Phase estimation of phase shifts in two arms for an SU(1,1) interferometer with coherent and squeezed vacuum states*

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(Received 10 March 2017; revised manuscript received 9 May 2017; published online 31 July 2017)

We theoretically study the quantum Fisher information (QFI) of the SU(1,1) interferometer with phase shifts in two arms by coherent ⊗ squeezed vacuum state input, and give the comparison with the result of phase shift only in one arm. Different from the traditional Mach–Zehnder interferometer, the QFI of single-arm case for an SU(1,1) interferometer can be slightly higher or lower than that of two-arm case, which depends on the intensities of the two arms of the interferometer. For coherent ⊗ squeezed vacuum state input with a fixed mean photon number, the optimal sensitivity is achieved with a squeezed vacuum input in one mode and the vacuum input in the other.

Keywords: phase estimation, quantum Fisher information, SU(1,1) interferometer

PACS: 42.50.St, 42.50.Lc, 07.60.Ly, 42.65.Yj **DOI:** 10.1088/1674-1056/26/9/094205

1. Introduction

Quantum enhanced metrology which has received a lot of attention in recent years is the use of quantum measurement techniques to obtain higher statistical precision than purely classical approaches.[1–16] Mach–Zehnder interferometer (MZI) and its variants were used as a generic model to realize highly precise estimation of phase. In order to achieve the ultimate lower bounds,[17,18] much work has been devoted to finding the methods to improve the sensitivity of phase estimation, such as (i) using the nonclassical input states (quantum resources): squeezed states[3,19,20] and NOON states,[21,22] (ii) using the new detection methods: homodyne detection[23,24] and parity detection,[25–28] and (iii) using the nonlinear processes: amplitude amplification[29] and phase magnification.[13] Here we focus on the nonlinear amplitude amplification process to improve the sensitivity. In 1986, Yurke *et al.*[29] introduced a new type of interferometer where two nonlinear beam splitters (NBSs) take the place of two linear beam splitters (BSs) in the traditional MZI. It is also called the SU(1,1) interferometer because it is described by the SU(1,1) group, as opposed to the traditional SU(2) MZI for BS. The detailed quantum statistics of the two-mode SU(1,1) interferometer was studied by Leonhardt.[30] The SU(1,1) phase states were also studied theoretically in quantum measurements for phase-shift estimation.[31,32] Furthermore, the SU(1,1)-type interferometers have been reported by different groups using different systems in theory and experiment, such as all optical arms,[33–36] all atomic arms,[37–39] atom–light hybrid arms,[40–45] light–circuit quantum electrodynamics system hybrid arms,[46] and all mechanical modes arms.[47] These SU(1,1)-type interferometers provide different methods for basic measurement.

At present, many researchers are focusing on how to measure the phase sensitivities, where several detection schemes have been presented.[23,28,36] In general, it is difficult to optimize all the detection schemes to obtain the optimal estimation protocol. However, the quantum Fisher information (QFI)[4,5] characterizes the maximum amount of information that can be extracted from quantum experiments about an unknown parameter (e.g., phase shift ϕ) using the best (and ideal) measurement device. Therefore, the lower bounds in quantum metrology can be obtained by using the method of the QFI. In recent years, many efforts were made to obtain the QFI of different measurement systems.[48–66] For the SU(1,1) interferometers with phase shift only in one arm, the QFI with coherent states input was studied by Sparaciari *et al.*,[60,62] and the QFI with coherent ⊗ squeezed vacuum state input was presented by some of us.[28] Nevertheless in some measurement schemes, phase shifts in two arms are required to measure. For example, the phase sensitivity of phase shifts in two arms for the SU(1,1) interferometer with coherent states input was experimentally studied by Linnemann *et al.*[37] Jarzyna *et al.* studied the QFIs of phase shifts in the two-arm case for a MZI, and presented the relationship with the result of phase shift in

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11474095, 11654005, and 11234003) and the National Key Research and Development Program of China (Grant No. 2016YFA0302000).

