Conditions on converting coherence into entanglement*

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The present studies show that any nonzero amount of coherence of a system can be converted into entanglement between the system and an incoherent ancillary system via incoherent operations. According to this conclusion, we study the process of converting coherence into entanglement via a unitary operation where the initial ancillary system is of different quantum state. We find that some other conditions should be satisfied in converting coherence into entanglement. We also study the conditions of coherence consumption of converting coherence into entanglement.

Keywords: coherence, entanglement, conditions

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1. Introduction

Quantum coherence arising from quantum superposition is a fundamental feature of quantum mechanics. It is a key component in various quantum information and is primarily accountable for the advantage offered by quantum tasks versus classical ones.[1,2] Coherence as an important physical resource is used in thermodynamics,[3–5] biomolecular networks,[6–11] nanoscale physics,[12,13] and quantum metrology.[14,15] But the rigorous characterization of coherence in the framework of resource theories has been a rather recent development,[16] and a subsequent stream of work has identified some coherence measures.[17–22] Examples of coherence measures include the l_1-norm and the relative entropy of coherence.[16]

Nonclassical correlations have long been regarded as a key quantum resource. Entanglement was the first concept to be known about. Both coherence and entanglement capture the quantumness of a physical system, and it is well known that entanglement stems from the superposition principle, which is also the essence of coherence.[23] It is then legitimate to ask what relations they are. Recently, the relation between coherence and entanglement was studied in Refs. [19], [24], [25], and [26]. Streltsov *et al.*[19] utilized a common frame to quantify quantumness in terms of coherence and entanglement. In particular, they showed that any nonzero amount of coherence of a system A can be converted into entanglement between A and an initially incoherent ancilla B, by means of incoherent operations. Then we ask whether any nonzero amount of coherence can be converted to entanglement via incoherent operations. If not, what conditions should be satisfied under which the coherence can be converted into entanglement via incoherent operations? We do the work in this paper.

The rest of this paper is organized as follows. In Section 2, we briefly review the resource theory of coherence and entanglement. In Section 3, we study the necessary conditions of the initial state of converting coherence into entanglement via incoherent operation. In Section 4, we study the necessary conditions of the coherence consumption of converting coherence into entanglement via incoherent operation, and demonstrate the reasons for those conditions. Section 5 contains the concluding remarks.

2. Review of the resource theory of coherence and entanglement

We first review the information-theoretic definitions of coherence and entanglement. The resource theory of coherence characterizes the free resources, i.e., incoherent states and incoherent operations, and the criteria identifying coherence measures.[16] The framework has been extended to the multipartite scenario.[19,21,24] In an N-partite system, the incoherent states can be represented as $\varepsilon = \sum_{\boldsymbol{k}} p_{\boldsymbol{k}} |\boldsymbol{k}\rangle \langle \boldsymbol{k}|$, where $|\boldsymbol{k}\rangle = |k_1\rangle \otimes \cdots \otimes |k_N\rangle$, $|k_i\rangle$ is the pre-fixed local basis of the i-th subsystem, $\{p_{\boldsymbol{k}}\} \geq 0$, and $\sum_{\boldsymbol{k}} p_{\boldsymbol{k}} = 1$. The set of incoherent states τ is the set of quantum states with diagonal density matrices with respect to this basis. Any other state has nonzero coherence. An incoherent operation is any quantum operation that maps an incoherent state to an incoherent state. A measure of coherence $f_{\mathrm{C}}(\rho)$ is a nonnegative function which vanishes for incoherent states and is a non-increasing monotone under incoherent operations, $f_{\mathrm{C}}(\rho) \geq f_{\mathrm{C}}(\Lambda_{\mathrm{IC}}(\rho))$. This ensures that

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the coherence cannot be increased through incoherent operations. A notable coherence measure is the relative entropy of coherence, which is given by $C(\rho) = \min_{\sigma\in\tau} S(\rho||\sigma)$, where τ denotes the set of incoherent states. For a quantum state ρ, the closest incoherent state is $\rho_{\rm d}$, where $\rho_{\rm d}$ is the matrix containing only the leading diagonal elements of ρ in the reference basis, so the relative entropy of coherence[14] is

$C_{\rm r}(\rho) = S(\rho||\rho_{\rm d}) = S(\rho_{\rm d}) - S(\rho)$, where $S(\rho) = -\operatorname{Tr}(\rho\log\rho)$ is the von Neumann entropy.

