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Quantum speed limit time of a two-level atom under different quantum feedback control*

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We investigate the quantum speed limit time (QSLT) of a two-level atom under quantum-jump-based feedback control or homodyne-based feedback control. Our results show that the two different feedback control schemes have different influences on the evolutionary speed. By adjusting the feedback parameters, the quantum-jump-based feedback control can induce speedup of the atomic evolution from an excited state, but the homodyne-based feedback control cannot change the evolutionary speed. Additionally, the QSLT for the whole dynamical process is explored. Under the quantum-jump-based feedback control, the QSLT displays oscillatory behaviors, which implies multiple speed-up and speed-down processes during the evolution. While, the homodyne-based feedback control can accelerate the speed-up process and improve the uniform speed in the uniform evolution process.

Keywords: quantum speed limit time, quantum feedback control, speedup

PACS: 03.65.Yz, 03.67.Lx, 42.50.–p **DOI:** 10.1088/1674-1056/27/1/010303

1. Introduction

Quantum speed limit time (QSLT), defined as the minimum evolution time for a quantum system evolving from an initial state to a target state, plays a vital role in many fields such as quantum communication,[1] quantum computation,[2] quantum state transfer,[3] quantum metrology,[4,5] and quantum optimal control protocols.[6,7] QSLT, which originates from the Heisenberg uncertainty relation for energy and time, is a key factor in characterizing the maximal speed of evolution of quantum systems. For closed systems with unitary evolution, the Mandelstam–Tamm type bound[8] and the Margolus–Levitin type bound[9] have been proposed, which are respectively based on the variance of energy of the initial state and the mean energy with respect to the ground state. Since every practical quantum system inevitably interacts with its surrounding environments, it is necessary to investigate the QSLT for open quantum systems. Based on different distance measures, there are many bounds on the evolution time for open quantum systems. By employing a geometric approach, a unified bound on the evolution time including both Mandelstam–Tamm and Margolus–Levitin types was formulated[10] for open quantum systems from a pure initial state to a target state. Later, a generic bound on the evolution time of open quantum systems was provided by using relative purity,[11] which is applicable to both mixed and pure initial states. Since the appearance of definitions for QSLT in open quantum systems, the QSLT of quantum systems interacting with various environments has been widely investigated.[12–14] Recently, in order to obtain a faster speed of evolution, a lot of efforts have been made to short the QSLT. It has been suggested that memory environments can speed up quantum evolution and the mechanism of the acceleration in open quantum systems is related not only to the non-Markovianity but also to the population of excited states.[15] Meanwhile, in Ref. [16], the authors proposed a method of accelerating the speed of evolution for an open system by external classical driving. Additionally, quantum speedup under dynamical decoupling pulses[17] and an atomic quantum speedup in a photonic-band-gap reservoir[18] have been carefully investigated.

On the other hand, as a typical method to steer the dynamics of quantum systems by feeding back the measurement results to the systems, Markovian quantum feedback control introduced by Wiseman and Milburn[19,20] has been widely applied. According to different measurement schemes, the Markovian quantum feedback control is categorized as quantum-jump-based and homodyne-based feedback controls. In Ref. [21], the effects of the two kinds of different Markovain feedback control schemes on protecting entangled states against decoherence have been compared, and the results showed that the prior outperforms the latter. Recently, quantum-jump-based feedback control and classical driving have been combined to generate the three-atom singlet state,[22,23] W state, and GHZ state.[24,25] We also obtained steady and optimal entropy squeezing by using the quantum-jump-based feedback control and classical driving in Ref. [26]. Besides, the homodyne-based feedback control has been applied to produce steady entanglement[27] and manipu-

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late quantum discord of a two-atom system.[28] However, how to control the evolutionary speed of open quantum systems by quantum feedback control has never been reported. In this paper, we mainly use either the quantum-jump-based feedback control or the homodyne-based feedback control to manipulate the QSLT of an open two-level system and compare their effects on speeding up the evolution of the system.

This paper is structured as follows. In Section 2, we introduce the physical model and respectively describe the dynamics of the system under two different feedback control schemes by master equations. The definitions of the QSLT for pure and mixed initial states are reviewed in Section 3. We devote Section 4 to exploring the evolution of the QSLT under quantum-jump-based feedback control or homodyne-based feedback control and comparing their effects on controlling the evolutionary speed by adjusting the feedback parameters. Finally, we provide a conclusion in Section 5.