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the single-arm case.[52] Since phase shift in the single arm is not simply equivalent to the phase shifts in two arms where one phase shift of them is 0, the QFIs of phase shifts in two arms for an SU(1,1) interferometer needed to be researched. In this paper, we study the QFI of SU(1,1) interferometer of phase shifts in two arms with two coherent states input and coherent $\otimes$ squeezed vacuum state input, and give the comparison with the result of phase shift only in one arm. These results should be useful for some phase measurement processes.

2. The QFI of phase shifts in two arms for an SU(1,1) interferometer

The QFI $\mathcal{F}$ is the intrinsic information in the quantum state and is not related to actual measurement procedure as shown in Fig. 1. It establishes the best precision that can be attained with a given quantum probe.[4,5] In this section, we study the QFIs of SU(1,1) interferometer of phase shifts in two arms, and compare them with the results of phase shift only in one arm.

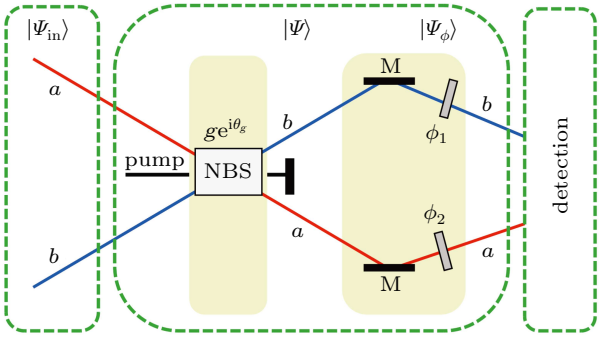


Fig. 1. (color online) Schematic diagram of the parameter estimation process based on the SU(1,1) interferometer. g and θ_g describe the strength and phase in the first NBS process, respectively. a and b denote two light modes in the interferometer. In the Schrödinger picture, the initial state $|\Psi_{\rm in}\rangle$ injecting into a NBS results in the output $|\Psi\rangle$, and the state is transformed as $|\Psi_\phi\rangle$ after phase shifts. NBS: nonlinear beam splitter. ϕ_1, ϕ_2: phase shifts. M: mirrors.

2.1. NBS and phase shifts

In an SU(1,1) interferometer, the NBSs take the place of the BSs in the traditional MZI shown in Fig. 1. Firstly, we theoretically describe the NBS briefly, which can be completed by the optical parameter amplifier (OPA) or four-wave mixing (FWM) process. The annihilation operators of the two modes a, b, and the pump field are $\hat{a}$, $\hat{b}$, and $\hat{c}$, respectively. The interaction Hamiltonian for NBS is

$$\hat{H} = i\hbar\eta^* \hat{a}\hat{c}^\dagger \hat{b}\hat{c}^\dagger - i\hbar\eta \hat{a}^\dagger \hat{c}\hat{b}^\dagger \hat{c}. \tag{1}$$

Because the pump field is very strong and the intensity of the pump field is not significantly changed in the mixing process, the initial and final states of the pump field are the same as the coherent state $|\alpha_{\rm pump}\rangle$. Under the undepleted pump approximation, the Hamiltonian is written as

$$\hat{H} = i\hbar\eta^* \alpha_{\rm pump}^{*2} \hat{a}\hat{b} - i\hbar\eta \alpha_{\rm pump}^2 \hat{a}^\dagger \hat{b}^\dagger. \tag{2}$$

The corresponding time-evolution operator is $\hat{U}(t) = e^{-i\hat{H}t/\hbar} = \exp(-\xi \hat{a}^\dagger \hat{b}^\dagger + \xi^* \hat{a}\hat{b})$, where $\xi = \eta \alpha_{\rm pump}^2 t = g e^{i\theta_g}$ is the two-mode squeezing parameter. In the Schrödinger picture, the initial state $|\Psi_{\rm in}\rangle$ injecting into an NBS results in the output $|\Psi\rangle = \hat{U}(t)|\Psi_{\rm in}\rangle$, where the transformation of the annihilation operators is

$$\begin{pmatrix} \hat{a}' \\ \hat{b}^{\dagger\prime} \end{pmatrix} = \begin{pmatrix} \cosh g & -e^{i\theta_g} \sinh g \\ -e^{-i\theta_g} \sinh g & \cosh g \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b}^\dagger \end{pmatrix}. \tag{3}$$

