Improved control for distributed parameter systems with time-dependent spatial domains utilizing mobile sensor–actuator networks*

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A guidance policy for controller performance enhancement utilizing mobile sensor–actuator networks (MSANs) is proposed for a class of distributed parameter systems (DPSs), which are governed by diffusion partial differential equations (PDEs) with time-dependent spatial domains. Several sufficient conditions for controller performance enhancement are presented. First, the infinite dimensional operator theory is used to derive an abstract evolution equation of the systems under some rational assumptions on the operators, and a static output feedback controller is designed to control the spatial process. Then, based on Lyapunov stability arguments, guidance policies for collocated and non-collocated MSANs are provided to enhance the performance of the proposed controller, which show that the time-dependent characteristic of the spatial domains can significantly affect the design of the mobile scheme. Finally, a simulation example illustrates the effectiveness of the proposed policy.

Keywords: distributed parameter systems, time-dependent spatial domains, mobile actuator–sensor networks, Lyapunov stability

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1. Introduction

The wireless sensor networks (WSNs) are very frequently used in many networked control systems (NCSs) for the economic efficiency and flexibility in modularization of the networks, such as distributed parameter estimation in sensor networks,[1–3] diffusion source tracking,[4] and some special missions taking full advantages of WSNs.[5–7] Among the theoretical researches for the sensor networks, one of the fundamental problems is how to design controllers or filters to improve the control performance or estimation performance under some environment. The performance improvement here means faster convergence rate of the system state or lower computational complexity of the algorithm. These problems on NCSs were studied by many researchers with aid of various tools such as linear matrix inequalities, T-S fuzzy model, and sum-of-squares method.[8] In addition, another effective way to deal with these problems is the use of mobile sensor–actuator networks (MSANs), which are WSNs composed of mobile agents with sensing or actuating capabilities, as it provides an extra dimension to make the best use of the mobile agents.[9,10]

However, it is noted that in the majority of the NCSs mentioned above, the target systems to be estimated or controlled are the finite dimensional systems described by ordinary differential equations (ODEs). Nevertheless, in practice, most of the physical processes are infinite dimensional systems described by PDEs, which are commonly called the DPSs with their dynamics depending both on spatial position and time.[11,12] Specifically, many industrial control problems, crystal growth,[13] liquid solidification,[14] and gas–solid reaction systems,[15] can be depicted by parabolic PDEs with time-dependent spatial domains (i.e., moving boundaries). Such time-dependent characteristics of the boundaries may cause variation of observability and controllability with time and space, which is not encountered in DPSs with fixed boundaries. These characteristics are usually described by convective transport owing to the variation of the length of the domain with time. And thus, such characteristics, if not appropriately accounted for in the synthesis of the observer or controller, may lead to poor performance of the systems.[11,16,17] However, although the aforementioned literatures demonstrated that the moving controller can lead to better performance of the controller, they gave neither a specific moving strategy of the mobile actuating-sensing devices to enhance the controller performance, nor the relationship between the moving strategy of the actuating-sensing devices and the moving boundaries.

In the past two decades, a lot of related studies in DPSs with fixed boundaries using MSANs have already been conducted, such as performance enhancement in control or estimation,[18,19] reduction in power consumption,[20] and mobile robot's collision avoidance.[21] Many earlier works that make use of MSANs in DPSs were presented in Refs. [22]–[25]. The optimality criteria for estimation and control scheme to enhance the supervisory control scheme were built in

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Ref. [23]. Pioneering work that considered spatially point-wise distributed moving agents was presented in Ref. [24]. Most of the theoretic properties of DPSs with mobile controls were basically investigated by Khapalov in Refs. [25]– [27]. Subsequently, a new algorithm called central voronoi tessellations (CVT) was proposed to solve the dual problem of parameter estimation for parabolic PDEs based on MSANs by Chao *et al*. in Refs. [28]– [30]. While the most relevant works were explored by Demetriou *et al.* in Refs. [31]– [37]. A new methodological framework was presented by means of a group of closely placed point-wise sensors and a single point-wise actuator in Ref. [31]. Based on the Lyapunov stability theory, Demetriou first integrated MSANs for DPS in Ref. [33], in which the sensing and actuating devices were assumed to be collocated and a guidance law for the MSANs was presented to enhance the controller performance. The optimal switching policy and estimation for performance enhancement of distributed parameter systems were discussed in a similar fashion in Refs. [18] and [38], respectively. However, to the best of the authors' knowledge, few authors have considered the control problems for DPSs with moving boundaries based on MSANs in an abstract framework so far.

Motivated by the previous work, in this paper, we use an integrating abstract structure for the repositioning of mobile actuating and sensing devices (collocated and non-collocated), which are viewed as an integral part of the process. Specifically, the PDEs with moving boundaries that describe the process dynamics are viewed as an abstract evolution equation in an appropriate Hilbert space. The Lyapunov stability arguments for infinite dimensional systems are used to analytically achieve the main purpose that provides the guidance policy for each mobile agent so that the closed loop system performance is enhanced. The performance enhancement means that the system state converges to the equilibrium point faster with the help of moving agents satisfying the proposed guidance policy, compared with the agents dissatisfying the proposed policy (fixed in the domain, for example). In order to verify the proposed guidance policy and avoid on-line computations of the dynamic output feedback controller, a static output feedback control law is used in this paper, and by means of which there will be no need to implement a real-time state estimator for its computationally expensive observer gains.

The remainder of this paper is organized as follows. First, an abstract framework of the considered system model is formulated and the problem under consideration is redescribed under this framework, which is to be addressed with the aids of MSANs in Section 2. In Section 3, the guidance law for each moving agent is proposed under the conditions of collocated and non-collocated MSANs respectively. Several sufficient conditions to enhance the performance based on the proposed static output feedback controller are obtained. Finally, the effectiveness of the proposed guidance policy is successfully evaluated through an example of a diffusion process in Section 4 and conclusions follow in Section 5.

2. Preliminaries and problem formulation

Consider a class of DPS with time-dependent spatial domain described by the following m-input, m-output parabolic PDE in one spatial dimension:

$$\frac{\partial x(t,z)}{\partial t} = \frac{\partial}{\partial z}\left(\alpha(z)\frac{\partial x(t,z)}{\partial z}\right) + \sum_{i=1}^{m} b_i(z;z_i^{\mathrm{a}}(t))u_i(t), \quad (1)$$

$$y_i(t) = \int_0^{h(t)} c_i(z;z_i^{\mathrm{s}}(t))x(t,z)\mathrm{d}z, \quad i = 1,2,\ldots,m, \quad (2)$$

subjected to the initial condition

$$x(0,z) = x_0(z), \quad (3)$$

and the boundary conditions

$$\begin{aligned} &c_1 x(t,0) + d_1\frac{\partial x}{\partial z}(t,0) = r_1,\\ &c_2 x(t,h(t)) + d_2\frac{\partial x}{\partial z}(t,h(t)) = r_2. \end{aligned} \quad (4)$$

In the aforementioned DPS, $x(t,z) \in \mathbf{R}$ denotes the state variable of the process, in which $t \in \mathbf{R}^+$ and $z \in [0,h(t)]$ are the time and the spatial coordinates, respectively, and $h(t)$ is the moving boundary of the time-dependent spatial domain of DPS, which is denoted as $\Omega(t)$. The function $b_i(z;z_i^{\mathrm{a}}(t))$ is a known smooth function of (t,z), which describes how the i-th control action $u_i(t)$ is distributed in the spatial domain $\Omega(t)$, while $u_i \in \mathbf{R}^m$ denotes the associated manipulated input. The spatial point $z_i^{\mathrm{a}}(t) \in \Omega(t)$ denotes the time varying centroid of the i-th actuator. Similarly, $c_i(z;z_i^{\mathrm{s}}(t))$ is a known smooth function of (t,z) which describes how the i-th moving sensor is distributed in the same domain $\Omega(t)$, and is determined by the location and type of the measurement sensor (e.g., point or distributed sensing), and the spatial point $z_i^{\mathrm{s}}(t) \in \Omega(t)$ denotes the time varying centroid of the i-th moving sensor. The time dependence of both $z_i^{\mathrm{a}}(t)$ and $z_i^{\mathrm{s}}(t)$ describes the time variation of their locations within the spatial domain. $y_i \in \mathbf{R}$ denotes the measured output of the i-th moving sensor. $x_0(z)$ is the initial condition, and $c_1, d_1, c_2, d_2, r_1, r_2$ are real numbers.