2. The physical model

The system consists of a two-level atom coupled resonantly to a single cavity mode with coupling strength g. The cavity mode a is damped with decay rate κ, and the atom with excited state $|1\rangle$ and ground state $|0\rangle$ can spontanously decay with rates γ_0. A quantum feedback control conditioned on the measurement of the leaky cavity is applied to the atom. In the absence of feedback control, the master equation describing the system in the interaction picture is of the form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H,\rho] + \kappa\mathcal{D}[a]\rho + \gamma_0\mathcal{D}[\sigma_-]\rho, \tag{1}$$

where

$$H = g(\sigma_+ a + \sigma_- a^\dagger), \tag{2}$$

$$\mathcal{D}[c]\rho = c\rho c^\dagger - \frac{1}{2}(c^\dagger c\rho + \rho c^\dagger c). \tag{3}$$

Here H is the Hamiltonian of the system in the interaction picture; superoperators $\mathcal{D}[a]$ and $\mathcal{D}[\sigma_-]$ describe the cavity and atomic decays; a is the cavity-mode annihilation operator, and $\sigma_- = |0\rangle\langle 1|$ is the lowering operator of the atom.

In the limit that the decay rate κ of the cavity is very large, the cavity mode can be adiabatically eliminated,[29] and we obtain an effective damping rate relevant to the atom–cavity interaction strength and the cavity damping rate, $\gamma = 4g^2/\kappa$. Then, under the assumption that the effective decay rate is much larger than the atomic spontaneous emission rate, the following master equation is obtained:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \gamma\left[\sigma_-\rho\sigma_+ - \frac{1}{2}(\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-)\right]. \tag{4}$$

Here we will consider the Markovian feedback control in which the feedback Hamiltonian is proportional to the measurement signal. According to different measurement strategies, we discuss two different feedback control schemes, quantum-jump-based feedback control and homodyne-based feedback control. In the quantum-jump-based feedback control, the measurement stage is a direct photodetection of the output field, and the control is only triggered after a detection click, i.e., a quantum jump occurs. While in the homodyne-based feedback control, the detector registers a continuous photocurrent, and the feedback Hamiltonian is constantly applied to the atom.

The dynamics of the atom under the quantum-jump-based feedback control can be described by the master equation[30]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \gamma\left[U\sigma_-\rho\sigma_+U^\dagger - \frac{1}{2}(\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-)\right], \tag{5}$$

where $U = \exp(-\mathrm{i}F\delta t)$ represents the finite amount of evolution imposed by the control Hamiltonian F on the system. Once a quantum jump event $\sigma_-\rho\sigma_+$ occurs, the unitary operator U is applied, resulting in $U\sigma_-\rho\sigma_+U^\dagger$.

Differently, the master equation including the homodyne-based feedback control reads[31–33]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}\left[\frac{1}{2}(A^\dagger F + FA), \rho\right] + \mathcal{D}[A - \mathrm{i}F]\rho, \tag{6}$$

where $A = \sqrt{\gamma}\sigma_-$, and F is the feedback Hamiltonian.

In this paper, we will choose the feedback Hamiltonian $F = \lambda\sigma_x$ for the two types of feedback controls. Then, in the quantum-jump-based feedback control, the unitary operator is $U = \exp(-\mathrm{i}\lambda\sigma_x)$, where $\delta t = 1$ is set for simplicity.

3. Definitions of quantum speed limit time

In this section, we will briefly review the definitions of the QSLT for an open quantum system. A unified bound for the minimal evolution time between an initial pure state $\rho_0 = |\phi_0\rangle\langle\phi_0|$ and its target state ρ_{τ_D}, governed by the master equation $\dot{\rho}_t = L_t\rho_t$, was derived by Deffner and Lutz.[10] With the aid of the von Neumann trace inequality and the Cauchy–Schwarz inequality, the QSLT is obtained as follows:

$$\tau_\mathrm{D} \geq \tau_\mathrm{QSL} = \max\left\{\frac{1}{\Lambda^1_{\tau_\mathrm{D}}}, \frac{1}{\Lambda^2_{\tau_\mathrm{D}}}, \frac{1}{\Lambda^\infty_{\tau_\mathrm{D}}}\right\}\sin^2[\mathcal{B}(\rho_0,\rho_{\tau_\mathrm{D}})], \tag{7}$$

