Collective motion of active particles in environmental noise*

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(Received 26 April 2017; revised manuscript received 16 May 2017; published online 18 July 2017)

We study the collective motion of active particles in environmental noise, where the environmental noise is caused by noise particles randomly diffusing in two-dimensional space. We show that active particles in a noisy environment can self organize into three typical phases: polar liquid, band, and disordered gas states. In our model, the transition between band and disordered gas states is discontinuous. Giant number fluctuation is observed in the polar liquid phase. We also compare our results with the Vicsek model and show that the interaction with noise particles can stabilize the band state to very low noise condition. This band structure could recruit most of the active particles in the system, which greatly enhances the coherence of the system. Our findings of complex collective behaviors in environmental noise help us to understand how individuals modify their self-organization by environmental factors, which may further contribute to improving the design of collective migration and navigation strategies.

Keywords: active matter, soft matter, self-organization

PACS: 89.75.Fb, 87.18.Tt **DOI:** 10.1088/1674-1056/26/9/098903

1. Introduction

Collective behavior of active matter has attracted many physicists' attention recently. It displays various fascinating patterns at every scale, down to molecular motors in the cell, up to large animal groups.[1] For example, actin filaments perform persistent random walk, wave-like structures, spirals and swirls,[2–5] the bacillus subtilis grows a peculiar concentric ring-like pattern,[6,7] E. coli in the lab self organizes into a highly ordered phase through growth and division in a dense colony.[8] Similarly, large living organisms such as locusts perform a disorder to order transition,[9] fish schools and bird flocks provide some complex patterns such as travelling band, milling, and cluster.[10–13] The reason why active matters form evolutionary patterns remains unclear. One possible explanation would be that there exists some inherent benefit to overcome environmental perturbations or other distractions.[14–17] However, there is scarce empirical information about the precise interaction rules between the components of active matters because of the technological difficulties.[18] Physicists are trying to find a minimal model to study these unexpected collective properties.[19,20]

Vicsek and collaborators provided a metric interaction self-propelled particles model that exhibits a disorder to order phase transition.[21] In this model, particles move in a constant speed and interact locally with their neighbors within a certain radius to keep alignment with the group members. It is interesting that a collective pattern occurs with large particle density and small noise intensity. Some theoretical description of the dynamics of the flocking behavior for self-propelled particles was proposed by Toner and Tu.[22,23] Beyond the previous investigation on the nature of the original Vicsek model, people proposed many variants to describe other kinds of active matter systems. Most of these models only change the angular interaction rules with their neighbors, and usually these models can exhibit spectacular collective behaviors that are reminiscent to fascinating dynamic patterns.[24–33] Indeed, the collective behavior of organisms is responsive to two kinds of interactions: social interaction with their neighbors and interactions with the surrounding environment.[14,15] However, most previous studies in literature mainly focus on the collective motion in response to nearby neighbors, less concentrating on the environmental factor. Individuals may adopt appropriate moving patterns that facilitate group motion in an environmentally dependent way.[15] For example, under environmental stimuli or threat, bird flocks would align more strongly with their neighbors to keep cohesion.[14] Fish schools form a large size group in the alarm treatment and a small one in the food treatment.[34] Bacteria in colonies forms various patterns on artificial surfaces.[35] In particular, E. coli. employs run and tumble locomotion upon environmental stimuli.[36–38] In summary, environmental factors have a great effect on individual and collective behaviors.

In this paper, we introduce a Vicsek-like model that contains two kinds of particles to investigate the collective motion of the system: active particles that keep alignment with their neighbors and the noise particles randomly diffusing in two dimensional space that lead to the environmental noise introduced here. We study the generic phase behavior of the active particles interacting with the noise particles, in competition with inherent noise and alignment interaction. We show that,

*Project supported by the National Natural Science Foundation of China (Grant Nos. 91427302, 91027040, and 11474155) and the National Basic Research Program of China (Grant No. 2012CB821500).

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under a noisy environment, the system exhibits three typical phases: polar liquid, band and disordered gas states. We also compare our results with the original Vicsek model. In our system, the band state can exist even in a relative low noise region. This effect could help to increase the spatial coherence in collective motion.

