Turbulence modulation model for gas–particle flow based on probability density function approach*

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The paper focuses on the turbulence modulation problem in gas–particle flow with the use of probability density function (PDF) approach. By means of the PDF method, a general statistical moment turbulence modulation model without considering the trajectory difference between two phases is derived from the Navier–Stokes equations. A new turbulence production term induced by the dispersed-phase is analyzed and considered. Furthermore, the trajectory difference between two media is taken into account. Subsequently, a new k–ε turbulence modulation model in dilute particle-laden flow is successfully set up. Then, the changes to several terms, including the turbulence production, dissipation, and diffusion terms, are well described consequently. The promoted model provides a more probable explanation for the modification of particles on the turbulence. Finally, we applied the model to simulate a gas–particle turbulence flow case in a wall jet, and found that the simulation results agree well with the experimental data.

Keywords: turbulence modulation model, PDF approach, gas–particle flow, turbulence flow

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1. Introduction

The modification of particles on the turbulence has been studied extensively and is one of the central topics in the field of the particle-laden turbulence dynamics. Up to now, much attention has been paid to their theory, modelling, and experimental studies. The experimental and direct numerical simulation (DNS) results, most of these studies are qualitative, have shown that the turbulence flow can be affected by several important factors, including the particle diameter,[1] the Stokes number,[2,3] the particle Reynolds number,[4,5] and the ratio of the particle diameter to the turbulence length scale (d_p/l_e).[6] Although great efforts have been made to understand the effects of particles on turbulence, due to the extremely complex interactions between the particles and turbulent structures, the mechanism of the turbulence modulation is still not clear at present.

It is difficult to carry out fully resolved simulations containing a large number of particles. Presently, the progress of quantitative studies on turbulence modulation is slowly. Principally, three types models describing the turbulence modulation have been put forward. The first one is the Reynolds averaging method,[7,8] in which the source term of the Reynolds stress equation is derived from multiplying the momentum source term by u_i and applying a Reynolds averaging procedure. It can be found that the kinetic energy source term from the emergence of particles is always negative, therefore the models based on this approach are only able to predict an attenuation of the carrier phase turbulence. In order to overcome the deficiency of the Reynolds averaging approach, the semi-empirical approach was proposed to include the effect of the particle wake. Consequently, some semi-empirical models,[9–11] with additional turbulence production terms were introduced. Although this method is simple and effective in dealing with the turbulence modulation problems, it was criticized for its lack of a theoretical basis. The third approach, namely the volume averaging method,[12] was introduced by means of the volume-averaged equations for the kinetic energy of the carrier phase. This approach yields a turbulent energy equation which contains the turbulence production and redistribution terms resulted from particles. However, the model in terms of this approach is only able to reflect the particle's enhancement effect on the turbulence. Later, Mandø,[13] divided the momentum source term into two parts. Correspondingly, he applied the Reynolds averaging method on one part and applied the volume averaging method on the other part. The hybrid method is able to predict the attenuation and augmentation of the carrier phase turbulence. However, it could not predict turbulence damping of the carrier phase for small particles.

Therefore, the turbulence modulation problem is still an open topic. Given most previous studies applies the PDF method to establish discrete models,[14–16] the PDF approach has played a prominent role in dealing with the two-phase turbulence flows. Moreover, Pope *et al.*[17,18] proposed a continuous phase PDF model for turbulent flows. In particular, Minier *et al.*[19] attempted to establish a unified PDF model of the two-phase turbulent flow. Unfortunately, the models do not give full consideration for the turbulence modulation prob-

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lem and the attenuation and augmentation of the carrier phase turbulence influenced by particles have not been explained in the PDF theory.

From the description mentioned above, one can find that, the PDF theory has been widely used to study the two-phase turbulence flow. However, utilizing it in the investigations of turbulence modulation problem has not been explored so far, and its advantages deserve to be further exhibited, which makes the studies meanwhile. The outline of the paper is follows: in Section 2, a general turbulence modulation model is established and then the trajectory differences between phases is taken into consideration. In Section 3, a brief description of the computational setup is presented, and the two-way coupling numerical analysis are discussed with a concrete comparison with the traditional methods. The conclusions of the paper are given in Section 4.

2. Turbulence modulation model based on PDF theory

In this section, we provided a general statistical moment turbulence modulation model based on the PDF approach, and build upon this to build a new k–ε turbulence modulation model. For the present research, we consider Newtonian viscous incompressible fluid flows. The fluid is assumed to be composed of a large number of 'fluid elements' or 'fluid particles' (each 'particle' is a very small amount of fluid with the same densities), and the turbulence is considered as a stochastic process containing Gaussian colored-noise. The dispersed-phase is treated as rigid spherical particles. For heavy particles where $\rho_{\rm p} \gg \rho_{\rm f}$, any interphase forces other than the drag force are ignored.

