Identifying the closeness of eigenstates in quantum many-body systems*

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We propose a quantity called modulus fidelity to measure the closeness of two quantum pure states. We use it to investigate the closeness of eigenstates in one-dimensional hard-core bosons. When the system is integrable, eigenstates close to their neighbor or not, which leads to a large fluctuation in the distribution of modulus fidelity. When the system becomes chaos, the fluctuation is reduced dramatically, which indicates all eigenstates become close to each other. It is also found that two kind of closeness, i.e., closeness of eigenstates and closeness of eigenvalues, are not correlated at integrability but correlated at chaos. We also propose that the closeness of eigenstates is the underlying mechanism of eigenstate thermalization hypothesis (ETH) which explains the thermalization in quantum many-body systems.

Keywords: quantum chaos, thermalization, fidelity

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1. Introduction

Statistical mechanic has succeeded in describing quantum many-body systems. But how to understand thermodynamics from quantum mechanics remains unsolved in physics. One of the fundamental problems is the thermalization of isolated quantum many-body systems, which refers to the relaxation of system into a steady state that can be described by an equilibrium thermodynamic ensemble. Recently, the experiment realization of ultracold quantum gas weakly coupled with environment[1–6] provides an ideal test bed for the investigation of relaxation and thermalization. It is found that the thermalization surely takes place[5] in boson system trapped in a (quasi-)one-dimensional geometry but fails to be approached in another boson system trapped in a different (quasi-)one-dimensional geometry.[6] It is believed that integrability is the intrinsic mechanism of the failure of thermalization and the breaking of integrability caused by a strong perturbation is related to the onset of thermalization in quantum many-body systems.[7–12] As is well known, quantum chaos will emerge in the breaking of integrability. Then, it is also shown that quantum chaos plays a key role in thermalization of quantum systems.

Quantum chaos is used to address some properties of quantum system whose classic counterpart is chaotic. It is found that the properties of spectra, eigenstates and dynamics of quantum system can indicate the emergence of quantum chaos. Most studies deal with the statistical properties.[13–18] An important property of energy level is level spacing distribution $P(s)$ which is an intrinsic indicator of integrable-chaos transition. At integrability, $P(s)$ is a typically Poisson distribution $P(s) = \mathrm{e}^{-s}$ while at chaos the distribution takes the Wigner–Dyson form.[12,13] Contrary to eigenvalues, the quantities used to measure the properties of eigenstates, like inverse participation ratio and Shannon entropy, are not intrinsic indicators of integrable-chaos transition because of basis-dependence. However, they still show important information of quantum chaos. Considering eigenstates of Hamiltonian, usually expanded in a certain basis, the distribution of coefficients was shown to be localized or extended, which is related to the integrable-chaos transition to some degree. In latter case, eigenstates are chaotic wavefunction[19–23] and they are similar because of similar structure,[15,16] which is a kind of statistical similarity. However, despite intense studies, what is the nature of the integrable-chaos transition is still unclear.

For isolated quantum many-body systems in chaos, the eigenstate thermalization hypothesis (ETH)[24,25] states that thermalization occurs in individual eigenstates, which has been verified numerically in a wide variety of quantum many-body systems which are far from integrability.[15,16,26–30] That is to say, the expectation value of an observable is the smooth function of eigenenergy, which shows a kind of closeness of eigenstate in term of observable. Then, a question arises that what is the relation between the statistical similarity and the closeness of expectation value of observables between eigenstates.

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The closeness of quantum states is also concerned in quantum mechanics and quantum information theory, in which the intrinsic and widely used tool to measure the closeness of two states, including pure state and mixed state, is fidelity F.[31] In recent decade, fidelity was applied to the identifying the quantum phase transition.[32–36] As previously discussed, experiment setup can prepare isolated quantum many-body system which is described by a single wavefunction according to quantum mechanics. Therefor, in the following, we just consider pure state. Assuming two pure states denoted by $|\Phi\rangle$ and $|\Psi\rangle$, the fidelity of these two states is defined as $F(|\Phi\rangle,|\Psi\rangle) = |\langle\Phi|\Psi\rangle|$. If these two states are identical, $|\Phi\rangle = |\Psi\rangle$ or differ by a global phase as $|\Phi\rangle = \mathrm{e}^{\mathrm{i}\theta}|\Psi\rangle$, then $F = 1$. If they are orthogonal, $F = 0$. As all eigenstates of one quantum many-body system are orthogonal, the fidelity of any pair of eigenstates is zero no matter what this system is, integrable or nonintegrable, then we cannot use fidelity to define and study the closeness of eigenstates of quantum many-body system in this sense. To study the substantial properties of quantum many-body system, for example, the transition from integrability to chaos in which thermalization emerges, we should build a new understanding of closeness of quantum states.

