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Transitionless driving on local adiabatic quantum search algorithm*

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We apply the transitionless driving on the local adiabatic quantum search algorithm to speed up the adiabatic process. By studying quantum dynamics of the adiabatic search algorithm with the equivalent two-level system, we derive the transitionless driving Hamiltonian for the local adiabatic quantum search algorithm. We found that when adding a transitionless quantum driving term $H_{\mathrm{D}}(t)$ on the local adiabatic quantum search algorithm, the success rate is 1 exactly with arbitrary evolution time by solving the time-dependent Schrödinger equation in eigen-picture. Moreover, we show the reason for the drastic decrease of the evolution time is that the driving Hamiltonian increases the lowest eigenvalues to a maximum of $O(\sqrt{N})$.

Keywords: transitionless driving, local adiabatic quantum search algorithm

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1. Introduction

The task of the search problem is locating the marked item out of N unsorted items. In the classical algorithm, on average $O(N)$ steps are required, while Grover proposed a quantum search algorithm which can find the marked item in a time of $O(\sqrt{N})$,[1] giving rise to a quadratic speed up. Moreover, it has been proved that there is no quantum algorithm in the circuit model which can perform an unstructured search faster since Grover's algorithm saturates the lower bound found by Bennett *et al.*[2] On the other hand, a new alternative type of quantum computation, an adiabatic quantum algorithm, was proposed by Farhi *et al.*[3] The adiabatic model is computationally equivalent to the quantum circuit model,[4] therefore, it is attracting more and more attention.[5–8] Naturally, the adiabatic version of the Grover algorithm, called the adiabatic quantum search algorithm, was proposed.[9] At first, it needs a complexity of order N, which is no better than the classical algorithm. Afterwards, Roland improved it by adjusting the evolution rate of the Hamiltonian so as to fulfill the adiabatic condition locally.[10] The improved algorithm, called local adiabatic quantum search algorithm, decreases the scaling of the complexity to $O(\sqrt{N})$, which is consistent with the Grover algorithm.

However, the adiabatic quantum algorithm requires that its dynamics evolves slowly enough to stay on its instantaneous ground state. To overcome the slowness, a technique, named "shortcut to adiabaticity"[11,12] and designing non-adiabatic protocols to speed up the quantum adiabatic process, has been developed and attracted a great deal of attention.[13,14] Demirplack and Rice[15,16] and Berry[17] proposed that adding a suitable "transitionless quantum driving" term $H_{\mathrm{D}}(t)$ to an original time-dependent Hamiltonian $H(t)$ can suppress the transitions between instantaneous eiegnbases of $H(t)$. This is an important technique of shortcut to adiabaticity. Oh *et al.*[18] applied it on a global adiabatic quantum search algorithm to study the non-adiabatic transition probability and showed that a constant driving Hamiltonian could reduce the non-adiabatic transition probability. In this paper, we focused on the complexity and utilized the transitionless quantum driving on the local adiabatic quantum search algorithm to speed up the search. We found that when adding a transitionless quantum driving term $H_{\mathrm{D}}(t)$ on the local adiabatic quantum search algorithm, the success rate can be 1 exactly with arbitrary evolution time by solving the time-dependent Schrödinger equation. We also analyzed the reason for the drastic improvement.

The paper is organized as follows. In Section 2, we introduce the local adiabatic quantum search algorithm. The transitionless quantum driving is used in Section 3, and then, we analyze the success rate and the complexity in Section 4 and draw the conclusion of this work in the last section.