2.1. The statistical moments turbulence modulation model

In the PDF approach, the turbulence was generally described as a stochastic process. For any random variable $\boldsymbol{R}$, whose components can be expressed as R_i (or R_j, R_k), the mean and fluctuating values of the component R_i are denoted as $\overline{R_i}$ and r_i. The fluid-particle state vector is defined as $\boldsymbol{Z}(\boldsymbol{t}) = (\boldsymbol{x}(t), \boldsymbol{U}(t))$ (or $\boldsymbol{Z} = (\boldsymbol{x}, \boldsymbol{U})$), where $\boldsymbol{x}$ and $\boldsymbol{U}$ are the fluid positions and fluid velocities, respectively. The fluid Lagrange model was generally characterized at the stochastic level in the previous works.[20,21] In this paper, the Lagrangian model from Navier–Stokes equations for 'fluid particles' are written as

$$\frac{{\rm d}x_i}{{\rm d}t} = U_i, \tag{1}$$

$$\frac{{\rm d}U_i}{{\rm d}t} = \underbrace{-\frac{1}{\rho_{\rm f}}\frac{\partial P}{\partial x_i} + \nu \triangle U_i}_{A_{{\rm f},i}} + A_{{\rm p},i}. \tag{2}$$

In Eq. (2), the first two terms on the right-hand side are the accelerations of the fluid particles due to the pressure and shear force. The drift vectors $A_{{\rm p},i}$, which is induced by the presence of particles, is given by[19]

$$A_{{\rm p},i} = \begin{cases} 0, & \text{with a probability } 1-\alpha_{\rm p}, \\ \chi\dfrac{V_i - U_i}{\tau_{\rm p}}, & \text{with a probability } \alpha_{\rm p}, \end{cases} \tag{3}$$

where $\chi = \alpha_{\rm p}\rho_{\rm p}/\alpha_{\rm f}\rho_{\rm f}$, P and ν are the fluid pressure and the kinematic viscosity, V_i and $\tau_{\rm p}$ are the particle velocity and particle response time, $\alpha_{\rm f}$ and $\alpha_{\rm p}$ refer to the phase volume fractions of the fluid and particles, $\rho_{\rm f}$ and $\rho_{\rm p}$ are the fluid and particle densities. According to the Liouville's theorem, the fluid transitional PDF $P_{\rm f}(t;\boldsymbol{x},\boldsymbol{U})$ verifies the following equation corresponding to Lagrange model (Eq. (1) and Eq. (2)):

$$\begin{aligned} &\frac{\partial P_{\rm f}}{\partial t} + \frac{\partial (U_i P_{\rm f})}{\partial x_i} \\ &= -\frac{\partial (A_{{\rm f},i} P_{\rm f})}{\partial U_i} - \frac{\partial (\langle A_{{\rm p},i} \mid \boldsymbol{x}, \boldsymbol{U}\rangle P_{\rm f})}{\partial U_i}, \end{aligned} \tag{4}$$

where $\langle A_{{\rm p},i} \mid \boldsymbol{x}, \boldsymbol{U}\rangle$ represents the conditional expectation of the random variable $A_{{\rm p},i}$ for a given value of $(\boldsymbol{x}, \boldsymbol{U})$. By changing the coordinates in the fluid velocity space, $\boldsymbol{u} = \boldsymbol{U} - \overline{\boldsymbol{U}}$, the differential equation of $p_{\rm f}(t;\boldsymbol{x},\boldsymbol{u})$ can be obtained from the differential equation of $P_{\rm f}(t;\boldsymbol{x},\boldsymbol{U})$. During the derivation, the following equations need to be used[19]

$$\frac{\partial P(\boldsymbol{U})}{\partial t} = \frac{\partial P(\boldsymbol{u})}{\partial t} - \frac{\partial \overline{U_i}}{\partial t}\frac{\partial P(\boldsymbol{u})}{\partial u_i}, \tag{5}$$

$$\frac{\partial P(\boldsymbol{U})}{\partial x_i} = \frac{\partial P(\boldsymbol{u})}{\partial x_i} - \frac{\partial \overline{U_j}}{\partial x_i}\frac{\partial P(\boldsymbol{u})}{\partial u_j}. \tag{6}$$

In terms of Eq. (4), combined with Eq. (5) and Eq. (6), it is straightforward to obtain the Fokker–Planck equation

$$\begin{aligned} &\frac{{\rm d}p_{\rm f}}{{\rm d}t} + u_i\frac{\partial p_{\rm f}}{\partial x_i} \\ &= -\frac{\partial (A_{ij}p_{\rm f})}{\partial u_i} + \frac{{\rm d}\overline{U_i}}{{\rm d}t}\frac{\partial p_{\rm f}}{\partial u_i} + u_i\frac{\partial \overline{U_j}}{\partial x_i}\frac{\partial p_{\rm f}}{\partial u_j} \\ &\quad - \frac{\partial}{\partial u_i}\left[\frac{\chi}{\tau_{\rm p}}(\overline{V_i} - U_i)p_{\rm f}\right] - \frac{\partial}{\partial u_i}\left[\frac{\chi}{\tau_{\rm p}}\langle v_i|\boldsymbol{x},\boldsymbol{u}\rangle p_{\rm f}\right]. \end{aligned} \tag{7}$$