It is observed that the moving boundary $h(t)$ plays a powerful role in the above DPS. A schematic of a typical process with moving boundary can be found in Ref. [16], in the case of moving actuators and sensors. To simplify the discussion in this manuscript, it is necessary to assume that the moving boundary $h(t)$ is known and smooth. Precisely, the assumption on the properties of $h(t)$ and $\dot{h}(t)$ is given below.

Assumption 1 $h(t)$ is a known and smooth function of time which satisfies $\dot{h}(t) > 0$ and $h(t) \in (0, h_{\max}]$, $\forall t \in [0,\infty)$, where $h_{\max}$ is finite and denotes the maximum length of the one dimensional spatial domain $\Omega(t)$.

To simplify the presentation and employ Lyapunov stability methods for the DPS with time-dependant spatial domain, following Refs. [16] and [33], the PDE systems of Eqs. (1)–(4) are formulated as an evolution equation in an appropriate Hilbert space $\mathscr{H}_{2,\Omega(t)}$, which is defined on $\Omega(t)=[0,h(t)]$, with the inner product

$$\langle \phi_1,\phi_2\rangle=\int_{\Omega(t)}\phi_1(z)\phi_2(z)\,\mathrm{d}z,$$

and norm

$$\|\phi\|_2=\langle\phi,\phi\rangle^{1/2},$$

where

$$\mathscr{H}_{2,\Omega(t)}\triangleq\{\phi:\Omega(t)\mapsto\mathbf{R}\ \text{ and }\ \|\phi\|_2<\infty\},$$

and the space derivatives' definitions are defined in the space $\mathscr{H}_{2,\Omega(t)}$.

Define the state $x(t)$ on $\mathscr{H}_{2,\Omega(t)}$ as

$$x(t)=x(t,z),\ \ t>0,\ \ z\in[0,h(t)],$$

the system's second order time-varying operator $\mathscr{A}(t)$ and its domain as

$$\mathscr{A}(t)x=\frac{\partial}{\partial z}\left(\alpha(z)\frac{\partial x}{\partial z}\right)+\dot{h}(t)\frac{z}{h(t)}\frac{\partial x}{\partial z},$$
$$\alpha(z)>0,\ \ x\in\mathrm{Dom}(\mathscr{A}),$$

$\mathrm{Dom}(\mathscr{A}(t))=\{x\in\mathscr{H}_{2,\Omega(t)}|\ x,\ \partial x/\partial z$ abs. continuous, $\partial^2x/\partial z^2\in\mathscr{H}_{2,\Omega(t)}$, and $c_1x(t,0)+d_1(\partial x/\partial z)(t,0)=r_1$, $c_2x(t,h(t))+d_2(\partial x/\partial z)(t,h(t))=r_2\}$. The m input operators are

$$\langle\mathscr{B}_i(z_i^{\mathrm{a}}(t))u_i(t),x\rangle=\int_0^{h(t)}b_i(z;z_i^{\mathrm{a}}(t))x(z)u_i(t)\,\mathrm{d}z,$$
$$\text{or}\ \ \mathscr{B}_i(z_i^{\mathrm{a}}(t))u_i(t)=b_i(z;z_i^{\mathrm{a}}(t))u_i(t),\ \ i=1,\ldots,m,$$

and the output operators are

$$\mathscr{C}_i(z_i^{\mathrm{s}}(t))x=\langle\mathscr{C}_i(z_i^{\mathrm{s}}(t)),x\rangle=\int_0^{h(t)}c_i(z;z_i^{\mathrm{s}}(t))x(z)\,\mathrm{d}z,\ \ i=1,\ldots,m.$$

It is straightforward to observe that the output operator is equal to the adjoint of the input operator, i.e., $\mathscr{C}=\mathscr{B}^*$, when $b_i(z)=c_i(z)$. Then, in Hilbert space $\mathscr{H}_{2,\Omega(t)}$, the DPS of Eqs. (1)–(4) may easily be expressed in the following abstract form:

$$\begin{aligned}&\dot{x}(t)=\mathscr{A}(t)x(t)+\mathscr{B}(z^{\mathrm{a}}(t))u(t),\ \ x(0)=x_0,\\&y=\mathscr{C}(z^{\mathrm{s}}(t))x(t),\end{aligned}\tag{5}$$

where $x_0=x_0(z)$.

Remark 1 Referring to Assumption 1, smoothness of $h(t)$ is needed for the well-posedness of the solution for the initial value problem (5) in $\mathscr{H}_{2,\Omega(t)}$. $\dot{h}(t)\geq 0$ implies that the time-dependent spatial domain $\Omega(t)$ is always moving "outwards", which is also in accordance with some actual situations. More details can be found in Ref. [16].

Remark 2 For the case in which the spatial coordinate is more than one dimension, two and three spatial dimensions for instance, the same infinite dimensional abstract framework technique may be employed as for the system represented by Eq. (5), and hence no generality is lost by considering one dimension system in this paper.

Additionally, for the employment of Lyapunov stability arguments, it is assumed that the state operator $\mathscr{A}(t)$ satisfies the following properties of boundedness, coercivity, and symmetry.[33] Furthermore, another assumption is imposed on state operator $\mathscr{A}(t)$ due to its time-varying characteristic.

Assumption 2 The state operator $\mathscr{A}(t)$ has the following properties:

(A1) bounded: the operator $\mathscr{A}(t)$ is bounded, i.e., there exists $\alpha>0$ such that

$$|\langle\mathscr{A}(t)\phi,\phi\rangle|\leq\alpha\|\phi\|^2,\ \ \text{for}\ \ \phi\in\mathscr{H}_{2,\Omega(t)};$$

(A2) coercive: the operator $-\mathscr{A}(t)$ is coercive, i.e.,

$$\langle-\mathscr{A}(t)\phi,\phi\rangle\geq\mu\|\phi\|^2,\ \text{for some positive}\ \mu\ \text{and}\ \phi\in\mathscr{H}_{2,\Omega(t)};$$

(A3) symmetric: the operator $\mathscr{A}(t)$ is symmetric, i.e.,

$$\langle\mathscr{A}(t)\phi,\phi\rangle=\overline{\langle\phi,\mathscr{A}(t)\phi\rangle}\ \ \text{for all}\ \ \phi\in\mathscr{H}_{2,\Omega(t)};$$

(A4) differentiable: the operator $\mathscr{A}(t)$ is differentiable and $\langle\dot{\mathscr{A}}(t)\phi,\phi\rangle\geq 0$, for $\phi\in\mathscr{H}_{2,\Omega(t)}$, where $\dot{\mathscr{A}}(t)$ is the derivative of the operator $\mathscr{A}(t)$ with respect to time.

Remark 3 The above four assumptions of state operator $\mathscr{A}(t)$ can naturally simplify the Lyapunov stability analysis in Section 3. It should be pointed out that the differentiability of operator $\mathscr{A}(t)$ in (A4) is necessary for the well-posedness of evolution equation (5) when $\mathscr{A}(t)$ is time variant. And $\langle x(t),\dot{\mathscr{A}}(t)x(t)\rangle\geq 0$ guarantees the asymptotic stability of the open-loop DPS system, which overwhelmingly simplifies the Lyapunov stability analysis. In fact, if the inequality $\langle x(t),\dot{\mathscr{A}}(t)x(t)\rangle\geq 0$ is not satisfied, one may still obtain similar conclusions as the derived theorems 1 and 2 in this paper, providing that the assumption of boundedness is made on $\dot{\mathscr{A}}(t)$.