This article is organized as follows. In Section 2, we introduce a new quantity called modulus fidelity. In Section 3, we use modulus fidelity to investigate the transition from integrability to chaos in one-dimensional hard-core boson system. In Section 4, based on the numerical results, we propose an understanding of the origin of ETH. Finally, we summarize our results in Section 5.

2. The definition of modulus fidelity

In this paper, we propose a new quantity to define and identify the closeness of two quantum pure states in a same Hilbert space. We still denote them by $|\Phi\rangle$ and $|\Psi\rangle$ and let $|n\rangle$ denote a complete orthogonal set in this Hilbert space with dimension D. This orthogonal basis is arbitrary and can be assumed as the eigenstates of an operator A with eigenvalue a_n. Then $|\Phi\rangle$ and $|\Psi\rangle$ can be expanded as

$$|\Phi\rangle = \sum_{n=1}^{D} c_n|n\rangle, \quad |\Psi\rangle = \sum_{n=1}^{D} d_n|n\rangle,$$

where c_n (d_n) is expansion coefficient. The fidelity of these two states is $F = \sum_{n=1}^{D} c_n^* d_n$. If we measure A on the state $|\Phi\rangle$, then the probability of obtaining eigenvalue a_n is $|c_n|^2$. After repeating measurement, one can obtain all eigenvalues with a sequence of probability $\{|c_1|^2,|c_2|^2,\ldots,|c_D|^2\}$. Doing same measurement on state $|\Psi\rangle$, we can obtain another sequence $\{|d_1|^2,|d_2|^2,\ldots,|d_D|^2\}$. To identify the closeness of two states by comparing these two sequence, we define a quantity in the form of fidelity as

$$F_m = \sum_{n=1}^{D} |c_n||d_n|, \tag{1}$$

where the replacing square modulus by modulus itself is to make the value of F_m in $[1,0]$ as well as fidelity and we call F_m modulus fidelity. A quantity defined as

$$C = \sum_{n=1}^{D} |c_n|^2|d_n|^2$$

was proposed to study closeness of eigenstates in quantum many-body systems in Refs. [17] and [37] where the square of modulus of coefficient were used. However, it can be shown easily that this quantity cannot measure the overlap or closeness of two quantum states. For example, consider a two-level system, two basis states are represented as $|1\rangle$ and $|0\rangle$, then assume two states which can be represented as $|\psi\rangle = \cos\alpha|1\rangle + \sin\alpha|0\rangle$, $|\phi\rangle = \cos\beta|1\rangle + \sin\beta|0\rangle$, one can find $C = (1/2)(1+\cos 2\alpha\cos 2\beta)$, which may be larger than $C = (1/2)(1+\cos 2\alpha\cos 2\alpha)$ when $\alpha = \beta$. This result means one state can be more close to a different state than to itself, which is unreasonable.

For clarity, we construct a new kind of vectors in the same Hilbert space $|n\rangle$ by setting its coefficients as the modulus of the coefficients of a vector like $|\Phi\rangle$. Then the new vectors built on $|\Phi\rangle$ and $|\Psi\rangle$, denoting by $|\widetilde{\Phi}\rangle$ and $|\widetilde{\Psi}\rangle$, are defined as

$$|\widetilde{\Phi}\rangle = \sum_{n=1}^{D} |c_n||n\rangle, \quad |\widetilde{\Psi}\rangle = \sum_{n=1}^{D} |d_n||n\rangle. \tag{2}$$

Then F_m of two pure states $|\Phi\rangle$ and $|\Psi\rangle$ is the fidelity of $|\widetilde{\Phi}\rangle$ and $|\widetilde{\Psi}\rangle$ as

$$F_m(|\Phi\rangle,|\Psi\rangle) = |\langle\widetilde{\Phi}|\widetilde{\Psi}\rangle| = \langle\widetilde{\Phi}|\widetilde{\Psi}\rangle. \tag{3}$$

In the last equality, the modulus operation vanishes due to the definition of new state (2). Note that F_m has the same definition as fidelity, but it is defined in new constrained space, describing the closeness of two quantum pure states in terms of modulus of coefficient. However, modulus fidelity is different from fidelity. It is dependent on the choice of basis, while fidelity is independent. Such replacing coefficients by their modulus was also proposed recently in Ref. [38] to study its effect on entanglement entropy.

