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Citation: Chin. Phys. B . 2018, 27(1): 010307. **doi:** 10.1088/1674-1056/27/1/010307

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Soliton excitations in a polariton condensate with defects

Abderahim Mahmoud Belounis[†] and Salem Kessal

SNIRM Laboratory, Faculty of Physics, Universite des Sciences et de la Technologie Houari Boumediene (USTHB), P. O. Box 32 El Alia, Bab Ezzouar, 16111 Algiers, Algeria

(Received 12 May 2017; revised manuscript received 17 August 2017; published online 17 November 2017)

To study soliton excitations in a polariton condensate with defects, we use the Gross–Pitaevskii equation and its hydrodynamic form. An extra term is added to take into account the non-equilibrium nature of the polariton condensate and the presence of defects. The reductive perturbation method transforms these hydrodynamic equations into a modified Korteweg–de Vries equation in the long wavelength limit. We linearize this equation and study the soliton linear excitations. We give an analytic expression of traveling excitations using the variation of constants method. In the more general form, we show numerically that the excitations are oscillations, i.e., the amplitude and the width of the dark soliton oscillate simultaneously but in an opposite way.

Keywords: solitons, polariton condensates, reductive perturbation method

PACS: 03.75.Kk, 05.45.Yv, 47.37.+q, 71.36.+c **DOI:** 10.1088/1674-1056/27/1/010307

1. Introduction

The polariton condensation (Bose–Einstein condensate of polaritons) in a microcavity[1,2] is presently one of the most active research topics. The non-equilibrium nature of these systems (caused by the finite lifetime of polaritons) attracts the interest of physicists and provides them with a better insight into some fundamental problems, like the superfluidity,[3,4] the formation of vortices,[5] the vortex–antivortex pair,[6] the BKT transition,[7] the BEC-BCS crossover,[8] and the formation of dark and bright solitons.[9–11] Solitons are localized waves which propagate in a medium without deformation or attenuation.[12] These waves result from the competition between the dispersion and the nonlinearity. Their study is still a very active research topic. For example in optics, bright and dark solitons are examined under perturbations in the nonlinear Schrödinger equation framework.[13–16] It becomes even more interesting with the occurrence of Bose–Einstein condensates (atomic condensate).[17,18] These systems give a large variety of sets which allow more in-depth studies of solitons[19–23] and now with the polariton condensates, even dissipative solitons are studied.[24,25] This work focuses on the theoretical investigation of linear excitations of solitons in polariton condensates with defects.

This paper is divided into four parts. In Section 2, we present the model which describes a polariton condensate. In Section 3, we give some solutions which correspond to specific physical situations. In Section 3, we derive a modified Korteweg–de Vries equation to investigate the influence of the finite lifetime of polaritons and the presence of defects on the soliton excitations. We solve this equation both analytically and numerically when the finite lifetime of polaritons and the presence of defects are considered as perturbations.

2. The model

We consider a polariton condensate flow. It is polaritons cooled down to a temperature of the order ~ 10 K.[26] We recall that a polariton condensate is a non-equilibrium system, i.e., a continuous pumping from an external reservoir is vital to its existence. This flow propagates in an environment with defects (for simplicity we consider that the flow interacts with only one defect which is modeled by an external potential V). We ignore all the polarization effects and choose a configuration where the condensate is a one-dimensional system. We describe the Bose–Einstein condensate using the Gross–Pitaevskii equation (GP equation).[27,28] A modified form of the GP equation was proposed in Refs. [29] and [30] to take into account the specificity of the polariton condensate. Its dimensionless form is given by[31]

$$\mathrm{i}\Psi_t = -\frac{1}{2}\Psi_{xx} + V\Psi + |\Psi|^2\Psi + \mathrm{i}\eta(1-|\Psi|^2)\Psi, \qquad (1)$$

where $\Psi(x,t)$ is the condensate wave function, $-\frac{1}{2}\Psi_{xx}$ describes the kinetic energy of the system, $V(x)$ is the external potential, and $|\Psi|^2\Psi$ is a term resulting from the polariton self-interaction, $\mathrm{i}\eta\Psi$ represents the global gain and loss of the condensate, i.e., it expresses the gain of polaritons from the reservoir and their loss due to their finite lifetime. $-\mathrm{i}\eta|\Psi|^2\Psi$ is a term describing the saturation of the gain. The term $\mathrm{i}\eta(1-|\Psi|^2)\Psi$ is added in a phenomenological way to consider the non-equilibrium character of the polariton condensate.