The method of solution for the unclosed term $\langle v_i|\boldsymbol{x},\boldsymbol{u}\rangle p_{\rm f}$ is one of the key issues in the PDF approach. Many successful methods have been developed[22–24] by assuming that the stochastic term had the white noise property. Here, the unclosed term is treated by applying the Gaussian integration by parts[25]

$$\begin{aligned} \langle v_i|\boldsymbol{x},\boldsymbol{u}\rangle p_{\rm f}(t;x_i,u_i) &= \int v_i p_{\rm f}(t;x_i,u_i,v_i)\,{\rm d}v_i \\ &= -\overline{v_i u_j}\frac{\partial p_{\rm f}}{\partial u_j} - \overline{v_i x_j}\frac{\partial p_{\rm f}}{\partial x_j}. \end{aligned} \tag{8}$$

The statistical moments equation for the fluid phase is then obtained by multiplying Eq. (7) with $\mathscr{H}_{\rm f}$ and integrating it over the fluid velocity $\boldsymbol{U}$, that is:

$$\frac{{\rm d}(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{\mathscr{H}_{\rm f}})}{{\rm d}t} + \frac{\partial (\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{u_i\mathscr{H}_{\rm f}})}{\partial x_i}$$

$$= \overline{\alpha_{\rm f}}\rho_{\rm f}\overline{A_{ij}\frac{\partial \mathscr{H}_{\rm f}}{\partial u_i}} - \overline{\alpha_{\rm f}}\rho_{\rm f}\frac{{\rm d}\overline{U_i}}{{\rm d}t}\overline{\frac{\partial \mathscr{H}_{\rm f}}{\partial u_i}} - \overline{\alpha_{\rm f}}\rho_{\rm f}\frac{\partial \overline{U_j}}{\partial x_i}\overline{\frac{\partial (u_i \mathscr{H}_{\rm f})}{\partial u_j}}$$
$$+ \overline{\alpha_{\rm p}}\rho_{\rm p}\overline{\frac{V_i - U_i}{\tau_{\rm p}}\frac{\partial \mathscr{H}_{\rm f}}{\partial u_i}} + \overline{\alpha_{\rm p}}\rho_{\rm p}\overline{\frac{v_i u_j}{\tau_{\rm p}}\frac{\partial^2 \mathscr{H}_{\rm f}}{\partial u_i \partial u_j}}$$
$$- \overline{\alpha_{\rm p}}\rho_{\rm p}\frac{\partial \overline{U_k}}{\partial x_j}\overline{\frac{v_i x_j}{\tau_{\rm p}}\frac{\partial^2 \mathscr{H}_{\rm f}}{\partial u_i \partial u_k}}$$
$$- \frac{\overline{\alpha_{\rm p}}\rho_{\rm p}}{\overline{\alpha_{\rm f}}\rho_{\rm f}}\overline{\frac{v_i x_j}{\tau_{\rm p}}}\frac{\partial}{\partial x_j}\left(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{\frac{\partial \mathscr{H}_{\rm f}}{\partial u_i}}\right). \tag{9}$$

By replacing the variable $\mathscr{H}_{\rm f}$ with 1, u_i, and $u_i u_j$, respectively, the mean continuity equation, momentum equations, and Reynolds stress equation can be obtained as follows:

$$\frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f})}{\partial t} + \frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f})\overline{U_i}}{\partial x_i} = 0, \tag{10}$$

$$\frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{U_i})}{\partial t} + \frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{U_i}\overline{U_j})}{\partial x_j}$$
$$= -\frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{u_i u_j})}{\partial x_j} - \overline{\alpha_{\rm f}}\frac{\partial \overline{P}}{\partial x_i} + \overline{\alpha_{\rm f}}\rho_{\rm f}\nu\frac{\partial^2 \overline{U_i}}{\partial x_j \partial x_j} + I^{\rm M}_{{\rm pf},i}, \tag{11}$$

$$\frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{u_i u_j})}{\partial t} + \frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{U_k}\overline{u_i u_j})}{\partial x_k}$$
$$= P_{ij} + \Pi_{ij} + D_{ij} + \varepsilon_{ij}, \tag{12}$$

where

$$I^{\rm M}_{{\rm pf},i} = \overline{\alpha_{\rm p}}\rho_{\rm p}[\frac{\overline{V_i} - \overline{U_i}}{\tau_{\rm p}} - \lambda_{ij}\frac{\partial \ln(\overline{\alpha_{\rm f}}\rho_{\rm f})}{\partial x_j}], \tag{13}$$

