Effect of gravity gradient in weak equivalence principle test*

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A high accuracy test of the weak equivalence principle (WEP) is of great scientific significance no matter whether its result is positive. We analyze the gravity gradient effect which is a main systematic error source in the test of WEP. The result shows that the uncompensated gravity gradient effect from the coupling term of the dominated gravity gradient multipole moment component q_{21} and the relative multipole field component Q_{21} contributes to an uncertainty of 1×10^{-11} on the Eötvös parameter. We make a Q_{21} compensation to reduce the effect by about 20 times, and the limit of the test precision due to this coupling is improved to a level of a part in 10^{13}.

Keywords: weak equivalence principle test, gravity gradient effect, test precision, gravity gradient compensation

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1. Introduction

Weak equivalence principle (WEP) states that the ratios of gravitational mass to inertial mass for all matters are the same.[1] That is to say, all objects have an identical acceleration in the uniform gravitational field.[2] In Einstein's gravitational theory of general relativity, the WEP is a natural result.[3] Although the general relativity, Einstein's elegant theory of gravity, has been verified by all experimental tests,[4–7] it is a classical theory that cannot be quantized, and much theoretical effort is now devoted to developing a realistic quantum theory that would reduce to Einstein's theory in an appropriate limit.[8] The theories which want to unify four fundamental interactions such as the string theory[9,10] and the supersymmetry theory[11] demand that the WEP is violated to some degrees. Therefore, a highly accurate result for testing whether or not the WEP is correct, is of important scientific significance.[12]

There are many famous experiments for testing the WEP. The earliest experiment could be traced back to the Galileo's Leaning Tower of Pisa experiment and later Newton's simple pendulum experiment,[13] and Newton's result shows that the WEP is correct while the Eötvös parameter $\eta_{12} < 10^{-3}$.[14] Eötvös ingeniously applied a precision torsion pendulum to the experiment in about 1900, and in this way the test precision of the WEP was improved remarkably.[15] The result of the Eötvös's experiment is $\eta_{12} < 10^{-9}$.[15] Roll *et al.*[16] and Braginsky *et al.*[17] took the sun as a gravitational source for experiments in 1960 and 1970 respectively. They modulated the possible violative signal into a periodic signal which has a period of 24 hours.[16,17] Therefore, the effect from external environment could be reduced, and their experimental precisions of the Eötvös parameter reached to the 10^{-11} and 10^{-12} levels, respectively.[18] Since 1990, the group of University of Washington (Eot–Wash) began to test WEP by using a rotating torsion pendulum and modulating the possible violative signal into an adjustable-periodic signal,[3,4] its advantages were the same as those of Dicke *et al.*'s experiment, and they concluded that the WEP is correct while the η_{12} is under the 10^{-13} level.[19]

The group of Huazhong University of Science and Technology (HUST) began the experiment of testing WEP in 2003. The initial experimental method we used is similar to Dicke's, and the result is that the Eötvös parameter is less than 3.1×10^{-10}, in which Al_2O_3 and Pb were as the test mass.[20] The initial method has a main limit that the period of the modulated signal is too long, and at the same time, the $1/f$ noise effect, temperature fluctuation effect etc. could be enlarged and later limit the test precision. Hence, the HUST group performed a new experiment, whose principle is the same as that of the Eot–Wash, in order to obtain a higher measurement precision. The main error sources of the HUST group's experiment were the external environmental effects and the thermal noise relating to the dissipative structure of the fiber.[19,21] For the external influence, the effect of the gravity gradient is the most important systematic error.[2,3,22]

In this work, we briefly discuss the experimental principle of the WEP test by the HUST group. For the dominant effect from the gravity coupling $q_{21}Q_{21}$, we give its theoretical ex-

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pression relating to the η_{12}. After that, we make a q_{21} torsion pendulum to measure the uncompensated multipole field Q_{21} of the external gravity source. By processing a typical data set of the measurement of Q_{21}, we obtain the value of Q_{21}. Then we measure the multipole moment $\bar{q}_{21}$ of the pendulum used in formal experiment and further estimate the effect of the gravity coupling in our WEP test. Finally, in order to achieve the demanded test precision, we make a Q_{21} compensation to compensate for the gravity coupling and then reduce the effect from the coupling $q_{21}Q_{21}$, and thus make contribution to correcting the effect of the gravity gradient.