3. Numerical results

In the following, we use modulus fidelity F_m to study the closeness of eigenstates of a quantum many-body system in both integrable and nonintegrable domain. We consider one-dimensional hard-core boson model(HCB) with dimensionless Hamiltonian

$$H = \sum_{i=1}^{N}\{-t(\hat{b}_i^{\dagger}b_{i+1} + \mathrm{H.c.}) - t'(\hat{b}_i^{\dagger}b_{i+2} + \mathrm{H.c.})$$

$$+V\hat{n}_i\hat{n}_{i+1}+V'\hat{n}_i\hat{n}_{i+2}\}, \quad (4)$$

where t and t' are the nearest-neighbor hopping and the next-nearest-neighbor hopping, V and V' are the nearest-neighbor and the next-nearest-neighbor interactions respectively. Throughout this paper, t and V are set to be unit, $t = V = 1$, and t' and V' are set to be equal, $t' = V'$. $\hat{n}_i = \hat{b}_i^{\dagger}\hat{b}_i$ is density operator. As is well known, this model is integrable when $t' = V' = 0$ but nonintegrable when $t' = V' \neq 0$. It has been verified that the thermalization can be achieved in this system and the underlying mechanism is ETH when system is far from integrability.[15,26,27]

We use full exact diagonalization method to calculate all eigenstates and eigenvalues of present system. We study the lattice up to 25 sites and 6 hard-core bosons. Under period boundary condition, the system preserves translational symmetry, by which the Hilbert space of Hamiltonian can be decomposed into different independent subspaces with different total momentum k. We can diagonalize each subspace. As discussed above, modulus fidelity is dependent on the choice of basis. That is to say, for two eigenstates we consider, the values of modulus fidelity are different in different bases. However, as the number of eigenstates of the many-body quantum system is large, we can focus on the statistical properties of modulus fidelity of eigenstates. We investigate the modulus fidelity in two different bases, site basis (Fock state) and k basis (Bloch state). These two bases were used to study the localization of a single eigenstate of quantum many-body systems in integrable-chaos transition.

We compute the eigenstates in subspace with momentum $k = 1$ rather than $k = 0$ to avoid a parity symmetry. Then by transformation, we can get the eigenstates in site basis. Given the above results, we can find the modulus fidelity F_m of each pair of neighbor eigenstates in subspace at different values of the next-nearest neighbor hopping and interaction which determine the breaking of integrability in the system.

The numerical results in site basis are plotted in Fig. 1. Note that when $t' = V' = 0$, i.e., the system is integrable, as shown in Fig. 1(a), the modulus fidelity F_m of each pair of neighbor eigenstates is non-zero. For comparison, we take $F'_m = 0.5$ as a threshold value in a common sense. If F_m is larger than 0.5, this two states are close, otherwise, they are not close. Then we find that at integrability, most eigenstates are close to their neighbor, i.e., $F_m > 0.5$, while a number of eigenstates are far from their neighbor, i.e., $F_m < 0.5$. This makes a most significant feature that the function $F_m(i, i-1)$, where i is the index of eigenvalues, has a large fluctuation and distributes like random number. When t' and V' are nonzero, the system becomes nonintegrable. As is well known, the level spacing distribution changes from Poisson to Wigner–Dyson form, indicating the transition from integrability to chaos and the emergence of thermalization. In Figs. 1(b)–1(f), when t' and V' increase, we find more and more eigenstates, which are not close to their neighbor at integrability, but become close to each other. A dramatic change appears that the fluctuation of $F_m(i, i-1)$ is reduced. When t' and V are large enough, for example, $t' = V' = 2.0$ in Fig. 1(f), as a function of index of eigenvalues, F_m is no longer like random number, but tends to be a smooth constant. That is to say, when the system comes into chaos, all eigenstates of the system becomes close to their neighbor one than the case at integrability. Furthermore, the degree of closeness of each pair of eigenstates are to be the same.

We also calculate the average modulus fidelity of modulus fidelity between each pair of neighbor eigenstates,