The theory of entanglement has been the cornerstone of major developments in quantum information theory and has triggered the advancement of modern quantum technologies. An N-partite state is disentangled if it can be written in the separable form, $\rho = \sum_i p_i \rho_1^i \otimes \cdots \otimes \rho_N^i$. For pure quantum states, the separable states are in the form of $|\phi\rangle = |\phi\rangle_1 \otimes \cdots \otimes |\phi\rangle_N$. If a state of the quantum system cannot be written in these forms, it is called entangled state. To compare with the relative entropy of coherence, we focus on the relative entropy of entanglement, $E_{\rm r}(\rho) = \min_{\sigma\in SE} S(\rho||\sigma)$, where SE represents the sets of separable states.

3. Conditions of the initial state of converting coherence into entanglement

Considering the unitary operation

$$U = \sum_{i=1}^{d_A}\sum_{j=1}^{d_A} |i\rangle\langle i|^A \otimes |\bmod(i+j,d_A)\rangle\langle j|^B + \sum_{i=1}^{d_A}\sum_{j=d_A+1}^{d_B} |i\rangle\langle i|^A \otimes |j\rangle\langle j|^B, \quad (1)$$

where $\{|i\rangle\}$ and $\{|j\rangle\}$ are the reference bases of subsystems A and B, respectively. It can be seen by inspection that this unitary is incoherent (i.e., the state $\Lambda^{AB}(\rho_{AB}) = U\rho_{AB}U^\dagger$ is incoherent for any incoherent state ρ_{AB}). Note that for two qubits this unitary is equivalent to the generalized controlled-not gate (CNOT) $U_{\rm CN}$, with A serving as the control qubit and B the target qubit.[19] It can convert any pure incoherent state $|i\rangle\otimes|j\rangle$ into another pure incoherent state, $U_{\rm CN}(|i\rangle\otimes|j\rangle) = |i\rangle\otimes|\bmod(i+j),2\rangle$. It is well known that the state $U_{\rm CN}(|\psi\rangle\otimes|j\rangle)$ is entangled for any coherent state $|\psi\rangle$.[2] But if the target qubit is also a coherent state, the state $U_{\rm CN}(|\psi\rangle\otimes|\varphi\rangle)$ is always entangled anyway.

Using the CNOT gate as an incoherent operation, we analyze the conditions under which a product state can be converted into an entangled state via the incoherent operation. For some product pure states, the evolved states under the CNOT gate are as follows. We perform the CNOT gate on the incoherent control qubit and the coherent target qubit, the evolved state is

$$U_{\rm CN}(|0\rangle\otimes(a|0\rangle+b|1\rangle)) = |0\rangle\otimes(a|0\rangle+b|1\rangle), \quad (2a)$$

$$U_{\rm CN}(|1\rangle\otimes(a|0\rangle+b|1\rangle)) = |1\rangle\otimes(a|1\rangle+b|0\rangle), \quad (2b)$$

where $|a|^2+|b|^2=1$. We can see that the evolved state is also a separable state when the control qubit is an incoherent state. We perform the CNOT gate on the coherent control qubit A and the coherent target qubit B, the evolved state is

$$\begin{aligned} &U_{\rm CN}((\alpha|0\rangle+\beta|1\rangle)\otimes(a|0\rangle+b|1\rangle)) \\ &= \alpha a|00\rangle+\alpha b|01\rangle+\beta a|11\rangle+\beta b|10\rangle, \end{aligned} \quad (3)$$

where $|\alpha|^2+|\beta|^2=1$, and $|a|^2+|b|^2=1$. For a bipartite pure state, it is well known that the relative entropy of entanglement is equal to the entropy of the subsystem, $E_{\rm r}(\rho_{AB}) = S(\rho_A)$. We calculate the entanglement of the evolved state,

$$\begin{aligned} &E_{\rm r}(U_{\rm CN}(\rho_{AB})) \\ &= -\frac{1}{2}\left[1+\sqrt{1-4|\alpha|^2|\beta|^2\left(|a|^2-|b|^2\right)^2}\right] \\ &\times\log\frac{1}{2}\left[1+\sqrt{1-4|\alpha|^2|\beta|^2\left(|a|^2-|b|^2\right)^2}\right] \\ &-\frac{1}{2}\left[1-\sqrt{1-4|\alpha|^2|\beta|^2\left(|a|^2-|b|^2\right)^2}\right] \\ &\times\log\frac{1}{2}\left[1-\sqrt{1-4|\alpha|^2|\beta|^2\left(|a|^2-|b|^2\right)^2}\right]. \end{aligned} \quad (4)$$