2. Eötvös parameter and effect of coupling $q_{21}Q_{21}$

The WEP could be tested by measuring the differential accelerations due to the gravities for different test mass of the torsion pendulum. Our main instrument used to test WEP is a torsion pendulum which is extremely sensitive to the weak force from the horizontal differential acceleration $\Delta\bar{a}_{\perp}$. The $\Delta\bar{a}_{\perp}$ can be measured by the angular deflection of the torsion pendulum θ which we called WEP violative signal. However, the absolute value of the θ cannot be measured. Therefore, we change the phase of the θ and measure the relative angular deflection of the pendulum θ_{R}. The θ_{R} is the angular deflections of the pendulum which is measured in a rotating frame. In the experiment, the torsion pendulum is suspended from an air-bearing turntable by a tungsten fiber. The constant angular speed of the turntable is ω. Hence, the angle-time data θ_{R} is modulated into a periodic signal. The amplitude and angular speed of the θ_{R} are $|\Delta a_{\perp}||\boldsymbol{d}|/k$ and ω respectively, where $\boldsymbol{d}$ and k are the composition dipole of the pendulum and the torsional constant of the fiber, respectively.[2] We can obtain the amplitude of the measured θ_{R} and later the $|\Delta a_{\perp}|$. Therefore, the violative coefficient of the WEP or the well-known Eötvös parameter η_{12} could be expressed as[1]

$$\eta_{12} \simeq \frac{|\Delta a_{\perp}|}{g_{\mathrm{h}}}, \tag{1}$$

where g_{h} is the horizontal acceleration component of the gravitational field g, which in Wuhan, Hubei Province, China at a latitude of 30° N is 1.46 cm·s^{-2}.

Based on Su Y's work[3] and our experimental condition, the main gravity gradient effect due to the coupling $q_{21}Q_{21}$ can exert the influence on the Eötvös parameter $\delta\eta_{12}$ that is expressed as

$$\delta\eta_{12} = \frac{8\pi G}{5|\boldsymbol{d}|}\frac{|\bar{q}_{21}||Q_{21}|}{g_{\mathrm{h}}}, \tag{2}$$

where G is the gravitational constant, the $\bar{q}_{21}$ and Q_{21} are called the lmth order multipole moment and lmth order multipole field,[2] respectively, where $l = 2$ and $m = 1$. The $\bar{q}_{21}$ and Q_{21} only contain the properties of the pendulum of the formal experiment and the source mass distribution, both of which can be calculated in theory. In fact, since the pendulum is made regular and the source mass distribution is usually complex, the $\bar{q}_{21}$ is easy to estimate and the Q_{21} is hard to calculate. So we make a q_{21} pendulum to measure the Q_{21} and later obtain the $\delta\eta_{12}$.

3. Measure Q_{21}

Our laboratory is sited at Yu-Jia mountain with an irregular shape, so it is hard to build some ideal models to calculate the Q_{21}. Hence, we use a q_{21} pendulum to measure the Q_{21}, and the multipole moment q_{21} of the pendulum is predominant. As figure 1 shows, the typical q_{21} pendulum has a dominant multipole term q_{21}, and the other higher terms vanish naturally or approximate to zero due to the imperfect pendulum. That is to say, the multipole terms of the pendulum only couple with the Q_{21}, and other higher terms can be ignored.

