ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理
Chinese Physics B

Solution of the spin-one DKP oscillator under an external magnetic field in noncommutative space with minimal length

Bing-Qian Wang(王炳乾), Zheng-Wen Long(隆正文), Chao-Yun Long(龙超云), Shu-Rui Wu(吴淑蕊)
Citation: Chin. Phys. B . 2018, 27(1): 010301. **doi:** 10.1088/1674-1056/27/1/010301

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Analytical and numerical investigations of displaced thermal state evolutions in a laser process

Chuan-Xun Du(杜传勋), Xiang-Guo Meng(孟祥国), Ran Zhang(张冉), Ji-Suo Wang(王继锁)
Chin. Phys. B . 2017, 26(12): 120301. **doi:** 10.1088/1674-1056/26/12/120301

Comparative analysis of entanglement measures based on monogamy inequality

P J Geetha, Sudha, K S Mallesh
Chin. Phys. B . 2017, 26(5): 050301. **doi:** 10.1088/1674-1056/26/5/050301

Kraus operator solutions to a fermionic master equation describing a thermal bath and their matrix representation

Xiang-Guo Meng(孟祥国), Ji-Suo Wang(王继锁), Hong-Yi Fan(范洪义), Cheng-Wei Xia(夏承魏)
Chin. Phys. B . 2016, 25(4): 040302. **doi:** 10.1088/1674-1056/25/4/040302

Monogamous nature of symmetric *N*-qubit states of the W class: Concurrence and negativity tangle

P. J. Geetha, K. O. Yashodamma, Sudha
Chin. Phys. B . 2015, 24(11): 110302. **doi:** 10.1088/1674-1056/24/11/110302

Explicit solution of diffusion master equation under the action of linear resonance force via the thermal entangled state representation

Yao Fei, Wang Ji-Suo, Xu Tian-Niu
Chin. Phys. B . 2015, 24(7): 070304. **doi:** 10.1088/1674-1056/24/7/070304

Solution of the spin-one DKP oscillator under an external magnetic field in noncommutative space with minimal length*

Bing-Qian Wang(王炳乾)[1], Zheng-Wen Long(隆正文)[2,†], Chao-Yun Long(龙超云)[2], and Shu-Rui Wu(吴淑蕊)[2]

[1] College of Mathematics and Statistics, Guizhou University, Guiyang 550025, China
[2] College of Physics, Guizhou University, Guiyang 550025, China

(Received 10 July 2017; revised manuscript received 17 September 2017; published online 28 November 2017)

The spin-one Duffin–Kemmer–Petiau (DKP) oscillator under a magnetic field in the presence of the minimal length in the noncommutative coordinate space is studied by using the momentum space representation. The explicit form of energy eigenvalues is found, and the eigenfunctions are obtained in terms of the Jacobi polynomials. It shows that for the same azimuthal quantum number, the energy E increases monotonically with respect to the noncommutative parameter and the minimal length parameter. Additionally, we also report some special cases aiming to discuss the effect of the noncommutative coordinate space and the minimal length in the energy spectrum.

Keywords: DKP oscillator, noncommutative space, minimal length, momentum space representation

PACS: 03.65.Pm, 03.65.Ge **DOI:** 10.1088/1674-1056/27/1/010301

1. Introduction

The Duffin–Kemmer–Petiau (DKP) equation[1–4] is a first-order relativistic wave equation that describes spin-0 and spin-1 particles in the description of the standard model.[5–7] As typical examples, the equation has been studied in connection with various aspects including α-nucleus elastic scattering, meson–nuclear interaction,[8,9] quantum chromodynamics (QCD),[10,11] five dimensional Galilean invariance,[12] Aharonov–Bohm potential,[13,14] and so on.

The noncommutative (NC) geometry has drawn much attention in both quantum mechanics and field theory in the past few years. At first, the coordinate noncommutativity was introduced by Snyder[15] as a regularized mechanism aiming to improve the infinite self-energy in quantum field theory. Afterwards, this concept was put forward by Connes[16] in the study of Yang–Mills theories. In recent years, various aspects of both NC classical[17] and quantum mechanics[18] have been extensively studied to explore the role of NC parameter in the physical observables. Typical examples of such cases include a particle confined by a quadratic potential in the generalized noncommutative plane,[19] the noncommutative harmonic oscillators,[20–22] classical Newton mechanics in the noncommutative space,[23,24] and the Aharonov–Bohm effect in a class of noncommutative theories.[25]

The minimal length uncertainty relation is another appealing case of the research, in connection with which various topics have been studied.[26–30] Motivated by the intense interest in the study of the minimal length uncertainty relation combination with the noncommutative coordinate space commutation relations in nonrelativistic wave equation and relativistic wave equation,[31–33] we study the DKP oscillator for spin-one particle under a magnetic field in the presence of a minimal length in the noncommutative coordinate space in the present work.