$$P_{ij} = \underbrace{-\overline{\alpha_{\rm f}}\rho_{\rm f}\left(\overline{u_i u_k}\frac{\partial \overline{U_j}}{\partial x_k} + \overline{u_j u_k}\frac{\partial \overline{U_i}}{\partial x_k}\right)}_{P_{{\rm I},ij}}$$
$$\underbrace{- \overline{\alpha_{\rm p}}\rho_{\rm p}\left(\frac{\overline{v_i x_k}}{\tau_{\rm p}}\frac{\partial \overline{U_j}}{\partial x_k} + \frac{\overline{v_j x_k}}{\tau_{\rm p}}\frac{\partial \overline{U_i}}{\partial x_k}\right)}_{P_{{\rm II},ij}}, \tag{14}$$

$$\varepsilon_{ij} = \underbrace{-2\overline{\alpha_{\rm f}}\rho_{\rm f}\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}}_{\varepsilon_{{\rm I},ij}} + \underbrace{\overline{\alpha_{\rm p}}\rho_{\rm p}\frac{\overline{v_i u_j} + \overline{u_i v_j} - 2\overline{u_i u_j}}{\tau_{\rm p}}}_{\varepsilon_{{\rm II},ij}}, \tag{15}$$

$$\Pi_{ij} = \overline{\alpha_{\rm f}}\overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}. \tag{16}$$

$$D_{ij} = -\frac{\partial}{\partial x_k}\left[\overline{\alpha_{\rm f}}\left(\overline{pu_i}\delta_{jk} + \overline{pu_j}\delta_{ik} + \rho_{\rm f}\overline{u_i u_j u_k}\right.\right.$$
$$\left.\left. - \rho_{\rm f}\nu\frac{\partial \overline{u_i u_j}}{\partial x_k}\right)\right]. \tag{17}$$

Here, Π_{ij} and D_{ij} can be found in Ref. [26]. The dissipation of $\overline{u_i u_j}$ is approximated as $\varepsilon_{{\rm I},ij} = 2\delta_{ij}\varepsilon/3$, where ε is the turbulent dissipation rate, that will be discussed in Subsection 2.2. A new unclosed term $\overline{v_i x_j}/\tau_{\rm p}$ (denoted as λ_{ij}) needs to be modelled. We predit that the ratio of the fluid displacement to the particle response time is proportional to the fluid fluctuating velocity, that is $(x_j/\tau_{\rm p}) \propto u_j$. Then a semi-empirical formula based on the dimension analysis theory can be written as follows:

$$\lambda_{ij} = \frac{\overline{v_i x_j}}{\tau_{\rm p}} = C_{\rm fp}\overline{v_i u_j}, \tag{18}$$

where $C_{\rm fp}$ is a model constant. The generation and dissipation of the turbulence are composed of four parts: $P_{{\rm I},ij}$ and $\varepsilon_{{\rm I},ij}$ are inherent production and dissipation, $P_{{\rm II},ij}$ and $\varepsilon_{{\rm II},ij}$ indicate the influence of particles. The previous studies found that the term $\varepsilon_{{\rm II},ij}$ is always negative, so it reflects the weakening effect of the particles on the fluid turbulence. The new term, $P_{{\rm II},ij}$, which has a similar form with the inherent production of the fluid in the case without particles, is of great significance to the turbulence modulation. The condition $P_{{\rm II},ij} > 0$, means the particles enhanced the turbulence, it induces an opposite effect with comparing with the case $P_{{\rm II},ij} < 0$. The differences between the new model with the previous works are shown in Table 1.

Table 1. Compression of different turbulence modulation models.

Model	Production term due to particles	Dissipation term due to particles
Reynolds averaging method	0	$\varepsilon_{{\rm II},ij}$
Volume averaging method	$\frac{\overline{\alpha_{\rm p}}\rho_{\rm p}}{\tau_{\rm p}}[2(\overline{U_i} - \overline{V_i})(\overline{U_j} - \overline{V_j}) + 2\overline{v_i v_j} - \overline{v_i u_j} - \overline{u_i v_j}]$	0
Hybrid method	$\frac{2\overline{\alpha_{\rm p}}\rho_{\rm p}}{\tau_{\rm p}}(\overline{U_i} - \overline{V_i})(\overline{U_j} - \overline{V_j})$	$\frac{2\overline{\alpha_{\rm p}}\rho_{\rm p}}{\tau_{\rm p}}(\overline{v_i v_j} - \overline{u_i u_j})$
Semi-empirical method	semi-empirical formulas	$\varepsilon_{{\rm II},ij}$
Present work	$P_{{\rm II},ij}$	$\varepsilon_{{\rm II},ij}$