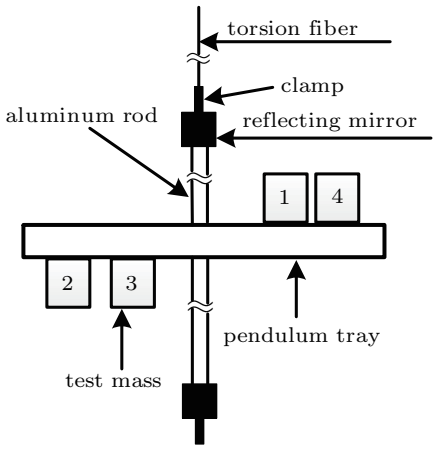


Fig. 1. The q_{21} pendulum consisting of four identical cylindrical test mass made of red copper, with essentially the same mass. They are positioned on a circular pendulum tray with a thickness of 2.5 mm and a radius of 40 mm. The numbers 1, 2, 3, and 4 mark the four-test mass, respectively. An aluminum rod with a length of 110 mm passes through the center of the tray, and two reflecting mirrors are installed on its bottom and top symmetrically. The main body of the pendulum is suspended by a torsion fiber with a length of 1000 mm and a diameter of 25 μm, which connects with the aluminum rod by a clamp. The torsional constant of the fiber k equals 6.2×10^{-9} Nm/rad. The total mass of the q_{21} pendulum is 70 g, and the $|q_{21}|$ equals 35.7(3) g·cm^2.

We let the q_{21} pendulum rotate about the fiber with a constant rotation angular velocity ω that equals 0.0052 rad/s in a high vacuum chamber ($\sim 10^{-5}$ Pa), and at the same time measure the angular deflection of the pendulum θ_{R} in the rotating frame. The recorded angle-time value θ_{R} is about 40 hours long. The typical angle-time data at the time from 15 to 18 hours are shown in Fig. 2(a). We find that θ_{R} is not a simple sine (or cosine) signal, and it contains other frequency signals. Then, we plot the power spectral density of the whole θ_{R} in Fig. 2(b). The natural torsional frequency ω_0 we obtain is 0.0108 rad/s. There are 1ω, 2ω, ω_0 signals, etc. due to the finite rotation angular speed. The amplitude of 1ω signal can

be written as[2,3]

$$\mathrm{Amp}(1\omega) = \frac{1}{k}\frac{\omega_0^2}{|\omega_0^2-\omega^2|}\frac{8\pi G}{5}|q_{21}||Q_{21}|. \quad (3)$$

Only the 1ω signal we should consider since the angular speed of the signal is the same as the WEP violative signal.[2,3,22] In other words, we can estimate the amplitude of the 1ω signal and later $|Q_{21}|$ for calculating $\delta\eta_{12}$.

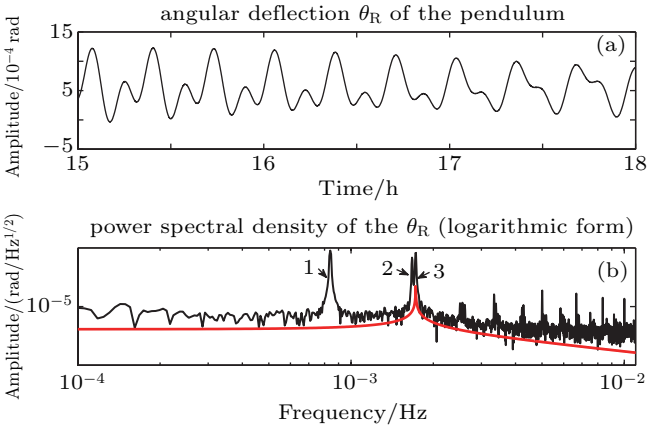


Fig. 2. (color online) (a) Black line marks the angular deflection θ_R. The horizontal and vertical axes are the time and the amplitude of the θ_R, respectively. (b) Black and red lines denote the power spectral densities of the θ_R and thermal noise,[23] respectively. The peaks 1, 2, and 3 mark 1ω, 2ω, and ω_0 signal, respectively. The horizontal and vertical axes are the frequency and the amplitude, respectively.