2. Spin-one DKP oscillator under an external magnetic field in noncommutative space with minimal length

We first review the DKP equation, which is a relativistic wave equation for a free boson of mass m given by[1–3]

$$\left(\mathrm{i}\hbar\beta_\mu\partial^\mu - mc\right)\psi = 0, \quad \mu = 0,1,2,3, \tag{1}$$

where m is the mass, ψ is the boson wave function, and β_μ are the DKP matrices which satisfy the algebra relation

$$\beta^\mu\beta^\nu\beta^\lambda + \beta^\lambda\beta^\nu\beta^\mu = g^{\mu\nu}\beta^\lambda + g^{\lambda\nu}\beta^\mu, \tag{2}$$

with $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ being the metric tensor in Minkowski space. For spin one particle, β^μ matrices are

$$\beta^0 = \begin{pmatrix} \mathbf{0} & \mathbf{0} & -I & \tilde{0}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \tilde{0}^{\mathrm{T}} \\ -I & \tilde{\mathbf{0}} & \mathbf{0} & \tilde{0} \\ 0 & \tilde{0}^{\mathrm{T}} & \tilde{0} & 0 \end{pmatrix},$$

$$\beta^i = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathrm{i}v^{i\mathrm{T}} \\ \mathbf{0} & \mathbf{0} & s^i & \tilde{0}^{\mathrm{T}} \\ \mathbf{0} & -s^i & \mathbf{0} & \tilde{0}^{\mathrm{T}} \\ \mathrm{i}v^i & 0 & 0 & 0 \end{pmatrix}, \quad i = 1,2,3, \tag{3}$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11465006 and 11565009) and the Project of Research Foundation for Graduate Students in Guizhou Province, China (Grant No. (2017)11108).

†Corresponding author. E-mail: zwlong@gzu.edu.cn

© 2018 Chinese Physical Society and IOP Publishing Ltd

with

$$\tilde{\mathbf{0}} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}, \quad s^1 = \mathrm{i}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$s^2 = \mathrm{i}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad s^3 = \mathrm{i}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$v^1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad v^2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \quad v^3 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.$$

The DKP oscillator system is introduced by using substitution $\boldsymbol{p}$ with $\boldsymbol{p} \to \boldsymbol{p} - \mathrm{i}mw\eta^0 r$, where the additional term is linear in r, w is the oscillator frequency, and $\eta^0 = 2\left(\beta^0\right)^2 - \boldsymbol{I}$ with $\left(\eta^0\right)^2 = \boldsymbol{I}$. In fact, it is a direct generalization to the Dirac particles of integer spin in which one replaces the γ matrices by β metrics but verifying a more complicated algebra.[34,35] Recently, it has been investigated by a large number of papers.[36–40] In addition, if we assume that there is a magnetic field B, which is along to z-axis direction, i.e. $\boldsymbol{A} = \left(-\frac{By}{2}\frac{Bx}{2}0\right)$, the Hamiltonian in DKP oscillator system turns into $H = c\boldsymbol{\beta}\cdot\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A} - \mathrm{i}mw\eta^0 r\right) + mc^2$.

In NC coordinate space, the canonical variables satisfy the commutation relations

$$[x_i, x_j] = i\alpha_{ij} \quad [x_i, p_j] = \mathrm{i}\hbar\delta_{ij}, [p_i, p_j] = 0 \quad (i, j = 1, 2), \quad (4)$$

where $\alpha_{ij} = \varepsilon_{ij}\alpha$ is the antisymmetric NC parameter, representing the noncommutativity of the space. By replacing the normal product with star product, the DKP oscillator system in commutation space will change into the DKP oscillator system in NC coordinate space, as

$$H\left(x_i, p_i\right) * \psi = H\left(x_i \to x_i\alpha_{ij}p_j, p_i\right)\psi = E\psi. \quad (5)$$

Normally, one way to deal with the problem of NC coordinate space is via the Bopp shift. In this case, the equation (5) becomes

$$\left[c\boldsymbol{\beta}\cdot\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A} - \mathrm{i}mw\eta^0 r\right) + mc^2\right]\psi = \beta^0 E\psi. \quad (6)$$

For a boson of spin one, the stationary state ψ is a vector with ten components wave function, which reads

$$\psi = \left(\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8, \psi_9, \psi_{10}\right)^{\mathrm{T}}. \quad (7)$$

Substituting Eq. (7) into Eq. (6), we have

$$\mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) + \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]\psi_{10} + mc^2\psi_1 + E\psi_7 = 0, \quad (8)$$

$$\mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) + \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]\psi_{10} + mc^2\psi_2 + E\psi_8 = 0, \quad (9)$$

$$mc^2\psi_3 + E\psi_9 = 0, \quad (10)$$

$$\mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) - \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]\psi_9 + mc^2\psi_4 = 0, \quad (11)$$

$$-\mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) - \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]\psi_9 + mc^2\psi_5 = 0, \quad (12)$$

$$\mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) - \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]\psi_8 + mc^2\psi_6 - \mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) - \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]\psi_7 = 0, \quad (13)$$

$$\mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) + \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]\psi_6 - E\psi_1 - mc^2\psi_7 = 0, \quad (14)$$

$$\mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) + \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]\psi_6 + E\psi_2 + mc^2\psi_8 = 0, \quad (15)$$

$$\mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) + \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]\psi_4 + E\psi_3 + mc^2\psi_9 - \mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) + \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]\psi_5 = 0, \quad (16)$$

$$\mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) - \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]\psi_1 + \mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) - \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]\psi_2 + mc^2\psi_{10} = 0. \quad (17)$$