Secondly, we describe the phase shifts process. Different from the BS, the NBS involves three light fields where the pump field is classical and with a classical reference phase. The uncertainty of classical pump field $1/\sqrt{|\alpha_{\rm pump}|^2}$ is very small and the phase uncertainties are from the modes a and b. After the first NBS, as shown in Fig. 1, the two beams sustain phase shifts, i.e., the mode a and mode b undergo the phase shifts of ϕ_1 and ϕ_2, respectively. Then we may write

$$\begin{aligned} &\exp\left(i\phi_1 \hat{a}^\dagger \hat{a}\right) \exp\left(i\phi_2 \hat{b}^\dagger \hat{b}\right) \\ &= \exp\left[i\frac{\phi_1+\phi_2}{2}(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b})\right] \exp\left[i\frac{\phi_1-\phi_2}{2}(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})\right] \\ &= \exp\left[i\frac{\phi}{\hbar}\hat{K}_z - i\frac{\phi}{2}\right] \exp\left[i\frac{(\phi_1-\phi_2)}{\hbar}\hat{J}_z\right], \end{aligned} \tag{4}$$

where $\phi = \phi_1 + \phi_2$, $\hat{K}_z = \hbar(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} + 1)/2$, and $\hat{J}_z = \hbar(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})/2$. In the Schrödinger picture, the transformation of the incoming state vector $|\Psi\rangle$ is given as

$$|\Psi_\phi\rangle = e^{-i\phi/2} e^{i(\phi_1-\phi_2)/\hbar \hat{J}_z} e^{i\phi/\hbar \hat{K}_z} |\Psi\rangle. \tag{5}$$

$\hat{J}_z$ is an invariant for the four-wave mixing process. The operator $e^{i(\phi_1-\phi_2)/\hbar \hat{J}_z}$ gives rise to phase factors which does not contribute to the expectation values of number operators.

2.2. QFI

The QFI $\mathcal{F}$ is defined as[4,5]

$$\mathcal{F} = {\rm Tr}[\rho(\phi) L_\phi^2], \tag{6}$$

where the Hermitian operator L_ϕ, called symmetric logarithmic derivative, is defined as the solution of the equation $\partial_\phi \rho(\phi) = [\rho(\phi) L_\phi + L_\phi \rho(\phi)]/2$. In terms of the complete basis $\{|k\rangle\}$ such that $\rho(\phi) = \sum_k p_k |k\rangle\langle k|$ with $p_k \geq 0$ and $\sum_k p_k = 1$, the QFI can be written as[4,5,48–50]

$$\mathcal{F} = \sum_{k,k'} \frac{2}{p_k + p_{k'}} \left|\langle k|\partial_\phi \rho(\phi)|k'\rangle\right|^2. \tag{7}$$

Under the condition of lossless, for a pure state the QFI is reduced to[49,52]

$$\mathcal{F} = 4\left(\langle \Psi_\phi'|\Psi_\phi'\rangle - \left|\langle \Psi_\phi'|\Psi_\phi\rangle\right|^2\right), \tag{8}$$

where $|\Psi_\phi'\rangle = \partial|\Psi_\phi\rangle/\partial\phi$. In general, the QFI bounds depend on the ways that the interferometer phase delay is modeled:

(i) phase shift only in the single arm and (ii) phase shifts distributed in two arms, which is shown in Figs. 2(a) and 2(b), respectively. Hereafter, we use the single-arm case (S) and two-arm case (T) to denote them.