In this paper, the following static output feedback control law is considered:

$$u_i(t)=-k_iy_i(t)=-k_i\int_0^{h(t)}c_i(z;z_i^{\mathrm{s}}(t))x(t,z)\,\mathrm{d}z,\tag{6}$$

for $k_i>0$, $i=1,\ldots,m$, implemented by the actuating devices, which may make the state close to zero in an appropriate norm. Comparing to the feedback controller fixed in space, the moving actuating devices will be able to enhance the system performance more effectively. That is because they

will be able to have more control authority than the devices fixed in the domain. Significant effects on performance enhancement via the use of moving or scheduled actuating devices in both thermal and structural systems were presented in Refs. [31] and [33]. Furthermore, when the spatial domains of the process are time-dependent, the feedback controller which is fixed in the time-dependent domains may lead to poor performance or closed-loop instability.[11,16] However, although it is proved in Ref. [16] that moving actuating devices have better effects than the controllers fixed in the spatial domains, it did not give a specific scheduled speed expression of the moving actuators–sensors.

Therefore, motivated by Refs. [31] and [33], the problem under consideration in this paper can now be stated as follows: Given the DPS with time-dependent spatial domains (1)–(4) and its abstract evolution expression (5), for a given static output feedback control law (6) implemented by collocated and non-collocated mobile sensing-actuating devices, respectively, find the trajectories (or velocities) of the mobile agents and the relationships among the derived velocities and the known conditions of the DPS with time-dependent spatial domains, so that the norm of the state $x(t,z)$ will converge to the equilibrium point faster than it would with the employment of fixed actuating-sensing devices.

In addition, for the purpose to further enhance the system performance with the aid of a network of mobile sensors and actuators, some additional assumptions on the network should be made to partially simplify the control problem of the DPS with moving boundaries.

Assumption 3[33] The network is homogeneous, that is, $b_i(z) = b(z)$ and $c_i(z) = c(z)$, $i = 1,\ldots,m$, which means that the mobile agents are different only at the locations. It is also assumed that each agent is massless and inertialess, so there is no need to consider its kinematic equations in its travel within $\Omega(t)$.

Assumption 4 (Network with simple topology structure) It is assumed that $\Omega(t)$ is simply connected and there is no exchange of information between the i-th agent and the j-th agent for $i \neq j$, which implies that the i-th sensor only conveys information to the i-th controller/actuator to implement the corresponding control behavior.

Remark 4 Assumption 3 and Assumption 4 are mainly to simplify the proof in Section 3. However, there are random failures and various network-induced limitations at the device layer in any practical network-based industrial processes.[8,39] Such questions over the design of controllers/filters of DPSs with time-dependent spatial domains based on MSANs will be studied by the authors in the future.

3. Main results

In this section, two theorems will be presented and accurately proved, which mainly contain the guidance policy for the improved control of DPS with time-dependent spatial domains. First, the following basic identity named Leibniz–Reynolds transport theorem (one dimension) is introduced which is of importance in the ensuing development.

Lemma 1[40] Let $\Omega(t) = [a(t), b(t)]$, and for all $z \in \Omega(t)$, the function $t \to F(t,z)$ is continuously differentiable for arbitrary $t < \infty$, it holds

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\Omega(t)} F(t,z)\mathrm{d}z\right)\\ &= \int_{\Omega(t)} F_t(t,z)\mathrm{d}z + F(t,b(t))\frac{\partial b(t)}{\partial t} - F(t,a(t))\frac{\partial a(t)}{\partial t}.\end{aligned} \tag{7}$$

3.1. Guidance of collocated MSANs

Definition 1 (Collocated actuator–sensor networks) The MSANs are said to be collocated if

$$z_i^{\mathrm{a}}(t) = z_i^{\mathrm{s}}(t) \ \text{ with } \ b(z; z_i^{\mathrm{a}}(t)) = c(z; z_i^{\mathrm{s}}(t)), \ \ i = 1,\ldots,m,$$

where $z_i^{\mathrm{s}}(t)$ and $z_i^{\mathrm{a}}(t)$ denote the spatial coordinates of the i-th actuating device and the i-th sensing device, respectively.

As mentioned in Ref. [33], the above collocated condition can significantly simplify the stability analysis, the controller design, and the proposed mobile sensor–actuator guidance scheme. Specifically, in this paper, to analytically obtain the Lyapunov-based guidance scheme, a representative spatial distribution is given by

$$\begin{aligned}b(z; z^{\mathrm{c}}) &= H_{z^{\mathrm{c}}-\varepsilon}(z) - H_{z^{\mathrm{c}}+\varepsilon}(z)\\ &= \begin{cases}1, & \text{if } z \in [z^{\mathrm{c}}-\varepsilon, z^{\mathrm{c}}+\varepsilon],\\ 0, & \text{otherwise,}\end{cases}\end{aligned} \tag{8}$$

where $H_a(z) \triangleq H(z-a)$ is the Heaviside function, $\varepsilon > 0$ is a small positive constant, and z^{c} denotes the spatial coordinate of the collocated actuator–sensor agent. In this case, each sensing device can both provide the measurement information by measurement function $y_i(t)$ and the state information $x(t, z^{\mathrm{c}}-\varepsilon)$ and $x(t, z^{\mathrm{c}}+\varepsilon)$ at the two edges of the sensor's range. In practice, the spatial distribution of the actuator may not be the same as the spatial distribution of the sensing device, such a case will be shown in Theorem 2.

The following theorem provides an explicit formula and conditions that ensure controller performance enhancement by the collocated MSANs.

Theorem 1 Consider the DPS system of Eqs. (1)–(4) with infinite-dimensional representation of Eq. (5) satisfying Assumptions 1-4, where the collocated sensor–actuator spatial distribution is given by Eq. (8). Then the proposed mobile policy for each agent enhances the controller performance in

the sense that the static output feedback control law (6) asymptotically stabilizes the spatial process, if the proposed mobile policy for each agent is given by

$$\dot{z}_i^{\mathrm{c}}(t) = \gamma_i \frac{\beta c_0^2}{\Delta x(t, z_i^{\mathrm{c}}(t))} \frac{\dot{h}(t)}{h(t)}, \tag{9}$$

where $\gamma_i \geq 1/(2mk_i y_i(t))$, $i = 1, \ldots, m$, is the velocity gain for each agent, $\Delta x(t, z_i^{\mathrm{c}}(t)) = (x(t, z_i^{\mathrm{c}}(t) + \varepsilon) - x(t, z_i^{\mathrm{c}}(t) - \varepsilon)$ and y_i are supposed to be nonzero, and $c_0 > 0, \beta > 0$ are constants.

Proof If the static output feedback law (6) is considered, then system (5) becomes

$$\begin{aligned}
\dot{x}(t) &= \mathscr{A}(t)x(t) - \sum_{i=1}^{m} k_i \mathscr{B}_i(z_i^{\mathrm{c}}(t)) \mathscr{B}_i^*(z_i^{\mathrm{c}}(t)) x(t) \\
&= (\mathscr{A}(t) - \mathscr{B}(z^{\mathrm{c}}(t)) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t))) x(t) \\
&= \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t),
\end{aligned} \tag{10}$$

where the positive scalars k_i are user-defined feedback gains and $K = \mathrm{diag}\{k_i\}$. Choose the following parameter-dependent time variant Lyapunov functional:

$$V(t) = -\langle x(t), \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t) \rangle.$$

First, the positive definiteness of $V(t)$ is directly obtained from the boundedness and coercivity of the closed loop operator $\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))$, which is similar to the proof in Ref. [33]. For the following derivation of the guidance policy for each agent, we only reprove the boundedness of the operator $\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))$ here. Indeed, using assumption (A1), we have

$$\begin{aligned}
|V(t)| &= |-\langle x(t), \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t) \rangle| \\
&= |\langle x(t), (\mathscr{A}(t) - \mathscr{B}(z^{\mathrm{c}}(t)) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t))) x(t) \rangle| \\
&\leq \alpha \|x(t)\|^2 + |\langle x(t), \mathscr{B}(z^{\mathrm{c}}(t)) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t)) x(t) \rangle| \\
&\leq \alpha \|x(t)\|^2 + \lambda_{\max}(\boldsymbol{K}) |\langle \mathscr{B}^*(z^{\mathrm{c}}(t)) x(t), \mathscr{B}^*(z^{\mathrm{c}}(t)) x(t) \rangle| \\
&\leq \alpha \|x(t)\|^2 + \lambda_{\max}(\boldsymbol{K}) \|\mathscr{B}^*(z^{\mathrm{c}}(t))\|^2 \|x(t)\|^2 \\
&= (\alpha + \lambda_{\max}(\boldsymbol{K}) \|\mathscr{B}^*(z^{\mathrm{c}}(t))\|^2) \|x(t)\|^2 \\
&= \beta \|x(t)\|^2,
\end{aligned} \tag{11}$$

where $\lambda_{\max}(\boldsymbol{K})$ denotes the maximum eigenvalue of matrix $\boldsymbol{K}$ and $\beta = \alpha + \lambda_{\max}(\boldsymbol{K}) \|\mathscr{B}^*(z^{\mathrm{c}}(t))\|^2 > 0$.