We apply the Madelung transformation[32]

$$\Psi(x,t) = \sqrt{\rho(x,t)}\,\mathrm{e}^{\mathrm{i}\int u(x,t)\,\mathrm{d}x} \qquad (2)$$

[†]Corresponding author. E-mail: ambelounis@usthb.dz

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to Eq. (1) and obtain a hydrodynamic system for the density $\rho(x,t)$ and the phase velocity $u(x,t)$:

$$\begin{cases} \rho_t + (\rho u)_x = 2\eta(1-\rho)\rho, \\ u_t + uu_x + \rho_x + \left(\dfrac{(\rho_x)^2}{8\rho^2} - \dfrac{\rho_{xx}}{4\rho}\right)_x = -V_x, \end{cases} \tag{3}$$

where $\rho(x,t) = |\Psi(x,t)|^2$ is the condensate density.

The system (3) is an alternative formulation of the GP equation (1). Solving the nonlinear partial differential equations (3) will allow us to study the flow dynamics.

3. Hydraulic approximation and linear case

In some cases, we can solve the system (3) in a simple way. For instance, we use the hydraulic approximation when the flow is stationary and the space-variation of the density is very slow. We then ignore the dispersion term[33] and simplify the system (3) as

$$\begin{cases} (\rho u)_x = 2\eta(1-\rho)\rho, \\ uu_x + \rho_x = -V_x. \end{cases} \tag{4}$$

In this case, the flow density follows the differential equation

$$[\rho\sqrt{(M^2 + 2(1-\rho-V))}]_x = 2\eta(1-\rho)\rho, \tag{5}$$

where M is the Mach number. This approximation describes well the physical system considered above.[34] It is analytically solvable if the external potential V is constant.[31]

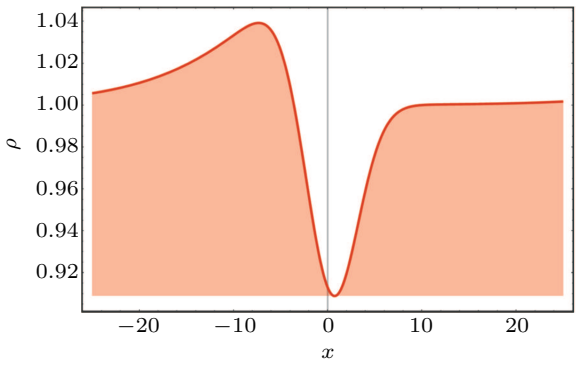


Fig. 1. (color online) Density variation of a colliding polariton condensate flow with a Gaussian external potential ($V = V_0\exp(-x^2/\sigma^2)$) in the hydraulic approximation. Equation (5) is solved numerically when the mach number $M = 0.5$, the damping $\eta = 0.1$, the potential width $\sigma = 4$, and the potential intensity $V_0 = 0.1$. The density varies essentially in the region where the external potential is important ($x \sim 0$).