2.2. The k–ε turbulence modulation model with different trajectories

The typical two-phase trajectories can be represented in Fig. 1. Suppose a discrete particle is located in the center of the fluid particle. The displacement vector is $\boldsymbol{r} = (\boldsymbol{V} - \boldsymbol{U})\delta t$ after a time step δt. Thus, the influence of the discrete particles on the fluid can be expressed as follows:

$$A_{{\rm p},i} = \begin{cases} 0, & \text{with probability } 1 - \alpha_{\rm p}, \\ \chi\left[\frac{V_i - U_i}{\tau_{\rm p}} + (V_j - U_j)\frac{\partial U_i}{\partial x_j}\right], & \text{with probability } \alpha_{\rm p}. \end{cases} \tag{19}$$

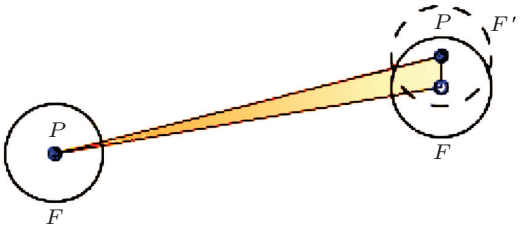


Fig. 1. (color online) The fluid and particle trajectories.

Similar to the model described by Eqs. (10)–(18), a turbulence modulation model with different trajectories can be achieved when the equation (3) is replaced by Eq. (19). The Reynolds stress is modelled by using the Boussinesq approximation,[27] and the volume fraction gradient is ignored for the dilute particle-laden flows. A generally accepted method to model the turbulent dissipation rate ε is the analogy method, which consider that the turbulent energy and dissipation equations have similar form of expressions.[28] In this case, the k–ε turbulence modulation model for different trajectories can be expressed as:

$$\frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f})}{\partial t}+\frac{\partial(\overline{\alpha_{\rm f}}\rho_{\rm f}\overline{U_i})}{\partial x_i}=0, \tag{20}$$

$$\begin{aligned}&\frac{\partial\overline{U_i}}{\partial t}+\overline{U_j}\frac{\partial\overline{U_i}}{\partial x_j}\\&=\frac{\partial}{\partial x_j}\left[(\nu+f_{\rm p}\nu_T)\left(\frac{\partial\overline{U_i}}{\partial x_j}+\frac{\partial\overline{U_j}}{\partial x_i}\right)\right]-\frac{1}{\rho_{\rm f}}\frac{\partial\overline{P}}{\partial x_i}+I^{\rm M}_{{\rm pf},i},\end{aligned} \tag{21}$$

$$\begin{aligned}&\frac{\partial k}{\partial t}+\overline{U_j}\frac{\partial k}{\partial x_j}\\&=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{f_{\rm p}\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k-\varepsilon+I^k_{\rm pf},\end{aligned} \tag{22}$$

$$\begin{aligned}&\frac{\partial\varepsilon}{\partial t}+\overline{U_j}\frac{\partial\varepsilon}{\partial x_j}\\&=\frac{\partial}{\partial x_j}\left[(\nu+\frac{f_{\rm p}\nu_T}{\sigma_\varepsilon})\frac{\partial\varepsilon}{\partial x_j}\right]+\left(C_{\varepsilon1}\frac{P_k}{\rho}-C_{\varepsilon2}\varepsilon+I^k_{\rm pf}\right)\frac{\varepsilon}{k},\end{aligned} \tag{23}$$

where

$$I^{\rm M}_{{\rm pf},i}=\chi\left[\frac{\overline{V_i}-\overline{U_i}}{\tau_{\rm p}}+(\overline{V_j}-\overline{U_j})\frac{\partial\overline{U_i}}{\partial x_j}\right], \tag{24}$$

$$I^k_{\rm pf}=\chi\left[\frac{\overline{u_iv_i}-2k}{\tau_{\rm p}}+(\overline{V_j}-\overline{U_j})\frac{\partial k}{\partial x_j}\right], \tag{25}$$

$$P_k=-(f_{\rm p}\overline{u_iu_k}+\chi\lambda^*_{ik})\frac{\partial\overline{U}_i}{\partial x_k}, \tag{26}$$

$$f_{\rm p}=1+\frac{\chi\tau_{\rm p}}{\tau^*+\tau_{\rm p}}, \tag{27}$$

$$\lambda^*_{ij}=C^*_{\rm fp}\overline{v_iu_j}. \tag{28}$$

Here, σ_k, σ_ε, $C_{\varepsilon1}$, $C_{\varepsilon2}$, and $C^*_{\rm fp}$ are model constants equal to 1.0, 1.3, 1.44, 1.92, and 1.2, respectively. As mentioned in Subsection 2.1, the term P_k contains the production of the turbulence from the effect of the dispersed phase. In the present model, the correction factor $f_{\rm p}$ modified the diffusion and production terms, and it is obvious that the correction factor approaches to 1, namely $f_{\rm p}\to1$ when $\tau_{\rm p}\to0$ (with $d_{\rm p}\to0$). It means that the model expressed in Eqs. (21)–(29) is changed to the standard k–ε model when $\tau_{\rm p}\to0$. Furthermore, the additional items $I^{\rm M}_{{\rm pf},i}$ and $I^k_{\rm pf}$ contain the contributions of the velocity and turbulent kinetic energy gradient, which reflect the effect of the different trajectories. The correlation of the two-phase fluctuation velocity in Eq. (28) is modelled by Lightstone's semi-empirical correlations[29]