$$\overline{F}_m = \frac{1}{L-1}\sum_{i=2}^{L} F_m(i, i-1),$$

where L is the size of subspace, and the standard deviation. The results are also plotted in Fig. 1. We can see obvious changes in both measures. When t' and V' increase from zero, i.e., integrability point, average modulus fidelity increases quickly. In Figs. 1(a)–1(f), we have found all eigenstates become close to their neighbor in the same degree when system is far from integrability. Here, we find that the average value of closeness of eigenstates will also increase in this process. After t' an V' reach a certain value, the average modulus fidelity will not increase so quickly and almost tends to be saturated. For the parameters of the model investigated here, this value is about 0.25. This scenario can also be seen in the result of standard deviation of modulus fidelity. It decreases very quickly when t' and V' increase from zero, which indicates the reduction of fluctuation seen in Figs. 1(a)–1(f). The reduction of fluctuation was also found in Ref. [17] by using a similar measure as discussed above. But except for this result, the closeness are measured differently. In Ref. [17], the maximal value of closeness decreases when system becomes nonintegrable, which is contrary to our result. We have explained the reason in last section. When t' and V' are large enough, the system comes into chaos, the fluctuation is inhibited to be small and decreases slowly as confirmed in Fig. 1(h). The values of t' and V' at which transition occurs are the same as that of the average modulus fidelity. The existence of such critical values has been found[7,15,27] and they will decrease with the increase of size of system. So it is believed that in the thermodynamic limit an infinitesimal integrability breaking would lead to chaos. On the other hand, it is also confirmed[7,15,27] that in the chaotic regime, the thermalization is achieved in this kind of model. This may indicate a relation between the closeness of eigenstates and the mechanism of thermalization, which will be discussed in next section.