System (3) can also be simplified in the case of linear waves; we take the damping η and the external potential V equal to zero and then linearize the system (3) with respect to u and $\rho' = \rho - \rho_0$, where ρ_0 is a steady state density (we set the system in a density ρ_0 when $V = \eta = 0$) and $\rho' \ll \rho_0$. We obtain

$$\begin{cases} \rho'_t + \rho_0 u_x = 0, \\ u_t + \rho'_x - \dfrac{1}{4\rho_0}\rho'_{xxx} = 0. \end{cases} \tag{6}$$

We look for solutions in the form of plane waves ($\sim \exp[\mathrm{i}(kx - \omega t)]$) and the system (6) leads to the dispersion relation

$$\omega^2(k) = {v_s}^2 k^2 + \frac{k^4}{4}, \tag{7}$$

where $v_s = \pm\sqrt{\rho_0}$ is the dimensionless sound velocity according to the steady state density ρ_0.[35] In the long wavelength limit ($k \ll v_s$), the dispersion relation becomes

$$\omega(k) = v_s k + \frac{k^3}{8v_s}. \tag{8}$$

This relation corresponds to the following linear partial differential equation:

$$\rho'_t + v_s\rho'_x - \frac{1}{8v_s}\rho'_{xxx} = 0, \tag{9}$$

which describes the evolution of the linear waves in the long wavelength case. We suppose here the changes of ρ' so small that we can ignore all the nonlinear effects.

To permit soliton study, the equation must contain dispersion and nonlinearity terms. These terms must be of the same order of magnitude.

4. Weakly nonlinear case: effect of the damping and the defects

To investigate the soliton excitations in a polariton condensate with defects, we consider the external potential $V(x)$ and the damping η as perturbations (they cause a small change in the steady state density ρ_0). In fact, it is demonstrated[31] that long wavelength waves can propagate in polariton condensates only for low damping η. We choose then a small enough η to ensure that all waves propagate in this system. This choice allows also the use of the reductive perturbation method,[36,37] which permits us to simplify the hydrodynamic equations and to investigate the effect of the damping on the waves behavior. $V(x)$ is seen like an unwanted defect in the system produced by the experimental conditions. It can also be created by a laser beam for a fundamental study.

We ignore the dispersion terms and rewrite the system (3) as follows:

$$\begin{cases} \rho'_t + (\rho' u)_x + \rho_0 u_x = 2\eta(\rho_0 - \rho_0^2 - 2\rho_0\rho' + \rho' - \rho'^2), \\ u_t + uu_x + \rho'_x = -V_x. \end{cases} \tag{10}$$

We use the stretched variables

$$X = \varepsilon^{1/2}(x - vt), \quad T = \varepsilon^{3/2}t; \tag{11}$$

and set V and η as follows:

$$V = \varepsilon^2\hat{V}, \quad \eta = \varepsilon^{5/2}\hat{\eta}, \tag{12}$$

where ε is the perturbation parameter. We obtain the system

$$\begin{cases} \varepsilon\rho'_T - v\rho'_X + (\rho' u)_X + \rho_0 u_X \\ = \varepsilon^2 2\hat{\eta}(\rho_0 - \rho_0^2 - 2\rho_0\rho' + \rho' - \rho'^2), \\ \varepsilon u_T - v u_X + u u_X + \rho'_X = -\varepsilon^2 \hat{V}_X. \end{cases} \tag{13}$$

We expand ρ' and u in the form

$$\rho' = \varepsilon\rho^{(1)} + \varepsilon^2\rho^{(2)} + \cdots, \quad u = \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \cdots. \tag{14}$$

This procedure is the reductive perturbation method. It is used to simplify a large class of complicated systems.[36] At the first order of perturbation, we have

$$\begin{cases} -v\rho_X^{(1)} + \rho_0 u_X^{(1)} = 0, \\ -v u_X^{(1)} + \rho_X^{(1)} = 0. \end{cases} \tag{15}$$

The system (15) gives the relations

$$u_X^{(1)} = \frac{v}{\rho_0}\rho_X^{(1)} \tag{16}$$

and $v = v_{\rm s}$. At the second order of perturbation, we find

$$\begin{cases} \rho_T^{(1)} - v\rho_X^{(2)} + (\rho^{(1)}u^{(1)})_X + \rho_0 u_X^{(2)} = 2\hat{\eta}(1-\rho_0)\rho_0, \\ u_T^{(1)} - v u_X^{(2)} + u^{(1)}u_X^{(1)} + \rho_X^{(2)} = -\hat{V}_X. \end{cases} \tag{17}$$

