Invariants-based shortcuts for fast generating Greenberger–Horne–Zeilinger state among three superconducting qubits*

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As one of the most promising candidates for implementing quantum computers, superconducting qubits (SQs) are adopted for fast generating the Greenberger–Horne–Zeilinger (GHZ) state by using invariants-based shortcuts. Three SQs are separated and connected by two coplanar waveguide resonators (CPWRs) capacitively. The complicated system is skillfully simplified to a three-state system, and a GHZ state among three SQs is fast generated with a very high fidelity and simple driving pulses. Numerical simulations indicate the scheme is insensitive to parameter deviations. Besides, the robustness of the scheme against decoherence is discussed in detail.

Keywords: Greenberger–Horne–Zeilinger state, superconducting qubits, shortcuts to adiabaticity

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1. Introduction

With the rapid development of quantum information processing, the concept of quantum entanglement is increasingly considered to be essential. Generations of entangled states are heavily relied on in many important applications, such as quantum teleportation,[1] quantum communication,[2] quantum secret sharing,[3] fault-tolerant computing,[4,5] and so on.[6] Greenberger–Horne–Zeilinger (GHZ) states are a group of well-known entangled states because they provide a possibility for testing quantum mechanics against local hidden theory without using Bell's inequality.[7,8] Therefore, various schemes for generating GHZ states have been proposed during the past ten years or more in various physical systems, such as trapped ions systems,[9] photons systems,[1,10] atoms systems,[11] and solid systems.[12] In addition, many schemes for generating GHZ states have been implemented by using superconducting qubits (SQs) in superconducting circuits.[13–17] Compared with other systems, SQ systems possess a natural superiority of scalability for implementing quantum computers.[18–21] Devices of SQs used in quantum computers can be created more easily for large-scale integration and scale with the mature microelectronic technique. Besides, coupling strengths among SQs can be controlled well up to desired values through modulating electromagnetic signals and designing level configurations. In this paper, as one of the most promising candidates for implementing quantum computers, SQs are adopted for fast generating GHZ state.

Stimulated Raman adiabatic passage (STIRAP) is one technique related to adiabatic evolution and has been widely used for robust quantum state transfer.[22–24] STIRAP has also been used to generate GHZ states in cavity quantum electrodynamics.[25–28] In an SQ system, Wu *et al.* generated n-qubit GHZ states based on STIRAP in 2016.[29] As is well known, however, STIRAP usually requires a relatively long interaction time for restraining non-adiabatic transitions, which may accumulate decoherence and errors leading to useless dynamics. Therefore, a set of techniques of shortcuts to adiabaticity (STA), which aim to accelerate an adiabatic evolution process, arose at the historic moment.[30–44] Take some of the most recent works as examples; Kang *et al.* proposed two schemes to speed up adiabatic evolutions by reverse engineering of a Hamiltonian,[42,43] and Chen *et al.* presented arbitrary quantum state engineering in three-state systems.[44] By using STA, lots of remarkable achievements have been made in quantum information processing.[45–55] Also, many schemes have been proposed for speeding up the generations of GHZ states.[56–61]

Recently, some schemes have been proposed by combining the technique of STA and the SQ system to generate entangled states. For example, Zhang *et al.* proposed a scheme for fast reparation of the three-qubit GHZ state in 2016;[59] Kang *et al.* proposed two schemes to fast generate W states in 2016;[62,63] Yu *et al.* fast generated a W state recently.[64] There are generally two kinds of techniques of STA widely used. One is to construct a Hamiltonian by means of instantaneous eigenstates given, and the other is to chase down the desired instantaneous eigenstates based on the given Hamiltonian. Invariants-based shortcuts we will employ in this paper belong to the second kind. Different from Refs. [56]–[59] using the first kind of technique, the scheme needs to neither interpolate the system Hamiltonian nor add counter-diabatic driving pulses, but find a pair of suitable Rabi frequencies by constructing a Hermitian invariant of the system. Compared with Refs. [60] and [61] also using the invariants-based short-

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cuts, apart from advantages of the SQ system, the scheme we proposed has simpler energy level configurations of qubit carriers and fewer driving pulses.