Due to fact that both $\mathscr{A}(t)$ and $\mathscr{B}(z^{\mathrm{c}}(t)) \mathscr{B}^*(z^{\mathrm{c}}(t))$ are self-adjoint, which implies $\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))$ is self-adjoint, and with the aids of Lemma 1, the derivative of the Lyapunov functional along the trajectories of Eq. (10) is

$$\begin{aligned}
\dot{V}(t) &= -\int_0^{h(t)} \frac{\partial}{\partial t} \left[x(t) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t) \right] \mathrm{d}z \\
&\quad - x(t) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t)|_{z=h(t)} \dot{h}(t) \\
&\quad + x(t) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t)|_{z=0} \dot{0} \\
&= -\langle \dot{x}(t), \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t) \rangle - \langle x(t), \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) \dot{x}(t) \rangle \\
&\quad - \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} (\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))) x(t) \rangle \\
&\quad - x(t) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t)|_{z=h(t)} \dot{h}(t) \\
&= -\langle \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t), \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t) \rangle \\
&\quad - \langle x(t), \mathscr{A}_{\mathrm{cl}}(z^{\mathrm{c}}(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t) \rangle \\
&\quad - \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} (\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))) x(t) \rangle \\
&\quad - x(t, h(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t, h(t)) \dot{h}(t) \\
&= -2 \|\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t)\|^2 \\
&\quad - \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} (\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))) x(t) \rangle \\
&\quad - x(t, h(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t, h(t)) \dot{h}(t) \\
&= -2 \|\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t)\|^2 - \langle x(t), \dot{\mathscr{A}}(t) x(t) \rangle \\
&\quad + \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} \left(\mathscr{B}(z^{\mathrm{c}}(t)) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t)) \right) x(t) \rangle \\
&\quad - x(t, h(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t, h(t)) \dot{h}(t).
\end{aligned} \tag{12}$$

To draw the final result, due to the fact that time and spatial dimensions in the DPS are two independent variables, the last term $x(t, h(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t, h(t)) \dot{h}(t)$, which is merely the function of time, can be written as

$$\begin{aligned}
&x(t, h(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t, h(t)) \dot{h}(t) \\
&= \frac{\dot{h}(t)}{h(t)} \int_0^{h(t)} x(t, h(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t, h(t)) \, \mathrm{d}z \\
&= \frac{-V(t, h(t)) \dot{h}(t)}{h(t)}.
\end{aligned} \tag{13}$$

Thus, equation (12) becomes

$$\begin{aligned}
\dot{V}(t) &= -\langle \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t), \mathscr{A}_{\mathrm{cl}}(z^{\mathrm{c}}(t)) x(t) \rangle \\
&\quad - \langle x(t), \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) \mathscr{A}_{\mathrm{cl}}(z^{\mathrm{c}}(t)) x(t) \rangle \\
&\quad - \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} (\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))) x(t) \rangle \\
&\quad - \frac{\dot{h}(t)}{h(t)} \int_0^{h(t)} x(t, h(t)) \mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t, h(t)) \, \mathrm{d}z \\
&= -2 \|\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t)\|^2 - \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} (\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t))) x(t) \rangle \\
&\quad + \frac{V(t, h(t)) \dot{h}(t)}{h(t)} \\
&= -2 \|\mathscr{A}_{\mathrm{cl}}(t, z^{\mathrm{c}}(t)) x(t)\|^2 - \langle x(t), \dot{\mathscr{A}}(t) x(t) \rangle \\
&\quad + \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} \left(\mathscr{B}(z^{\mathrm{c}}(t)) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t)) \right) x(t) \rangle \\
&\quad + \frac{V(t, h(t)) \dot{h}(t)}{h(t)}.
\end{aligned} \tag{14}$$

The first two terms and the last term in Eq. (14) are definite non-positive and positive based on assumption (A4) and Assumption 1, respectively. The third term when written explicitly in terms of the integral representation becomes

$$\begin{aligned}
&\langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} \left(\mathscr{B}(z^{\mathrm{c}}(t)) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t)) \right) x(t) \rangle \\
&= 2 \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t} (\mathscr{B}(z^{\mathrm{c}}(t))) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t)) x(t) \rangle \\
&= 2 \langle x(t), \dot{z}^{\mathrm{c}}(t) \frac{\partial}{\partial z^{\mathrm{c}}(t)} (\mathscr{B}(z^{\mathrm{c}}(t))) \boldsymbol{K} \mathscr{B}^*(z^{\mathrm{c}}(t)) x(t) \rangle
\end{aligned}$$

$$
\begin{aligned}
&= 2\int_0^{h(t)} \dot{z}^{\mathrm{c}}(t)\frac{\partial}{\partial z^{\mathrm{c}}(t)}(\mathscr{B}(z;z^{\mathrm{c}}(t)))x(t,z)\boldsymbol{K}\mathscr{B}^{*}\\
&\quad\times(z;z^{\mathrm{c}}(t))x(t,z)\,\mathrm{d}z\\
&= 2\sum_{i=1}^{m}\dot{z}_i^{\mathrm{c}}(t)\int_0^{h(t)}\frac{\partial}{\partial z_i^{\mathrm{c}}(t)}(H_{z_i^{\mathrm{c}}-\varepsilon}(z)-H_{z_i^{\mathrm{c}}+\varepsilon}(z))\\
&\quad\times x(t,z)k_iy_i(t)\,\mathrm{d}z\\
&= 2\sum_{i=1}^{m}\dot{z}_i^{\mathrm{c}}(t)k_iy_i(t)\int_0^{h(t)}(\delta_{z_i^{\mathrm{c}}-\varepsilon}(z)-\delta_{z_i^{\mathrm{c}}+\varepsilon}(z))x(t,z)\,\mathrm{d}z\\
&= 2\sum_{i=1}^{m}\dot{z}_i^{\mathrm{c}}(t)k_iy_i(t)(x(t,z_i^{\mathrm{c}}(t)-\varepsilon)-x(t,z_i^{\mathrm{c}}(t)+\varepsilon)). \qquad (15)
\end{aligned}
$$

So, by Eqs. (14) and (15), if the following inequality holds:

$$
\begin{aligned}
&2\sum_{i=1}^{m}\dot{z}_i^{\mathrm{c}}(t)k_iy_i(t)(x(t,z^{\mathrm{c}}(t)-\varepsilon)-x(t,z^{\mathrm{c}}(t)+\varepsilon))\\
&\quad+\frac{V(t,h(t))\dot{h}(t)}{h(t)}\le 0\\
\Leftrightarrow\ &2\sum_{i=1}^{m}\dot{z}_i^{\mathrm{c}}(t)k_iy_i(t)(x(t,z^{\mathrm{c}}(t)-\varepsilon)-x(t,z^{\mathrm{c}}(t)+\varepsilon))\\
&\quad\le-\frac{V(t,h(t))\dot{h}(t)}{h(t)}, \qquad (16)
\end{aligned}
$$

the derivative of the Lyapunov functional will be negative semi-definite. From Eq. (11), we can obtain

$$
\begin{aligned}
V(t,h(t)) &= -\langle\mathscr{A}_{\mathrm{cl}}(t,z^{\mathrm{c}}(t))x(t,h(t)),x(t,h(t))\rangle\\
&\le \beta\|x(t,h(t))\|^2, \qquad (17)
\end{aligned}
$$

and using the boundary conditions (4) with absolutely continuous $\partial x/\partial z$, which implies $\partial x/\partial z$ has an upper bound $M>0$, we have

$$
\begin{aligned}
\|x(t,h(t))\| &= \frac{1}{|c_2|}\|r_2-d_2\frac{\partial x}{\partial z}(t,h(t))\|\\
&\le \left|\frac{r_2}{c_2}\right|+\left|\frac{d_2}{c_2}\right|\left|\frac{\partial x}{\partial z}(t,h(t))\right|\\
&\le \frac{|r_2|+|d_2|M}{|c_2|}.
\end{aligned}
$$