2. Local adiabatic quantum search algorithm

In the Grover algorithm, the objective is to find the marked item in an unsorted database. More formally, suppose that there is a function $f:\{0,1\}^n \mapsto \{0,1\}$ with the promise

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that $f(m)=1$ and $f(x)=0, \forall x \neq m$, then the goal is to find the unknown index m by calling the function f. This is an oracle problem, in which the algorithm can make queries to an oracle that recognizes the marked items. In an adiabatic quantum search algorithm, we use a set of orthonormal basis $|1\rangle, |2\rangle, \ldots, |N\rangle$ to denote the N unsorted items and use $|m\rangle$ to denote the marked item. The oracle is defined in terms of the problem Hamiltonian as

$$H_1 = I - |m\rangle\langle m|, \tag{1}$$

thus, the target state $|m\rangle$ is the ground state of H_1. The initial state can be prepared in an equal superposition of all could-be solutions as

$$|\varphi_0\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |i\rangle, \tag{2}$$

and the initial state is the ground state of the following initial Hamiltonian:

$$H_0 = I - |\varphi_0\rangle\langle\varphi_0|. \tag{3}$$

The adiabatic quantum search algorithm is built from the time-dependent Hamiltonian

$$H(s(t)) = (1-s(t))H_0 + s(t)H_1, \tag{4}$$

where $s(t)$ is a monotonic function with $s(0)=0$ and $s(T)=1$. Here, T is the total run time. Suppose that the system begins with the ground state of H_0 and evolves slowly enough according to $H(s(t))$, the quantum adiabatic theorem[19] ensures that the system will evolve into the ground state of H_1, and we can find the marked item by measuring the final state.

As mentioned above, $s(t)$ is a function that can be optimized. Generally, we choose a linear function $s(t)=t/T$, which is called global adiabatic. In the local adiabatic quantum search algorithm,[10] we do not use the linear function, but adapt the evolution rate $\mathrm{d}s/\mathrm{d}t$ to the local adiabatic condition. The corresponding $s(t)$ satisfies

$$s(t) = \frac{\tan\left((2t/T-1)\arctan\sqrt{N-1}\right)+\sqrt{N-1}}{2\sqrt{N-1}}. \tag{5}$$

The local adiabatic quantum search algorithm decreases the scaling of the complexity to $O(\sqrt{N})$, consistent with the Grover's algorithm, but the local adiabatic quantum search algorithm still requires that its dynamics evolves slowly enough to stay on its instantaneous ground state. To overcome the slowness, some techniques, which aim at designing non-adiabatic protocols to speed up the quantum adiabatic process, have been developed. Next, we introduce the transitionless driving and apply it on the local adiabatic quantum search algorithm.

3. Transitionless driving on the local adiabatic quantum search algorithm

The main idea of transitionless driving is to suppress transitions by adding a counterdiabatic term to the original time-dependent Hamiltonian. Here, we introduce its main step briefly. Suppose a time-dependent Hamiltonian $H(t)$ with N eigenstates $\{|\phi_n(t)\rangle\}$ as

$$H(t)|\phi_n(t)\rangle = E_n(t)|\phi_n(t)\rangle. \tag{6}$$

The dynamics of a system governed by $H(t)$ is described by

$$\mathrm{i}\hbar\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle. \tag{7}$$

When the system is transformed to another picture (eigen-picture), whose bare states are the instantaneous eigenstates of $H(t)$, the system is described by

$$\mathrm{i}\hbar\partial_t|\psi^{\mathrm{e}}(t)\rangle = H^{\mathrm{e}}(t)|\psi^{\mathrm{e}}(t)\rangle, \tag{8}$$

and the transform equation is expressed as $|\psi^{\mathrm{e}}(t)\rangle = R|\psi(t)\rangle$. Substituting this relationship to Eqs. (7) and (8), we have

$$H^{\mathrm{e}}(t) = RH(t)R^{\dagger} - \mathrm{i}\hbar R\dot{R}^{\dagger}, \tag{9}$$

where the overdot means time derivative. The first term $RH(t)R^{\dagger}$ is the diagonalization matrix of Hamiltonian $H(t)$ and the second term contains the nonadiabatic coupling. The convenient way is adding a Hamiltonian $H_{\mathrm{D}}^{\mathrm{e}}(t) = \mathrm{i}\hbar R\dot{R}^{\dagger}$, to counteract the nonadiabatic coupling. Returning to the original picture, we obtain