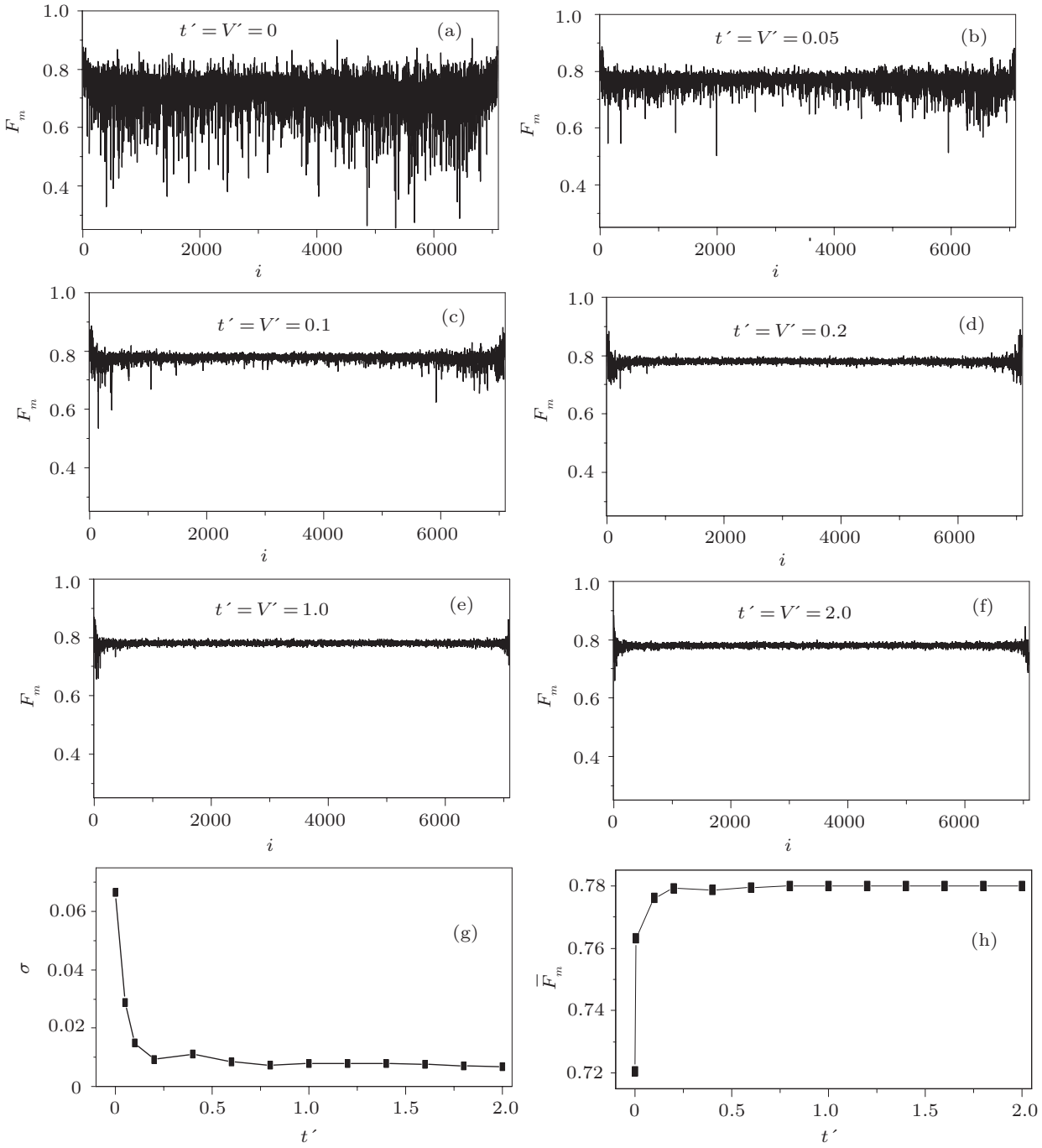


Fig. 1. The modulus fidelity $F_m(i,i-1)$ of i-th eigenstates and $(i-1)$-th eigenstate in one-dimensional hard-core model. The data are obtained in the site basis and in the condition that 6 bosons on 25 lattices and the next-nearest-neighbor parameters are changed from (a) integrable case to nonintegrable case [panels (b)–(f)]. The dimension of subspace is $L = 7085$. Average modulus fidelity (g) and the standard deviation of modulus fidelity (h) of neighbor eigenstates in the present model as a function of the next-nearest-neighbor hopping parameters $t' = V'$.

In Fig. 2, we also show the properties of F_m in the k space, including the individual value of $F_m(i,i-1)$, the average of $F_m(i,i-1)$, and the standard derivation. We note that the value of F_m between the two neighbor eigenstates with energy E_i and E_{i-1} in this space is surely different from the one between the two eigenstates in the site space with the same energy E_i and E_{i-1}. However, the statistical properties of $F_m(i,i-1)$ in this k basis is the same as those in the site space. The reduction of the fluctuation of F_m emerges in the destroying of integrability as shown in Figs. 2(a)–2(f). The average of $F_m(i,i-1)$ in Fig. 2(g) and the standard derivation in Fig. 2(h) also show the transition from integrability to chaos. Furthermore, it can be seen that the values of parameters at which the transition occurs are the same as the one in the site basis. This basis-independent property also presents in our further studying in the following discussions, so we just illustrate the results in the site basis.