2. Methods and models

We consider a modified version of the Vicsek model for N_a active particles moving off lattice in a two-dimensional space of linear size L, with periodic boundary conditions, interacting via polar alignment with their neighbors[21] in competition with environmental noise. The noise intensity is proportional to the local density of noise particles. The form of environmental noise in the update rule is analogous to the vector noise introduced in Ref. [39] and inherent scalar noise is also considered. We express the evolution of the j-th particle according to

$$\theta_j^{t+1} = \arg\left(\Sigma_k e^{i\theta_k^j} + \gamma n_n^t e^{i\varepsilon_n^t}\right) + \eta\varepsilon_j^t, \quad (1)$$

$$r_j^{t+1} = r_j^t + v e^{i\theta_k^{t+1}}, \quad (2)$$

where r_j^t and θ_j^t correspond to the position and orientation of the j-th particle respectively. The sum Σ denotes the neighbors of the j-th particle within a certain radius R, γ is the strength of environmental noise introduced by noise particles, n_n^t represents the current number of noise particles that are located within a radius of R from r_j^t. If no noise particle is within this range, it returns a null vector. η is the strength of the noise in the interval [0,1]. η =1 means that the moving direction is totally controlled by the noise, while $\eta = 0$ represents the system without noise. The terms ε_n^t and ε_j^t correspond to the environmental noise and inherent noise respectively, uniformly distributed in $[-\pi, \pi]$, with $\langle\varepsilon_i^t\rangle = 0$ and $\langle\varepsilon_i^t\varepsilon_j^{t'}\rangle = \delta_{i,j}\delta(t-t')$. In Eq. (2), v is a constant moving speed of the active particle. For instance, if $\gamma = 0$, our model reduces to the original Vicsek model. There is another scenario with $\eta = 0$, which corresponds to the Vicsek model with vector-form environmental noise. Finally, we regard the noise particle as a Brownian particle with constant speed v but randomly orientated in discrete time. Our model has two main control parameters: the active particle density $\rho_a = N_a/L^2$ and the noise particle density $\rho_n = N_n/L^2$, where N_a and N_n correspond to the total active particles' number and noise particles' number, respectively.

3. Results

Here, we mostly report on the system with $v = 0.5$, $\rho_a = N_a/L^2 = 1$, $R = 1$, $\gamma = 5$, and time interval $\Delta t = 1$. To characterize the global degree of orientational order, we consider the following order parameter, which is defined as $\varphi = \langle\varphi(t)\rangle = \frac{1}{N_a}|\Sigma_i e^{i\theta(t)}|$, where N_a is the total number of active particles in the system.

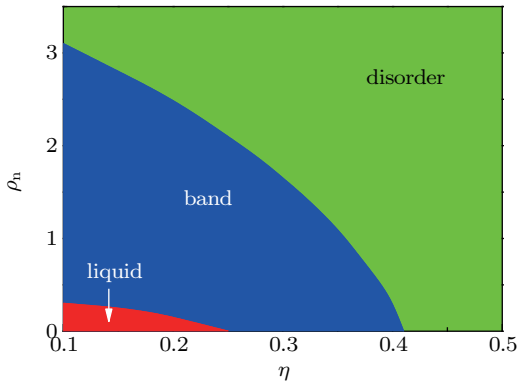


Fig. 1. (color online) Phase diagram in the $(\rho_n\eta)$ plane. The red region corresponds to the polar liquid state. The blue region corresponds to the band state. The green region corresponds to the disordered gas state.

As we change the noise level η and noise particles' density ρ_n, we find three typical phases as shown in Fig. 1: the polar ordered liquid state with no band structures in the small η and small ρ_n region (Fig. 2(a)), the band state with the coexistence of a polar liquid band and disordered gas in an intermediate region (Figs. 2(b)–2(d)), and the disordered gas state in the large η and large ρ_n region.