$$H_{\mathrm{D}}(t) = R^{\dagger}H_{\mathrm{D}}^{\mathrm{e}}(t)R = \mathrm{i}\hbar\dot{R}^{\dagger}R, \tag{10}$$

so when applying the transitionless driving on the local adiabatic quantum search algorithm, the main step is to find the rotation matrix R. Actually, this step is analogous to what Oh *et al.* did in the global adiabatic quantum search algorithm.[18] We restrict the analysis to the space spanned by $|m\rangle$ and $|m_\perp\rangle = \sum_{i\neq m}^{N} i/\sqrt{N-1}$ and represent $H(s(t))$ with the orthonormal basis $\{|m\rangle, |m_\perp\rangle\}$ as

$$H(s) = \begin{pmatrix} 1-\dfrac{1-s}{N}-s & -\dfrac{(1-s)\sqrt{N-1}}{N} \\ -\dfrac{(1-s)\sqrt{N-1}}{N} & 1-\dfrac{(1-s)(N-1)}{N} \end{pmatrix}. \tag{11}$$

The Hamiltonian in Eq. (11) can be written in a convenient form as

$$H(s) = \frac{I}{2} - \frac{1}{2N}\begin{pmatrix} Z(s) & X(s) \\ X(s) & -Z(s) \end{pmatrix}, \tag{12}$$

where $Z(s) = 2(1-s) + N(2s-1)$ and $X(s) = 2(1-s)\sqrt{N-1}$. The first item amounts to a common phase, which can be dropped, so the Hamiltonian can be expressed as

$$H(s) = -\frac{\hbar\omega(s)}{2}\begin{pmatrix} \cos\theta(s) & \sin\theta(s) \\ \sin\theta(s) & -\cos\theta(s) \end{pmatrix}, \tag{13}$$

where the mixing angle θ is defined by $\tan\theta(s) \equiv X(s)/Z(s)$ and $\hbar\omega(s) \equiv \sqrt{Z^2+X^2}/N$. Hereafter, we set $\hbar = 1$. It is easy to calculate the eigenvalues of $H(s)$ that $e_{\pm}(s) = \pm\omega(s)/2$, with the corresponding instantaneous eigenstates

$$|e_-(s)\rangle = \begin{bmatrix} \cos\dfrac{\theta}{2} \\ \sin\dfrac{\theta}{2} \end{bmatrix}, \quad |e_+(s)\rangle = \begin{bmatrix} -\sin\dfrac{\theta}{2} \\ \cos\dfrac{\theta}{2} \end{bmatrix}. \tag{14}$$

The evolved state in the interaction picture can be written as

$$|\varphi(s)\rangle = a(s)|m\rangle + b(s)|m_\perp\rangle. \tag{15}$$

We rewrite the evolved state in the eigen-picture as

$$|\varphi^{\mathrm{e}}(s)\rangle = a_-(s)|e_-(s)\rangle + b_+(s)|e_+(s)\rangle. \tag{16}$$

The relation between the eigen-picture and the interaction picture is expressed as

$$\begin{bmatrix} a_-(s) \\ b_+(s) \end{bmatrix} = R \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}, \tag{17}$$

and thus, we obtain the rotation matrix

$$R = \begin{pmatrix} \cos\dfrac{\theta}{2} & \sin\dfrac{\theta}{2} \\ -\sin\dfrac{\theta}{2} & \cos\dfrac{\theta}{2} \end{pmatrix}. \tag{18}$$

It is easy to calculate the transitionless driving term

$$H_{\mathrm{D}}(t) = \mathrm{i}\hbar\dot{R}^{\dagger}R = \frac{\dot{\theta}}{2}\hbar\sigma_y. \tag{19}$$

Here, $\theta = \arctan(X(s)/Z(s))$, then

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} = \frac{1}{1+(X/Z)^2}\frac{\dot{X}Z-\dot{Z}X}{Z^2} = -\frac{2\sqrt{N-1}}{N\omega^2}. \tag{20}$$