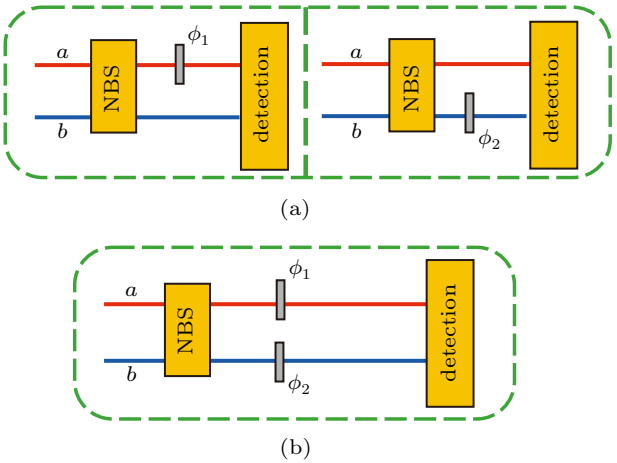


Fig. 2. (color online) Different phase delay ways of the interferometer. (a) Phase shift in the single arm is divided into single upper arm and single lower arm due to the fact that the intensities of the two arms are not equal. (b) Phase shifts in the two arms.

Now, we give the QFIs with different input states under the condition of phase shifts in two arms. From Eq. (5), $|\Psi_\phi\rangle$ is the state vector just before the detection process of the SU(1,1) interferometer and $|\Psi'_\phi\rangle = -\mathrm{i}(1/2 - \hat{K}_z/\hbar)\,|\Psi_\phi\rangle$. Then from Eq. (8), the QFI can be worked out as

$$\mathcal{F} = \frac{4}{\hbar^2}\Delta^2\hat{K}_z, \tag{9}$$

where $\Delta^2\hat{K}_z = \langle\Psi|\hat{K}_z^2|\Psi\rangle - \langle\Psi|\hat{K}_z|\Psi\rangle^2$. Using the transforms of Eqs. (3) and (5) and with two coherent states $|\alpha\rangle\otimes|\beta\rangle$ ($j = |j|\mathrm{e}^{\mathrm{i}\theta_j}$, $N_j = |j|^2$, $j = \alpha, \beta$) input case, for the SU(1,1) interferometer we have

$$\begin{aligned}\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&coh}} &= (N_\alpha + N_\beta)\cosh(4g) + \sinh^2(2g)\\ &\quad + 2\sinh(4g)\sqrt{N_\alpha N_\beta}\cos(\theta_\alpha + \theta_\beta - \theta_g - \pi).\end{aligned} \tag{10}$$

When $\theta_\alpha + \theta_\beta - \theta_g = \pi$, the maximal QFI $\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&coh}}$ is reduced to

$$\begin{aligned}\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&coh}} &= (N_\alpha + N_\beta)\cosh(4g) + \sinh^2(2g)\\ &\quad + 2\sinh(4g)\sqrt{N_\alpha N_\beta}.\end{aligned} \tag{11}$$

When $N_\alpha = N_\beta = 0$ (vacuum input) and $N_\alpha \neq 0$, $N_\beta = 0$ (one coherent state input), from Eq. (11) the corresponding QFIs are given by $\mathcal{F}^{\mathrm{T}}_{\mathrm{vac}} = \sinh^2(2g)$ and $\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&vac}} = N_\alpha\cosh 4g + \sinh^2(2g)$, respectively.

Next, we consider a coherent light combined with a squeezed vacuum light as the input $|\psi_{\mathrm{in}}\rangle = |\alpha\rangle_a \otimes |0,\varsigma\rangle_b$ ($\alpha = |\alpha|\mathrm{e}^{\mathrm{i}\theta_\alpha}$, $N_\alpha = |\alpha|^2$, and $|0,\varsigma\rangle_b = \hat{S}_b(r)|0\rangle_b$ is the single-mode squeezed vacuum state in the b-mode where $\hat{S}_b(r) = \exp[(\varsigma^*\hat{b}^2 - \varsigma\hat{b}^{\dagger 2})/2]$ with $\varsigma = r\exp(\mathrm{i}\theta_\varsigma)$ the single-mode squeezing parameter), and the QFI can be worked out as