2. Physical model and effective dynamics

The physical model for generating the GHZ state among three SQs is shown in Fig. 1. There are three SQs separated by two single-mode coplanar waveguide resonators (CPWRs) capacitively. The three SQs have identical Λ-type level configuration with one upper level $|e\rangle$ and two lower levels $|g_L\rangle$ and $|g_R\rangle$. Transitions $|e\rangle_1 \leftrightarrow |g_L\rangle_1$ of SQ 1 and $|e\rangle_2 \leftrightarrow |g_L\rangle_2$ of SQ 2 are resonantly coupled to CPWR-L with corresponding coupling strengths $\lambda_{1,L}$ and $\lambda_{2,L}$, respectively. $|e\rangle_2 \leftrightarrow |g_R\rangle_2$ of SQ 2 and $|e\rangle_3 \leftrightarrow |g_R\rangle_3$ of SQ 3 are resonantly coupled to CPWR-R with corresponding coupling strengths $\lambda_{2,R}$ and $\lambda_{3,R}$, respectively. In addition, $|e\rangle_1 \leftrightarrow |g_R\rangle_1$ of SQ 1 and $|e\rangle_3 \leftrightarrow |g_L\rangle_3$ of SQ 3 are resonantly driven by two time-dependent classical laser fields with corresponding Rabi frequencies $\Omega_R(t)$ and $\Omega_L(t)$, respectively. Then, the interaction Hamiltonian of the whole system is ($\hbar = 1$):

$$\begin{aligned} H(t) &= \Omega_R(t)|e\rangle_1\langle g_R| + \Omega_L(t)|e\rangle_1\langle g_L| \\ &+ \sum_{j=1,2} \lambda_{j,L} a_L |e\rangle_j\langle g_L| \\ &+ \sum_{k=2,3} \lambda_{k,R} a_R |e\rangle_k\langle g_R| + \text{H.c.}, \end{aligned} \tag{1}$$

where $a_{L(R)}$ is the annihilation operator of CPWR-$L(R)$.

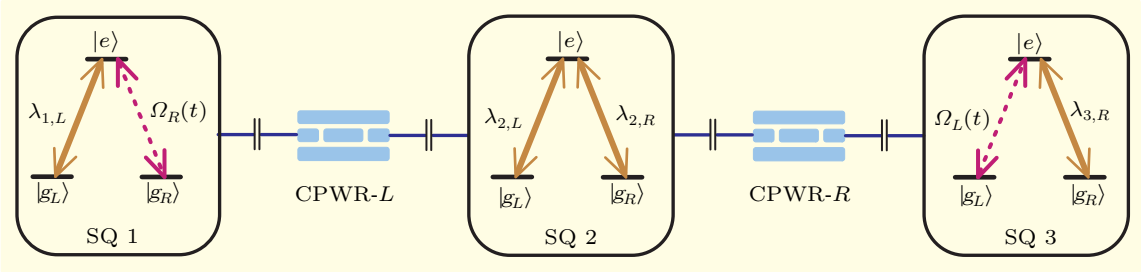


Fig. 1. (color online) The diagrammatic sketch of the SQ system, level configurations, and related transitions of three SQs.

The system initial state is $|\phi_1\rangle = |g_R\rangle_1|g_L\rangle_2|g_R\rangle_3|0\rangle_L|0\rangle_R$, denoting the three SQs in states $|g_R\rangle$, $|g_L\rangle$, and $|g_R\rangle$, respectively, and the two CPWRs both in the vacuum state. The SQ system will evolve in the subspace spanned by

$$\begin{aligned} |\phi_1\rangle &= |g_R\rangle_1|g_L\rangle_2|g_R\rangle_3|0\rangle_L|0\rangle_R, \\ |\phi_2\rangle &= |e\rangle_1|g_L\rangle_2|g_R\rangle_3|0\rangle_L|0\rangle_R, \\ |\phi_3\rangle &= |g_L\rangle_1|g_L\rangle_2|g_R\rangle_3|1\rangle_L|0\rangle_R, \\ |\phi_4\rangle &= |g_L\rangle_1|e\rangle_2|g_R\rangle_3|0\rangle_L|0\rangle_R, \\ |\phi_5\rangle &= |g_L\rangle_1|g_R\rangle_2|g_R\rangle_3|0\rangle_L|1\rangle_R, \\ |\phi_6\rangle &= |g_L\rangle_1|g_R\rangle_2|e\rangle_3|0\rangle_L|0\rangle_R, \\ |\phi_7\rangle &= |g_L\rangle_1|g_R\rangle_2|g_L\rangle_3|0\rangle_L|0\rangle_R, \end{aligned} \tag{2}$$

with $|1\rangle_{L(R)}$ denoting a single photon in CPWR-$L(R)$. Then Hamiltonian (1) can be rewritten as $H(t) = H_{\text{a-1}}(t) + H_{\text{a-c}}$ with