As mentioned in Section 2, in the local adiabatic quantum search algorithm,

$$s(t) = \frac{\tan\left((2t/T-1)\arctan\sqrt{N-1}\right)+\sqrt{N-1}}{2\sqrt{N-1}}, \tag{21}$$

so

$$\begin{aligned} \frac{\mathrm{d}s}{\mathrm{d}t} &= \frac{\arctan\sqrt{N-1}}{T\sqrt{N-1}} \\ &\quad\times\left(1+\tan^2\left((2t/T-1)\arctan\sqrt{N-1}\right)\right). \end{aligned} \tag{22}$$

Substituting $\tan\left((2t/T-1)\arctan\sqrt{N-1}\right) = \sqrt{N-1}(2s-1)$ into Eq. (22), we have

$$\begin{aligned} \frac{\mathrm{d}s}{\mathrm{d}t} &= \frac{\arctan\sqrt{N-1}}{T\sqrt{N-1}}\left(1+(N-1)(2s-1)^2\right) \\ &= \frac{\arctan\sqrt{N-1}}{T\sqrt{N-1}}N\omega^2, \end{aligned} \tag{23}$$

and then

$$\dot{\theta} = \frac{\mathrm{d}\theta}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = -\frac{2\arctan\sqrt{N-1}}{T},$$

thus, we obtain the transitionless driving term as

$$H_{\mathrm{D}}(t) = \frac{\dot{\theta}}{2}\hbar\sigma_y = -\frac{\hbar\arctan\sqrt{N-1}}{T}\sigma_y. \tag{24}$$

Usually, we need to add the driving Hamiltonian $H_{\mathrm{D}}(t)$ on the original $H(t)$. Note that, we have represented $H(t)$ with $H(s)$ for convenience. Hence, in the next section, we add the driving term

$$H_{\mathrm{D}}(s) = -\frac{\sqrt{N-1}}{N\omega^2}\sigma_y, \tag{25}$$

on $H(s)$ to make the analysis more simple.

4. Efficiency analysis

As mentioned above, the evolved state can be written as $|\varphi(s)\rangle = a(s)|m\rangle + b(s)|m_\perp\rangle$, where $a(s)$ is the probability amplitude of $|m\rangle$, so the probability $p(s) = \langle\varphi(s)|m\rangle = |a(s)|^2$ is the instantaneous fidelity and $p(1) = |a(1)|^2$ corresponding to the success rate. The goal of the adiabatic search algorithm is changing $p(0) = 1/N$ toward $p(1) = 1$, utilizing the dynamics evolution controlled by a time-dependent Hamiltonian. We can obtain the expression of $a(s)$ by solving the following time-dependent Schrödinger equation:

$$\begin{bmatrix} \dfrac{\mathrm{d}a(s)}{\mathrm{d}s} \\ \dfrac{\mathrm{d}b(s)}{\mathrm{d}s} \end{bmatrix} = -\frac{\mathrm{i}}{\hbar}H(s)\begin{bmatrix} a(s) \\ b(s) \end{bmatrix}. \tag{26}$$

However, it is difficult to solve Eq. (26) in the basis $\{|m\rangle, |m_\perp\rangle\}$. On the other hand, we know the evolved state can be expressed by $|\varphi^{\mathrm{e}}(s)\rangle = a_-(s)|e_-(s)\rangle + b_+(s)|e_+(s)\rangle$ in the eigen-picture and

$$\begin{bmatrix} a_-(s) \\ b_+(s) \end{bmatrix} = R\begin{bmatrix} a(s) \\ b(s) \end{bmatrix}, \tag{27}$$

so if we obtain $a_-(s)$, we can obtain $a(s)$ by using Eq. (27). In the eigen-picture, the dynamics of the system is described by the Schrödinger equation

$$\mathrm{i}\hbar\partial_t|\psi^{\mathrm{e}}(s)\rangle = H^{\mathrm{e}}(s)|\psi^{\mathrm{e}}(s)\rangle, \tag{28}$$