where $\Lambda^p_{\tau_\mathrm{D}} = \tau_\mathrm{D}^{-1}\int_0^{\tau_\mathrm{D}}\|L_t\rho_t\|_p\,\mathrm{d}t$, and $\|A\|_p = (\sigma_1^p + \cdots + \sigma_n^p)^{1/p}$ denotes the Schatten p norm, $\sigma_1,\ldots,\sigma_n$ are the singular values of A. When $p = 1$, $\|A\|_1$ corresponds to the trace norm $\|A\|_\mathrm{tr} = \sum_i\sigma_i$. When $p = 2$, $\|A\|_2$ corresponds to the Hilbert–Schmidt norm, which is defined by $\|A\|_\mathrm{hs} = \sqrt{\sum_i\sigma_i^2}$. When $p = \infty$, $\|A\|_\infty$ corresponds to the operator norm, which is given by the largest singular value $\|A\|_\infty = \sigma_\mathrm{max}$. Owing to the relationship $\|A\|_\mathrm{op} \leq \|A\|_\mathrm{hs} \leq \|A\|_\mathrm{tr}$, the ML-type bound based on the operator norm ($p = \infty$) provides the sharpest bound on the QSLT. $\mathcal{B}(\rho_0,\rho_{\tau_\mathrm{D}}) = \arccos\sqrt{\langle\phi_0|\rho_{\tau_\mathrm{D}}|\phi_0\rangle}$ is the Bures angle between the initial and the target states of the quantum system.

However, for the initial mixed states, the definition of QSLT in Eq. (7) fails. Fortunately, Zhang *et al.*[11] proposed a time lower bound for any mixed initial state ρ_τ based on the relative purity, which is defined as

$$\tau_{\rm D} \geq \tau_{\rm QSL} = \max\left\{\frac{1}{\overline{\sum_{i=1}^{n}\sigma_i\rho_i}}, \frac{1}{\overline{\sum_{i=1}^{n}\sigma_i^2}}\right\} \times |f(\tau+\tau_{\rm D})-1|\,{\rm tr}(\rho_\tau^2), \tag{8}$$

where $\overline{X} = \tau_{\rm D}^{-1}\int_\tau^{\tau+\tau_{\rm D}} X\,{\rm d}t$, σ_i and ρ_i are the singular values of $L_t\rho_t$ and the mixed initial state ρ_τ, respectively. $f(\tau+\tau_{\rm D}) = {\rm tr}(\rho_\tau\rho_{\tau+\tau_{\rm D}})/{\rm tr}(\rho_\tau^2)$ denotes the relative purity between the initial state and the final state with the driving time $\tau_{\rm D}$. For a pure initial state $\rho_{\tau=0} = |\phi_0\rangle\langle\phi_0|$, expression (8) can recover to the unified lower bound given in Eq. (7).

4. Quantum speed limit under different quantum feedback control

By solving the master equations (5) and (6) respectively, the density matrix elements of the atom under the quantum-jump-based feedback control at time t read

$$\begin{aligned}
\rho_{11}(t) &= {\rm e}^{-\gamma t\cos^2(\lambda)}\rho_{11}(0),\\
\rho_{12}(t) &= {\rm e}^{-\frac{1}{2}\gamma t}\big[\rho_{12}(0) - {\rm e}^{\frac{\pi}{2}{\rm i}}(1-{\rm e}^{-\frac{1}{2}\gamma t\cos 2\lambda})\tan(2\lambda)\rho_{11}(0)\big],\\
\rho_{21}(t) &= \rho_{12}^*(t),\\
\rho_{22}(t) &= 1-{\rm e}^{-\gamma t\cos^2(\lambda)}\rho_{11}(0),
\end{aligned} \tag{9}$$

and the density matrix elements under the homodyne-based feedback control are written as

$$\begin{aligned}
\rho_{11}(t) &= \frac{\lambda^2+{\rm e}^{-t(\gamma+2\lambda^2)}[\gamma\rho_{11}(0)+\lambda^2(\rho_{11}(0)-\rho_{22}(0))]}{\gamma+2\lambda^2},\\
\rho_{12}(t) &= \frac{{\rm e}^{-\frac{1}{2}t(\gamma+4\lambda^2)}\{[{\rm i}\sqrt{\gamma}(1-{\rm e}^{2\lambda^2 t})+\lambda\,{\rm e}^{2\lambda^2 t}]\,{\rm Re}[\rho_{12}(0)]+\lambda\,{\rm Im}[\rho_{12}(0)]\}}{\lambda},\\
\rho_{21}(t) &= \rho_{12}^*(t),\\
\rho_{22}(t) &= \frac{(\gamma+\lambda^2)+{\rm e}^{-t(\gamma+2\lambda^2)}[\lambda^2(\rho_{22}(0)-\rho_{11}(0))-\gamma\rho_{11}(0)]}{\gamma+2\lambda^2}.
\end{aligned} \tag{10}$$