By using the relation (16), system (17) becomes

$$\rho'_T + \frac{3v_{\rm s}}{2\rho_0}\rho'\rho'_X = f_X(X), \tag{18}$$

where $f_X(X) = \hat{\eta}(1-\rho_0)\rho_0 - \dfrac{\rho_0}{2v_{\rm s}}\hat{V}_X(X)$ is a function which depends on the external potential derivative and the damping. $\hat{\eta}$, ρ_0, $v_{\rm s}$ are constants and $\rho' \equiv \rho^{(1)}$. Equation (18) describes a dispersionless polariton condensate with defects in the weak nonlinearity limit. When $f_X(X)$ is zero, equation (18) reduces to the inviscid Burgers equation.[38] It governs gas dynamics. It is a nonlinear equation with non-continuous solutions, which implies the occurrence of shock waves.

We consider now the case of a non-negligible dispersion. We rewrite system (3) with respect to ρ' and obtain

$$\begin{cases} \rho'_t + (\rho' u)_x + \rho_0 u_x = 2\eta(\rho_0 - \rho_0^2 - 2\rho_0\rho' + \rho' - \rho'^2), \\ u_t + u u_x + \rho'_x - \dfrac{1}{4\rho_0}\rho'_{xxx} + \dfrac{1}{2\rho_0^2}\rho'_x\rho'_{xx} - \dfrac{1}{4\rho_0^3}(\rho'_x)^3 = -V_x. \end{cases} \tag{19}$$

Using the reductive perturbation technique, we obtain

$$\begin{cases} \varepsilon\rho'_T - v\rho'_X + (\rho' u)_X + \rho_0 u_X \\ = \varepsilon^2 2\hat{\eta}(\rho_0 - \rho_0^2 - 2\rho_0\rho' + \rho' - \rho'^2), \\ \varepsilon u_T - v u_X + u u_X + \rho'_X - \varepsilon\dfrac{1}{4\rho_0}\rho'_{XXX} \\ + \varepsilon\dfrac{1}{2\rho_0^2}\rho'_X\rho'_{XX} - \varepsilon\dfrac{1}{4\rho_0^3}(\rho'_X)^3 = -\varepsilon^2\hat{V}_X. \end{cases} \tag{20}$$

At the first order of perturbation, nothing changes from the previous case (Eqs. (15) and (16)). At the second order of perturbation, we find

$$\begin{cases} \rho_T^{(1)} - v\rho_X^{(2)} + (\rho^{(1)}u^{(1)})_X + \rho_0 u_X^{(2)} = 2\hat{\eta}(1-\rho_0)\rho_0, \\ u_T^{(1)} - v u_X^{(2)} + u^{(1)}u_X^{(1)} + \rho_X^{(2)} - \dfrac{1}{4\rho_0}\rho_{XXX}^{(1)} = -\hat{V}_X. \end{cases} \tag{21}$$

By using the relation (16), this system reduces to

$$\rho'_T + \frac{3v_{\rm s}}{2\rho_0}\rho'\rho'_X - \frac{1}{8v_{\rm s}}\rho'_{XXX} = f_X(X). \tag{22}$$

This equation describes a polariton condensate in the long wavelength and weak nonlinearity limits. When $f_X(X)$ equals to zero, it is the Korteweg–de Vries equation (KdV) which was first proposed to describe the wave propagation in shallow water.[12,39,40] By using the inverse scattering technique,[41] it was demonstrated that the KdV equation can have soliton solutions. The presence of damping and defects in Eq. (22) will perturb the soliton dynamics.

We consider in this work a uniform damping all along the polariton condensate; in other words, the density gain and loss are constant. This approximation is valid for weak damping. As the defect is localized in space, the external potential acts only in a limited area.

