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Finite-time robust control of uncertain fractional-order Hopfield neural networks via sliding mode control*

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The finite-time control of uncertain fractional-order Hopfield neural networks is investigated in this paper. A switched terminal sliding surface is proposed for a class of uncertain fractional-order Hopfield neural networks. Then a robust control law is designed to ensure the occurrence of the sliding motion for stabilization of the fractional-order Hopfield neural networks. Besides, for the unknown parameters of the fractional-order Hopfield neural networks, some estimations are made. Based on the fractional-order Lyapunov theory, the finite-time stability of the sliding surface to origin is proved well. Finally, a typical example of three-dimensional uncertain fractional-order Hopfield neural networks is employed to demonstrate the validity of the proposed method.

Keywords: fractional-order neural networks, finite-time, sliding mode control, parameters estimation

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1. Introduction

Neural networks have a long development history and have received more and more attention recently.[1–4] Neural networks have so many successful applications, ranging from image processing, combinatorial optimization, associative memories, pattern recognition, and many other fields. Fractional calculus is an extension of integer calculus, which can be extended to arbitrary order. Fractional calculus is better than integer one in many fields. Fractional calculus provides an excellent tool for us to describe hereditary and memory properties of kinds of materials and processes. It is natural for us to take advantage of this process to study neural networks.[5–8] There are two main reasons that the fractional-order formulation of neural network models are used to introduce biological neurons. Firstly the fractional-order parameter enlarges the system performance by heightening one degree of freedom, and secondly the fractional-order formulation of neural networks has infinite memory. Fractional-order neural networks are hoped to play an important role in the field of parameter estimation. Neural networks approximation showed higher rates of approximation in fractional level.[9–13] Taking these facts into consideration, it is necessary for us to study fractional-order neural networks and combine their infinite memory term into the model of neural networks.

Recently, the dynamics analysis of fractional-order neural networks has become a very hot research topic, and some important results have been gained. For example, the bifurcations, stability, and chaos of fractional-order neural networks of Hopfield type were studied.[14–18] Fractional-order Hopfield neural networks and their stability were studied through energy-like function analysis.[19–24] Control and synchronization have become a hot topic for nonlinear systems.[25–28] The LMI technique for the stability region was proposed.[29,30] However, there are few results about the finite-time control of fractional-order Hopfield neural networks with unknown parameters.

Fractional-order Hopfield neural networks belong to a special form of fractional-order nonlinear systems. It is important to establish some control criteria for fractional-order neural networks. The sliding mode control (SMC) will be used to analyze conditions for finite-time robust control of fractional-order Hopfield neural networks with unknown parameters. SMC is a robust variable structure control method, which is suitable for nonlinear complex uncertain systems. SMC has several useful features, such as fast response, low sensitivity to external noises, easy realization, and parameters unknown. It has been studied for controlling the nonlinear dynamic systems, especially for uncertain systems.[31,32] The low sensitivity to system parameter variations and external noises were shown.[33,34]

Motivated by the above discussion, firstly a switched sliding mode surface is proposed. Secondly, a robust controller is established. Based on fractional-order Lyapunov theory, finite-time robust control is achieved for the fractional-order Hopfield neural networks with unknown parameters. Besides, for the unknown parameters, some estimations are made.

2. Preliminaries

In this section, several basic definitions, lemmas, and properties will be given. In this paper, ${}_{t_0}D_t^{\alpha}$ denotes the

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Riemann–Liouville fractional-order calculus.

Definition 1[35] **Riemann–Liouville fractional-order integration.** The Riemann–Liouville fractional-order integration of order $\alpha > 0$ for a function $f(t)$ is defined as

$$ {}_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} \mathrm{d}\tau, \tag{1} $$

where $\Gamma(\cdot)$ denotes the Gamma function and $\alpha \in \boldsymbol{R}^+$.

Definition 2[35] **Riemann–Liouville fractional derivative.** The Riemann–Liouville fractional derivative of order $\alpha > 0$ of a continuous function $f(t)$ is defined as the n-th derivative of fractional integral (1) of order $n-\alpha$:

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} \mathrm{d}\tau, \tag{2} $$

where n is the smallest integer larger than or equal to α, and $\Gamma(.)$ denotes the Gamma function.

