] and we can further study the molecular basis of their motion directions. In addition, there are still the electrophoretic separation of microparticles, DNA molecules, proteins, viruses, cells[40] and so on. And the results may also have potential applications in physical systems. The nonequilibrium transitions of two global coupled models driven by noise are studied according to their motion.[42] We can predict the trajectory of particles motion based on whether the sign of the correlated noise intensity is in agreement with the sign of the external force,[28] etc. We expect that this novel mechanism will be able to control molecular motors and other micro scale phenomena and be widely applied to science and technology.

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