To study the effect of damping and the presence of defects on solitons, we look for a solution as a sum of a soliton function $\rho^{\rm s}$ (when $f_X(X)$ is zero) and a response function r

$$\rho'(X,T) = \rho^{\rm s}(X,T) + r(X,T), \tag{23}$$

with $r \ll \rho^{\rm s}$. r takes into account the effect of the damping and the external potential and then allows us to study the modifications on the initial solution $\rho^{\rm s}$. We insert this ansatz into Eq. (22) and obtain

$$r_T + \frac{3v_{\rm s}}{2\rho_0}(\rho^{\rm s} r)_X - \frac{1}{8v_{\rm s}}r_{XXX} = f_X(X). \tag{24}$$

r is the correction to the soliton function; it obeys a linear partial differential equation. In the actual form, equation (24) is not easy to solve.

Note that $\rho^{\rm s}$ is the solution of

$$\rho^{\rm s}_T + \frac{3v_{\rm s}}{2\rho_0}\rho^{\rm s}\rho^{\rm s}_X - \frac{1}{8v_{\rm s}}\rho^{\rm s}_{XXX} = 0. \tag{25}$$

In fact, the soliton is a traveling wave solution. We look for a traveling excitation ($\rho^{\rm s}$ and r are of the form $\rho^{\rm s}(X+v_{\rm s}T)$ and $r(X+v_{\rm s}T)$; we take $\xi = (X+v_{\rm s}T)$). One can demonstrate that equation (24) can be written as

$$v_{\rm s} r + \frac{3v_{\rm s}}{2\rho_0}(\rho^{\rm s} r) - \frac{1}{8v_{\rm s}}r_{\xi\xi} = f(\xi), \tag{26}$$

and equation (25) becomes

$$\int \frac{{\rm d}\rho^{\rm s}}{\sqrt{-c_1 + 8v_{\rm s}^2(\rho^{\rm s})^2 + 4(\rho^{\rm s})^3}} = \xi - c_2. \tag{27}$$

To solve the linear differential equation (26), we use the variation of constants method.[42] The solution is given by

$$r = A(\xi)\frac{\partial \rho^{\rm s}}{\partial c_1} + B(\xi)\frac{\partial \rho^{\rm s}}{\partial c_2}, \tag{28}$$

where $A(\xi)$ and $B(\xi)$ satisfy the conditions

$$\begin{aligned} &A_\xi(\xi)\frac{\partial \rho^{\rm s}}{\partial c_1} + B_\xi(\xi)\frac{\partial \rho^{\rm s}}{\partial c_2} = 0,\\ &A_\xi(\xi)\left(\frac{\partial \rho^{\rm s}}{\partial c_1}\right)_\xi + B_\xi(\xi)\left(\frac{\partial \rho^{\rm s}}{\partial c_2}\right)_\xi = 8v_{\rm s}f(\xi). \end{aligned} \tag{29}$$

One can show using Eq. (27) that

$$\frac{\partial \rho^{\rm s}}{\partial c_2} = -\rho^{\rm s}_\xi = -\sqrt{-c_1 + 8v_{\rm s}^2(\rho^{\rm s})^2 + 4(\rho^{\rm s})^3}, \tag{30}$$

$$\left(\frac{\partial \rho^{\rm s}}{\partial c_1}\right)_\xi = \frac{\partial}{\partial c_1}\rho^{\rm s}_\xi = \frac{\left(8v_{\rm s}^2(\rho^{\rm s}) + 6(\rho^{\rm s})^2\right)\left(\partial\rho^{\rm s}/\partial c_1\right) - 1/2}{\sqrt{-c_1 + 8v_{\rm s}^2(\rho^{\rm s})^2 + 4(\rho^{\rm s})^3}}. \tag{31}$$

In the case of $c_1 = 0$, the solution of Eq. (27) is the well known dark soliton. Its expression is given by

$$\rho^{\rm s} = -2v_{\rm s}^2\,\mathrm{sech}^2\left(\xi'\right), \tag{32}$$

with $\xi' = \sqrt{2}v_{\rm s}(\xi - c_2)$. The use of dark solitons as solutions of the KdV equation is acceptable since several theoretical and experimental works have shown that dark solitons exist in polariton condensates,[9,43,44] even in one dimension.[10]