Property 1[35] For the Riemann–Liouville derivative, we have

$$ \begin{aligned} &{}_{t_0}D_t^{-\alpha}({}_{t_0}D_t^{\beta} f(t)) \\ &= {}_{t_0}D_t^{\beta-\alpha} f(t) - \sum_{j=1}^{m} [{}_{t_0}D_t^{\beta-j} f(t)]_{|t=t_0} \frac{(t-t_0)^{\alpha-j}}{\Gamma(1+\alpha-j)}, \\ &m-1 \le \beta < m. \end{aligned} \tag{3} $$

Lemma 1[36] For an integrable function $f(t)$, if there is at least one $t_1 \in (0,t)$ such that $f(t_1) \neq 0$, there is a positive constant L such that ${}_{t_0}D_t^{-\alpha}|f(t)| \ge L$.

Lemma 2[37] Let $x(t) \in \boldsymbol{R}$ be a continuous and differentiable function. For any time instant $t \ge t_0$, $\forall \alpha \in (0,1)$, it has

$$ |\sum_{k=1}^{\infty} \frac{\Gamma(1+\alpha)}{\Gamma(1+k)\Gamma(1-k+\alpha)} {}_{t_0}D_t^{k} x_i \, {}_{t_0}D_t^{\alpha-k} x_i| \le \delta_i |x_i|, \tag{4} $$

where δ_i is a positive constant.

Consider the non-autonomous fractional-order system

$$ \begin{cases} {}_{t_0}D_t^{\alpha} x(t) = f(x,t), \\ x(t_0) = x_{t_0}, \end{cases} \tag{5} $$

where $f(x,t)$ satisfies the Lipschitz condition with Lipschitz constant $l > 0$ and $\alpha \in (0,1)$, $x \in \boldsymbol{R}^n$, and let $x = 0$ be an equilibrium point of the system.

Definition 3[38] (Mittag–Leffler stability) If $\bar{x} = 0$ is an equilibrium point of system (5), the solution of system (5) is said to be Mittag–Leffler stable if

$$ \|x(t)\| \le [m(x_0)(t-t_0)^{-\gamma} E_{\alpha}(-\lambda(t-t_0)^{\alpha})]^b, \tag{6} $$

where $\lambda > 0$, $b > 0$, $m(0) = 0$, $m(x) \ge 0$ and $m(x)$ satisfies locally Lipschitz condition on $x \in \boldsymbol{R}^n$ with Lipschitz constant m_0, and $E_{\alpha}(z) = \Sigma_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}$ is the Mittag–Leffler function.

Lemma 3[39] For $t_0 = 0$, let $x^* = 0$ be an equilibrium point for the fractional-order system (5). $x^* = 0$ is Mittag–Leffler stable, if there exists a continuous function $V(t,x(t))$ satisfies

$$ \begin{aligned} &\alpha_1 \|x(t)\|^a \le V(t,x(t)) \le \alpha_2 \|x(t)\|^{ab}, \\ &{}_{t_0}D_t^{\beta} V(t,x(t)) \le -\alpha_3 \|x(t)\|^{ab}, \end{aligned} \tag{7} $$

where $V(t,x(t)) : [0,\infty) \times D \to \boldsymbol{R}$ satisfies locally Lipschitz condition on x, $V(t,x(t))$ is a continuously differentiable function, $D \in \boldsymbol{R}^n$ is a domain containing the origin, $t \ge 0, \beta \in (0,1), \alpha_1, \alpha_2, \alpha_3, a$, and b are arbitrary positive constants.

3. Main results

The fractional-order Hopfield neural networks with constant external inputs and controller are described by

$$ {}_{t_0}D_t^{\alpha} x_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) + I_i + u_i(t), \tag{8} $$

where $i = 1,2,\ldots,n$, n is the number of units in a neural network, $x_i(t)$ is the state of the ith unit at time t, f_j denotes the activation function of the jth neuron, b_{ij} denotes the constant connection weight of the jth neuron on the ith neuron, $a_i > 0$ represents the rate with which the ith neuron resets its potential to the resting state when disconnected from the network and $a_i > 0$ is unknown, I_i denotes the constant external inputs, and $u_i(t)$ is the control input.

Assumption 1 There exists an equilibrium point of system (8), which is zero, i.e, $x^* = 0$.