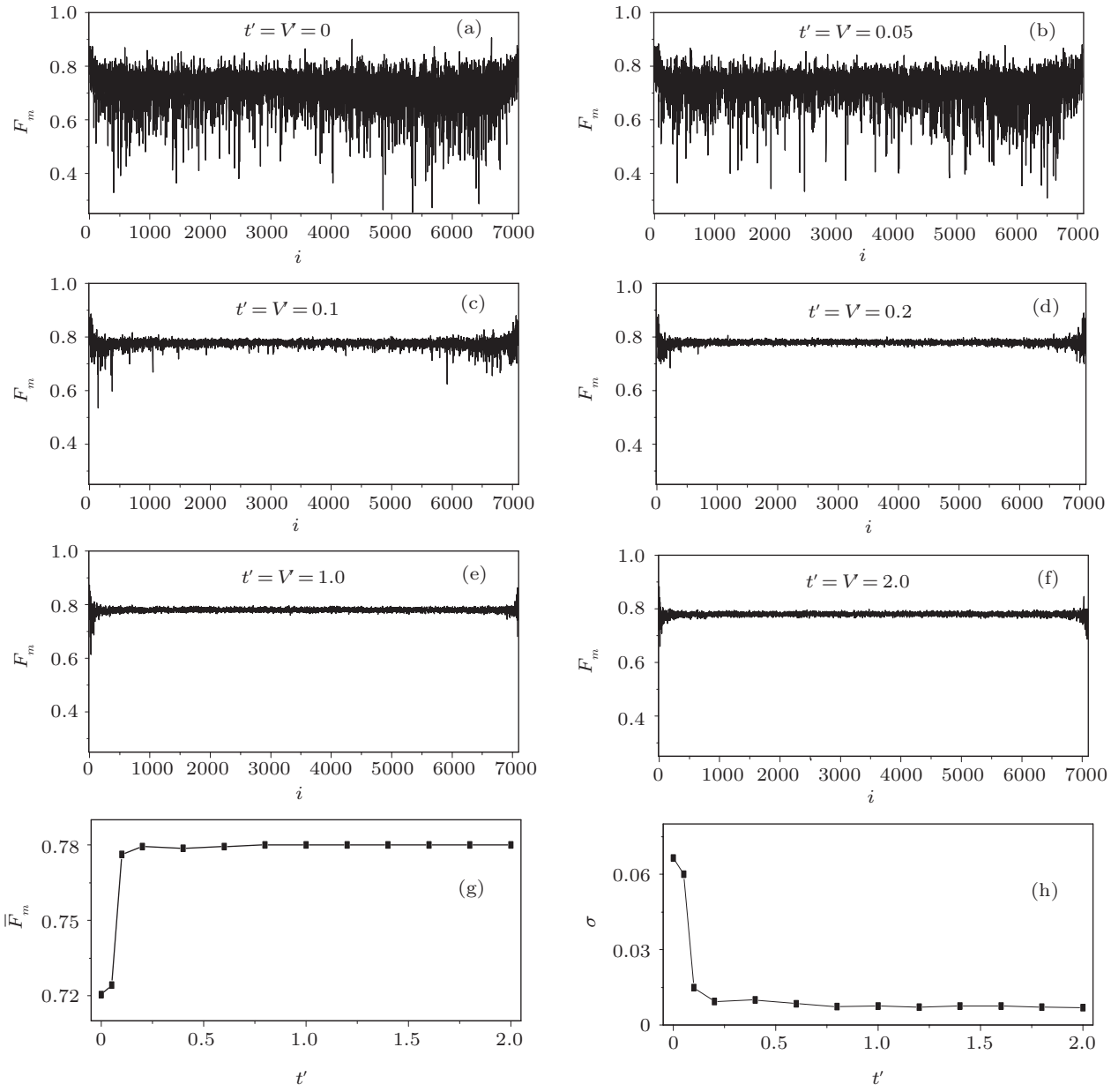


Fig. 2. Same as in Fig. 1 but in the momentum k basis.

As shown previously, when the system is in the integrable case and in the nonintegrable case, the statistical properties of the modulus fidelity of neighbor eigenstates behave differently. Next, we provide further evidence for this statement. We choose ground state with energy E_1 as a reference state and calculate modulus fidelity of excited eigenstates with respect to it. We plot such results of the modulus fidelity, i.e., $F_m(i,1)$, as a function of the index of eigenvalues in Fig. 3 at different parameters. Both in the integrable case and in the nonintegrable case, the function $F_m(i,1)$ has the same profile which decreases when the index of the excited state becomes large. However, the local details in the two cases are very different. We also notice a reduction of the fluctuation appearing in such the modulus fidelity as well as the modulus fidelity of neighbor eigenstates as shown in Fig. 1 and Fig. 2. In the integrable case, there is also a large fluctuation presenting in the function $F_m(i,1)$. We consider two excited states denoted by index i and j. $i < j$ means the eigenenergy $E_i < E_j$. The i-th eigenstate is close to the ground state than the j-th eigenstate in energy. But the closeness of the i-th eigenstate to the ground state maybe smaller or larger than the closeness of the j-th to the ground state, which means these two kinds of closeness are not correlated. When the system is nonintegrable, the fluctuation is reduced sharply and the modulus fidelity of the excited state with respect to the ground state becomes a smooth function of the index of the eigenstate. That is to say, the closeness of the two states is related to the closeness of energy, i.e, the small level spacing is, the more close the two states become. Such the correlation also indicates the transition from integrability to chaos and the emergence of thermalization.

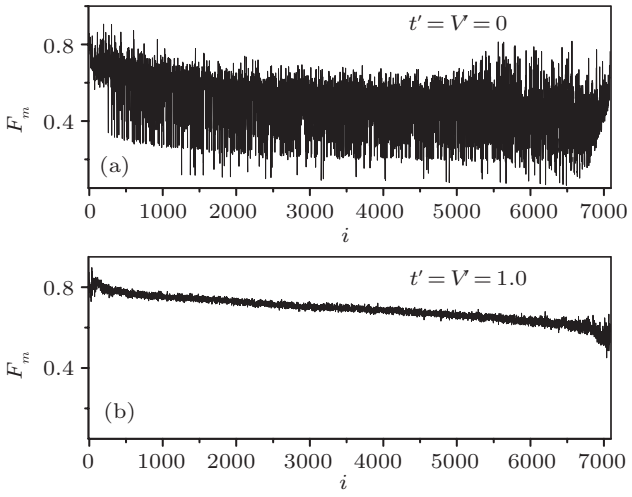


Fig. 3. The modulus fidelity of the excited states with the ground state. (a) $t' = V' = 0.0$ and (b)$t' = V' = 1.0$. The other parameters are the same as those in Figs. 1 and 2.