In this case, equations (30) and (31) give

$$\frac{\partial \rho^{\rm s}}{\partial c_2} = -4\sqrt{2}v_{\rm s}^3\,\mathrm{sech}(\xi')^2\tanh(\xi'), \tag{33}$$

$$\begin{aligned} \left(\frac{\partial \rho^{\rm s}}{\partial c_1}\right)_\xi &= \sqrt{2}v_{\rm s}[3\,\mathrm{sech}^2(\xi') - 2]\coth(\xi')\frac{\partial \rho^{\rm s}}{\partial c_1}\\ &\quad + \frac{1}{8\sqrt{2}v_{\rm s}^3}\cosh^2(\xi')\coth(\xi'), \end{aligned} \tag{34}$$

after integration, we find

$$\begin{aligned} \frac{\partial \rho^{\rm s}}{\partial c_1} &= \frac{1}{16v_{\rm s}^4}\,\mathrm{sech}^2(\xi')\tanh(\xi')\\ &\quad \times \left[\sinh^3(\xi')\cosh(\xi') - \coth(\xi') + 3\xi'\right]. \end{aligned} \tag{35}$$

By using the conditions (29), $A_\xi(\xi)$ and $B_\xi(\xi)$ are expressed as

$$A_\xi(\xi) = 4v_{\rm s}f(\xi)\frac{\partial \rho^{\rm s}}{\partial c_2} \tag{36}$$

$$= -16\sqrt{2}v_{\rm s}^4 f(\xi)\,\mathrm{sech}^2(\xi')\tanh(\xi'), \tag{37}$$

$$B_\xi(\xi) = -4v_{\rm s}f(\xi)\frac{\partial \rho^{\rm s}}{\partial c_1} \tag{38}$$

$$\begin{aligned} &= \frac{1}{4v_{\rm s}^3}f(\xi)\,\mathrm{sech}^2(\xi')\tanh(\xi')\\ &\quad \times \left[-\sinh^3(\xi')\cosh(\xi') + \coth(\xi') - 3(\xi')\right]. \end{aligned} \tag{39}$$

The global solution of Eq. (22) is given by

$$\begin{aligned} \rho' &= \rho^{\rm s} + A(\xi)\frac{\partial \rho^{\rm s}}{\partial c_1} + B(\xi)\frac{\partial \rho^{\rm s}}{\partial c_2}\\ &= -2v_{\rm s}^2\,\mathrm{sech}^2(\xi') - \frac{1}{v_{\rm s}}\,\mathrm{sech}(\xi')^2\tanh(\xi')\\ &\quad \times \Bigg\{\left[\sinh^3(\xi')\cosh(\xi') - \coth(\xi') + 3\xi'\right]\\ &\quad \times \int f\left(\xi'\right)\mathrm{sech}^2(\xi')\tanh(\xi')\,\mathrm{d}\xi'\\ &\quad + \int f\left(\xi'\right)\mathrm{sech}^2(\xi')\tanh(\xi')\\ &\quad \times \left[-\sinh^3(\xi')\cosh(\xi') + \coth(\xi') - 3\xi'\right]\mathrm{d}\xi'\Bigg\}. \end{aligned} \tag{40}$$

The solution (40) is valid when $f_\xi(\xi)$ is small enough to permit a direct soliton perturbation method (Eqs. (24) and (25)); both the solution and its response function are in the form of traveling waves. This method works for any perturbation $f_\xi(\xi)$. The right choice of $f_\xi(\xi)$ can give an analytical solution for Eq. (22).