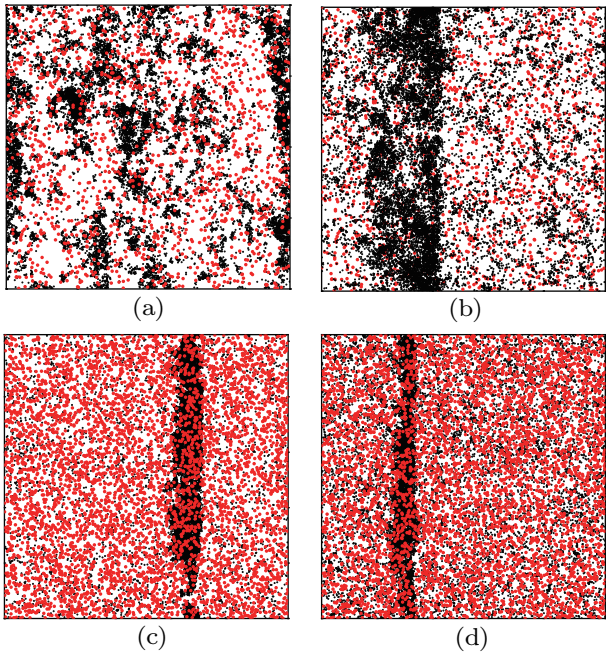


Fig. 2. (color online) (a) The snapshot of polar liquid state with ρ_n= 0.1 and η = 0.1. The snapshots of band state for different parameters: (b) ρ_n= 0.1 and η = 0.3, (c) ρ_n= 1 and η = 0.1, (d) ρ_n= 1 and η = 0.3, respectively. The black dots represent the active particles, while the red dots represent the noise particles.

We study the phase transition between different phases on the phase diagram in detail. Firstly, we show the phase transition from the band state to the disordered gas state by increasing noise η. The transition could be well characterized by the

measurement of the polar order parameter. At the low noise region, as the noise intensity increases, the order parameter decreases slowly. Further increasing the noise intensity, when it is close to the transition point, the order parameter φ drops sharply to zero, as shown in Fig. 3(a). This is the famous discontinuous phase transition from band state to disordered gas state in the original Vicsek model because of the existence of band structure.[39,40] For stronger environmental noise, i.e, the density of noise particles increases, the transition line of the order parameter moves down and the transition point moves left. As we show, the environmental noise decreases the polar order of active particles.

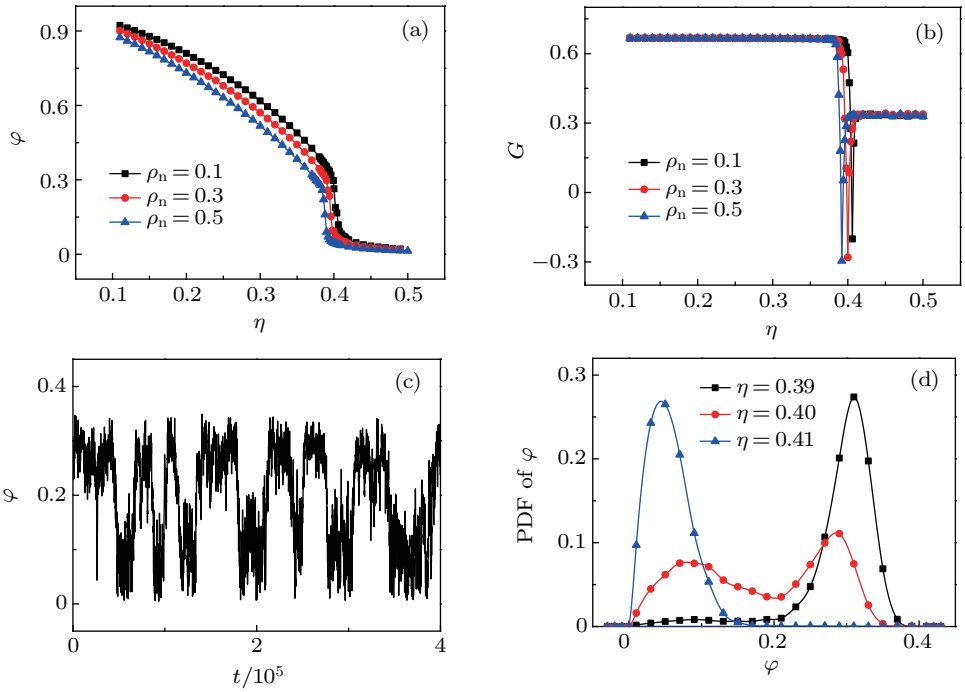


Fig. 3. (color online) (a) The time-averaged order parameter φ vs noise strength η for various densities of noise particles. (b) Binder cumulant value G. (c) Piece of order parameter time series around the transition point ($\rho_n = 0.3$ and $\eta = 0.4$). (d) Order parameter distribution around the transition point ($\rho_n = 0.3$).