3.1. Controller design

In general, it can be divided into two steps by designing the process of sliding mode control. Firstly, select an appropriate sliding surface which represents the required system dynamic characteristics. In this paper, a switching fractional-order sliding surface is proposed as

$$ s_i(t) = {}_{t_0}D_t^{\alpha-1} x_i + {}_{t_0}D_t^{-1}(p_i x_i + q_i \tanh(x_i)), \tag{9} $$

where $i = 1,2,\ldots,n$, x_i is the system states, and p_i and q_i are positive constants. $\tanh(\cdot)$ is the hyperbolic tangent function.

According to the sliding mode control theory, when the system operates in sliding mode surface, the sliding surface and its derivative must satisfy

$$ s_i(t) = 0, \quad \dot{s}_i(t) = 0. \tag{10} $$

As a result, considering Eqs. (9) and (10), one obtains

$$ \dot{s}_i(t) = {}_{t_0}D_t^{\alpha} x_i + p_i x_i + q_i \tanh(x_i) = 0. \tag{11} $$

Then, one gets

$$ {}_{t_0}D_t^{\alpha} x_i = -(p_i x_i + q_i \tanh(x_i)). \tag{12} $$

Once we establish an appropriate sliding surface, the next part is to construct an input signal $u_i(t)$ to guarantee that the

state trajectories reach the sliding surface $s_i(t)=0$ and stay on it forever according to the sliding mode method. The sliding mode control law is presented as

$$\begin{aligned} u_i(t) &= -((-\hat{a}_i+p_i)x_i+\sum_{j=1}^{n}b_{ij}f_i(x)+I_i \\ &\quad +q_i\tanh(x_i)+\zeta_i^a s_i+\zeta_i^b\tanh(s_i))), \end{aligned} \tag{13}$$

where $\zeta_i^a>0$ and $\zeta_i^b>0$ are constant gains. $\hat{a}_i$ is the estimation of a_i. The unknown parameters a_i are estimated as

$$\dot{\hat{a}}_i=-\eta_i s_i x_i, \tag{14}$$

where $i=1,2,\ldots,n$ and $\eta_i<0$ are adaptation gains.

In order to get the results, we have two steps to complete the proof. Firstly, we need to control the system trajectories to reach the sliding surface $s_i(t)=0$, which is shown in the Theorem 1. After that, when the system operates in sliding mode surface, we should prove the stability of the system and convergence to zero in finite time, which is shown in Theorem 2.

Theorem 1 Consider the fractional-order Hopfield neural networks (8). If this system is controlled by the control signal obtained by Eq. (13), the system trajectories will converge to the sliding surface $s_i(t)=0$.

Proof Choose the positive definite Lyapunov function candidate

$$V_i(t)=\frac{1}{2}s_i^2+\frac{1}{2\eta_i}(\hat{a}_i-a_i)^2. \tag{15}$$

Taking the integer-order derivative of $V(t)$, one has

$$\dot{V}_i=s_i\dot{s}_i+\tfrac{1}{\eta_i}(\hat{a}_i-a_i)\dot{\hat{a}}_i. \tag{16}$$

Inserting $\dot{s}_i(t)$ from Eq. (11) into the above equation, one obtains

$$\begin{aligned} \dot{V}_i &= s_i({}_{t_0}D_t^{\alpha}x_i+p_ix_i+q_i\mathrm{sign}(x_i)) \\ &\quad +\frac{1}{\eta_i}(\hat{a}_i-a_i)\dot{\hat{a}}_i. \end{aligned} \tag{17}$$

Based on ${}_{t_0}D_t^{\alpha}x_i=-a_ix_i(t)+\sum_{j=1}^{n}b_{ij}f_j(x_j(t))+I_i+u_i(t)$, it has

$$\begin{aligned} \dot{V}_i &= s_i(-a_ix_i(t)+\sum_{j=1}^{n}b_{ij}f_j(x_j(t)) \\ &\quad +I_i+u_i(t)+p_ix_i+q_i\tanh(x_i)) \\ &\quad +\frac{1}{\eta_i}(\hat{a}_i-a_i)\dot{\hat{a}}_i. \end{aligned} \tag{18}$$