We first examine the QSLT under the quantum-jump-based feedback control of the atom with the pure initial state $|\phi_0\rangle = |1\rangle$. By using Eqs. (7) and (9), the QSLT can be written as

$$\tau_{\rm QSL} = \frac{1-p(\tau_{\rm D})}{\int_0^{\tau_{\rm D}}\sigma(t)\,{\rm d}t/\tau_{\rm D}}, \tag{11}$$

with

$$\begin{aligned}
&\sigma(t)\\
&= \sqrt{\cos 4\lambda + 2\cosh\left(\frac{1}{2}\gamma t\cos 2\lambda\right) - 2\cos(2\lambda)\sinh\left(\frac{1}{2}\gamma t\cos 2\lambda\right) - 1}\sqrt{\gamma^2\cos^2\lambda\sec^2 2\lambda\exp\left[-\frac{1}{2}\gamma t(2+\cos 2\lambda)\right]}/\sqrt{2},
\end{aligned} \tag{12}$$

where $p(\tau_{\rm D}) = \rho_{11}(\tau_{\rm D})$ represents the population of the atomic excited state at time $\tau_{\rm D}$.

In Fig. 1, we plot the QSLT (the black solid curve) under the quantum-jump-based feedback control as a function of the feedback parameter λ. It is known from Eq. (9) that the quantum-jump-based feedback control can influence the dynamics of the atom on a π cycle, so we set $\lambda \in [0,\pi]$ in Fig. 1. We can find that when there is no feedback control, e.g., $\lambda = 0$ or π, the QSLT is equal to the actual driving time $\tau_{\rm D} = 1$. While as the feedback parameter increases, the QSLT first decreases and then gradually increases, which implies that the evolution of the atom experiences a process from acceleration to deceleration by adjusting the feedback parameter. Besides, we can find that the critical point is $\lambda = \pi/2$, where the dynamical map reduces to a phase-damping channel, and the QSLT becomes zero. This result is consistent with the consequences in Refs. [11] and [34].

From Eq. (11), one can see that the QSLT is related to the population of the excited state. In order to explore the internal mechanism on the speed of evolution, we also depict the excited state population $p(\tau_{\rm D})$ (the red dashed curve) as a function of the feedback parameter in Fig. 1. It is obvious that the excited state population has a negative relation with the QSLT. At the beginning, the population of the excited state increases as the feedback parameter increases, until it reaches the maximum which corresponds to the critical point in QSLT from acceleration to deceleration. Then the excited state population decreases, which causes the deceleration of the evolution.

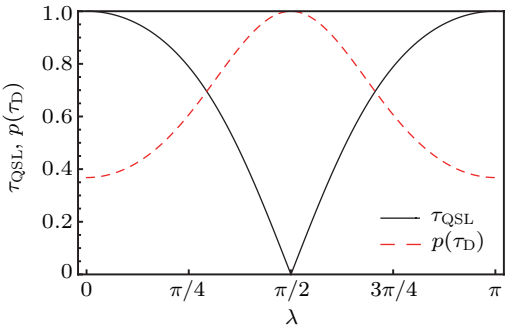


Fig. 1. (color online) The QSLT (black solid curve) and excited state population $p(\tau_{\tau_{\rm D}})$ (red dashed curve) under the quantum-jump-based feedback control as a function of the feedback parameter λ for the pure initial state $|1\rangle$. Other parameters are $\gamma = 1$ and $\tau_{\rm D} = 1$.

For comparison, we also study the behavior of the QSLT under the homodyne-based feedback control when the atom is initially in the excited state. By numerical calculation, it is found that the QSLT is always equal to the actual driving time $\tau_{\rm D}$ for any feedback parameter, which indicates that the homodyne-based feedback control cannot induce acceleration or deceleration of the atomic evolution. Thus, we conclude that the evolution of the atom from the excited state can be accelerated by appropriately adjusting the quantum-jump-based feedback parameter, whereas the homodyne-based feedback control has no effect on the speed of the atomic evolution. That is to say, the quantum-jump-based feedback control outperforms the homodyne-based feedback control in inducing speedup of the atomic evolution from the excited state.