We turn our attention to the question whether the order to disorder phase transition is discontinuous, the same as that in the Vicsek model. A direct method to distinguish first-order phase transition and second-order phase transition is to measure Binder cumulant value G, which is defined as $G = 1 - \langle\varphi^4\rangle/3\langle\varphi^2\rangle^2$. If the phase transition is of first-order, G exhibits minimum near the threshold, while for the second-order phase transition, G does not have minimum near the threshold. As we show in Fig. 3(b), in different environmental noise conditions, all of the G curves fall to negative values near the transition point, suggesting a discontinuous transition. The minimum points in this figure indicate the transition points. It is interesting that G keeps a constant value of 0.66 in order state and 0.33 in disordered state, for different ρ_n values. This is the property of G distinguishing different states.

We also show the time series of the order parameter around the transition point in Fig. 3(c). The band state and the disordered gas state are bistable near the transition point. $\varphi(t)$ exhibits strong fluctuations between these two states. High $\varphi(t)$ values correspond to the band state, while low $\varphi(t)$ values correspond to the disordered gas state. The system can stay in both states for a long time and suddenly take a transition to the other state. We further characterize the probability distribution function (PDF) of the order parameter φ near the transition point in Fig. 3(d). The blue and black curves with one hump indicate the disordered state and the ordered state, respectively. The red φ curve for $\eta = 0.4$ demonstrates strongly a bimodal distribution, indicating a discontinuous phase transition.

Secondly, we study the phase transition from the polar ordered liquid state to the band state on the phase diagram. It is difficult to observe this transition in the measurement of the polar order parameter, because both of these states are of high orientational order. The main difference between these two phases is the existence of the well organized band structures. A clear travelling band is observed in the band state in Figs. 2(b)–2(d), while in the polar liquid state the particles are homogeneously distributed without any clear band-like structures (Fig. 2(a)). We can distinguish these two states by measuring density fluctuations $\langle\Delta\rho_{||}^2(t)\rangle$ and order fluctuations $\langle\Delta\varphi_{||}^2(t)\rangle$ with the increase of noise strength,[40] where $\Delta\rho_{||}^2(t) = \langle[\rho_{||}(x,t) - \langle\rho_{||}(x,t)\rangle]^2\rangle$ and $\Delta\varphi_{||}^2(t) = \langle[\varphi_{||}(x,t) - \langle\varphi_{||}(x,t)\rangle]^2\rangle$. As we show in

Figs. 4(a) and 4(b), the polar liquid state in the small η region has relative small density fluctuations and order fluctuations with the change of η. When $\eta > 0.22$, both density and order fluctuations grow rapidly with the increase of η. As we show in Fig. 2(a), near the transition, the system has no clear band structures but are globally polar ordered. Further increasing the noise strength η, the system self-organizes into high density and high order band structures (Figs. 2(b)–2(d)). We locate the transition point as the point that the fluctuations curve starts to increase; however, this point is hard to locate accurately. For the lower η region, the density fluctuation slowly increases, indicating strong density inhomogeneity created by non-band structures. While the order fluctuation decreases to zero, suggesting a homogeneous profile of order parameter field. The decouple of density and polar fluctuations is because of the formation of many dense clusters which increases the inhomogeneity of the number density. At the same time, the system is highly ordered with small fluctuation of order parameter.