To our knowledge, it is difficult to find the general solution to the above equations. As a result, we pay our attention to the component ψ_9. In this case, according to the usual methods,[41–44] we try to find the specific solutions of the system. Therefore, in order to obtain the same form of spectrum of energy as for ψ_9, we may assume that $\psi_1 = \psi_2 = 0$. It leads to $\psi_6 = \psi_7 = \psi_8 = \psi_{10} = 0$. At this stage, from Eqs. (10)–(12) we have

$$\psi_3 = -\frac{E}{mc^2}\psi_9, \quad (18)$$

$$\psi_4 = -\frac{\mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) - \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]}{mc^2}\psi_9, \quad (19)$$

$$\psi_5 = \frac{\mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) - \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]}{mc^2}\psi_9. \quad (20)$$

Taking these equations into Eq. (16), we obtain

$$\mathrm{i}c\left[p_y - \frac{eB}{2c}\left(x - \frac{\alpha}{2\hbar}p_y\right) + \mathrm{i}mw\left(y + \frac{\alpha}{2\hbar}p_x\right)\right]\psi_4 + E\psi_3 + mc^2\psi_9 - \mathrm{i}c\left[p_x + \frac{eB}{2c}\left(y + \frac{\alpha}{2\hbar}p_x\right) + \mathrm{i}mw\left(x - \frac{\alpha}{2\hbar}p_y\right)\right]\psi_5 = 0. \quad (21)$$

Simplifying Eq. (21) leads to

$$\left\{\left[p_x+\frac{eB}{2c}\left(y+\frac{\alpha}{2\hbar}p_x\right)+\mathrm{i}mw\left(x-\frac{\alpha}{2\hbar}p_y\right)\right]\times\left[p_x+\frac{eB}{2c}\left(y+\frac{\alpha}{2\hbar}p_x\right)-\mathrm{i}mw\left(x-\frac{\alpha}{2\hbar}p_y\right)\right]+\left[p_y-\frac{eB}{2c}\left(x-\frac{\alpha}{2\hbar}p_y\right)+\mathrm{i}mw\left(y+\frac{\alpha}{2\hbar}p_x\right)\right]\times\left[p_y-\frac{eB}{2c}\left(x-\frac{\alpha}{2\hbar}p_y\right)-\mathrm{i}mw\left(y+\frac{\alpha}{2\hbar}p_x\right)\right]+\frac{m^2c^4-E^2}{c^2}\right\}\psi_9=0. \tag{22}$$

For the sake of simplicity, we bring the problem into the momentum space. In this case, the expressions of the position vector and momentum vector in the presence of the minimal length are given by

$$x_{\mathrm{i}}=\mathrm{i}\hbar\left[\left(1+\gamma p^2\right)\frac{\partial}{\partial p_{\mathrm{i}}}\right],\quad p_{\mathrm{i}}=p_{\mathrm{i}}, \tag{23}$$

where γ is the minimal length positive parameter. By using a transformation of the form $\psi_9(p,\varphi)=\mathrm{e}^{\mathrm{i}l\varphi}\psi(P)$, $l=0,1,2,$ we rewrite Eq. (22) as

$$\left\{\left(1+\frac{eB\alpha}{4\hbar c}\right)^2+\frac{m^2w^2\alpha^2}{4\hbar^2}-2\hbar\gamma mw\left(1+\frac{eB\alpha}{2\hbar c}\right)-\gamma l\hbar\left[\frac{eB}{c}\left(1+\frac{eB\alpha}{4\hbar c}\right)+\frac{m^2w^2\alpha}{\hbar}-\frac{2mweB\hbar\gamma}{c}\right]\right\}p^2\psi+\left(\frac{e^2B^2}{4c^2}+m^2w^2\right)\left\{\left(1+\gamma p^2\right)^2\frac{\partial^2}{\partial p^2}+\left[\frac{1}{p}\left(1+\gamma p^2\right)^2+2\gamma p\left(1+\gamma p^2\right)\right]\frac{\partial}{\partial p}-\frac{l^2}{p^2}\left(1+\gamma p^2\right)^2\right\}\psi+\left\{\frac{m^2c^4-E^2}{c^2}-2\hbar mw\left(1+\frac{eB\alpha}{2\hbar c}\right)-\hbar l\left[\frac{eB}{c}\left(1+\frac{eB\alpha}{4\hbar c}\right)+\frac{m^2w^2\alpha}{\hbar}-\frac{2mweB\hbar\gamma}{c}\right]\right\}\psi=0. \tag{24}$$

A transformation of the form $\chi=\gamma^{-\frac{1}{2}}\arctan(\gamma p)$ brings Eq. (24) into

$$\left[\frac{\partial^2}{\partial\chi^2}+\gamma^{\frac{1}{2}}\left(\frac{1}{\tan(\chi\sqrt{\gamma})}+\tan(\chi\sqrt{\gamma})\right)\frac{\partial}{\partial\chi}-\gamma l^2\frac{1}{\tan^2(\chi\sqrt{\gamma})}+\gamma\Sigma\tan^2(\chi\sqrt{\gamma})+\gamma\Gamma'\right]\psi(\chi)=0, \tag{25}$$