Let $c_0:=(|r_2|+|d_2|M)/|c_2|$, we obtain

$$
V(t,h(t))\le\beta c_0^2. \qquad (18)
$$

Hence, with both Eqs. (16) and (18), if only the velocity of each agent satisfies the following inequality:

$$
\begin{aligned}
&2\sum_{i=1}^{m}\dot{z}_i^{\mathrm{c}}(t)k_iy_i(t)(x(t,z^{\mathrm{c}}(t)-\varepsilon)-x(t,z^{\mathrm{c}}(t)+\varepsilon))\\
&\le-\frac{\beta c_0^2\dot{h}(t)}{h(t)}, \qquad (19)
\end{aligned}
$$

the derivative of the Lyapunov functional is negative semi-definite.

Therefore, if $\Delta x(t,z_i^{\mathrm{c}}(t))\triangleq x(t,z^{\mathrm{c}}(t)+\varepsilon)-x(t,z^{\mathrm{c}}(t)-\varepsilon)$ and $y_i(t)$ are assumed to be nonzero, and the velocity of each mobile agent is chosen as

$$
\dot{z}_i^{\mathrm{c}}(t)=\gamma_i\frac{\beta c_0^2}{\Delta x(t,z_i^{\mathrm{c}}(t))}\frac{\dot{h}(t)}{h(t)}, \qquad (20)
$$

where $\gamma_i\ge 1/(2mk_iy_i(t))$, $i=1,\ldots,m$, it is easy to verify that inequality (19) holds and thus we obviously obtain $\dot{V}(t)\le 0$ without identically vanishing for any nonzero solution of the initial condition. This completes the proof.

Remark 5 Equation (9) shows that the deduced guidance velocity of each agent based on DPS with moving boundary has connection with user defined control gain k_i, the boundary $h(t)$ and its moving rate $\dot{h}(t)$, which is different from Ref. [33]. It is the main originality of this article. It should be mentioned that when $\dot{h}(t)\equiv 0$, i.e., the boundary of the process is fixed, the guidance law (9) reduces to the result in Ref. [33]. Thus, the time-varying property of the boundary can be exactly exhibited by the proposed guidance policy, and the result in Theorem 1 is more general than that in Ref. [33].

Remark 6 (Evaluation of the parameters in Eq. (9) Since the closed-loop operator $\mathscr{A}_{\mathrm{cl}}(t,z^{\mathrm{c}}(t))$ satisfies $-\langle\mathscr{A}_{\mathrm{cl}}(t,z^{\mathrm{c}}(t))x(t),x(t)\rangle\le\beta\|x(t)\|^2$, the constant β can be evaluated by off-line numerical simulations

$$
\begin{aligned}
\beta &= \sup_{\|x\|\neq 0}\frac{\langle-\mathscr{A}_{\mathrm{cl}}(t,z^{\mathrm{c}}(t))x,x\rangle}{\|x\|^2}\\
&= \sup_{\|x\|=1}\langle-\mathscr{A}_{\mathrm{cl}}(t,z^{\mathrm{c}}(t))x,x\rangle. \qquad (21)
\end{aligned}
$$

The parameter c_0 is determined by the boundary condition $x(t,h(t))$, i.e., $c_0\ge\|x(t,h(t))\|$, which is supposed to be nonzero here. And one may notice that the velocity gain for each agent, which must satisfy $\gamma_i\ge 1/(2mk_iy_i(t))$, chosen to be $\gamma_i=1/(mk_iy_i(t))$ for example, is a decentralized adaptation since each γ_i uses the output information only from its own sensing device.

Remark 7 According to Fermat's lemma, Assumption 1 means that $\dot{h}(t)\to 0$ as $t\to\infty$ which renders the proposed policy to be ultimately stable. And we claim that the nonzero conditions for $\Delta x(t,z_i^{\mathrm{c}}(t))$ and $y_i(t)$ are quite conservative, so, the proposed policy $\dot{z}_i^{\mathrm{c}}(t)$ can be modified to

$$
\dot{z}_i^{\mathrm{c}}(t)=\begin{cases}\gamma_i\dfrac{\beta c_0^2}{\Delta x(t,z_i^{\mathrm{c}}(t))}\dfrac{\dot{h}(t)}{h(t)}, & \text{if } \Delta x(t,z_i^{\mathrm{c}}(t))y_i(t)\neq 0,\\ 0, & \text{otherwise.}\end{cases} \qquad (22)
$$

In this case, if $\dot{z}_i^{\mathrm{c}}(t)=0$ for all $i=1,\ldots,m$ occur only at the time when the system is ultimately stabilized, i.e., $\dot{z}_i^{\mathrm{c}}(t)$ are not equal to zero simultaneously when the system is not stabilized, inequality (19) will still hold, which guarantees that the proposed guidance law (22) is effective.

3.2. Guidance of non-collocated MSANs

The above collocated condition significantly simplifies the system analysis and controller design. However, as mentioned in the above subsection, one may not be able to attain such a collocated case, because in practice one often uses the

sensing device to measure information from external environment and transmit the collected data to controllers/actuators, then the controllers/actuators perform actions to change the behavior of the physical systems, and this leads to the spatial distribution of the actuating device being different from the spatial distribution of the sensing device. Inspired by the previous work, in this subsection, a guidance law for the optimization of mobile actuating and sensing devices which are not collocated is developed to control DPSs with moving boundaries.

Definition 2 (Non-collocated actuator–sensor networks) The non-collocated networks in this article mean that there are differences in position and distribution for actuators and sensors, i.e.,

$$z_i^{\mathrm{a}}(t) \neq z_i^{\mathrm{s}}(t) \ \text{ and } \ b_i(z;z_i^{\mathrm{s}}(t)) \neq c_i(z;z_i^{\mathrm{s}}(t)), \ \ i=1,\ldots,m.$$

Specifically, for simplifying the guidance scheme of the mobile non-collocated actuator–sensor agents, the spatial distribution of the actuating device is taken to be a spatial delta function

$$\begin{aligned} b(z;z^{\mathrm{a}}) &= \delta_{z^{\mathrm{a}}}(z) \\ &\approx \begin{cases} \dfrac{1}{2\varepsilon_{\mathrm{a}}}, & \text{if } z\in\Omega(z^{\mathrm{a}})=[z^{\mathrm{a}}-\varepsilon_{\mathrm{a}},z^{\mathrm{a}}+\varepsilon_{\mathrm{a}}], \\ 0, & \text{otherwise} \end{cases} \\ &= \frac{H_{z^{\mathrm{a}}-\varepsilon_{\mathrm{a}}}(z)-H_{z^{\mathrm{a}}+\varepsilon_{\mathrm{a}}}(z)}{2\varepsilon_{\mathrm{a}}}, \end{aligned} \tag{23}$$

and the spatial distribution of the sensing device is taken to be the boxcar function

$$\begin{aligned} c(z;z^{\mathrm{s}}) &= H_{z^{\mathrm{s}}-\varepsilon_{\mathrm{s}}}(z)+H_{z^{\mathrm{s}}-\varepsilon_{\mathrm{s}}}(z) \\ &= \begin{cases} 1, & \text{if } z\in\Omega(z^{\mathrm{s}})=[z^{\mathrm{s}}-\varepsilon_{\mathrm{s}},z^{\mathrm{s}}+\varepsilon_{\mathrm{s}}], \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{24}$$

Similar to Theorem 1, Theorem 2 provides a sufficient condition that ensures controller performance enhancement by the non-collocated MSANs. We still use k_i and γ_i as the guidance gains without causing confusion.