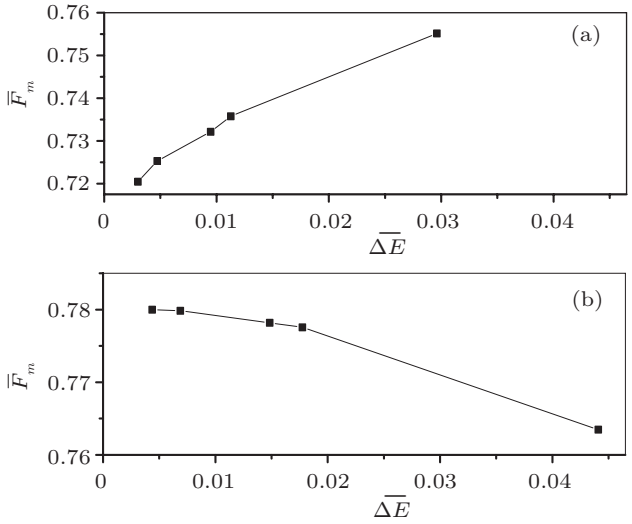


Fig. 4. Average modulus fidelity versus average level spacing (energy density) for 1D hard-core boson system with (a) $t' = V' = 0.0$ and (b)$t' = V' = 1.0$. Average level spacings $\overline{\Delta E}$ are obtained by $(E_{\max} - E_{\min})/N$ where $E_{\max}$ and $E_{\min}$ are the two ends of energy spectrum. The size of the subspace N is taken to be 506, 1463, 1771, 4389, and 7084.

Next, we investigate the effort of system size on the closeness of the neighbor eigenstates and their relation to the closeness of energy. As is well known , when system size increases, both individual level spacing and average level space become small. And in the thermodynamic limit, level spacing of quantum many-body is expected to be zero. However, for a finite system, level spacings between neighbor eigenstates are not the same and obey a distribution. To compare the closeness at different system sizes, we calculate average modulus fidelity of neighbor eigenstates. Accordingly, level spacings are also averaged. In Fig. 4, we plot numerical results of the system in both integrable and nonintegrable cases. When system is integrable, average modulus fidelity decreases with the decrease of energy level, which means the closeness of energy level leads to the separation of corresponding eigenstates. But in nonintegrable case, average modulus fidelity increases with the decrease of level spacing, showing that the closeness in energy corresponds to the closeness in eigenstates as well as the conclusion from Fig. 3. The standard derivation of modulus fidelity of neighbor eigenstates in different system sizes are plotted in Fig. 5. When the system is nonintegrable, standard derivation decreases with the decrease of the level spacing, i.e., the increase of the system size. That is to say, when the system size becomes large, the closeness of neighbor eigenstates tends to be the smooth function of energy. When the system is integrable, the standard derivation also decreases as the system size increase for most data, except for the one when the system size takes maximal value in our study. In the future work, a large scale simulation should be done to find whether it is an error or an intrinsic character of the integrable system.

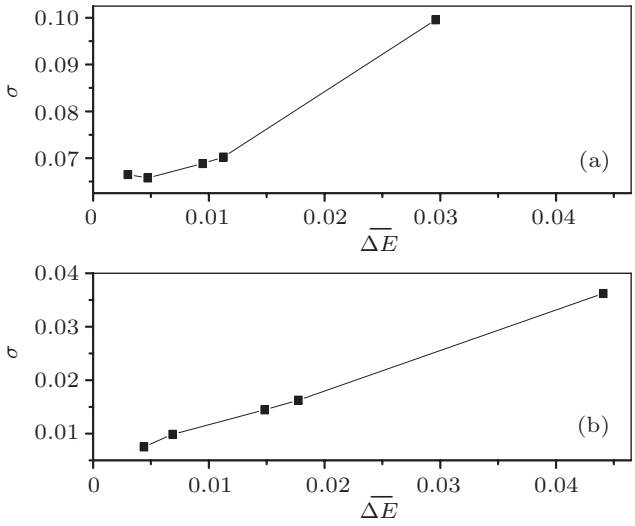


Fig. 5. Standard deviation versus average level spacing (energy density) for 1D hard-core boson system with (a)$t' = V' = 0.0$ and (b)$t' = V' = 1.0$. Average level spacings $\overline{\Delta E}$ are obtained by $(E_{\max} - E_{\min})/N$ where $E_{\max}$ and $E_{\min}$ are the two ends of energy spectrum. The size of subspace N is taken to be 506, 1463, 1771, 4389, and 7084.

4. The underlying mechanism of eigenstate thermalization hypothesis

We have identified the integrable-chaos transition in a quantum many-body system which is related to the mechanism of thermalization. Considering an isolated quantum many-body system with Hamiltonian H. If we let $|\Phi_i\rangle$ denote an eigenstate with eigenvalue E_i and A denote an observable, ETH states that the equation

$$\langle \Phi_i|A|\Phi_i\rangle = \langle A\rangle_{\text{micro}}(E_i) \tag{5}$$

is satisfied. The subscript "micro" denotes the microcanonical thermal average over the energy interval $[E_i - \triangle E/2, E_i + \triangle E/2]$, where $\triangle E$ is the energy interval.