where

$$\Sigma=-l^2-\frac{\left(1+\frac{eB\alpha}{4\hbar c}\right)^2+\frac{m^2w^2\alpha^2}{4\hbar^2}-2\hbar\gamma mw\left(1+\frac{eB\alpha}{2\hbar c}\right)-\gamma l\hbar\left[\frac{eB}{c}\left(1+\frac{eB\alpha}{4\hbar c}\right)+\frac{m^2w^2\alpha}{\hbar}-\frac{2mweB\hbar\gamma}{c}\right]}{\gamma^2\hbar^2\left(m^2w^2+\frac{e^2B^2}{4c^2}\right)},$$

$$\Gamma'=\frac{\frac{E^2-m^2c^4}{c^2}+2\hbar mw\left(1+\frac{eB\alpha}{2\hbar c}\right)+\hbar l\left[\frac{eB}{c}\left(1+\frac{eB\alpha}{4\hbar c}\right)+\frac{m^2w^2\alpha}{\hbar}-\frac{2mweB\hbar\gamma}{c}\right]}{\gamma^2\hbar^2\left(m^2w^2+\frac{e^2B^2}{4c^2}\right)}-2l^2.$$

With the help of an auxiliary function given by

$$\psi(\chi)=\left[1-\sin^2(\chi\sqrt{\gamma})\right]^{\frac{5}{2}}\sin^l(\chi\sqrt{\gamma})S(\chi), \tag{26}$$

the equation (25) takes the form

$$\left\{\frac{\partial^2}{\partial\chi^2}+\gamma^{\frac{1}{2}}\left[(2l+1)\frac{\sqrt{1-\sin^2(\chi\sqrt{\gamma})}}{\sin(\chi\sqrt{\gamma})}+(1-2\zeta)\frac{\sin(\chi\sqrt{\gamma})}{\sqrt{1-\sin^2(\chi\sqrt{\gamma})}}\right]\frac{\partial}{\partial\chi}+\gamma\left[\Gamma'-2\zeta(l+1)\right]\right\}S(\chi)=0, \tag{27}$$

where ζ verifies

$$\gamma[\zeta(\zeta-2)+\Sigma]=0, \tag{28}$$

whose solutions can be written as

$$\zeta=1\pm\sqrt{1-\Sigma}. \tag{29}$$

Considering Eq. (26), the auxiliary function should be nonsingular at $\chi=\pm1$, which implies that the accepted value of ζ is

$$\zeta=1+\sqrt{1-\Sigma}. \tag{30}$$

In order to transform Eq. (27) into a class of known differential equations with a polynomial solution, we introduce another auxiliary function defined by $y=2\sin^2(\chi\sqrt{\gamma})-1$ and impose the condition

$$\frac{1}{4}\left[\Gamma'-2\zeta(l+1)\right]=n(n+l+\zeta), \tag{31}$$

where n is a non-negative integer. Then the equation (27) takes the form

$$\left(1-y^2\right)\frac{\partial^2S(y)}{\partial y^2}+\left[(l+1-\zeta)-(l+1+\zeta)y\right]\frac{\partial S(y)}{\partial y}$$

$$+\frac{1}{4}\left[\Gamma'-2\zeta\left(l+1\right)\right]S\left(y\right)=0, \tag{32}$$

whose solution can be expressed in terms of Jacobi polynomials as

$$S\left(y\right)=\mathrm{P}_{n}^{\zeta-1,l}\left(y\right). \tag{33}$$

Then the energy eigenvalue of the system can be derived from Eq. (31) as

$$\begin{aligned}E^2 \;=\; & \left(2n^2+2nl+l^2\right)\gamma\hbar^2c^2\left(2m^2w^2+\frac{e^2B^2}{2c^2}\right)\\ & -\hbar c^2mw\left(2+\frac{eB\alpha}{\hbar c}\right)-\hbar lc^2\\ & \times\left[\frac{eB}{c}\left(1+\frac{eB\alpha}{4\hbar c}\right)+\frac{m^2w^2\alpha}{\hbar}-\frac{eB2\hbar\gamma mw}{c}\right]+m^2c^4\\ & +\left(2n+l+1\right)\gamma\hbar^2c^2\left(2m^2w^2+\frac{e^2B^2}{2c^2}\right)\\ & \times\left(1+\sqrt{1-D}\right),\quad n,l=0,1,2,\ldots,\end{aligned} \tag{34}$$

with

$$D=-\left\{l^2+\frac{\left(1+\frac{eB\alpha}{4\hbar c}\right)^2+\frac{m^2w^2\alpha^2}{4\hbar^2}-\hbar\gamma mw\left(2+\frac{eB\alpha}{\hbar c}\right)-\gamma\hbar l\left[\frac{eB}{c}\left(1+\frac{eB\alpha}{4\hbar c}\right)+\frac{m^2w^2\alpha}{\hbar}-\frac{eBm2\hbar\gamma w}{c}\right]}{\gamma^2\hbar^2\left(m^2w^2+\frac{e^2B^2}{4c^2}\right)}\right\}.$$

In view of the obscurity and complexity of Eq. (34), we decide to depict the numerical results aiming to show the effect of the noncommutativity and the minimal length relation in the energy spectra. In Figs. 1 and 2, the energy spectra E versus the magnetic field parameter B for different values of azimuthal quantum number, the noncommutative parameter, and the minimal length parameter are plotted. Positive and negative energy levels correspond to the case of particle and antiparticle, respectively. Figures 1 and 2 show that for the same noncommutative parameter, the energy E increases monotonically with the increase of the minimal length parameter, and for the same minimal length parameter, the energy E also increases monotonically with the increase of the noncommutative parameter. Remarkably, for the same noncommutative parameter and the minimal length parameter in both figures, the energy E increases with the increase of the azimuthal quantum number.