$$\overline{u_iv_j}=\frac{\tau^*}{\tau^*+\tau_{\rm p}}\overline{u_iu_j}, \tag{29}$$

where

$$\begin{aligned}&\nu_{\rm T}=C_\mu\frac{k^2}{\varepsilon},\\&\frac{1}{\tau^*}=\frac{|\,U_i-V_i\,|}{LI}+\frac{1}{\tau_{LI}},\\&LI=2\tau_{LI}\sqrt{\frac{2k}{3}},\\&\tau_{LI}=0.135\frac{k}{\varepsilon},\end{aligned}$$

and $C_\mu=0.09$ is a model constant.

3. Numerical results and discussions

The wall jet case[30] has certain representativeness as a test of the gas–particle turbulence flow. The sketch of the geometry is shown in the following Fig. 2, and the corresponding parameters describing the case are represented in Table 2.

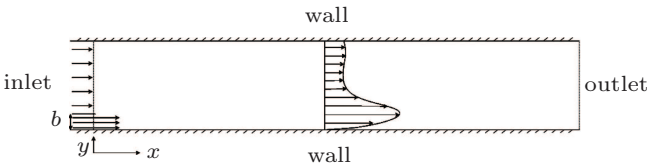


Fig. 2. Geometry of the Wall jet test case.

Table 2. Characteristics of the case.

Dimensions of the case	Width of jet	Jet velocity at nozzle exit	Freestream velocity
150 mm×100 mm×300 mm	5 mm	10 m·s^{-1}	2 m·s^{-1}
Jet Reynolds number	Particle diameter	Particle density	Particle mass loading
3300	49.4 μm	2590 kg·m^{-3}	0.1

Based on the two-dimensional flow equations, the computer program for the simulation of wall jet case is developed. The mesh is made up of 170×100 sites. The two-way coupled Eulerian/Lagrangian method is employed to predict the flow fields. The fluid phase is simulated by the new model as shown in Eqs. (20)–(29), and the Lagrangian model (30) is adopted to describe the particle phase.[31]

$$\begin{cases} \mathrm{d}\overline{y_i} = \overline{V_i}\,\mathrm{d}t, \\ \mathrm{d}\overline{V_i} = \dfrac{1}{\tau_\mathrm{p}}(\overline{U_i} - \overline{V_i})\,\mathrm{d}t, \\ \mathrm{d}\overline{v_i v_j} = \dfrac{2}{\tau_\mathrm{p}}(\overline{u_i v_j} - \overline{v_i v_j})\,\mathrm{d}t, \end{cases} \tag{30}$$

where, y_i is the particle positions. A brief description of the procedure developed in this work is presented here: (i) The Euler equations for fluid phase are solved initially by the SIMPLEC algorithm. (ii) The Lagrangian solver is adopted to simulate the track of particles and the source terms to the gas-phase governing equations are calculated. (iii) The Eulerian solver is run again to solve the turbulence modulation model by considering the presence of particles. (iv) The above Step (ii) to Step (iii) are repeated until final convergence. A more detailed description for solving the two-phase Eulerian/Lagrangian model can be found in Refs. [29].

Since the model in Eqs. (20)–(29) becomes the standard k–ε model when $\tau_\mathrm{p} \to 0$, the single-phase flow can be simulated by this model, which is shown in Fig. 3. One major deficiency of the standard k–ε model is that it cannot analyze the anisotropy of the turbulence because the turbulent viscosity coefficient is assumed to be isotropic. For the wall jet flow case, the dynamical characteristics in the free shear layer is more complicated, besides, the turbulence fluctuation shows a strong anisotropy. One can see that the simulative data have little difference with the actually measured data in the free shear layer, but on the whole the satisfactory result is achieved by using the k–ε model.