Substituting $u_i(t)$ from Eq. (13) into Eq. (18), it yields

$$\begin{aligned} \dot{V}_i &= s_i(-a_ix_i(t)+\sum_{j=1}^{n}b_{ij}f_j(x_j(t))+I_i+p_ix_i+q_i\tanh(x_i) \\ &\quad -((-\hat{a}_i+p_i)x_i+\sum_{j=1}^{n}b_{ij}f_i(x)+I_i+q_i\tanh(x_i) \\ &\quad +\zeta_i^a s_i+\zeta_i^b\tanh(s_i)))+\frac{1}{\eta_i}(\hat{a}_i-a_i)\dot{\hat{a}}_i. \end{aligned} \tag{19}$$

Then, we get

$$\begin{aligned} \dot{V}_i &= s_i((\hat{a}_i-a_i)x_i(t)-\zeta_i^a s_i-\zeta_i^b\tanh(s_i))) \\ &\quad +\frac{1}{\eta_i}(\hat{a}_i-a_i)\dot{\hat{a}}_i. \end{aligned} \tag{20}$$

Inserting $\dot{\hat{a}}_i=-\eta_i s_i x_i$ and using $\tanh(s_i)s_i\le|s_i|$, one obtains

$$\dot{V}_i=-\zeta_i^a s_i^2-\zeta_i^b|s_i|\le-\zeta_i^b|s_i|, \tag{21}$$

where $\zeta_i^b>0$. Therefore, based on Lyapunov theory, the system states will converge to the sliding surface $s_i(t)=0$. Hence, the proof is achieved completely.

Theorem 2 Consider the sliding mode dynamics (12). This system is stable and its states converge to zero in finite time.

Proof Choose the positive definite Lyapunov function

$$V_i(t)=\frac{1}{2}x_i^2. \tag{22}$$

Taking the fractional-order derivative of $V_i(t)$ and using Lemma 2, one has

$$\begin{aligned} &{}_{t_0}D_t^{\alpha}V_i(t) \\ &= x_i\,{}_{t_0}D_t^{\alpha}x_i \\ &\quad +\sum_{k=1}^{\infty}\frac{\Gamma(1+\alpha)}{\Gamma(1+k)\Gamma(1-k+\alpha)}\,{}_{t_0}D_t^{k}x_i\,{}_{t_0}D_t^{\alpha-k}x_i \\ &\le x_i\,{}_{t_0}D_t^{\alpha}x_i \\ &\quad +\left|\sum_{k=1}^{\infty}\frac{\Gamma(1+\alpha)}{\Gamma(1+k)\Gamma(1-k+\alpha)}\,{}_{t_0}D_t^{k}x_i\,{}_{t_0}D_t^{\alpha-k}x_i\right| \\ &\le x_i(-(p_ix_i+q_i\tanh(x_i)))+\delta_i|x_i|. \end{aligned} \tag{23}$$

Using the $\tanh(x_i)x_i\le|x_i|$, we have

$${}_{t_0}D_t^{\alpha}V_i(t)\le-(p_ix_i^2+q_i|x_i|)+\delta_i|x_i|. \tag{24}$$

Then

$${}_{t_0}D_t^{\alpha}V_i\le-(q_i-\delta_i)|x_i|, \tag{25}$$

where δ_i is set as $q_i-\delta_i>0$. As a result, based on Lemma 3, the system states will converge to $x_i(t)=0$ asymptotically. In the following, we will prove that the convergence to the origin happens in finite time.

Taking the fractional integral of both sides of Eq. (25) from the reaching time t_i^{r} to the settling time t_i^s and using Property 1, one obtains

$$\begin{aligned} &V_i(t_i^s)-V_i^{(\alpha-1)}(t_i^{\mathrm{r}})\frac{(t_i^s-t_i^{\mathrm{r}})^{\alpha-1}}{\Gamma(\alpha)} \\ &\le -(q_i-\delta_i)\,{}_{t_0}D_t^{-\alpha}|x_i|. \end{aligned} \tag{26}$$

Based on Lemma 1 and ${}_{t_0}D_t^{-\alpha}|x_i| \geq L_i$, since $x_i(t_i^s) = 0$, one gets

$$-V_i^{(\alpha-1)}(t_i^{\mathrm{r}})\frac{(t_i^s - t_i^{\mathrm{r}})^{\alpha-1}}{\Gamma(\alpha)} \leq -(q_i - \delta_i)L_i. \tag{27}$$

After some simple manipulations, one has

$$(t_i^s) \leq \left(\frac{V_i^{\alpha-1}(t_i^{\mathrm{r}})}{(q_i - \delta_i)L_i\Gamma(\alpha)}\right)^{\frac{1}{1-\alpha}} + t_i^{\mathrm{r}}, \tag{28}$$

where t_i^{r} is the time for reaching sliding surface.