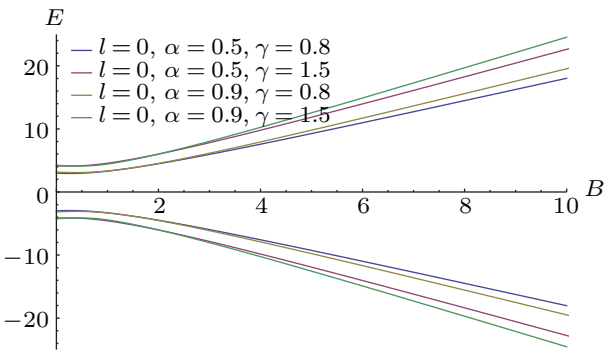


Fig. 1. (color online) The profile of the energy spectra E versus B ($\hbar=c=n=m=w=e=1$).

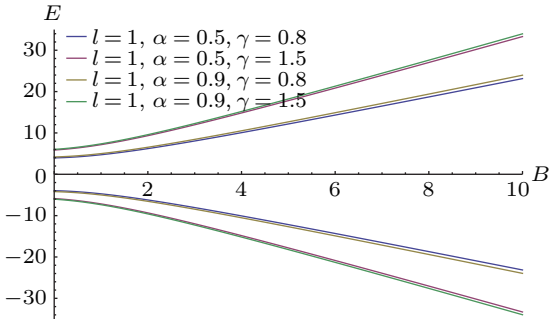


Fig. 2. (color online) The profile of the energy spectra E versus B ($\hbar=c=n=m=w=e=1$).

In addition, we try to quantify the influence of the two parameters on the energy spectrum. For the sake of convenience, simplifying Eq. (34) gives

$$\begin{aligned}& E\left(\alpha,\gamma\right)\\ &= E\left(0,0\right)+\alpha\frac{\partial E\left(0,0\right)}{\partial\alpha}+\gamma\frac{\partial E\left(0,0\right)}{\partial\gamma}\\ &= E\left(0,0\right)\pm\frac{\alpha}{2\sqrt{\left|E\left(0,0\right)\right|}}\left[-cmweB-l\left(\frac{e^2B^2}{4}+m^2w^2c^2\right)\right.\\ &\quad\left.+\left(2n+l+1\right)\frac{ecB\sqrt{m^2w^2+\frac{e^2B^2}{4c^2}}}{2}\right]\\ &\quad\pm\frac{\gamma}{2\sqrt{\left|E\left(0,0\right)\right|}}\left[\left(2n^2+2nl+l^2\right)\hbar^2c^2\left(2m^2w^2+\frac{e^2B^2}{2c^2}\right)\right.\\ &\quad+2mwceBl\hbar^2+\left(2n+l+1\right)\left(2m^2w^2+\frac{e^2B^2}{2c^2}\right)\hbar^2c^2\\ &\quad\left.\times\left(1-\frac{2mwc-leB}{2c\sqrt{m^2w^2+\frac{e^2B^2}{4c^2}}}\right)\right],\end{aligned} \tag{35}$$

with

$$E(0,0)=\pm\sqrt{m^2c^4-2mw\hbar c^2-\hbar lceB+2\hbar c^2(2n+l+1)\sqrt{m^2w^2+\frac{e^2B^2}{4c^2}}}.$$

Obviously, the equation (35) is the result of Taylor's expansion, where the second term and the third term are corrections from the noncommutative parameter and the minimal length parameter on the energy spectrum respectively. With the help of the upper bound,[45–47] after some algebraic operations, we may conclude that the corrections from the noncommutative parameter and minimal length parameter on the energy spectrum are very small. At present, the precision of the corrections has not been detected by experiments.

Next, we discuss the non-relativistic limit of the system. As in the usual case, a Taylor expansion of Eq. (34) turns into

$$E\approx mc^2+E_{P0}-\frac{E_{P0}^2}{2mc^2}, \tag{36}$$

with

$$\begin{aligned}E_{P0}=&\frac{1}{2m}\left\{(2n^2+2nl+l^2)\gamma\hbar^2\left(2m^2w^2+\frac{e^2B^2}{2c^2}\right)\right.\\&-\hbar mw\left(2+\frac{eB\alpha}{\hbar c}\right)\\&-\hbar l\left[\frac{eB}{c}\left(1+\frac{eB\alpha}{4\hbar c}\right)+\frac{m^2w^2\alpha}{\hbar}-\frac{eB2\hbar\gamma mw}{c}\right]\\&\left.+(2n+l+1)\gamma\hbar^2\left(2m^2w^2+\frac{e^2B^2}{2c^2}\right)\left(1+\sqrt{1-D}\right)\right\}.\end{aligned}$$

In Eq. (36), the first term expresses the rest energy of the particle, and the second and third terms represent the energy of the non-relativistic oscillator and the relativistic correction in the presence of both the noncommutative coordinate space and the minimal length for the spin-one DKP oscillator respectively.