We can see that the entanglement is zero when $|a|=|b|$. If we set $a=b$, the evolved state is a product state,

$$\begin{aligned} &U_{\rm CN}(\alpha|0\rangle+\beta|1\rangle)\otimes(a|0\rangle+a|1\rangle) \\ &= (\alpha|0\rangle+\beta|1\rangle)\otimes(a|0\rangle+a|1\rangle). \end{aligned} \quad (5)$$

In terms of the equation $|a|^2+|b|^2=1$ and $|a|=|b|$, we obtain $|a|=|b|=1/\sqrt{2}$. Then we conclude that the entanglement cannot be generated when the target qubit is a maximally coherent state.

We now analyze the conditions of converting coherence into entanglement via the CNOT gate when the initial states are the mixed states. As is well known, a product state $\rho_{AB} = \rho_A\otimes|0\rangle_B\langle 0|$ can be converted into an entangled state via the CNOT gate if and only if the control qubit ρ_A is coherent.[19] The two-qubit CNOT gate can be rewritten as

$$U_{\rm CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (6)$$

with $U_{\rm CN}U_{\rm CN}^\dagger = I$, I is the identity operator. We perform the CNOT gate on the state $\rho_A\otimes\rho_B$, the state evolves into a final state,

$$\rho_{\rm CN} = U_{\rm CN}(\rho_A\otimes\rho_B)U_{\rm CN}^\dagger. \quad (7)$$

We now consider a general single-qubit state $\rho_A = \frac{1}{2}\left(I+\sum_j n_j\sigma_j\right)$ as the control qubit and another state $\rho_B =$

$\frac{1}{2}\left(\mathrm{I}+\sum_j n'_j\sigma_j\right)$ as the target qubit, where $j\in\{1,2,3\}$, and σ_j being the j-th Pauli matrix, with $n_1^2+n_2^2+n_3^2\leq 1$, and $(n'_1)^2+(n'_2)^2+(n'_3)^2\leq 1$. For convenience, we assume its Bloch vectors $\boldsymbol{n}=\{n_1,n_2=0,n_3\}$ and $\boldsymbol{n}'=\{n'_1,n'_2=0,n'_3\}$. In the reference basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$, the initial product state of the two qubits is

$$\rho_A\otimes\rho_B=\frac{1}{4}\begin{bmatrix}\rho_{11}&\rho_{12}&\rho_{13}&\rho_{14}\\ \rho_{21}&\rho_{22}&\rho_{23}&\rho_{24}\\ \rho_{31}&\rho_{32}&\rho_{33}&\rho_{34}\\ \rho_{41}&\rho_{42}&\rho_{43}&\rho_{44}\end{bmatrix}, \tag{8}$$

where $\rho_{11}=(1+n_3)(1+n'_3)$, $\rho_{12}=\rho_{21}=(1+n_3)n'_1$, $\rho_{13}=\rho_{31}=n_1(1+n'_3)$, $\rho_{14}=\rho_{41}=n_1n'_1$, $\rho_{23}=\rho_{32}=n_1n'_1$, $\rho_{33}=(1-n_3)(1+n'_3)$, $\rho_{24}=\rho_{42}=n_1(1-n'_3)$, $\rho_{22}=(1+n_3)(1-n'_3)$, $\rho_{34}=\rho_{43}=(1-n_3)n'_1$, $\rho_{44}=(1-n_3)(1-n'_3)$.

We perform the CNOT gate on the control qubit and the target qubit separately, the initial state of the qubits becomes

$$\begin{aligned}\rho_{\mathrm{CN}}&=U(\rho_A\otimes\rho_B)U^\dagger\\&=\frac{1}{4}\begin{bmatrix}\rho_{11}&\rho_{12}&\rho_{14}&\rho_{13}\\ \rho_{21}&\rho_{22}&\rho_{24}&\rho_{23}\\ \rho_{41}&\rho_{42}&\rho_{44}&\rho_{43}\\ \rho_{31}&\rho_{32}&\rho_{34}&\rho_{33}\end{bmatrix}.\end{aligned} \tag{9}$$

If $n_1=0$, the final state is a separable state,

$$\begin{aligned}\rho_{\mathrm{CN}}=&\frac{1}{4}(1-n_3)|1\rangle\langle 1|\left[\left(1-n'_3\right)|0\rangle\langle 0|\right.\\&+n'_1|0\rangle\langle 1|+n'_1|1\rangle\langle 0|+\left(1+n'_3\right)|1\rangle\langle 1|\left.\right]\\&+\frac{1}{4}(1+n_3)|0\rangle\langle 0|\left[\left(1-n'_3\right)|1\rangle\langle 1|+n'_1|0\rangle\langle 1|\right.\\&+n'_1|1\rangle\langle 0|+\left(1+n'_3\right)|0\rangle\langle 0|\left.\right].\end{aligned} \tag{10}$$

That is to say, if the control qubit is an incoherent state, the entanglement cannot be generated.