Theorem 2 Consider the DPS system of Eqs. (1)–(4) with infinite-dimensional representation of Eq. (5) satisfying Assumptions 1–4, where the non-collocated sensor–actuator spatial distributions are given by Eqs. (23) and (24). Then the proposed mobile policy for each agent enhances the controller performance in the sense that the static output feedback control law (6) asymptotically stabilizes the spatial process, if the proposed guidance law for each mobile agent satisfies the following inequality:

$$v_{\mathrm{s}}(\dot{z}_i^{\mathrm{s}}(t))+v_{\mathrm{a}}(\dot{z}_i^{\mathrm{a}}(t)) \leq -\frac{\beta c_0^2\dot{h}(t)}{h(t)}, \tag{25}$$

where $v_{\mathrm{s}}(\dot{z}_i^{\mathrm{s}}(t)) \triangleq \sum_{i=1}^m \dot{z}_i^{\mathrm{s}}(t)k_i x(t,z_i^{\mathrm{a}}(t))(-\Delta x(t,z_i^{\mathrm{s}}(t))$, $v_{\mathrm{a}}(\dot{z}_i^{\mathrm{a}}(t)) \triangleq \sum_{i=1}^m \dot{z}_i^{\mathrm{a}}(t)k_i y_i(t)(-x_z(t,z_i^{\mathrm{a}}))$, and $x_z(t,z_i^{\mathrm{a}}) = (\partial x(t,z)/\partial z)\,|_{z=z_i^{\mathrm{a}}}$.

Proof Consider the system with the static output feedback control law (6),

$$\begin{aligned} \dot{x}(t) &= \mathscr{A}(t)x(t)-\sum_i^m k_i\mathscr{B}_i(z_i^{\mathrm{a}}(t))\mathscr{C}_i(z_i^{\mathrm{s}}(t))x(t) \\ &= (\mathscr{A}(t)-\mathscr{B}(z^{\mathrm{a}}(t))\boldsymbol{K}\mathscr{C}(z^{\mathrm{s}}(t)))x(t) \\ &= \mathscr{A}_{\mathrm{nc}}(t,z^{\mathrm{a}}(t),z^{\mathrm{s}}(t))x(t), \end{aligned} \tag{26}$$

where the positive scalars k_i are user-defined feedback gains and $\boldsymbol{K}=\mathrm{diag}\{k_i\}$. Choose the following parameter-dependent time variant Lyapunov functional:

$$V(t) = -\langle x(t), \mathscr{A}_{\mathrm{nc}}(t,z^{\mathrm{a}}(t),z^{\mathrm{s}}(t))x(t)\rangle.$$

The verification for positive definiteness of $V(t)$ is similar to that in Theorem 1, and so does the derivative of the Lyapunov functional along the trajectories of Eq. (26). Hence, the Lyapunov derivative is negative semi-definite if only the following inequality holds:

$$\langle x(t), \frac{\mathrm{d}}{\mathrm{d}t}\big(\mathscr{B}(z^{\mathrm{a}}(t))\boldsymbol{K}\mathscr{C}(z^{\mathrm{s}}(t))\big)x(t)\rangle \leq -\frac{\beta c_0^2\dot{h}(t)}{h(t)}. \tag{27}$$

The left side of the above inequality (27) is calculated as

$$\begin{aligned} &\langle x(t), \frac{\mathrm{d}}{\mathrm{d}t}\big(\mathscr{B}(z^{\mathrm{a}}(t))\boldsymbol{K}\mathscr{C}(z^{\mathrm{s}}(t))\big)x(t)\rangle \\ &= \langle x(t), \frac{\mathrm{d}}{\mathrm{d}t}(\mathscr{B}(z^{\mathrm{a}}(t)))\boldsymbol{K}\mathscr{C}(z^{\mathrm{s}}(t))x(t)\rangle \\ &\quad + \langle x(t), \mathscr{B}(z^{\mathrm{a}}(t))\boldsymbol{K}\frac{\mathrm{d}}{\mathrm{d}t}(\mathscr{C}(z^{\mathrm{s}}(t)))x(t)\rangle \\ &= \langle x(t), \dot{z}^{\mathrm{a}}(t)\frac{\partial(\mathscr{B}(z^{\mathrm{a}}(t)))}{\partial z^{\mathrm{a}}(t)}\boldsymbol{K}\mathscr{C}(z^{\mathrm{s}}(t))x(t)\rangle \\ &\quad + \langle x(t), \dot{z}^{\mathrm{s}}(t)\mathscr{B}(z^{\mathrm{a}}(t))\boldsymbol{K}\frac{\partial(\mathscr{C}(z^{\mathrm{s}}(t)))}{\partial z^{\mathrm{s}}(t)}x(t)\rangle. \end{aligned} \tag{28}$$

In Eq. (28), the first term determines the actuator's velocity and is the same as that in Theorem 1 when $\mathscr{C}=\mathscr{B}^*$, so we obtain

$$\begin{aligned} &\langle x(t), \dot{z}^{\mathrm{a}}(t)\frac{\partial(\mathscr{B}(z^{\mathrm{a}}(t)))}{\partial z^{\mathrm{a}}(t)}\boldsymbol{K}\mathscr{C}(z^{\mathrm{s}}(t))x(t)\rangle \\ &= \sum_{i=1}^m \dot{z}_i^{\mathrm{a}}(t)k_i y_i(t)\frac{x(t,z_i^{\mathrm{a}}(t)-\varepsilon_{\mathrm{a}})-x(t,z_i^{\mathrm{a}}(t)+\varepsilon_{\mathrm{a}})}{2\varepsilon_{\mathrm{a}}}. \end{aligned} \tag{29}$$

Let $\varepsilon_{\mathrm{a}}\to 0$, the above equation becomes

$$\begin{aligned} &\langle x(t), \dot{z}^{\mathrm{a}}(t)\frac{\partial(\mathscr{B}(z^{\mathrm{a}}(t)))}{\partial z^{\mathrm{a}}(t)}\boldsymbol{K}\mathscr{C}(z^{\mathrm{s}}(t))x(t)\rangle \\ &= \sum_{i=1}^m \dot{z}_i^{\mathrm{a}}(t)k_i y_i(t)(-x_z(t,z_i^{\mathrm{a}})), \end{aligned} \tag{30}$$

where

$$x_z(t,z_i^{\mathrm{a}}) = \lim_{\varepsilon_{\mathrm{a}}\to 0}\frac{x(t,z_i^{\mathrm{a}}(t)+\varepsilon_{\mathrm{a}})-x(t,z_i^{\mathrm{a}}(t)-\varepsilon_{\mathrm{a}})}{2\varepsilon_{\mathrm{a}}}.$$