Furthermore, considering Eq. (33), the wave function of the system can be expressed as

$$\begin{aligned}\psi_9(p,\varphi)=&C\mathrm{e}^{\mathrm{i}l\varphi}(\gamma P^2)^{\frac{l}{2}}\left(\frac{1}{1+\gamma P^2}\right)^{\frac{\zeta+l}{2}}\mathrm{P}_n^{\zeta-1,l}\left(\frac{\gamma P^2-1}{1+\gamma P^2}\right),\\&n,l=0,1,2,3,4,\ldots.\end{aligned} \tag{37}$$

Therefore the other components can be written as

$$\psi_3=-\frac{E}{mc^2}\psi_9, \tag{38}$$

$$\psi_4=\frac{-\mathrm{i}CA_1}{mc}\mathrm{e}^{\mathrm{i}l\varphi}(\gamma P^2)^{\frac{l}{2}}\left(\frac{1}{1+\gamma P^2}\right)^{\frac{\zeta+l}{2}}, \tag{39}$$

$$\psi_5=\frac{\mathrm{i}CA_2\mathrm{e}^{\mathrm{i}l\varphi}}{mc}(\gamma P^2)^{\frac{l}{2}}\left(\frac{1}{1+\gamma P^2}\right)^{\frac{\zeta+l}{2}}, \tag{40}$$

with

$$\begin{aligned}A_1=&\left\{\left[\left[\left(1+\frac{eB\alpha}{4\hbar c}\right)\sin\varphi-\frac{\mathrm{i}mw\alpha\cos\varphi}{2\hbar}\right.\right.\right.\\&\left.+\frac{eB\mathrm{i}\hbar\gamma\zeta\cos\varphi}{2c}-\hbar mw\gamma\zeta\sin\varphi\right]p\\&+\left[-\frac{eB\mathrm{i}\hbar l\cos\varphi+e\hbar Bl\left(1+\gamma P^2\right)\sin\varphi}{2c}+\hbar mwl\sin\varphi\right.\\&\left.+\mathrm{i}lmw\hbar\left(1+\gamma P^2\right)\cos\varphi\right]\frac{1}{p}\Bigg]\mathrm{P}_n^{\zeta-1,l}\\&+\frac{\left(\hbar mw\sin\varphi-\frac{eB\mathrm{i}\hbar\cos\varphi}{2c}\right)2\gamma p(n+l+\zeta)}{1+\gamma P^2}\mathrm{P}_{n-1}^{\zeta,l+1}\Bigg\},\\A_2=&\left\{\left[\left[\left(1+\frac{eB\alpha}{4\hbar c}\right)\cos\varphi+\frac{\mathrm{i}mw\alpha\sin\varphi}{2\hbar}\right.\right.\right.\\&\left.-\frac{eB\mathrm{i}\hbar\gamma\zeta\sin\varphi}{2c}-\hbar mw\gamma\zeta\cos\varphi\right]p\\&+\left[\frac{eB\mathrm{i}\hbar l\sin\varphi}{2c}-\frac{e\hbar Bl\left(1+\gamma P^2\right)\cos\varphi}{2c}+\hbar mwl\cos\varphi\right.\\&\left.-\mathrm{i}lmw\hbar\left(1+\gamma P^2\right)\sin\varphi\right]\frac{1}{p}\Bigg]\mathrm{P}_n^{\zeta-1,l}\\&+\left(\frac{eB\mathrm{i}\hbar\sin\varphi}{2c}+\hbar mw\cos\varphi\right)\frac{2\gamma p(n+l+\zeta)}{1+\gamma P^2}\mathrm{P}_{n-1}^{\zeta,l+1}\Bigg\}.\end{aligned}$$

Before ending this part, we determine the normalization constant C by the normalization condition

$$\int_0^\infty\frac{\mathrm{d}P}{(1+\gamma P^2)}|\psi(P)|^2=1, \tag{41}$$

thus we have

$$C=c\left(\frac{\gamma m}{E}\right)^{\frac{1}{2}}\left[\frac{n!(2n+\zeta+l)\Gamma(n+\zeta+l)}{\Gamma(n+\zeta)\Gamma(n+l+1)}\right]^{\frac{1}{2}}. \tag{42}$$

3. Discussion

In this section, we discuss some special cases. First, when the minimal length parameter $\gamma\to 0$, the energy relation (34) reduces to

$$\begin{aligned}E^2=&-\hbar c^2mw\left(2+\frac{eB\alpha}{\hbar c}\right)\\&-\left[eBc\hbar l\left(1+\frac{eB\alpha}{4\hbar c}\right)+c^2lmw^2\alpha\right]+m^2c^4\\&+2(2n+l+1)\hbar^2c^2\sqrt{m^2w^2+\frac{e^2B^2}{4c^2}}.\end{aligned} \tag{43}$$

The energy eigenvalues versus B for different values of azimuthal quantum number and the noncommutative parameter are plotted in Fig. 3 and Fig. 4, respectively. Taking the positive energy levels as an example, in Figs. 3 and 4, it shows that for the case of ($\gamma=l=0$) or ($l=1$, $\gamma=0$), the energy E

increases monotonically with the increase of the noncommutative parameter. In addition, we show that the energy E increases monotonically with the increase of the magnetic field parameter B for both cases.