In the following, we explore effects of the two different feedback control schemes on the QSLT in the whole dynamical process. We also start from the excited state $|1\rangle$ and discuss the QSLT from an arbitrary time-evolution state ρ_τ to another $\rho_{\tau+\tau_{\rm D}}$. By making use of Eq. (8), the QSLT for mixed initial states can be calculated. Figures 2(a) and 2(b) present the variation of the QSLT respectively under quantum-jump-based feedback control and homodyne-based feedback control with respect to the initial time parameter τ for different choices of feedback parameters. It is observed that the QSLT without feedback control initially reduces to a minimum and then increases to a maximum (the green solid curves in Figs. 2(a) and 2(b)), which implies that the evolution of the whole dynamical process first exhibits a speed-up and then shows a speed-down. From Fig. 1(a), we can see that under the quantum-jump-based feedback control, the evolution of the QSLT for the whole dynamical process becomes more complicated, where the QSLT displays multiple decreases and increases when the feedback parameter is chosen to be $\pi/4$ or $\pi/3$. When $\lambda = \pi/4$ (the blue dotted curve), the QSLT infinitely approaches zero after a period of oscillations. However, when $\lambda = \pi/3$ (the red dashed curve) is set, it finally reaches a constant value 0.5, which indicates a uniform evolution process after oscillations. The effect of the homodyne-based feedback control on the QSLT is revealed in Fig. 2(b), where we choose the same feedback parameters as those in Fig. 1(a). One can observe that the homodyne-based feedback control does not change the evolutionary trend of the QSLT, but with the increase of the feedback parameter, the decay rate of the QSLT increases, which indicates a larger acceleration in the speed-up process. Another feature we find is that the final speed of the uniform evolution process after acceleration and deceleration relies on the feedback parameter. The larger the feedback parameter is, the smaller value the QSLT finally remains at, which corresponds to the faster uniform speed.

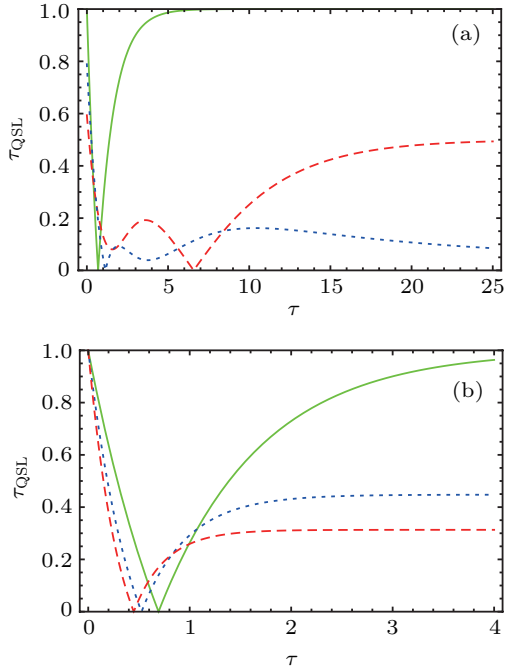


Fig. 2. (color online) The QSLT (a) under quantum-jump-based feedback control and (b) under homodyne-based feedback control as a function of the initial time parameter τ with different feedback parameters $\lambda = 0$ (green solid curve), $\lambda = \pi/4$ (blue dotted curve), and $\lambda = \pi/3$ (red dashed curve). Other parameters are chosen as $\gamma = 1$ and $\tau_{\rm D} = 1$.

Overall, for the whole dynamical process, the quantum-jump-based feedback control and the homodyne-based feedback control have different influences on the speed of evolution. The quantum-jump-based feedback control can change the evolution from the single speed-up and speed-down process to the multiple speed-up and speed-down process. However, the homodyne-based feedback control cannot change the whole tendency of evolution, but can induce a larger acceleration in the speed-up process and improve the uniform speed in the uniform evolution process.

5. Conclusion

In summary, we have studied the QSLT of a two-level atom under quantum-jump-based feedback control or homodyne-based feedback control. When the system evolves

from the excited state, the quantum-jump-based feedback control can induce speedup of systemic evolution by choosing appropriate feedback parameters, while the homodyne-based feedback control has no effect on the speed of systemic evolution. For the whole dynamical process, the quantum-jump-based feedback control causes multiple speed-up and speed-down processes, while the homodyne-based feedback control results in a larger acceleration in the speed-up process and improves the uniform speed in the uniform evolution process.

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