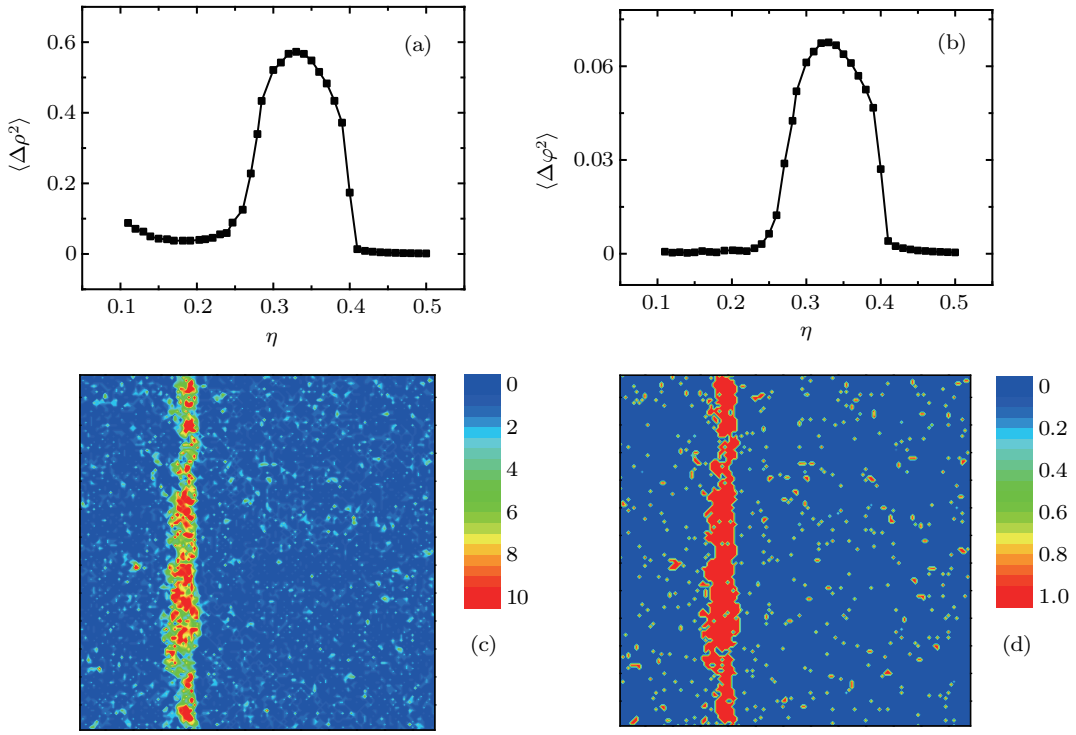


Fig. 4. (color online) (a) The time-averaged variances of density profiles as a function of noise strength η. (b) The time-averaged variances of order parameter profiles as a function of η. (c) The density profile in the band state. (d) Local order parameter profiles for panel (c). Only active particles are shown. Parameters: $L = 128$, $\rho_n = 1$, and $\eta = 0.3$.

For larger noise strength near $\eta = 0.33$, robust band structures lead to strong spatial inhomogeneity. As shown in Figs. 4(a) and 4(b), with the increase of η, a collective moving polar band emerges and the density fluctuation increases correspondingly in Fig. 4(a). Correspondingly, the fluctuation of order parameter is also strong as shown in Fig. 4(b). In this regime, particles inside the high density bands are highly ordered, while they are disordered in the background. That is why the spatial fluctuations of the order parameter follow the trend of fluctuations of density in the region $0.26 < \eta < 0.4$. We now focus on the internal structure of the travelling band. As we show in Figs. 4(c) and 4(d), the band does not consist of a single cluster that all particles inside the band move coherently. In fact, the band is a dynamical object made of many individual clusters. The band continuously absorbs clusters, at the same time, the clusters split and leave the band. The order parameter profile also shows that the local order inside the band is high, while it is lower in the background.

As we further increase the noise intensity η, the bands vanish, leaving a spatially homogeneous disordered phase (Fig. 3(a)). Both density fluctuations $\langle \Delta\rho_{||}^2(t) \rangle$ and order fluctuations $\langle \Delta\varphi_{||}^2(t) \rangle$ decrease to zero in Figs. 4(a) and 4(b). The noise dominates the dynamics of the system. In the polar liquid regime, as shown in Fig. 5(a), we measure the number fluctuation in a subsystem of various box sizes. For system sizes $L = 128$ and $L = 256$, we find the relationship $\Delta n \propto \langle n \rangle^{\alpha}$ with $\alpha = 0.75$, which is greater than a power law relation $\alpha = 0.5$ as expected in the equilibrium state. The giant density fluctuation is a typical phenomenon in an active system. The strong fluctuation is due to the formation of dense-packed clusters. The average-neighbor of each particle increases as we decrease the noise down to the polar liquid region, indi-

cating the emergence of locally high packed coherent clusters. The formation of a band structure from the polar liquid state to the band state by increasing noise could be understood as the following process. Strong fluctuation of density breaks the liquid clusters, then these clusters reconnect and stabilize into band structures. We look at the density distribution function in the liquid region as shown in Fig. 5(b), the probability distribution of high density clusters decreases as the cluster size increases. At the end of this curve, we observe an approximately exponential tail, which is in agreement with the Vicsek model.[41]