In the case where the damping is constant, and there is no defect [$f_X(X) = \hat{\eta}(1-\rho_0)\rho_0$], the solution (40) becomes

$$\begin{aligned} \rho' &= -2v_{\rm s}^2\,\mathrm{sech}^2(\xi') - \frac{\hat{\eta}(1-\rho_0)\rho_0}{16v_{\rm s}}\,\mathrm{sech}(\xi')^2\tanh(\xi')\\ &\quad \times \Big\{12\xi'^2 - 2\cosh(2\xi') + \cosh(4\xi') + 48\log[\cosh(\xi')]\\ &\quad + 8\xi'\left[\coth(\xi') - \sinh(2\xi') - 3\tanh(\xi')\right] - 5\Big\}. \end{aligned} \tag{41}$$

The solution (41) shows that the damping deforms slightly the dark soliton in an asymmetrical way. We recall that this study is limited to small $\hat{\eta}$.

In the case of a general $r(X,T)$, we solve Eq. (24) numerically. We consider the response function as negligible at great distance, where there is no damping or external potential. We use the Crank–Nicolson scheme on Eq. (24). The Newton iteration method is used to solve the set of linear equations obtained. We compute the Jacobian once and its LU decomposition will be used repeatedly to solve the set of linear equations.

In the numerical calculations, the potential (defect) is expressed as $\hat{V} = V_0\exp\left(-x^2/\sigma^2\right)$, where V_0 is its intensity and σ its width. ρ_0 is the steady state density, which is related to the flux velocity $v_{\rm s}$: $v_{\rm s} = \sqrt{\rho_0}$. $\hat{\eta}$ is the damping intensity. We have taken $m = \hbar = 1$; the time unit $t_{\rm p}$ and the distance unit $l_{\rm p}$ are then $t_{\rm p} = 0.24$ ps and $l_p = 0.53$ μm.

We notice that the response function always oscillates around an equilibrium value (see Figs. 2 and 3). These oscillations exist even if there are no defects. We have also noticed that the oscillation period and intensity depend on the velocity of the flow (the period is shorter and the intensity is lower for higher velocity). We notice also that the density variation induced by the modification of the damping is more important than the one caused by the external potential intensity; this is

more obvious when the velocity of the flow is important (the slow soliton interacts much longer with the defect and then its density changes more considerably). It means that the damping produces more change in the soliton density than does the external potential intensity. We have noticed also that for a narrow obstacle, the response function is confined around the soliton position (an effective potential is then created by the soliton (Fig. 3)). On the other hand, for a wide obstacle, the response function is confined around the defect position (see Fig. 4).

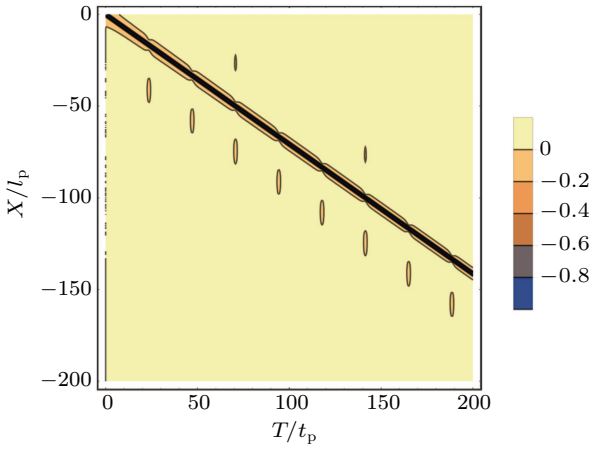


Fig. 2. (color online) Evolution of the dark soliton $\rho' = \rho^s + r$ when $\hat{\eta} = 0.0001$, $V_0 = 10$, $\sigma = 0.25$, and $\rho_0 = 0.5$. The dark soliton oscillates around an equilibrium position $X = -v_s T$. r is not affected by the external potential $\hat{V}$, then only the damping $\hat{\eta}$ perturbs the soliton. We notice that the amplitude and the width of the dark soliton oscillate simultaneously but in an opposite way (when the amplitude decreases, the width increases).