where

$$\begin{aligned} H^{\mathrm{e}}(s) &= R(H(s)+H_{\mathrm{D}}(s))R^{\dagger} - \mathrm{i}\hbar R\dot{R}^{\dagger} \\ &= RH(s)R^{\dagger} = \begin{bmatrix} \dfrac{\omega}{2} & 0 \\ 0 & -\dfrac{\omega}{2} \end{bmatrix}, \end{aligned} \tag{29}$$

and then the Schrödinger equation in the eigen-picture becomes

$$\begin{bmatrix} \dfrac{\mathrm{d}a_-(s)}{\mathrm{d}s} \\ \dfrac{\mathrm{d}b_+(s)}{\mathrm{d}s} \end{bmatrix} = -\mathrm{i}\begin{bmatrix} \dfrac{\omega}{2} & 0 \\ 0 & -\dfrac{\omega}{2} \end{bmatrix}\begin{bmatrix} a_-(s) \\ b_+(s) \end{bmatrix}. \tag{30}$$

It is easy to obtain the solution of Eq. (30) as

$$a_-(s) = a_-(0)\,\mathrm{e}^{\int_0^1 \frac{\mathrm{i}\omega}{2}\mathrm{d}s};\quad b_+(s) = b_+(0)\,\mathrm{e}^{\int_0^1 -\frac{\mathrm{i}\omega}{2}\mathrm{d}s},$$

where $a_-(0) = 1$ and $b_+(0) = 0$. Returning to the interaction picture, we have

$$a(s) = \cos\frac{\theta}{2}a_-(s) - \sin\frac{\theta}{2}b_+(s) = \cos\frac{\theta}{2}\,\mathrm{e}^{\int_0^1 \frac{\mathrm{i}\omega}{2}\mathrm{d}s}, \quad (31)$$

where $\mathrm{e}^{\int_0^1 \frac{\mathrm{i}\omega}{2}\mathrm{d}s}$ is an overall phase, which can be ignored. When $s = 1$, we have $\theta = 0$ and $\cos(\theta/2) = 1$, thus the success rate $p(1) = |a(1)|^2 = 1$.

Above all, when adding the transitionless driving term $H_{\mathrm{D}}(s)$ in Eq. (25) to the local adiabatic search algorithm, if the system begins with the ground state of $H(0)$, then the system will still stay on the ground state of $H(1)$ in the final time. Namely, the success rate will be 1 with the arbitrary evolution time theoretically. We numerically solve the Schrödinger equation controlled by $H(s) + H_{\mathrm{D}}(s)$. The results are shown in Fig. 1.

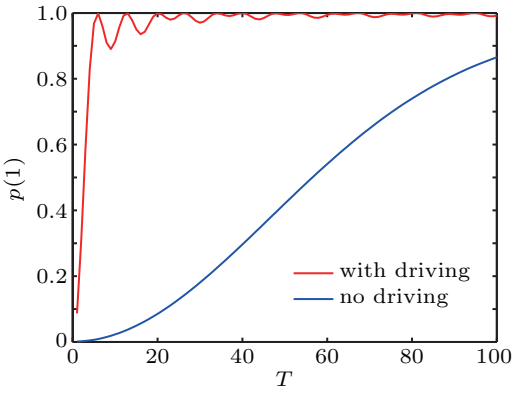


Fig. 1. (color online) Success rate as a function of evolution time T for $N = 1000$.