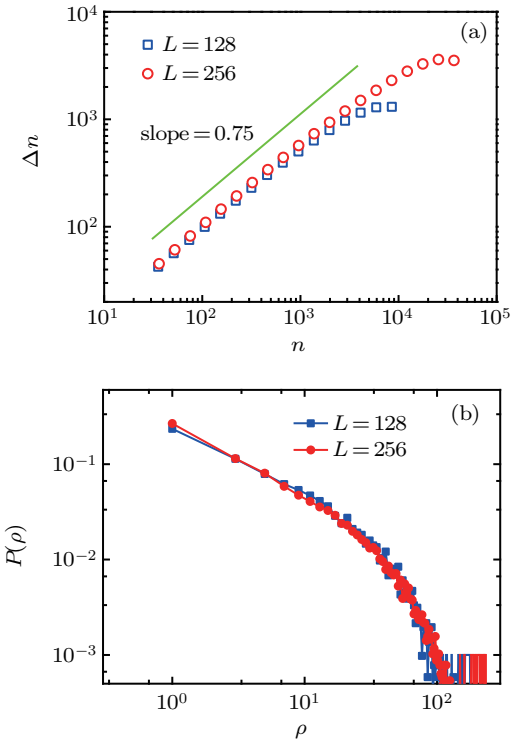


Fig. 5. (color online) (a) Giant number fluctuations that the root mean square Δn as a function of particles n contained in boxes of various linear sizes. The green line is a power-law of slope 0.75 ($\rho_n = 0.1$ and $\eta = 0.1$). (b) PDF of coarse-grained density ρ with measured box size $l = 4$ ($\rho_n = 0.1$ and $\eta = 0.1$).

Finally, we consider the limiting case when $\rho_n = 0$. In this case our model reduces to the original Vicsek model. Recently, the order–disorder transition in the original Vicsek model could be understood as a liquid–gas transition rather than an order–disorder phase transition.[40] Our findings are in agreement with such a scenario that as the noise intensity η increases, the system exhibits three phases: polar liquid at low noise, micro-phase separation with band structure at mediate noise, disordered gas at high noise. In our simulation, we show a phase transition from a disordered state to an ordered one. This order-disorder transition is also observed in experiments on locusts and fish schools. As the density of noise particles increases, there is a rapid transition from highly synchronized behavior to disordered state. Our model is useful for a qualitative understanding of such phenomena. For a low ρ_n condition, each particle only interacts with a small number of noise particles. Thus, their behaviors mainly depend on the alignment interaction. As the number of noise particles increases, each particle has more "noise" neighbors rather than "active" neighbors. The noise leads to a lower alignment order. In the band state, the particles form some clusters and rapidly aggregate into ordered bands, travelling in a disordered background. We also show that the band state could be extended into a finite ρ_n region, but the polar liquid phase as defined above would shrink with the increase of ρ_n. In general, the system may stay in the coherent moving band state at very low noise, with the introduction of noise particles. While in the original Vicsek model, the band state at the low noise condition already disappears because of the transition to a polar liquid state.

4. Conclusion

In this paper, we study the collective motion of self-propelled particles in environmental noise. We explore the (ρ_n, η) parameters plane and show three typical phases: polar liquid, band, and disordered states. When ρ_n approaches zero, the system reduces to the original Vicsek model. In comparison with the phase transition in the Vicsek model, we study the phase transition in noise particles condition and find that the transition from order to disorder state is strongly discontinuous, which is in agreement with the Vicsek model. For finite ρ_n, the disorder region becomes larger because the noise particle can be regarded as a noise source. At the same time, the transition point from band state to liquid phase also shifts to the low noise region. If η is low enough, for finite density of noise particles, active particles can recruit most of the particles into the band structure. This greatly enhances the spatial coherence of the system in the low η condition. However, in the original Vicsek model, the system is spatially homogeneous because of the transition to the polar liquid state. Our findings of complex collective behaviors in environmental noise help us to understand how individuals modify their self-organization by environmental factors, which may further contribute to improving the design of collective migration and navigation strategies.

Acknowledgment

We thank the soft matter laboratory in the department of physics, Nanjing University.

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