If $n_1\neq 0$ and $n'_1=0$, the control qubit is a coherent state, and the target qubit is an incoherent state. We find that the final state is a two-qubit X-state. A two-qubit X-state is entangled if and only if either of the following two conditions is satisfied:

$$\rho_{22}\rho_{33}<|\rho_{14}|^2\quad\text{and}\quad\rho_{11}\rho_{44}<|\rho_{23}|^2, \tag{11}$$

and these two conditions cannot hold simultaneously.[27,28] Here, the condition becomes $\rho_{22}\rho_{44}<|\rho_{13}|^2$ or $\rho_{11}\rho_{33}<|\rho_{24}|^2$. After calculation, we find that when the following condition is satisfied:

$$n'_3>\frac{\sqrt{1-n_3^2}-|n_1|}{\sqrt{1-n_3^2}+|n_1|}\quad\text{or}\quad n'_3<\frac{|n_1|-\sqrt{1-n_3^2}}{\sqrt{1-n_3^2}+|n_1|}, \tag{12}$$

the final state is an entangled state. Obviously, if $n'_3=1$ or $n'_3=-1$, the condition is fulfilled.

If $n_1\neq 0$, $n'_1=1$ and $n'_3=0$, the final state is identical to the initial state. Namely, when the control qubit is a coherent state and the target qubit is a maximally coherent state, the final state is a product state. The coherence cannot be converted into entanglement.

4. Conditions of the coherence consumption of converting coherence into entanglement

In order to demonstrate the variety of coherence of a quantum system under a unitary operation, we first prove the following theorem and define the concept of total coherence consumption.

Theorem 1 Unitary operation generates the coherence invariant, $C_{\mathrm{r}}(\rho)=C_{\mathrm{r}}\left(U\rho U^\dagger\right)$.

Proof Unitary transformation represents a change of basis (i.e., a change in our "perspective"), under which the distance between two states should not change. One property of the relative entropy is unitary operations generating $S(\rho||\sigma)$ invariant,[29] i.e.,

$$S(\rho||\sigma)=S\left(U\rho U^\dagger||U\sigma U^\dagger\right). \tag{13}$$

For a bipartite quantum state ρ_{AB} and its closest incoherent state $(\rho_{AB})_{\mathrm{d}}$, the relative entropy of ρ_{AB} and $(\rho_{AB})_{\mathrm{d}}$ are invariant under unitary operation, $S(\rho_{AB}||(\rho_{AB})_{\mathrm{d}})=S(U\rho_{AB}U^\dagger||U(\rho_{AB})_{\mathrm{d}}U^\dagger)$. Note that the application of a unitary operation to the incoherent state $(\rho_{AB})_{\mathrm{d}}$ brings it to another incoherent state $U(\rho_{AB})_{\mathrm{d}}U^\dagger$. Furthermore, $U(\rho_{AB})_{\mathrm{d}}U^\dagger=(U(\rho_{AB})U^\dagger)_{\mathrm{d}}$, so we obtain $S(\rho_{AB}||(\rho_{AB})_{\mathrm{d}})=S\left(U\rho_{AB}U^\dagger||\left(U(\rho_{AB})U^\dagger\right)_{\mathrm{d}}\right)$. The proof is complete.

Given an arbitrary quantum channel $\Lambda(\cdot)$, the coherence consumption[30] ΔC of a quantum system A under $\Lambda(\cdot)$ is $\Delta C(\rho_A)=C(\rho_A)-C(\Lambda(\rho_A))$. We note that this quantity is closely related to the decohering power of a channel introduced by Mani and Karimipour.[31] For a bipartite quantum system ρ_{AB} with subsystems A and B, which undergoes incoherent operations $\Lambda(\cdot)$, $\Delta C(\rho_A)=C(\rho_A)-C(\Lambda(\rho_A))$ denotes the coherence consumption of subsystem A, and $\Delta C(\rho_B)=C(\rho_B)-C(\Lambda(\rho_B))$ denotes the coherence consumption of subsystem B, where $\rho_{A(B)}=\mathrm{Tr}_{B(A)}(\rho_{AB})$ and $\Lambda\left(\rho_{A(B)}\right)=\mathrm{Tr}_{B(A)}(\Lambda(\rho_{AB}))$.