As shown in Fig. 1, when adding the transitionless driving term on the local adiabatic search algorithm, the success rate improved significantly. Moreover, the success rate approximately equals to 1 in a short evolution time, which is far less than $O(\sqrt{N})$. It sounds like an exciting conclusion, but we mention that the Hamiltonian we added in the local adiabatic search algorithm is

$$\begin{aligned} H_{\mathrm{D}}(s) &= -\frac{\sqrt{N-1}}{N\omega^2}\sigma_y \\ &= -\frac{\sqrt{N-1}}{N}\left((2s-1) + \frac{s(1-s)}{N}\right)^{-1}\sigma_y. \end{aligned} \quad (32)$$

When $s = 1/2$, $H_{\mathrm{D}}(s)$ reaches a maximum of $-\sqrt{N-1}\sigma_y$. It means that the energy scaling of the system gets $O(\sqrt{N})$ in a short time. Namely, we need to utilize the energy in exchange for the same scaling of less time. In fact, the role of $H_{\mathrm{D}}(s)$ is to make the gap between the lowest two eigenvalues wider.

Figure 2 shows that the gap between the lowest two instantaneous eigenvalues increases when the driving term $H_{\mathrm{D}}(s)$ is added to $H(s)$. More precisely, the eigenvalues of σ_y are ± 1, so the eigenvalues of $H_{\mathrm{D}}(s)$ are $\pm\sqrt{N-1}/(N\omega^2)$. If it is much larger than the eigenvalues of $H(s)$, then the eigenvalues of the system Hamiltonian are mainly decided by $H_{\mathrm{D}}(s)$. It means that when applying the transitionless driving on the local adiabatic search algorithm, the lowest two eigenvalues of the system Hamiltonian are temporarily increased to a maximum of $O(\sqrt{N})$, which can significantly reduce the evolution time.[20] On the other hand, in the local adiabatic search algorithm, the entire evolution time T is divided into infinitesimal time intervals and the evolutions are adiabatic in each of those intervals. Hence, the change of the lowest two eigenvalues produces a drastic decrease in the evolution time.

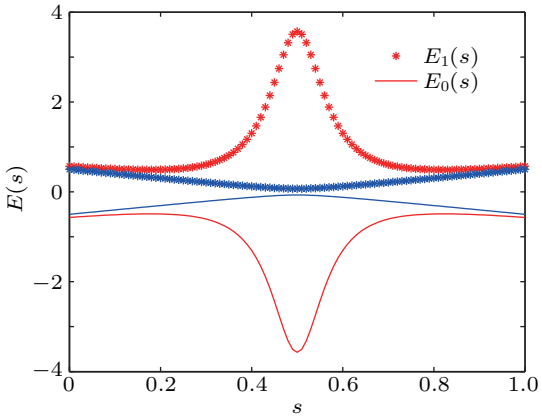


Fig. 2. (color online) Instantaneous eigenvalues $E_{0,1}(s)$ of $H(s) + H_{\mathrm{D}}(s)$ as a function of s for $N = 50, T = 10$. The blue line corresponding to $H_{\mathrm{D}}(s) = 0$.

5. Summary

In conclusion, we applied the transitionless driving on the local adiabatic quantum search algorithm to speed up the adiabatic process. We studied quantum dynamics of the adiabatic search algorithm with the equivalent two-level system, and derived the transitionless driving Hamiltonian for the local adiabatic quantum search algorithm. By solving the time-dependent Schrödinger equation in the eigen-picture, we found that when adding a transitionless quantum driving term $H_{\mathrm{D}}(s)$ on the local adiabatic quantum search algorithm, the success rate is 1 exactly with an arbitrary evolution time. The result of the numerical simulation shows that the transitionless driving can indeed lead to a drastic decrease of the evolution time in the local adiabatic quantum search algorithm. However, the system we considered is highly idealized, the total Hamiltonian is non-local in the sense that it needs all qubits to be coupled simultaneously. Such a Hamiltonian is hard to implement in a realistic physical system. We should therefore find simpler, more local, Hamiltonians that have the same ground states and hence can be used as the basis of a realis-

tic adiabatic quantum search algorithm. Moreover, the energy strength $\sqrt{N-1}/(N\omega^2)$ of the transitionless driving terms is difficult to control accurately in practice. One can use an approximate driving Hamiltonian, which is easier to control, to replace the exact time-dependent driving Hamiltonian.

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