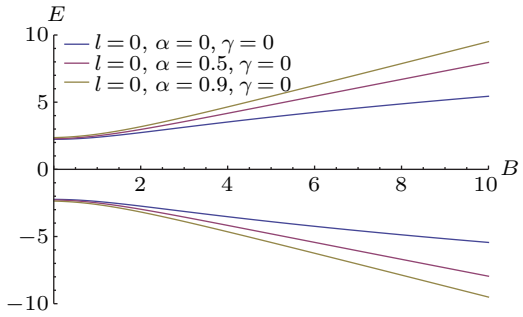


Fig. 3. (color online) The profile of the energy spectra E versus B ($\hbar = c = n = m = w = e = 1$, $\gamma = l = 0$).

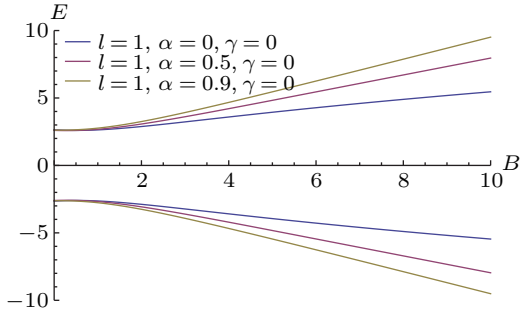


Fig. 4. (color online) The profile of the energy spectra E versus B ($\hbar = c = n = l = m = w = e = 1$, $\gamma = 0$).

Subsequently, when the noncommutative parameter $\alpha \to 0$, the energy relation (34) reduces to

$$\begin{aligned} E^2 \;=\; & \left(2n^2+2nl+l^2\right)\gamma\hbar^2c^2\left(2m^2w^2+\frac{e^2B^2}{2c^2}\right) \\ & -2\hbar c^2mw-\hbar cl\left(eB-eB2\hbar\gamma mw\right) \\ & +m^2c^4+(2n+l+1)\,\gamma\hbar^2c^2\left(2e^2B^2+\frac{e^2B^2}{2c^2}\right) \\ & \times\left(1+\sqrt{1-D'}\right), \end{aligned} \tag{44}$$

with

$$D' = -\left[l^2+\frac{1-2\hbar\gamma mw-\dfrac{\hbar l\gamma eB}{c}\left(1-2\hbar\gamma mw\right)}{\gamma^2\hbar^2\left(m^2w^2+\dfrac{e^2B^2}{4c^2}\right)}\right].$$

The energy eigenvalues versus B for different values of azimuthal quantum number and the minimal length parameter are plotted in Fig. 5 and Fig. 6, respectively. We also take the positive energy levels as an example. In Figs. 5 and 6, it shows that for the case of ($\alpha = l = 0$) or ($l = 1$, $\alpha = 0$), the energy E increases monotonically with the increase of the minimal length parameter. Besides, we find that the energy E also increases monotonically with the increase of the magnetic field parameter B for both cases.

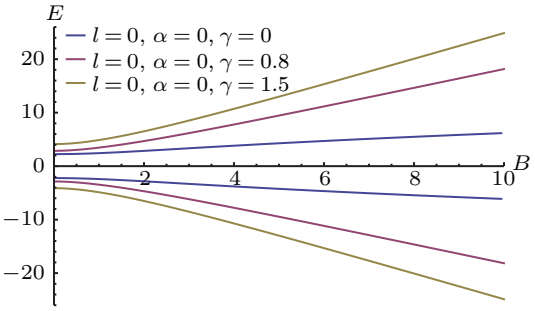


Fig. 5. (color online) The profile of the energy spectra E versus B ($\hbar = c = n = m = w = e = 1$, $\alpha = l = 0$).

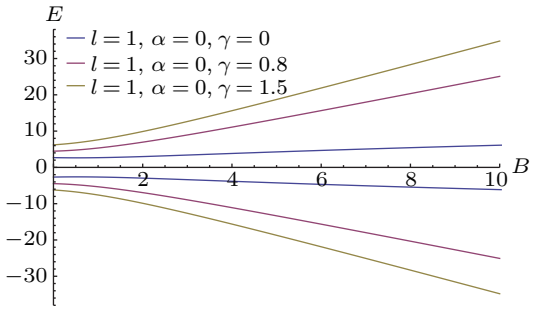


Fig. 6. (color online) The profile of the energy spectra E versus B ($\hbar = c = n = l = m = w = e = 1$, $\alpha = 0$).

Finally, when both the noncommutative and minimal length parameters are absent, i.e. $\alpha = \gamma = 0$, the energy relation (34) reduces to

$$\begin{aligned} E^2 \;=\; & m^2c^4-2mw\hbar c^2-\hbar lceB \\ & +2\hbar c^2\left(2n+l+1\right)\sqrt{m^2w^2+\frac{e^2B^2}{4c^2}}, \end{aligned} \tag{45}$$

obviously which is strictly consistent with the result from Boumali *et al.*[41]

4. Conclusion

This paper was devoted to the study of the noncommutative coordinate space spin-one DKP oscillator with minimal length. We first analyzed the DKP oscillator under a magnetic field in the presence of a minimal length in the noncommutative coordinate space and obtained energy spectrum as well as corresponding spinor ψ in terms of the Jacobi polynomials. Subsequently, the energy spectrum of the system is plotted, which shows that for the same azimuthal quantum number, the energy E increases monotonically with the increase of the noncommutative parameter and the minimal length parameter. Besides, the non-relativistic limit of the system and the numerical result of the correction for the energy spectrum are also reported. Finally, we analyzed some special cases in order

to discuss the effect of the noncommutative coordinate space and the minimal length in the energy spectrum, and the corresponding numerical results are depicted respectively. It shows that for the same azimuthal quantum number, the energy E increases monotonically with the increase of the noncommutative parameter or the minimal length parameter, and for both cases, the energy E increases monotonically with the increase of the magnetic field parameter B.