$$\begin{aligned} H_{\text{a-1}}(t) &= \Omega_R(t)|\phi_1\rangle\langle\phi_2| + \Omega_L(t)|\phi_6\rangle\langle\phi_7| + \text{H.c.}, \\ H_{\text{a-c}} &= \lambda_{1,L}|\phi_2\rangle\langle\phi_3| + \lambda_{2,L}|\phi_3\rangle\langle\phi_4| \\ &+ \lambda_{2,R}|\phi_4\rangle\langle\phi_5| + \lambda_{3,R}|\phi_5\rangle\langle\phi_6| + \text{H.c.} \end{aligned} \tag{3}$$

If we set $\lambda_{1,L} = \lambda_{2,L} = \lambda_{2,R} = \lambda_{3,R} = \lambda$ with λ being real, and choose the eigenstates of $H_{\text{a-c}}$,

$$\begin{aligned} |\Psi_{\text{D}}\rangle &= \frac{1}{\sqrt{3}}(|\phi_2\rangle - |\phi_4\rangle + |\phi_6\rangle), \\ |\Psi_1^{\pm}\rangle &= \frac{1}{2}[|\phi_2\rangle \pm (|\phi_3\rangle - \phi_5\rangle) - |\phi_6\rangle], \\ |\Psi_2^{\pm}\rangle &= \frac{1}{2\sqrt{3}}[|\phi_2\rangle \pm \sqrt{3}(|\phi_3\rangle + \phi_5\rangle) + 2|\phi_4\rangle + |\phi_6\rangle], \end{aligned} \tag{4}$$

with the corresponding eigenvalues 0, $\pm\lambda$, and $\pm\sqrt{3}\lambda$ as a set of transformations, then Hamiltonian (3) will become

$$\begin{aligned} H(t) &= H_0 + V(t), \\ H_0 &= \lambda(|\Psi_1^+\rangle\langle\Psi_1^+| - |\Psi_1^-\rangle\langle\Psi_1^-|) \\ &+ \sqrt{3}\lambda(|\Psi_2^+\rangle\langle\Psi_2^+| - |\Psi_2^-\rangle\langle\Psi_2^-|), \\ V(t) &= \frac{\Omega_R(t)}{\sqrt{3}}|\phi_1\rangle\left[\langle\Psi_{\text{D}}| + \frac{\sqrt{3}}{2}(\langle\Psi_1^+| + \langle\Psi_1^-|)\right. \\ &\left. + \frac{1}{2}(\langle\Psi_2^+| + \langle\Psi_2^-|)\right] \\ &+ \frac{\Omega_L(t)}{\sqrt{3}}\left[|\Psi_{\text{D}}\rangle - \frac{\sqrt{3}}{2}(|\Psi_1^+\rangle + |\Psi_1^-\rangle)\right. \\ &\left. + \frac{1}{2}(|\Psi_2^+\rangle + |\Psi_2^-\rangle)\right]\langle\phi_7| + \text{H.c.} \end{aligned} \tag{5}$$

Through performing the unitary transformation $U = \exp(-iH_0t)$ and disregarding high oscillating terms with the limit condition $\Omega_{L(R)}(t) \ll 2\lambda$, Hamiltonian (5) is simplified to an effective Hamiltonian

$$H_{\text{eff}}(t) = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & \Omega_R(t) & 0 \\ \Omega_R(t) & 0 & \Omega_L(t) \\ 0 & \Omega_L(t) & 0 \end{pmatrix}, \tag{6}$$

in the subspace $\{|\phi_1\rangle, |\Psi_D\rangle, |\phi_7\rangle\}$. The effective Hamiltonian (6) can be viewed as the Hamiltonian of a three-state system, and it will approximatively govern the evolution of the SQ system as long as the limit condition $\Omega_{L(R)}(t) \ll 2\lambda$ is satisfied very well.