The second term determines the sensors' velocities and is analytically discussed as follows:

$$\langle x(t), \mathscr{B}(z^{\mathrm{a}}(t))\boldsymbol{K}\frac{\mathrm{d}}{\mathrm{d}t}(\mathscr{C}(z^{\mathrm{s}}(t)))x(t)\rangle$$

$$
\begin{aligned}
&= \int_0^{h(t)} \dot{z}^{\mathrm{s}}(t)\mathscr{B}(z;z^{\mathrm{a}}(t))x(t,z)\boldsymbol{K}\frac{\partial(\mathscr{C}(z;z^{\mathrm{s}}(t)))}{\partial z^{\mathrm{s}}(t)}x(t,z)\mathrm{d}z \\
&= \int_0^{h(t)} \dot{z}^{\mathrm{s}}(t)b(z;z^{\mathrm{a}}(t))x(t,z)\boldsymbol{K} \\
&\quad \times \left[\int_0^{h(t)} \frac{\partial(c(z;z^{\mathrm{s}}(t)))}{\partial z^{\mathrm{s}}(t)}x(t,z)\mathrm{d}z\right]\mathrm{d}z \\
&= \sum_{i=1}^{m} \dot{z}_i^{\mathrm{s}}(t)k_i[x(t,z_i^{\mathrm{s}}(t)-\varepsilon_{\mathrm{s}})-x(t,z_i^{\mathrm{s}}(t)+\varepsilon_{\mathrm{s}})] \\
&\quad \times \int_0^{h(t)} x(t,z)\delta(z-z_i^{\mathrm{a}}(t))\mathrm{d}z \\
&= \sum_{i=1}^{m} \dot{z}_i^{\mathrm{s}}(t)k_i x(t,z_i^{\mathrm{a}}(t))[x(t,z_i^{\mathrm{s}}(t)-\varepsilon_{\mathrm{s}})-x(t,z_i^{\mathrm{s}}(t)+\varepsilon_{\mathrm{s}})] \\
&= \sum_{i=1}^{m} \dot{z}_i^{\mathrm{s}}(t)k_i x(t,z_i^{\mathrm{a}}(t))(-\Delta x(t,z_i^{\mathrm{s}}(t)). \qquad (31)
\end{aligned}
$$

Let

$$
v_{\mathrm{s}}(\dot{z}_i^{\mathrm{s}}(t)) \triangleq \sum_{i=1}^{m} \dot{z}_i^{\mathrm{s}}(t)k_i x(t,z_i^{\mathrm{a}}(t))(-\Delta x(t,z_i^{\mathrm{s}}(t)), \qquad (32)
$$

$$
v_{\mathrm{a}}(\dot{z}_i^{\mathrm{a}}(t)) \triangleq \sum_{i=1}^{m} \dot{z}_i^{\mathrm{a}}(t)k_i y_i(t)(-x_z(t,z_i^{\mathrm{a}})), \qquad (33)
$$

then, from Eqs. (30) and (31), it is easy to verify that inequality (27) holds if

$$
v_{\mathrm{s}}(\dot{z}_i^{\mathrm{s}}(t)) + v_{\mathrm{a}}(\dot{z}_i^{\mathrm{a}}(t)) \le -\frac{\beta c_0^2 \dot{h}(t)}{h(t)},
$$

which implies $\dot{V}(t) \le 0$. The proof is completed.

Remark 8 It can be clearly observed that, in Eqs. (32) and (33), $v_{\mathrm{s}}(\dot{z}_i^{\mathrm{s}}(t))$ is related to state value $x(t,z_i^{\mathrm{a}}(t))$ at i-th actuator location, and, $v_{\mathrm{a}}(\dot{z}_i^{\mathrm{a}}(t))$ is related to $y_i(t)$ determined by state measured value $x(t,z_i^{\mathrm{s}}(t))$ at the i-th sensor location, which implies that $v_{\mathrm{s}}(\dot{z}_i^{\mathrm{s}}(t))$ and $x(t,z_i^{\mathrm{a}}(t))$ are coupled. Such a case may increase the computational burden of the measuring sensors and is hard to implement, which results in a key challenge that is the optimal performance versus computational optimality with reduced controller design complexity.

As a special case, when the prior velocity weight is assigned for each mobile agent, it is easy to obtain the following corollary.

Corollary 1 If the user defined velocity weights $w_{a,i} \ge 0$, $w_{s,i} \ge 0$ with $\sum_{i=1}^{m}(w_{a,i}+w_{s,i}) = 1$, are prior assigned for each moving non-collocated agent, each actuating device implements the static output feedback control law (6), and the guidance policy for each moving agent is chosen as

$$
\dot{z}_i^{\mathrm{a}}(t) = w_{a,i}\gamma_{a,i}\frac{\beta c_0^2 \dot{h}(t)}{h(t)}\frac{1}{x_z(t,z_i^{\mathrm{a}}(t))}, \qquad (34)
$$

$$
\dot{z}_i^{\mathrm{s}}(t) = w_{s,i}\gamma_{s,i}\frac{\beta c_0^2 \dot{h}(t)}{h(t)}\frac{1}{\Delta x(t,z_i^{\mathrm{s}}(t))}, \qquad (35)
$$

where $\gamma_{a,i} \ge 1/(k_i y_i)$ and $\gamma_{s,i} \ge 1/(k_i x(t,z_i^{\mathrm{a}}(t)))$, then the proposed guidance policies (34) and (35) enhance the performance of the proposed output feedback controller.

Remark 9 Corollary 1 is directly derived from the conclusion of Theorem 2. And it is worth mentioning that Corollary 1 degenerates to Theorem 1 when the non-collocated distributions of the mobile actuator–sensors are changed to be collocated and $w_{\mathrm{s}} = w_{\mathrm{a}}$, $\gamma_{\mathrm{s}} = \gamma_{\mathrm{a}}$.

Remark 10 It should be noted that the proposed velocity policy in Theorem 2 depends on the gradient (spatial derivative) $x_z(t,z_i(t))$ at the point $z_i(t)$. In practical situations, one may approximatively obtain the gradient information by

$$
x_z(t,z_i(t)) \approx \frac{1}{2\varepsilon}(x(t,z_i(t)+\varepsilon)-x(t,z_i(t)-\varepsilon))
$$

under the aforementioned assumption that each agent can measure the state value at the two edges of the agent's range.

4. Numerical example

In this section, to verify the effectiveness of the proposed guidance scheme, we consider a diffusion process with time-dependent spatial domain described by one dimensional parabolic PDE (1)–(4), in which the moving boundary is considered in $\Omega(t) = [0,h(t)]$, and $h(t) = 1.5 - 0.4\,\mathrm{e}^{-0.02t^{2.7}}$ satisfies Assumption 1. The coefficient of the diffusion operator is $\alpha = 0.011$. The initial condition $x(0,z) = \sin(3z)\,\mathrm{e}^{-3z^2}$, the parameters $\beta = 0.2$ and $c_0 = 0.09$ which are determined by Remark 5 and the boundary condition value, respectively. The spatial process with collocated MSANs is considered here, and, as for the non-collocated case, it can also be verified to be correct as long as the locations and distribution functions of the non-collocated actuator/sensor devices are properly set. For the considered collocated case, the i-th controller has a gain of $k_i = k = 30$, $i = 1,\ldots,m$. The closed loop system is simulated in the time interval [0 s, 20 s] with 3 moving actuator-sensor agents, where the initial locations are $z_1(0) = 0.2$, $z_1(0) = 0.55$, and $z_3(0) = 1.1$. The adaptive velocity gains $\gamma_i = 3/2k_i y_i$, for $i = 1,2,3$.

For comparison, we also consider the cases of open-loop control and fixed control with three fixed actuators/controllers placed at the same initial positions. Hence, via MATLAB simulation technology, the open-loop profile of the state $x(t,z)$ and the closed-loop profiles with fixed control and moving control are depicted in Figs. 1(a)–1(c), respectively. Clearly, the controller implemented the proposed mobile scheme regulates the state profile $x(t,z)$ to zero faster than the fixed controller does.

Figure 2 shows the state $L_2(0,h(t))$ norms of the three cases aforementioned, by which the effects of the mobile agents on the controller performance improvement are well illustrated compared to the case of fixed-in-space agents. The point-wise convergence of the state distribution in space at four various time instances is examined for the above three cases in Fig. 3, which also shows that the distributed state controlled with the mobile actuator/sensor agents converges to zero much faster.