$$\begin{aligned}\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&squ}} &= \cosh^2(2g)\left[\frac{1}{2}\sinh^2(2r) + N_\alpha\right] + \sinh^2(2g)\\ &\quad \times\left[N_\alpha(\cosh 2r - \sinh 2r\cos\Phi) + \cosh^2 r\right],\end{aligned} \tag{12}$$

where $\Phi = \theta_\varsigma + 2\theta_\alpha - 2\theta_g$. When $\Phi = \pi$, the maximal QFI $\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&squ}}$ is given by

$$\begin{aligned}\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&squ}} &= \cosh^2(2g)\left[\frac{1}{2}\sinh^2(2r) + N_\alpha\right]\\ &\quad + \sinh^2(2g)\left[N_\alpha\,\mathrm{e}^{2r} + \cosh^2 r\right].\end{aligned} \tag{13}$$

When $r = 0$, $\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&squ}}$ is also reduced to $\mathcal{F}^{\mathrm{T}}_{\mathrm{coh\&vac}}$, which agrees with the above result. This input state was also used to improve the phase-shift measurement sensitivity in the SU(1,1) interferometer but only with the method of the error propagation in Ref. [23].

So far, we have given the QFI of SU(1,1) interferometer where the phase shifts in the two arms, and they as well as the QFIs with phase shift in the one arm case are summarized in the Table 1. The QFIs of phase shift in upper arm and in lower arm are also slightly different because the intensities in two arms of the interferometer are unbalanced. The QFI of single-arm case for an SU(1,1) interferometer can be slightly higher or lower than that of double arms case, which depends on the intensities of the two arms of the interferometer. Different from the SU(1,1) interferometer, the QFIs of the phase shifts in single upper arm and in single lower arm are the same due to the intensity balance of the two arms for the MZI.[52]

Table 1. The maximal QFIs of the SU(1,1) interferometer for different phase delay ways with different input states.

Input states	Single arm $\mathcal{F}^{\mathrm{S}}$		Phase shifts in two arms $\mathcal{F}^{\mathrm{T}}$
	Phase shift in upper arm	Phase shift in lower arm	
two coherent states $\|\alpha\rangle_a\otimes\|\beta\rangle_b$	$(N_\alpha+N_\beta)\cosh 4g+\sinh^2(2g)$ $+2\sqrt{N_\alpha N_\beta}\sinh 4g+N_\alpha+N_\beta$ $+2(N_\alpha-N_\beta)\cosh 2g$[a]	$(N_\alpha+N_\beta)\cosh 4g+\sinh^2(2g)$ $+2\sqrt{N_\alpha N_\beta}\sinh 4g+N_\alpha+N_\beta$ $-2(N_\alpha-N_\beta)\cosh 2g$	$(N_\alpha+N_\beta)\cosh 4g$ $+\sinh^2(2g)$ $+2\sqrt{N_\alpha N_\beta}\sinh 4g$
coherent $\otimes$ squee -zed vacuum states $\|\alpha\rangle_a\otimes\|\varsigma,0\rangle_b$ $(\varsigma = r\exp(\mathrm{i}\theta_\varsigma))$	$\cosh^2(2g)[\sinh^2(2r)/2+N_\alpha]$ $+\sinh^2(2g)[N_\alpha\mathrm{e}^{2r}+\cosh^2 r]$ $+N_\alpha(1+2\cosh 2g)$ $-\frac{1}{4}(\cosh 4r-1)(2\cosh 2g-1)$[b]	$\cosh^2(2g)[\sinh^2(2r)/2+N_\alpha]$ $+\sinh^2(2g)[N_\alpha\mathrm{e}^{2r}+\cosh^2 r]$ $+N_\alpha(1-2\cosh 2g)$ $+\frac{1}{4}(\cosh 4r-1)(2\cosh 2g+1)$[b]	$\cosh^2(2g)[N_\alpha$ $+\sinh^2(2r)/2]$ $+\sinh^2(2g)[N_\alpha\mathrm{e}^{2r}$ $+\cosh^2 r]$

[a]Ref. [62]. [b]Ref. [28].