3. Invariants-based shortcuts for fast generating the three-qubit GHZ state

In this section, we show the invariants-based shortcuts for fast generating the three-qubit GHZ state based on the effective Hamiltonian (6). First of all, we give a brief review concerning Lewis–Riesenfeld invariants theory.[65] A time-dependent Hermitian invariant $I(t)$ of a quantum system governed by a time-dependent Hamiltonian $H(t)$ satisfies

$$\mathrm{i}\partial_t I(t) - [H(t), I(t)] = 0. \tag{7}$$

The solution of the time-dependent Schrödinger equation $\mathrm{i}\partial_t|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$ can be expressed as a superposition of the instantaneous eigenvectors of $I(t)$

$$|\Psi(t)\rangle = \sum_n C_n \mathrm{e}^{\mathrm{i}\alpha_n}|\Phi_n(t)\rangle, \tag{8}$$

where $C_n = \langle\Phi_n(0)|\phi_1\rangle$ is the time-independent amplitude, α_n is the Lewis–Riesenfeld phase, and $|\Phi_n(t)\rangle$ is one of the orthogonal eigenvectors of $I(t)$. The Lewis–Riesenfeld phase is expressed by

$$\alpha_n(t) = \int_0^t \mathrm{d}t' \langle\Phi_n(t')|\mathrm{i}\partial_{t'} - H(t')|\Phi_n(t')\rangle. \tag{9}$$

For the effective three-state system with Hamiltonian (6), one of its Hermitian invariants can be chosen as

$$I(t) = \zeta \begin{pmatrix} 0 & \sin\gamma\sin\beta & -\mathrm{i}\cos\gamma \\ \sin\gamma\sin\beta & 0 & \sin\gamma\cos\beta \\ \mathrm{i}\cos\gamma & \sin\gamma\cos\beta & 0 \end{pmatrix}, \tag{10}$$

where ζ is an arbitrary constant with frequency unit making $I(t)$ have the dimension of energy. γ and β are two parameters determined later. The eigenvectors of $I(t)$ with the eigenvalues $\eta_0 = 0$ and $\eta_\pm = \pm 1$ are respectively

$$|\Phi_0(t)\rangle = \begin{pmatrix} \cos\beta\sin\gamma \\ -\mathrm{i}\cos\gamma \\ -\sin\beta\sin\gamma \end{pmatrix},$$
$$|\Phi_\pm(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \sin\beta \mp \mathrm{i}\cos\beta\cos\gamma \\ \pm\sin\gamma \\ \cos\beta \pm \mathrm{i}\sin\beta\cos\gamma \end{pmatrix}. \tag{11}$$

In order to insure that $I(t)$ is one of the Hermitian invariants of the effective system, based on Eq. (7), the two Rabi frequencies have to be written as

$$\Omega_R(t) = \sqrt{3}(\dot\beta\sin\beta\tan\gamma - \cos\beta\dot\gamma),$$
$$\Omega_L(t) = \sqrt{3}(\dot\beta\cos\beta\tan\gamma + \sin\beta\dot\gamma), \tag{12}$$

where the dot represents the time derivative.

In order to generate the GHZ state in a relatively easy way, we desire that Hamiltonian (6) could drive the initial state $|\phi_1\rangle$ to the target state $|\Psi_{\mathrm{GHZ}}\rangle = (|\phi_1\rangle + |\phi_7\rangle)/\sqrt{2}$ along the invariant eigenvector $|\Phi_0(t)\rangle$, which demands

$$\gamma(0) = \gamma(t_\mathrm{f}) = \pi/2, \quad \beta(0) = 0, \quad \beta(t_\mathrm{f}) = -\pi/4, \tag{13}$$

where we have assumed that the scheme starts at $t = 0$ and finishes at $t = t_\mathrm{f}$. In this way, $H_{\mathrm{eff}}(t)$ must have the same eigenvectors as $I(t)$ at the two boundary time, which requests $[H_{\mathrm{eff}}(0), I(0)] = 0$ and $[H_{\mathrm{eff}}(t_\mathrm{f}), I(t_\mathrm{f})] = 0$, i.e.,

$$\Omega_R(0) = 0, \quad \Omega_L(t_\mathrm{f}) = -\Omega_R(t_\mathrm{f}). \tag{14}$$

Equations (12)–(14) imply the additional boundary conditions

$$\dot\gamma(0) = \dot\gamma(t_\mathrm{f}) = 0. \tag{15}$$

Thus equations (13)–(15) give all the boundary conditions. However, it is worth noticing that the conditions $\gamma(0) = \gamma(t_\mathrm{f}) = \pi/2$ cause infinite Rabi frequencies $\Omega_R(t)$ and $\Omega_L(t)$, which implies that the evolution $|\phi_1\rangle \to |\Psi_{\mathrm{GHZ}}\rangle$ along $|\Phi_0(t)\rangle$ we desired is infeasible. Therefore, we slightly cut down the requirement and replace $\gamma(0) = \gamma(t_\mathrm{f}) = \pi/2$ by

$$\gamma(0) = \gamma(t_\mathrm{f}) = \pi/2 - \varepsilon, \tag{16}$$

with ε being a time-independent small value. Now the system evolution is not along $|\Phi_0(t)\rangle$ any more but along the superpositions of $|\Phi_0(t)\rangle$ and $|\Phi_\pm(t)\rangle$, and the finishing of the desired evolution $|\phi_1\rangle \to |\Psi_{\mathrm{GHZ}}\rangle$ strongly depends on the selection of ε.