References

[1] Duffin R J 1938 *Phys. Rev.* **54** 1114
[2] Kemmer N 1938 *Proc. R. Soc. A* **166** 127
[3] Petiau G 1936 *Acad. R. Belg. A. Sci. Mém. Collect.* **16** 2
[4] Unal N 2005 *Concepts Phys.* **2** 273
[5] Castro L B and de Castro A S 2014 *Phys. Rev. A* **90** 022101
[6] Nedjadi Y and Barrett R C 1993 *J. Phys. G-Nucl. Part. Phys.* **19** 87
[7] Nedjadi Y and Barrett R C 1994 *J. Math. Phys.* **35** 4517
[8] Gribov V N 1999 *Eur. Phys. J. C* **10** 71
[9] Keer L K, Clark B C, Hama S, Ray L and Hoffmann G W 1998 *Prog. Theor. Phys.* **103** 321
[10] Kanatchikov I V 2000 *Rep. Math. Phys.* **46** 107
[11] Boumali A 2008 *J. Math. Phys.* **49** 022302
[12] de Montigny M 2000 *J. Phys. A* **33** L273
[13] Boumali A 2004 *Can. J. Phys.* **82** 67
[14] Boumali A 2007 *Can. J. Phys.* **85** 1417
[15] Snyder H S 1947 *Phys. Rev.* **71** 38
[16] Connes A 1994 *Noncommutative Geometry* (San Diego: Academic Press)
[17] Djemai A E F 2004 *Int. J. Theor. Phys.* **43** 299
[18] Gamboa J, Loewe M, Mendez F and Rojas J C 2001 *Phys. Rev. D* **64** 067901
[19] Jing J, Liu F H and Chen J F 2008 *Europhys. Lett.* **84** 61001
[20] Kijanka A and Kosinski P 2004 *Phys. Rev. D* **70** 127702
[21] Muthukumar B and Mitra P 2002 *Phys. Rev. D* **66** 027701
[22] Jing J, Zhao S H, Chen J F and Long Z W 2008 *Eur. Phys. J. C* **544** 685
[23] Romero J M, Santiago J A and Vergara J D 2003 *Phys. Lett. A* **310** 9
[24] Daszkiewicz M and Walczyk C J 2008 *Phys. Rev. D* **77** 105008
[25] Das A, Falomir H, Nieto M, Gamboa J and Méndez F 2011 *Phys. Rev. D* **84** 045002
[26] Wang B Q, Long C Y, Long Z W and Xu T 2016 *Eur. Phys. J. Plus* **131** 378
[27] Wang L Z, Long C Y and Long Z W 2015 *Commun. Theor. Phys.* **63** 709
[28] Tawfik A and Diab A 2014 *Int. J. Mod. Phys. D* **23** 161
[29] Majumder B 2012 *Phys. Lett. B* **709** 133
[30] Chargui Y Chetouani L and Trabelsi A 2010 *Chin. Phys. B* **19** 43
[31] Hassanabadi H, Molaee Z and Zarrinkamar S 2014 *Adv. High Energy Phys.* **6** 459345
[32] Boumali A and Hassanabadi H 2015 *Z. Naturforsch. A* **70** 619
[33] Ikot A N, Obong H P and Hassanabadi H 2015 *Chin. Phys. Lett.* **32** 30201
[34] Lunardi J T, Pimentel B M, Teixeira R G and Valverde J S 2000 *Phys. Lett. A* **268** 165
[35] Nowakowski M 1998 *Phys. Lett. A* **244** 329
[36] Pimentel B M and Fainberg V Y 2000 *Phys. Lett. A* **271** 16
[37] Boztosun I, Karakoc M, Yasuk F and Durmus A 2006 *J. Math. Phys.* **47** 062301
[38] Castro A S D 2011 *J. Phys. A* **44** 035201
[39] Hassanabadi H, Forouhandeh S F, Rahimov H, Zarrinkamar S and Yazarloo B H 2012 *Can. J. Phys.* **90** 299
[40] Wang B Q, Long C Y, Long Z W and Wu S R 2017 *Adv. High Energy Phys.* **2017** 2843020
[41] Boumali A, Chetouani L and Hassanabadi H 2013 *Can. J. Phys.* **91** 1
[42] Hassanabadi S and Ghominejad M 2014 *Adv. High Energy Phys.* **2014** 185169
[43] Hassanabadi H, Molaee Z, Zarrinkamar S and Ghominejad M 2014 *Phys. Part. Nucl. Lett.* **10** 699
[44] Wu S R, Long Z W, Long C Y, Wang B Q and Liu Y 2017 *Eur. Phys. J. Plus* **132** 186
[45] Pedram P 2013 *Eur. Phys. J. C* **73** 2609
[46] Ma K, Ren Y J and Wang Y H 2017 arXiv:1703.10923 [hep-th]
[47] Ma K 2017 arXiv:1705.05789 [hep-th]