As one can expand eigenstate $|\Phi_i\rangle$ by another orthogonal complete basis which could be constructed by the eigenstates of observable A denoted by $|n\rangle$, with eigenvalues a_n,

i.e., $A|n\rangle = a_n|n\rangle$. If A commutes with Hamiltonian, its eigenstates are the same as that of Hamiltonian, so $|\Phi_i\rangle$ is expanded on itself. In this case, F_m is zero for any pair of eigenstates, which is trivial for studying the closeness of eigenstates as well as fidelity F. We focus on the case that observable A does not commuted with Hamiltonian. Let $|\Phi_i\rangle$ and $|\Phi_{i-1}\rangle$ denote two neighbor eigenstates of Hamiltonian , then they can be expanded by the eigenstates of observable A as

$$|\Phi_i\rangle = \sum_n c_{i,n}|n\rangle, |\Phi_{i-1}\rangle = \sum_n c_{i-1,n}|n\rangle.$$

Then the expectation value of A on these two eigenstates are

$$\begin{aligned}\langle\Phi_i|A|\Phi_i\rangle &= \sum_n |c_{i,n}|^2 a_n,\\ \langle\Phi_{i-1}|A|\Phi_{i-1}\rangle &= \sum_n |c_{i-1,n}|^2 a_n,\end{aligned} \tag{6}$$

where $|c_{k,n}|^2 = c^*_{k,n}c_{k,n}$ is the eigenstate occupation numbers which is the probability of obtaining the eigenvalue a_n in a measurement of A on eigenstate $|\Phi_i\rangle$. In nonintegrable case, due to Eq. (5) of ETH, the expectation value of A on an eigenstate equals microcanonical average, which means

$$\langle\Phi_i|A|\Phi_i\rangle \approx \langle\Phi_{i-1}|A|\Phi_{i-1}\rangle. \tag{7}$$

Due to Eq. (6), one can find the validation of Eq. (7) is contributed by two quantities, the eigenvalues of A and eigenstate occupation number. As the former is fixed for a certain quantum system, then there should be two possible scenarios for eigenstate occupation number ($|c_{k,n}|^2$) that they fluctuate between eigenstates close in energy or not. Now, our result that in nonintegrable system we studied the modulus fidelity has larger value and keeps almost constant suggests the later that for two eigenstates of Hamiltonian close in energy, the eigenstate occupation number of them are also close as

$$|c_{i,n}|^2 \approx |c_{i-1,n}|^2. \tag{8}$$

Then equation (7) will be satisfied. In this case, the modulus fidelity between each pair of neighbor eigenstate will be close to unit and equal to each other, showing a smooth function versus eigenvalues,as shown in Figs. 1 and 2. On the contrary, if the expansion coefficient ($|c_{k,n}|^2$) is fluctuating between eigenstates close in energy, i.e., equation (7) is violated, the modulus fidelity between neighbor eigenstates will have a fluctuation as shown previously. Recently, it was shown[39] that ETH is essentially equivalent to the basic assumption of von Neumann's quantum ergodic theorem (QET)[40] that the overlap between an eigenstate of energy and an eigenstate of observable A is exponential small and close to $1/D$, where D is the dimension of Hilbert space of the system. It can be found here that this assumption is an extreme point in our result that will also make modulus fidelity to be unit.

5. Conclusion

In conclusion, we found that the closeness of eigenstates of Hamiltonian of quantum many-body systems can be studied as an indicator of the transition from integrability to chaos and the emergence of thermalization. For this purpose, we proposed a new quantity, modulus fidelity, which is defined by replacing the expansion coefficients of one quantum state in a basis by their modulus. The full exact diagonalization of one-dimensional hard-core boson model showed that in the integrable and nonintegrable cases, the modulus fidelity of neighbor eigenstates has different properties, which indicates the integrable-chaos transition. The reduction of fluctuation of modulus fidelity and the increase of their average value in this transition show the eigenstates of quantum many-body system tend to be close to each other, which indicates a deeper uniformization of eigenstates than statistical similarity. We also studied the finite size effect on the correlation between the closeness of eigenstate and closeness of energy at integrable domain and nonintegrable domain. The basis-dependent properties of modulus fidelity was also discussed. Although the statistical properties of modulus fidelity did show same behavior in two different bases we studied, it is still necessary to investigate this scenario in the future work. Furthermore, our argument shows the closeness of eigenstate, more than statistical similarity, may guarantee the validation of ETH in nonintegrable system we studied and suggest a general understanding of the underlying mechanism of ETH and QET.

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