Now in order to meet all the boundary conditions, we choose the parameters as

$$\gamma = \pi/2 - \varepsilon, \quad \beta(t) = -\frac{\pi t}{4t_\mathrm{f}}. \tag{17}$$

Then we can obtain the Rabi frequencies based on Eq. (12) as

$$\Omega_R(t) = \Omega_0 \sin\frac{\pi t}{4t_\mathrm{f}}, \quad \Omega_L(t) = -\Omega_0\cos\frac{\pi t}{4t_\mathrm{f}}, \tag{18}$$

with $\Omega_0 = \sqrt{3}\pi\cot\varepsilon/4t_\mathrm{f}$. $\Omega_{R(L)}(t)$ is a harmonic pulse which can be obtained very easily in experiment. By using Eq. (8), we calculate the final fidelity for generating the GHZ state

$$\begin{aligned} F(t_\mathrm{f}) &= |\langle\Psi_{\mathrm{GHZ}}|\Psi(t_\mathrm{f})\rangle|^2 \\ &= \left|1 - \sin^2\varepsilon\left[1 - \cos\frac{\pi}{4\sin\varepsilon}\right]\right|^2, \end{aligned} \tag{19}$$

with the Lewis–Riesenfeld phases

$$\alpha_0 = 0, \quad \alpha_\pm = \pm\frac{\pi}{4\sin\varepsilon}. \tag{20}$$

Considering a pair of appropriate Rabi frequencies and the final fidelity $F(t_\mathrm{f}) = 1$, we choose

$$\frac{\pi}{4\sin\varepsilon} = 2\pi \;\Rightarrow\; \varepsilon = \arcsin(1/8) \approx 0.1253. \tag{21}$$

By the way shown above, we have constructed an invariant-based shortcut for fast generating the GHZ state among three SQs.

4. Numerical simulations and discussion

In this section, we perform numerical simulations to show the availability and robustness of the scheme for generating the GHZ state. First of all, for testing that the value $\varepsilon = 0.1253$ is right and picking a suitable final time t_f, we show a contour plot of the final fidelity versus ε and t_f in Fig. 2. Obviously, figure 2 shows that the value $\varepsilon = 0.1253$ may guarantee a very high final fidelity. Besides, t_f also heavily affects the final fidelity and a larger t_f will give a higher final fidelity. Because Ω_0 is inversely proportional to t_f, a larger t_f can satisfy the limit condition $\Omega_{L(R)}(t) \ll 2\lambda$ better.

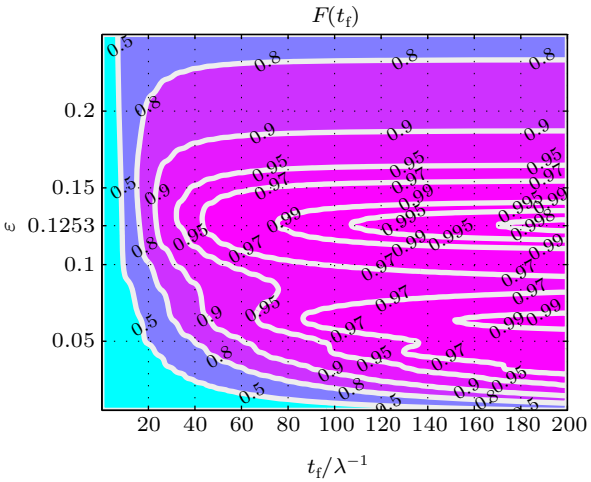


Fig. 2. (color online) Contour plot of the final fidelity versus ε and t_f.