3. Quantum Cramér–Rao bound (QCRB)

Whatever the measurement chosen, the QCRB can give the lower bound for the phase measurement[4,5,48–50]

$$\Delta\phi_{\mathrm{QCRB}} = \frac{1}{\sqrt{\mathcal{F}}}. \tag{14}$$

To describe the effect on the QCRB from the unbalanced input state, we introduce a parameter η defined by[24]

$$\eta = \frac{\text{mean photon number of } b \text{ mode}}{\text{total mean photon number of input}}. \tag{15}$$

For the two coherent states input, η is equal to N_β/N_{in} ($N_{\mathrm{in}} = N_\alpha + N_\beta$), and the optimal phase sensitivities $\Delta\phi_{\mathrm{QCRB}}$ as a function of η are shown in Fig. 3(a). When η is small, $\Delta\phi_{\mathrm{QCRB}}$ from the single upper arm case is the best. But when η is large, $\Delta\phi_{\mathrm{QCRB}}$ from the single lower arm case is the best. $\Delta\phi_{\mathrm{QCRB}}$ from the two-arm case is always an intermediate value. For a given fixed N_{in} and the two coherent states input case, the optimal value η is 0.5. Thus for the two coherent states input, the optimal input state is $|\sqrt{N_{\mathrm{in}}/2}\,\mathrm{e}^{\mathrm{i}\theta_\alpha}\rangle \otimes |-\sqrt{N_{\mathrm{in}}/2}\,\mathrm{e}^{-\mathrm{i}\theta_\alpha}\,\mathrm{e}^{\mathrm{i}\theta_g}\rangle$, and the corresponding optimal QFI is $\mathcal{F}^{\mathrm{T,opt}}_{\mathrm{coh\&coh}} = N_{\mathrm{in}}\,\mathrm{e}^{4g} + \sinh^2(2g)$. The optimal QFI $\mathcal{F}^{\mathrm{T,opt}}_{\mathrm{coh\&coh}}$ as a function of the total input mean photon number N_{in} is shown in Fig. 4 (the blue dotted-dashed line).

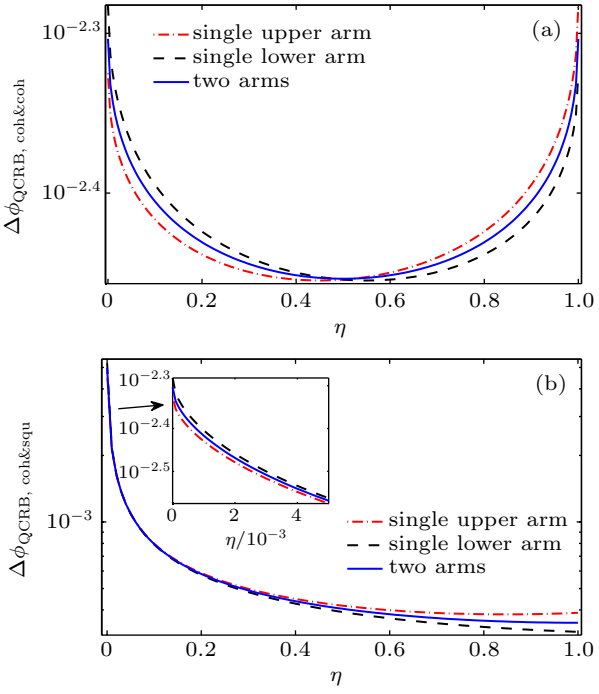


Fig. 3. (color online) The QCRB versus the η for (a) two coherent states input and (b) coherent $\otimes$ squeezed state input. The inset in panel (b) shows a zoom of the graph for small values of η. Here, $N_{\mathrm{in}} = 200$ and $g = 1.5$.