Therefore, the state trajectories of the system (12) will converge to $x_i = 0$ in finite time. This completes the proof.

4. Numerical simulations

In this section, the effectiveness of the obtained theoretical results is showed by the the following example.

Consider the three-dimensional fractional-order Hopfield neural networks

$$\begin{cases} {}_{t_0}D_t^{\alpha}x_1 = -a_1x_1 + 3\sin x_1 + 3\sin x_2 - 2\cos x_3 + 2, \\ {}_{t_0}D_t^{\alpha}x_2 = -a_2x_2 - 4\sin x_1 - 0.8\cos x_2 + 3\sin x_3 + 0.8, \\ {}_{t_0}D_t^{\alpha}x_3 = -a_3x_3 + 2\sin x_1 - 2.6\cos x_2 + 3.5\sin x_3 + 2.6. \end{cases} \tag{29}$$

Let the initial conditions be $x_1(0) = -4, x_2(0) = 4, x_3(0) = -3.7$, and the fractional order is $\alpha = 0.95$. Obviously, $[0,0,0]^{\mathrm{T}}$ is an equilibrium point of system (29).

Choosing $\eta_1 = -0.3, \eta_2 = -0.3$, and $\eta_3 = -0.3$, one gets

$$\begin{cases} \dot{\hat{a}}_1 = 0.3x_1s_1, \\ \dot{\hat{a}}_2 = 0.5x_2s_2, \\ \dot{\hat{a}}_3 = 0.2x_3s_3. \end{cases} \tag{30}$$

We use Eq. (11) and design the sliding surfaces

$$\begin{cases} s_1 = D^{-0.05}x_i + D^{-1}(x_i + 1.9\tanh(x_i)), \\ s_2 = D^{-0.05}x_i + D^{-1}(1.5x_i + 2.7\tanh(x_i)), \\ s_3 = D^{-0.05}x_i + D^{-1}(2.2x_i + 0.8\tanh(x_i)). \end{cases} \tag{31}$$

Thus, according to Eq. (13), the control input are obtained as

$$\begin{cases} u_1 = (\hat{a}_1 - 1)x_1 - 3\sin x_1 - 3\sin x_2 + 2\cos x_3 \\ \qquad -2 - 1.9\tanh(x_1) - s_1 - \tanh(s_1), \\ u_2 = (\hat{a}_2 - 1.5)x_2 + 4\sin x_1 + 0.8\cos x_2 - 3\sin x_3 \\ \qquad -0.8 - 2.7\tanh(x_2) - 2s_2 - 2\tanh(s_2), \\ u_3 = (\hat{a}_3 - 2.2)x_3 - 2\sin x_1 + 2.6\cos x_2 - 3.5\sin x_3 \\ \qquad -2.6 - 0.8\tanh(x_3) - 3s_3 - 3\tanh(s_3). \end{cases} \tag{32}$$

The state trajectories of the controlled fractional-order Hopfield neural networks are shown in Fig. 1. We can see that the states converge to zero in a given finite time. The sliding surface applied in the Hopfield fractional-order neural networks in Fig. 2 shows the validity of the proposed switched sliding surface. Figures 3–5 present the estimations of unknown parameters a_i $(i = 1,2,3)$. Obviously, the control inputs are feasible in practical applications.

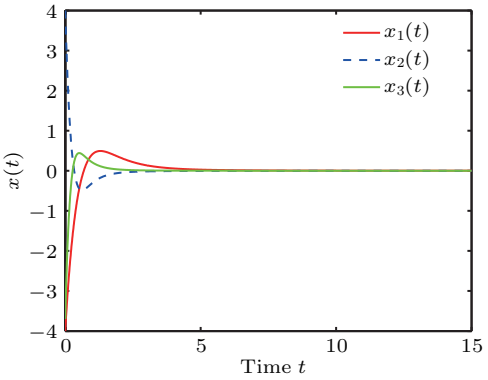


Fig. 1. (color online) Convergence of the solution $x(t)$ for Hopfield fractional-order neural networks (29).