In order to prove that the scheme we proposed is fast, in Fig. 3, we give a comparison between the STA scheme and the STIRAP scheme for the same GHZ state. For the STIRAP scheme, we choose the Rabi frequencies as[26–28]

$$\begin{aligned}\Omega'_R(t) &= \frac{1}{\sqrt{2}}\Omega'_0 \exp\left[-(t-t_f/2-t_0)^2/t_c^2\right],\\ \Omega'_L(t) &= \frac{1}{\sqrt{2}}\Omega'_0 \exp\left[-(t-t_f/2-t_0)^2/t_c^2\right]\\ &\quad -\Omega'_0 \exp\left[-(t-t_f/2+t_0)^2/t_c^2\right], \end{aligned} \tag{22}$$

with two related Gaussian parameters $t_0 = 0.14t_f$ and $t_c = 0.19t_f$. As we know, the adiabatic criterion in the STIRAP scheme requires either a very long operation time or very large differences among the system eigen energies,[66] and thus the only way to shorten the operation time is enhancing the amplitude Ω'_0, which requires more physical resources. Besides, as shown by the dotted green line of Fig. 3, Ω'_0 cannot be too much larger because of the limit condition $\Omega'_{L(R)}(t) \ll 2\lambda$. Numerically speaking, when the operation time is $t_f = 100/\lambda$, the final fidelity of the STA scheme is over 0.94 and the corresponding $\Omega_0 \approx 0.1\lambda$. For the STIRAP scheme, however, when $\Omega'_0 = 0.1\lambda$ and the final fidelity is up to 0.94, the corresponding operation time is near $t_f = 450/\lambda$, which is 4.5 times of that for the STA scheme. Therefore, the STA scheme we proposed for generating the GHZ state among three SQs is fast. For a relatively high final fidelity and a relatively short operation time, we choose $t_f = 100/\lambda$ for following discussions.

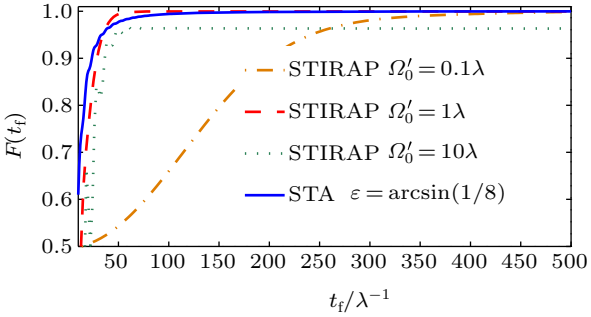


Fig. 3. (color online) Comparison between the STA scheme and the STIRAP scheme.

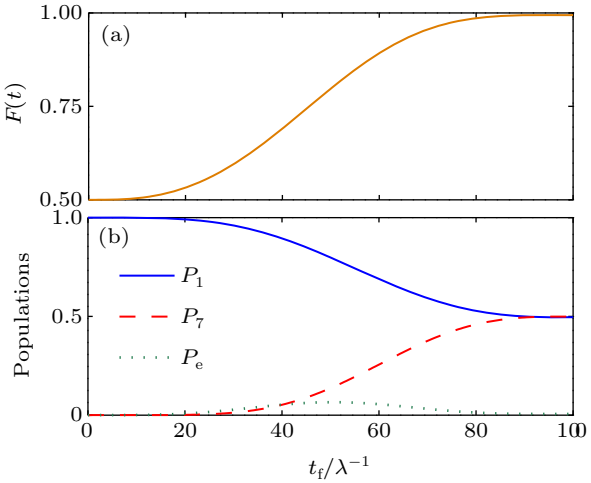


Fig. 4. (color online) (a) Time dependence of the fidelity. (b) Time evolution of the populations of $|\phi_1\rangle$, $|\phi_7\rangle$, and excited states.

For exhibiting the availability of the STA scheme, in Fig. 4, we plot the time dependence of the fidelity for generating the GHZ state and time evolution of the populations of the states (P_1 for $|\phi_1\rangle$, P_7 for $|\phi_7\rangle$, and $P_e = \sum_{n=2}^{6} P_n$ for all excited states including SQs excited states $|\phi_{2,4,6}\rangle$ and CPWRs excited states $|\phi_{3,5}\rangle$). From Fig. 4(a), we see that the fidelity gradually increases with the increase of time and reaches near unity at the final time $t_f = 100/\lambda$. Furthermore, figure 4(b) clearly shows that the GHZ state is near perfectly obtained at the final time and other states not involved in the GHZ state are hardly populated during the whole evolution. In a word, the STA scheme we proposed for fast generating the GHZ state among three SQs is quite valid.