For coherent $\otimes$ squeezed vacuum state input, η is equal to $\sinh^2 r/N_{\mathrm{in}}$ ($N_{\mathrm{in}} = N_\alpha + \sinh^2 r$), where the parameter η can be used to label the squeezing fraction of the mean photon number. When $\eta = 0$ or $\eta = 1$, the input state is only a coherent state $|\alpha\rangle_a$ or only a squeezed vacuum state $|0,\zeta\rangle_b$. When $0 < \eta < 1$, the input state is a coherent $\otimes$ squeezed vacuum state. For coherent $\otimes$ squeezed vacuum state input case, only the squeezed vacuum light as input and without the coherent state, the phase sensitivity is the highest shown in Fig. 3(b). The optimal input state is $|0\rangle \otimes |0,\zeta\rangle$, and the corresponding optimal QFI is $\mathcal{F}^{\mathrm{T,opt}}_{\mathrm{coh\&squ}} = (1+N_{\mathrm{in}})[2N_{\mathrm{in}}\cosh^2(2g) + \sinh^2(2g)]$, which is different from the commonly used optimal input state with $|\alpha|^2 \simeq \sinh^2(r) \simeq N_{\mathrm{in}}/2$ in MZI.[55,67,68] The reason is the number fluctuations and Pasquale *et al.*[69] have given the same result for generic two-mode interferometric setup recently. The optimal QFI $\mathcal{F}^{\mathrm{T,opt}}_{\mathrm{coh\&squ}}$ as a function of N_{in} is shown in Fig. 4 (the red dashed line). For a fixed mean photon number (with number fluctuations), Hofmann suggested the form of Heisenberg limit is $1/\langle\hat{N}^2\rangle^{1/2}$, which indicates averaging over the squared photon numbers.[70] In our proposal $\langle\hat{N}\rangle$ is defined as $\langle\Psi|(\hat{n}_a + \hat{n}_b)|\Psi\rangle$. In Fig. 4, the black solid line is the Hofman limit for coherent $\otimes$ squeezed vacuum state input under the optimal condition.

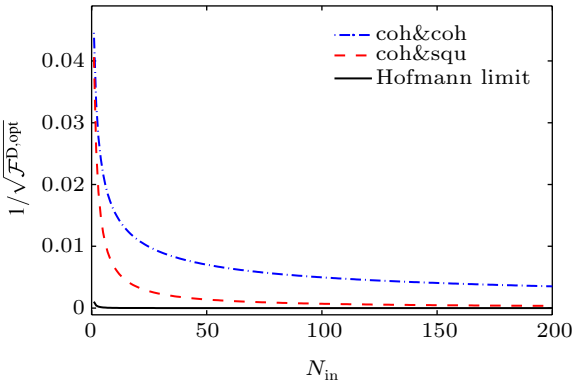


Fig. 4. (color online) The optimal QFIs versus the total input mean photon number N_{in}. The dotted-dashed line and the dashed line are the two coherent states input and coherent $\otimes$ squeezed state input, respectively. The solid line is the Hofmann limit with coherent $\otimes$ squeezed state input $g = 1.5$.

For the lossy interferometers, the pure states evolve into the mixed states and the QFI will be reduced. However, the QFI of pure state puts an upper bound on that of mixed state. Here, we focus on the maximal QFI of the SU(1,1) interferometer, and then we ignore the losses in the interferometer.

4. Conclusion

The analytical expressions of QFI for an SU(1,1) interferometer with two coherent states and coherent $\otimes$ squeezed vacuum state inputs have been derived. For single-arm case, the QCRBs of phase shift in upper arm and in lower arm are slightly different because the intensities in two interferometric arms are asymmetric. The phase sensitivities of phase shifts between the single-arm case and two-arm case are also compared. The QCRB of single-arm case can be slightly higher or

lower than that of two-arm case, which depends on the intensities of the two arms of the interferometer. For coherent state $\otimes$ squeezed vacuum state input with a definite input number of photons, the optimal condition to obtain the highest phase sensitivity is a squeezed vacuum in one mode and the vacuum state in the other mode.

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