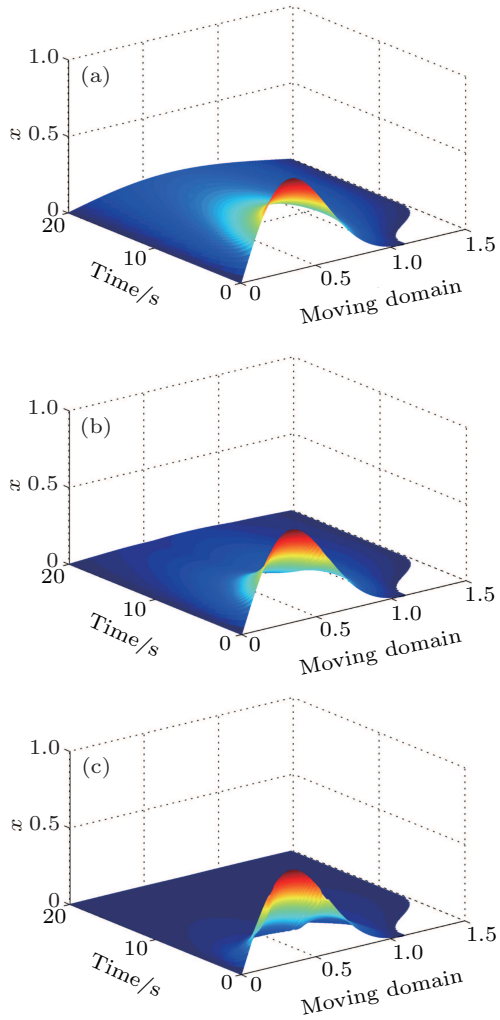


Fig. 1. (color online) Profiles of the state of the DPS with time-dependent spatial domains: (a) open-loop control, (b) closed-loop with fixed control, (c) closed-loop with moving control.

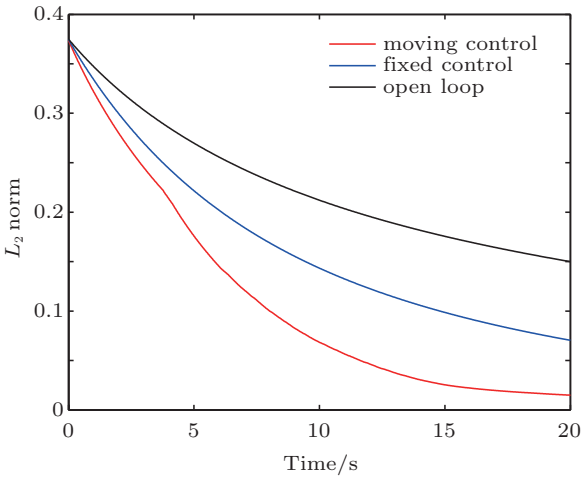


Fig. 2. (color online) Control of DPS with moving boundary: evolution of spatial L_2 norm.

Finally, the trajectories of the actuator/sensor agents for the fixed (the dotted line) and mobile (the solid line) cases are depicted in Fig. 4. It can be seen that the three agents move towards the center of the spatial domain under the guidance of the proposed moving policy, since the maximum value of the state $x(t,z)$ occurs at the location $z = h(t)/2$, (see Fig. 1(a)), and the three mobile agents are eventually stable in the domain when the system is stabilized.

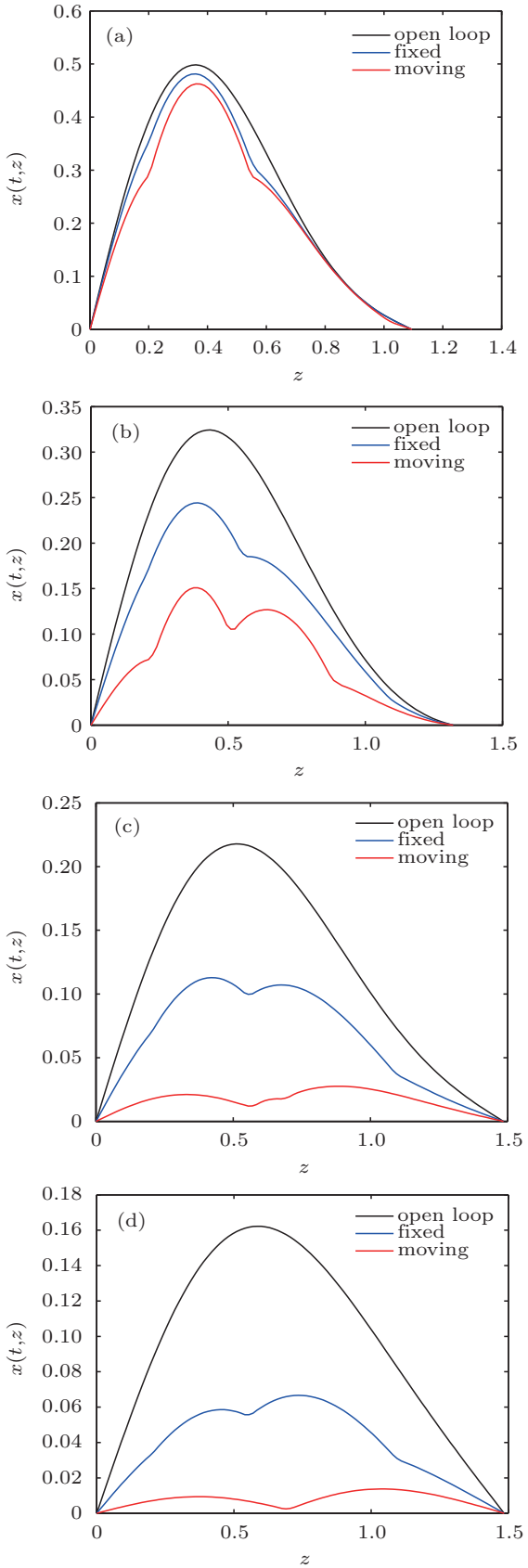


Fig. 3. (color online) Closed loop state distribution at various time instances: (a) $t = 1$ s, (b) $t = 4$ s, (c) $t = 8$ s, (d) $t = 12$ s.

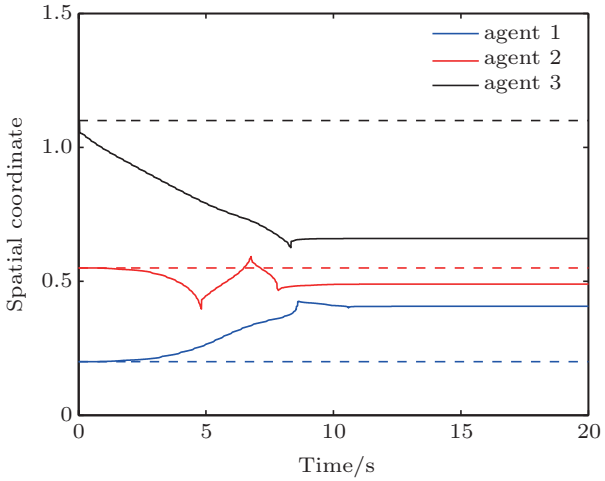


Fig. 4. (color online) Control of DPS with moving boundary: moving agents' trajectories.

5. Conclusion

This paper examines the effects of collocated and non-collocated MSANs on the state regulation for a diffusion process governed by PDEs with moving boundary. A repositioning policy is obtained for each moving device by means of the deduced velocity of each agent which is computed by using a static output feedback control strategy and Lyapunov stability arguments. The theoretical derivation and the results are based on a reasonable abstract framework, as well as some necessary assumptions which largely simplify the proof and make the Lyapunov-based guidance policy possible. It is observed that the guidance velocities of both collocated and non-collocated sensor–actuator agents are affected by the term $\dot{h}(t)/h(t)$, which exists in the operator $\mathscr{A}(t)$ owing to the variation of the length of the domain with time. Numerical simulations on a PDE system with moving boundary indicates that the proposed mobile output feedback control policy is effective to enhance the control performance of the spatial process.

It is noted that the agents are assumed to be massless and inertialess, the direct extension, agent collision avoidance for example, can be considered to relax these assumptions. The relaxations of Assumptions 3 and 4 have more practical significance as the asynchronous phenomena between the original plant and networked controllers/filters, or various network-induced limitations, are always existent in network-controlled processes. Meanwhile, it is also known that many industrial control processes have nonlinear characteristics, which may deteriorate the performance of controllers/filters or make the analysis and synthesis more difficult. Therefore, we will investigate the above issues in the context of the proposed abstract framework generated by parabolic DPS with time-dependent spatial domains utilizing MSANs.

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