The norm data processing is performed to estimate the amplitude of the 1ω signal. Firstly, we use a torsional filter to process the raw angle-time data in order to eliminate the ω_0 signal.[2] Secondly, the whole filtered data are cut into some segments, and each segment contains three periods of the turntable. Then, we adopt a similar nonlinear fitting method which was used in the test of the equivalence principle by Gundlach *et al.*[24] to determine the amplitude of the 1ω signal for each segment. The results of the fitting are shown in Fig. 3.

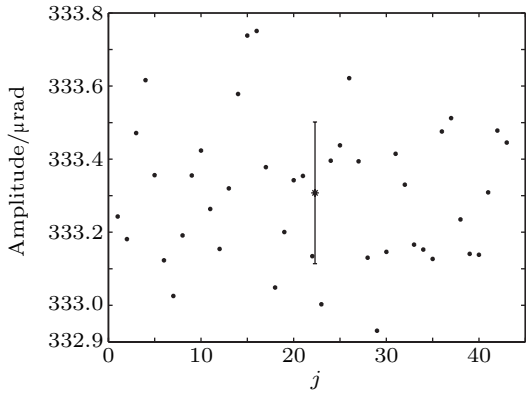


Fig. 3. Fitting amplitudes of the 1ω signal in the j-th segment (black points). The horizontal and vertical axes are the number of the segments and the amplitude of 1ω signal in the j-th segment, respectively. The asterisk and relative error bar denote the final value and uncertainty of the estimated amplitude, respectively. There are 43 segments in the figure.

The final value and uncertainty of the estimated amplitude are obtained as the average and scatter of the amplitude sequence in Fig. 3, and the result is given by

$$\mathrm{Amp}(1\omega) = 333.3(2)\ \mu\mathrm{rad}. \quad (4)$$

According to Eqs. (3) and (4), the $|Q_{21}|$ is given by

$$|Q_{21}| = 1.33(1)\ \mathrm{g\cdot cm^{-3}}. \quad (5)$$

4. Effect from coupling $q_{21}Q_{21}$ in formal experiment

The pendulum we used in the formal experiment consists of a long fused silica rod, four short fused silica rods, two left-handed silica crystals, two right-handed silica crystals and two reflecting mirrors as shown in Fig. 4. On the one hand, this pendulum is extremely sensitive to the torque due to the possible violation of the WEP or the potential 5th force. On the other hand, the shape and mass distribution of the pendulum is symmetrical, so the multipole moment q_{l1} and further the gravity gradient effect are minimized as little as possible.

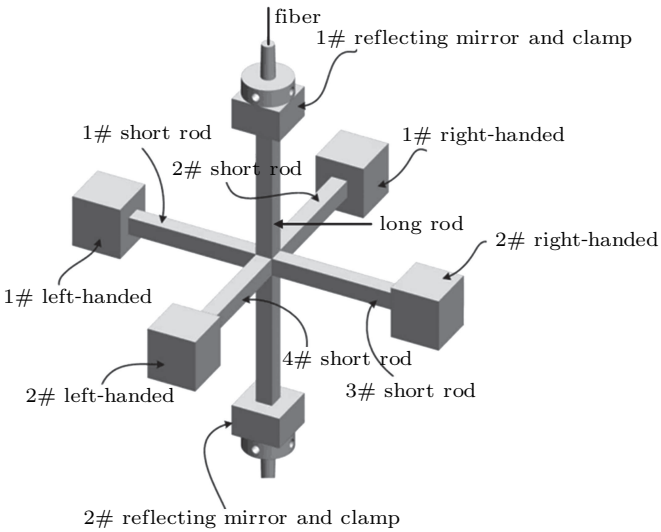


Fig. 4. Pendulum used in formal experiment. The 1# left-handed crystal connects with 2# right-handed crystal by the 1# short rod and 3# short rod, and 2# left-handed crystal connects with 1# right-handed crystal by the 2# short rod and 4# short rod. Four short rods are fixed on the long rod, and the whole pendulum is suspended from an air-bearing turntable by a tungsten fiber.