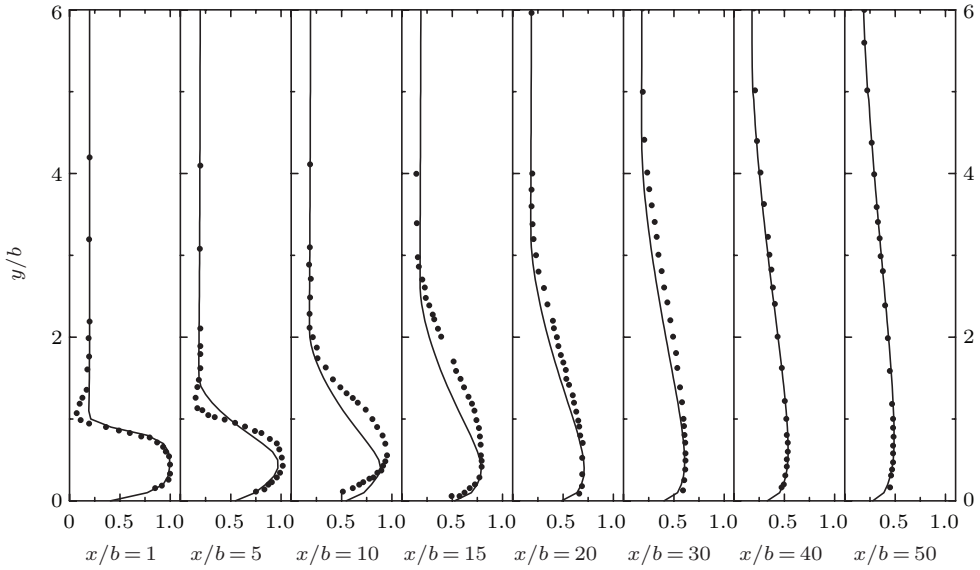


Fig. 3. Vertical mean fluid velocity profiles for the single-phase flow case. The solid line represents calculated results and is to be compared with experimental date (•).

Combined with the Lagrangian model for particles, the two-phase flow case is simulated in the use of the present turbulence modulation model. The simulation results are compared with that calculated from Reynolds averaging method. Correspondingly, the simulation results are shown in Fig. 4. It is found that the values of the mean fluid velocities calculated by using the Reynolds averaging method is slightly larger than that from experiment, and the results of the PDF method are better than those of the Reynolds averaging method near the wall and in fully developed turbulent flow region. Although the present model has not shown its obvious advantage in predicting fluid mean velocities, it performs very well in other respects. Figure 4(b) displays detailed comparisons of fluid fluctuating vertical velocities and the Reynolds stress, which shows that the calculation results from the present model are in accord with the experimental value.

The more accurate calculated results of the fluid phase will improve the simulation accuracy of the particle phase. The simulation results of particles are shown in Fig. 5. The simulation results of the Reynolds averaging method tender to be larger near the wall, while the results of the PDF method are much more accurate. Compared with the Reynolds averaging method and the experimental data, it indicates that our considered model is better than the Reynolds averaging model since it agrees well with the experimental data.

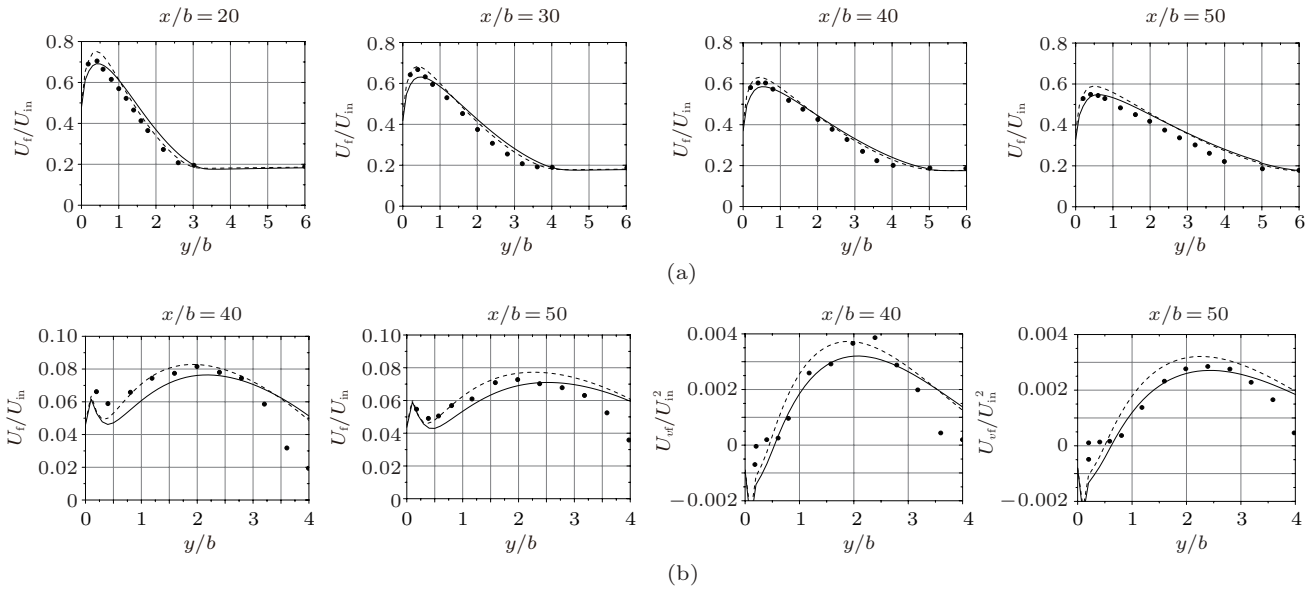


Fig. 4. Profiles for the two-phase flow case: (a) Fluid mean vertical velocity; (b) Fluid fluctuating vertical velocity and Reynolds stress. • experimental date; —— PDF method; ----- Reynolds averaging method.