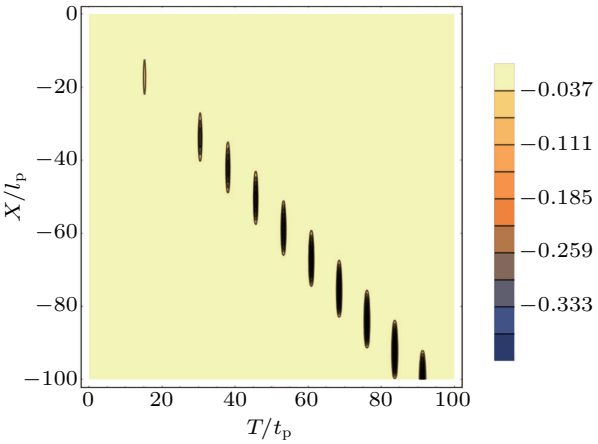


Fig. 3. (color online) Evolution of the dark soliton response function r when $\hat{\eta} = 0.001$, $V_0 = 0.1$, $\sigma = 0.25$, and $\rho_0 = 1.2$. r starts to oscillate around the defect position $X = 0$. This oscillation is confined in the $X = -v_s T$ direction. The oscillation amplitude depends on $\hat{\eta}$, V_0, and v_s (v_s is related to the steady state density ρ_0). Its period depends only on v_s. A modification of the external potential intensity V_0 causes a weak variation of r.

The oscillation of the dark soliton response functions suggests that these waves are stable. These oscillations exist even if there are no defects,i.e, they are caused by the continuous gain and loss of polaritons. The polariton self-interaction nature is now responsible for their stability. The repulsion between the polaritons promotes the creation of dark solitons in nonlinear systems and consequently helps also their stability. These results are consistent with those obtained by the soliton perturbations theory,[18,45] where it was found that dark solitons oscillate after interaction with an obstacle and maintain their shape and velocity. It was also demonstrated that dark solitons are stable in the case of a saturated gain.[15]

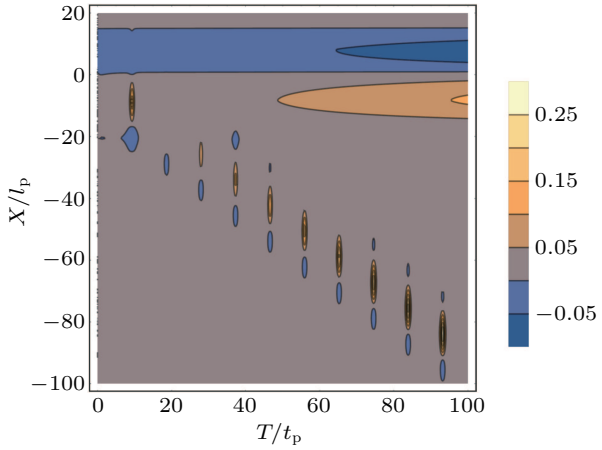


Fig. 4. (color online) Evolution of the dark soliton response function r when $\hat{\eta} = 0.001$, $V_0 = 0.1$, $\sigma = 5$, and $\rho_0 = 0.8$. We notice that r oscillates around the $X = -v_s T$ direction and it is also modified around the defect position $X = 0$. We see the competition between the external potential and the effective potential created by the soliton. For a wide enough obstacle, r will be modified only around the defect position.

5. Conclusion

We have described a polariton condensate using the Gross–Pitaevskii equation with some modifications to take into account the limited polariton lifetime and the presence of defects. We gave the shape of the polariton condensate flow in the hydraulic approximation. We transformed the GP equation into a modified KdV equation in the long wavelength limit using the reductive perturbation method. The resulting equation can have solitons as solutions when the damping and the external potential are zero. We have chosen a set in which the damping and the defects are small enough to be treated as perturbations; and we have written the solution of the system as a sum of a soliton and a response function.

In the case of traveling excitations, a full analytical solution is given using the variation of constants method. The result (Eq. (40)) is valid for any external perturbation $f_\xi(\xi)$. We calculated the solution for a constant $f_\xi(\xi)$ and noticed an asymmetric deformation of the soliton.

For general excitations, we have noticed that the response function always oscillates. So the external perturbations (damping and defects) disturb the solitons: they make dark solitons oscillate. We have found that the response function is more sensitive to the damping than to the external potential intensity.

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