Definition 1 The total coherence consumption of a bipartite system ρ_{AB} under an incoherent operation is

$$\Delta C(\rho_{AB})=\Delta C(\rho_A)+\Delta C(\rho_B). \tag{14}$$

For a product state $\rho_{AB}=\rho_A\otimes\rho_B$, The total coherence is equal to the sum of the coherence of subsystems, i.e., $C(\rho_{AB})=C(\rho_A)+C(\rho_B)$. As is well known, the CNOT gate is a unitary operation and also is an incoherent operation. We have proven that unitary operations generate the coherence invariant in a system, $C(\rho_{AB})=C(\Lambda(\rho_{AB}))$. So the total coherence consumption of a bipartite product state under the CNOT

gate is

$$\Delta C(\rho_{AB}) = C(\Lambda(\rho_{AB})) - C(\Lambda(\rho_A)) - C(\Lambda(\rho_B)), \quad (15)$$

which is the notion of correlated coherence demonstrated in Ref. [32]. Considering a bipartite state ρ_{AB}, the correlated coherence is the local coherence subtracted from the total coherence, i.e., $C_{cc}(\rho_{AB}) = C(\rho_{AB}) - C(\rho_A) - C(\rho_B)$, where ρ_A and ρ_B are the reduced density matrices of A and B, respectively. Tan *et al.*[32] have proven that the correlated coherence is always non-negative, i.e., $C_{cc}(\rho_{AB}) \geq 0$. We obtain

$$\Delta C(\rho_{AB}) = C_{cc}(\Lambda(\rho_{AB})) \geq 0. \quad (16)$$

A bipartite separable state is in the form of $\rho_{AB} = \sum_i p_i \rho_A^i \otimes \rho_B^i$. In terms of the property of entropy, $S(\sum_i p_i \rho_i) = H(p_i) + \sum_i p_i S(\rho_i)$, where $H(p_i) = -\sum_i p_i \log p_i$, the definition of the relative entropy of coherence. We can prove that the correlated coherence of a separable state is always zero, $C_{cc}(\rho_{AB}) = 0$. The converse is true for the relative entropy of coherence. Namely, if the total coherence consumption is zero, $\Delta C(\rho_{AB}) = 0$, the entanglement cannot be generated when the initial product state undergoes the CNOT gate. So the conditions of converting coherence to entanglement via the CNOT gate are the total coherence consumption always being positive,

$$\Delta C(\rho_{AB}) > 0. \quad (17)$$

According to the conditions of converting coherence into entanglement discussed above, we think that the reasons of converting coherence into entanglement may be as follows: when the product state $\rho_{AB} = \rho_A \otimes \rho_B$ undergoes the incoherent operations, the control qubit and the target qubit interact with each other, the coherence of the control qubit immerges into the target qubit and mixes it together, the total coherence of the system is consumed, then the entanglement is generated in the system. When the initial target qubit is in a maximally coherent state, the coherence is saturated. The coherence of the control qubit cannot immerge into the target qubit, so the total coherence cannot be consumed, and then the entanglement cannot be generated in the system either.

5. Conclusions

In this paper, we discuss the conditions of converting coherence into entanglement, demonstrating the formal potential of coherence for entanglement generation. According to the analysis of entanglement of the evolved state, we find the necessary conditions of converting coherence into entanglement via incoherent operations: firstly, the control qubit is a coherent state, secondly, the target qubit is not the maximally coherent state. In addition to these basic conditions, due to the methods of measuring entanglement and the property of logarithm function, there are some additional conditions for converting coherence into entanglement. According to the notion of coherence consumption, we derive the necessary conditions of converting coherence into entanglement via incoherent operations, that is, the total coherence consumption is always positive, the conditions have been proven as valid conditions to determine quantum correlation.[32] Finally, we provide a hypothesis for demonstrating the process of converting coherence into entanglement. Although converting coherence into entanglement via an incoherent operation may be not necessarily useful in practical applications, it, as a cheaper scheme for creating entanglement, might be available. But we hope our work can provide some clues to demonstrating the process of entanglement generation.

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