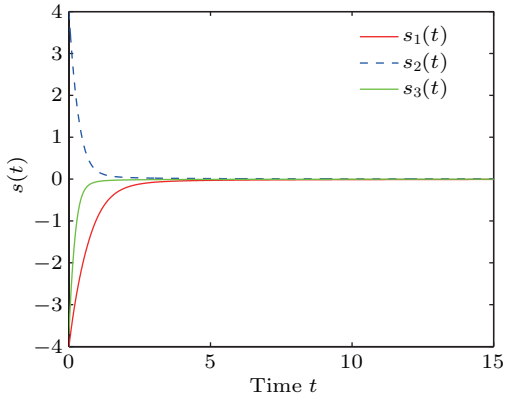


Fig. 2. (color online) The sliding surface applied in the Hopfield fractional-order neural networks (29).

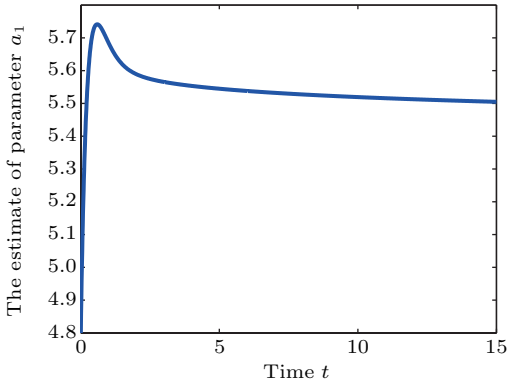


Fig. 3. (color online) The estimation of the unknown parameter a_1 for the Hopfield fractional-order neural networks (29).

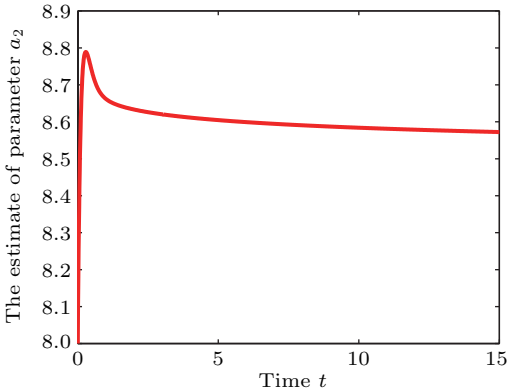


Fig. 4. (color online) The estimation of the unknown parameter a_2 for the Hopfield fractional-order neural networks (29).

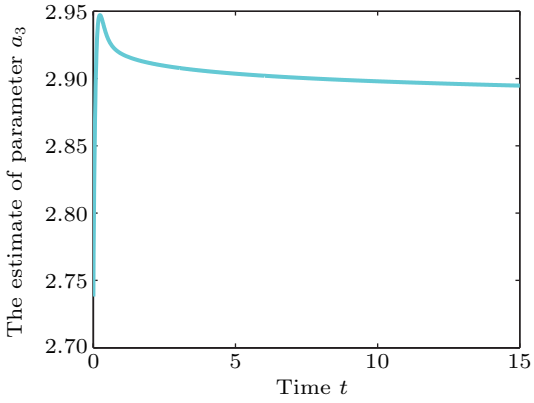


Fig. 5. (color online) The estimation of the unknown parameter a_3 for the Hopfield fractional-order neural networks (29).

5. Conclusion

We investigate the finite-time robust control of uncertain fractional-order Hopfield neural networks using sliding mode control. Some effective conditions are derived based on sliding mode control theory. Firstly, establishing a switched sliding surface, the finite-time stability of the sliding surface to origin is proved base on the fractional-order Lyapunov theory. Then, a robust controller is designed to force the system trajectories to reach the switching surface and remain on it forever. For the unknown parameters, some estimations are made. The effectiveness and feasibility of theoretical results are verified by the numerical simulations. Furthermore, taking advantage of the sliding mode control theory, the stability of fractional-order Hopfield neural networks with time-delay will be discussed in future works.

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