The composition dipole $|\boldsymbol{d}|$ equals 63.17 g·cm. Considering the imperfectness of the pendulum, an independent measurement of the $\bar{q}_{21}$ in the formal experiment is finished before we estimate the effect of the $q_{21}Q_{21}$ coupling. After that $\bar{q}_{21}$ is given as 0.135(2) g·cm^2. According to Eq. (2), we let $\bar{q}_{21}$ equal 0.135(2) g·cm^2, and then the influence on the Eötvös parameter is

$$\delta\eta_{12} = (6.54 \pm 0.11)\times 10^{-10}, \quad (6)$$

where the central value of the $\delta\eta_{12}$ (6.54×10^{-10}) can be corrected in the data analysis. The uncertainty (0.10×10^{-10}) cannot be corrected, which limits the test precision. The result means that the experimental precision could not be better than the order of 10^{-11} due to the $q_{21}Q_{21}$ coupling effect. The next work we should do is to compensate for the gravity gradient field with gravity gradient compensations for a higher test precision.

5. Compensation for Q_{21}

As figure 5 shows, we make a Q_{21} compensation in order to reduce the main effect from $q_{21}Q_{21}$ coupling. The two compensation mass blocks consist of 20 lead blocks and 8 stainless steels. Each 10 lead blocks and 4 stainless steels form a set, and these mass blocks constitute a semicircle compensation mass.

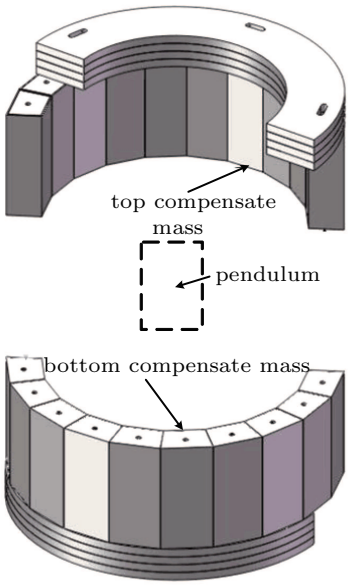


Fig. 5. Q_{21} compensations. The pendulum is set at the center of the compensation.

We calculate the multipole fields due to the Q_{21} compensations and the result is shown in Table 1. As Table 1 shows, the uncompensated multipole field Q_{21} could be offset and later reduced from 1.33(1) g·cm^{-1} to 0.01(1) g·cm^{-3} theoretically. We use the q_{21} pendulum to measure the compensated multipole field $|Q_{21}^{c}|$ again. The compensatory work consists of three steps. At the first step, we set the Q_{21} compensations around the q_{21} pendulum when the position of the compensation and uncompensated coupling are opposite roughly, and then we measure the amplitude of the 1ω signal to be 7.26(25) μrad; at the second step, we adjust the Q_{21} compensations to rotate a small angle, and then the amplitude of the 1ω signal extends to 13.25(14) μrad; finally, at the third step, we reverse the compensations, and let them rotate a small angle. Then the amplitude of the 1ω signal decreases at 3.53(25) μrad which we consider as an appropriate compensation. The results of the three steps are shown in Fig. 6.