Since most control parameters are impossible to be adjusted perfectly in experiment, we consider effects of variations in the control parameters on the final fidelity for the GHZ state generation in the STA scheme. Here we define $\delta x = x' - x$ as the deviation of x, where x denotes the ideal value and x' denotes the actual value. In Fig. 5, we show the effects of

the variations in two primary parameters t_f and Ω_0 on the final fidelity for generating the GHZ state among three SQs. As shown in Fig. 5, the STA scheme we proposed is robust against the variations in t_f and Ω_0, because the final fidelity is beyond 0.985 even when $|\delta t_f/t_f| = |\delta\Omega_0/\Omega_0| = 0.1$.

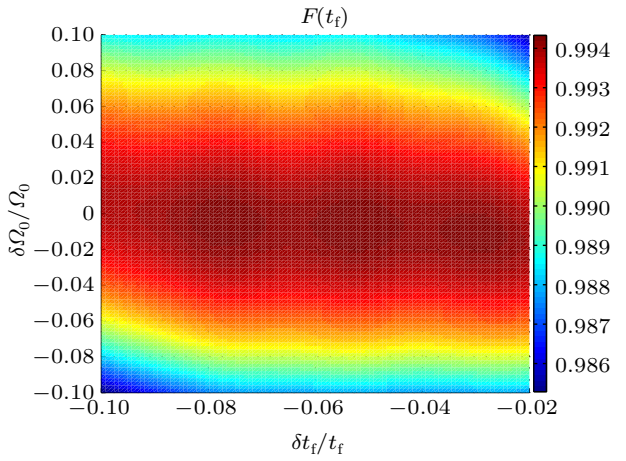


Fig. 5. (color online) Effects of variations in control parameters on the final fidelity.

Finally, by taking decoherence caused by SQs energy relaxations, CPWRs photon leakages, and SQs dephasing into account, the evolution of the whole system will be dominated by the master equation

$$\begin{aligned}\dot{\rho}(t) = & -\mathrm{i}[H(t),\rho(t)] + \sum_{k=L,R}\kappa_k L[a_k] \\ & + \sum_{k=L,R}\left\{\sum_{j=1}^{3}\frac{\Gamma_{\phi_{k,j}}}{2}[S^z_{k,j}\rho(t)S^z_{k,j} - \rho(t)] \right. \\ & \left. + \sum_{j=1}^{3}\Gamma_{k,j}L[S^-_{k,j}]\right\}, \end{aligned} \tag{23}$$

where $H(t)$ is Hamiltonian (1); $L[a_k] = a_k\rho(t)a_k^\dagger - a_k^\dagger a_k\rho(t)/2 - \rho(t)a_k^\dagger a_k/2$; $L[S^-_{k,j}] = S^-_{k,j}\rho(t)S^\dagger_{k,j} - S^\dagger_{k,j}S^-_{k,j}\rho(t)/2 - \rho(t)S^\dagger_{k,j}S^-_{k,j}/2$; $S^-_{k,j} = |g_k\rangle_j\langle e|$; $S^z_{k,j} = |e\rangle_j\langle e| - |g_k\rangle_j\langle g_k|$. κ_L is the photon leakage rate of CPWR-k, and $\Gamma_{\phi_{k,j}}$ ($\Gamma_{k,j}$) is the dephasing rate (energy relaxation rate) of the j-th SQ. For simplicity, we assume $\kappa_L = \kappa$, $\Gamma_{\phi_{k,j}} = \Gamma_\phi/2$, and $\Gamma_{k,j} = \Gamma/2$.

Based on the master equation above, in Figs. 6(a)–6(c), we plot the final fidelity versus every two decoherence factors of SQs energy relaxations, CPWRs photon leakages, and SQs dephasing. Set the effects of SQs energy relaxations, CPWRs photon leakages, and SQs dephasing on the final fidelity as E_Γ, E_κ, and E_{Γ_ϕ}, respectively. From Figs. 6(a)–6(c), we can clearly deduce $E_\kappa \ll E_\Gamma \ll E_{\Gamma_\phi}$, and we can say that compared with E_{Γ_ϕ}, E_Γ and E_κ are negligible. Moreover, we also plot the time dependence of the fidelity in the presence of three