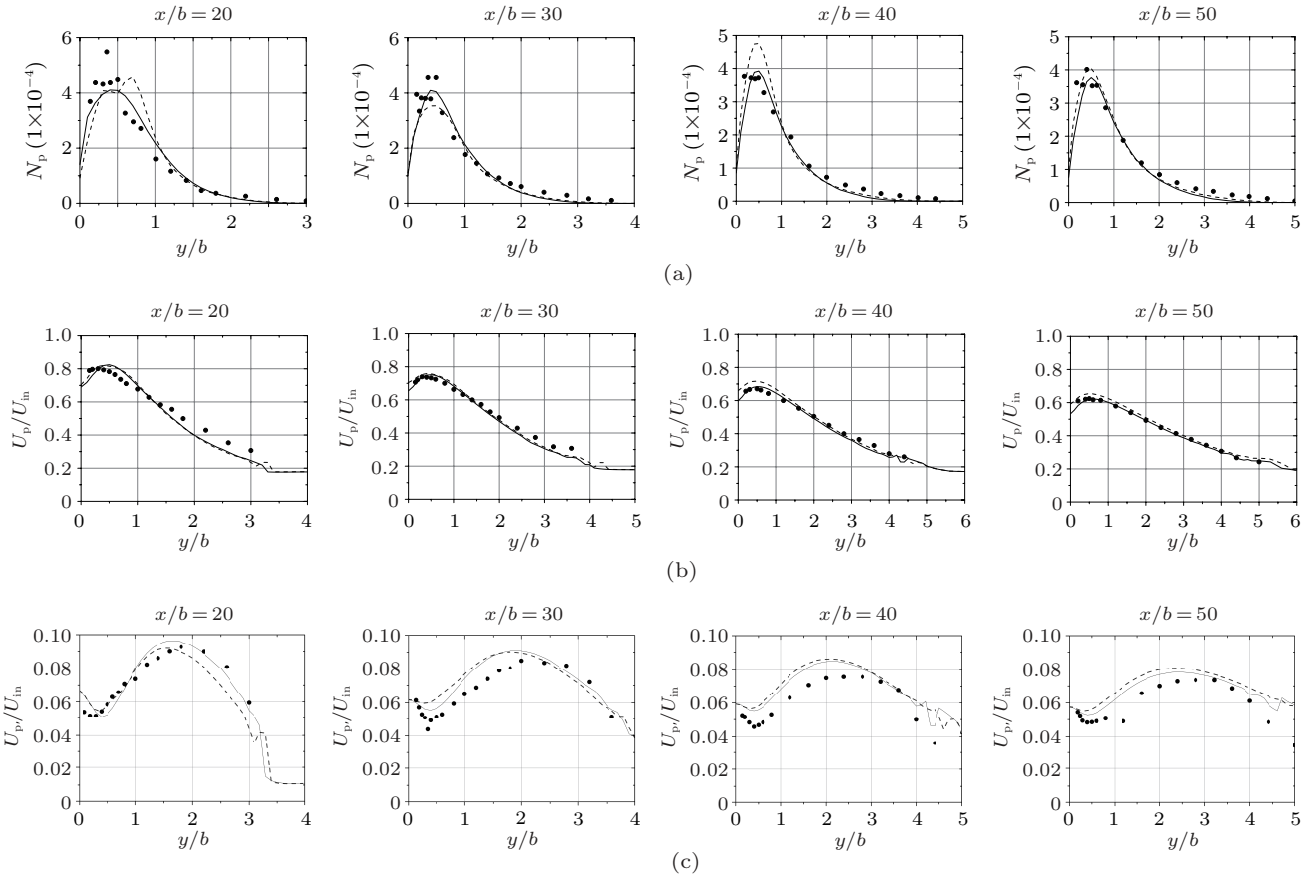


Fig. 5. Profiles for the particle phase: (a) Particle local volumetric fraction; (b) Particle mean vertical velocity; (c) Particle fluctuating vertical velocity. • experimental date; —— PDF method; ----- Reynolds averaging method.

4. Conclusion

In conclusion, we have developed models for the turbulence modulation in terms of the PDF approach. Detailed derivation of the turbulence modulation model without trajectory difference was introduced firstly and then the trajectory difference between phases was discussed and modelled. Compared with the Reynolds averaging model, our model contains a new production term of turbulence from the effect of the dispersed phase. The present model has been successfully applied to simulate a two-phase turbulence flow in a wall jet. Similar to the standard k–ε model, the limitations of the present

k–ε turbulence modulation model is that it has adopted the isotropic Boussinesq approximation, besides, the white Gaussian assumption is also used to close the PDF equation. Accordingly, the predictions in the fully developed turbulent flow region were more reasonable. Theoretical analyses and numerical simulations all indicate that the proposed model is more accordant with practical circumstances than the traditional models.

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