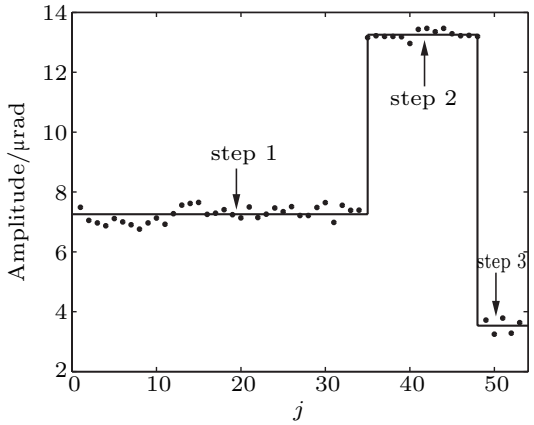


Fig. 6. Fitting amplitudes of the 1ω signal in the j-th segment. Horizontal and vertical axes denote the number of the segments and amplitude of 1ω signal in the j-th segment, respectively. There are 53 segments in the figure. The black points are fitting amplitude of the 1ω signal in the j-th segment. The vertical ordinate of the black line is the average value of the amplitude of 1ω signal in each step. The arrow steps 1, 2, and 3 mark the first, second, and third steps, respectively. The sum of segments for the first, second, and third steps are 34, 14, and 5 separately.

Table 1. Values and modes of the $l1$-th multipole fields duo to the Q_{21} compensations.

Q_{l1}/g·cm^{-l}	Top	Bottom	Sum	Mod
$Q_{21}/10^{-2}$ g·cm^{-3}	–51.9(7)+42.0(9)i	–51.9(7)+42.0(9)i	–103.8(1.0)+84.0(1.2)i	134(1)
$Q_{31}/10^{-3}$ g·cm^{-4}	–8.0(1)+6.8(1)i	8.0(1)–6.8(1)i	0.0(1)+0.0(1)i	0.0(1)
$Q_{41}/10^{-6}$ g·cm^{-5}	15.5(1.8)+9.1(4.9)i	15.5(1.8)+9.1(4.9)i	31.0(2.3)+18.2(6.9)i	35.9(7.2)

From the result of the third step, in the same way we estimate the $|Q_{21}|$, the appropriate compensated multipole field $|Q_{21}^{c}|$ is given by

$$|Q_{21}^{c}| = 0.014(1)\ \mathrm{g\cdot cm^{-3}} \tag{7}$$

and then the influence on the Eötvös parameter is given by

$$\delta\eta_{12} = (6.88 \pm 0.50) \times 10^{-12}. \tag{8}$$

From Eqs. (6) and (8), the bias of the $q_{21}Q_{21}$ coupling effect is reduced by about 95 times as a result of compensatory work. The effects from the gravity coupling $q_{21}Q_{21}$ before and after the compensatory work can be summarized in Table 2. The error contribution of the $q_{21}Q_{21}$ coupling effect is reduced by about 20 times. The limit of the test precision due to the $q_{21}Q_{21}$ coupling effect is improved to the 10^{-13} level.

Table 2. values of uncertainty $\delta\eta_{12}$ due to the gravity coupling $q_{21}Q_{21}$ before and after the compensatory work.

	Sum	Central value correction	Error contribution
Before compensating	$(6.54\pm0.11)\times10^{-10}$	6.54×10^{-10}	1.1×10^{-11}
After compensating	$(6.88\pm0.50)\times10^{-12}$	6.88×10^{-12}	5.0×10^{-13}

6. Summary

The test of WEP is of great physical significance. The HUST group has been performing the WEP test from 2003 on, and now expects the experimental precision 10^{-12} level.

In this paper, we analyzed the gravity gradient effect which is a main systematic error source in the test of WEP. For the experiment of the HUST group, the dominant gravity gradient effect comes from the $q_{21}Q_{21}$ coupling. Therefore, we make a q_{21} pendulum as the torsion pendulum to measure the multipole field Q_{21}. The experimental result shows the residual effect from the $q_{21}Q_{21}$ coupling will arrive at a 10^{-11} level. Based on the value of the Q_{21}, we make a Q_{21} compensation to compensate for the Q_{21} and the compensated multipole field Q_{21}^{c} is reduced by about 95 times. At the same time, the residual effect is reduced by about 20 times, and namely the limit of test precision due to the coupling is improved from a part in 10^{11} to a part in 10^{13} level. Since the effects from the other gravity couplings which also limit the accuracy of the test are not fully clear, there remains much work to be done in the future.

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