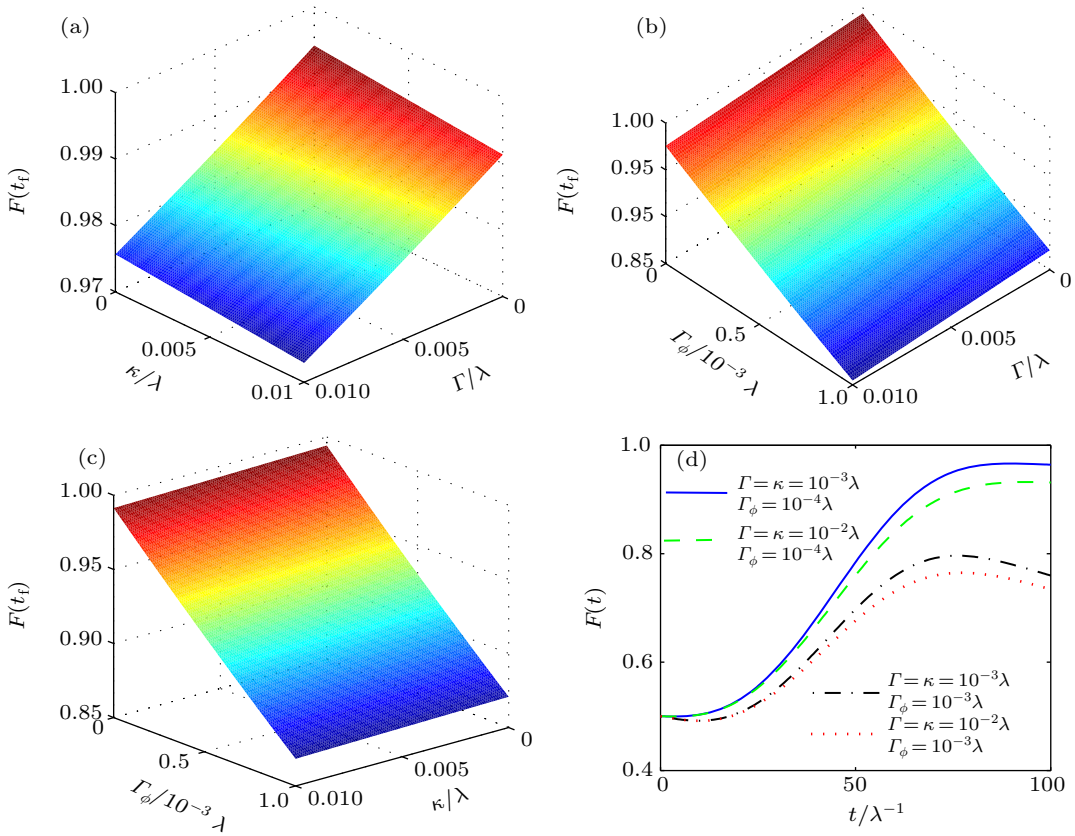


Fig. 6. (color online) (a) Final fidelity versus κ/λ and Γ/λ with $\Gamma_\phi = 0$. (b) Final fidelity versus Γ_ϕ/λ and Γ/λ with $\kappa = 0$. (c) Final fidelity versus Γ_ϕ/λ and κ/λ with $\Gamma = 0$. (d) Time dependence of the fidelity in the presence of three decoherence factors.

decoherence factors in Fig. 6(d). Through comparing the four curves and the corresponding parameters, we can also deduce that the STA scheme is sensitive to the decoherence caused by SQs dephasing. However, the scheme we proposed is still very robust against decoherence. As shown in Fig. 6(b), the final fidelity is beyond 0.85 even when $\Gamma = 0.01\lambda$ and $\Gamma_\phi = 0.001\lambda$. Combined with recent experimental parameters $\{T_{\Gamma_\phi} \sim 100\ \mu s, \lambda > 2\pi \times 300\ \text{MHz}, \lambda/\kappa > 10^2, \lambda/\Gamma > 10^2\}$ and $T_{\Gamma_\phi} = 1/\Gamma_\phi$,[67] i.e., $\{\kappa/\lambda < 10^{-2}, \Gamma/\lambda < 10^{-2}, \Gamma_\phi/\lambda < 10^{-5}\}$, the GHZ state among three SQs can be generated with a relatively high final fidelity over 97.16%.

5. Conclusion

We have implemented the fast generation of the GHZ state among three SQs by using invariants-based shortcuts. Compared with the STIRAP scheme, the STA scheme we proposed greatly shortens the operation time. The STA scheme needs to neither interpolate the system Hamiltonian nor add auxiliary driving pulses. Besides, the driving pulses are two harmonic pulses which can be obtained very easily in experiment. In addition, the adequate numerical simulations show that the STA scheme is robust against variations in control parameters and decoherence caused by SQs energy relaxations and CPWRs photon leakages. Although the scheme is somewhat sensitive to SQs dephasing, the GHZ state among three SQs can still be achieved with a relatively high final fidelity by adopting recent experimental parameters.

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