Coherent charge transport in ferromagnet/semiconductor nanowire/ferromagnet double barrier junctions with the interplay of Rashba spin–orbit coupling, induced superconducting pair potential, and external magnetic field

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By solving the Bogoliubov–de Gennes equation, the influence of the interplay of Rashba spin–orbit coupling, induced superconducting pair potential, and external magnetic field on the spin-polarized coherent charge transport in ferromagnet/semiconductor nanowire/ferromagnet double barrier junctions is investigated based on the Blonder–Tinkham–Klapwijk theory. The coherence effect is characterized by the strong oscillations of the charge conductance as a function of the bias voltage or the thickness of the semiconductor nanowire, resulting from the quantum interference of incoming and outgoing quasiparticles in the nanowire. Such oscillations can be effectively modulated by varying the strength of the Rashba spin–orbit coupling, the thickness of the nanowire, or the strength of the external magnetic field. It is also shown that two different types of zero-bias conductance peaks may occur under some particular conditions, which have some different characteristics and may be due to different mechanisms.

Keywords: Rashba spin–orbit coupling, induced superconducting pair potential, coherent charge transport, zero-bias conductance peaks

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1. Introduction

Spin–orbit interaction may play an important role in the spin-dependent transport in magnetic nanostructures. Stimulated by the pioneering theoretical work of Datta and Das on the ballistic spin field-effect transistor,[1] the utilization of spin–orbit coupling as an effective means of controlling coherent quantum charge transport properties of various magnetic nanostructures remains a hot research topic of enormous and persistent interest in the field of spintronics, and some significant progresses have been achieved in this direction.[2–5] These progresses have renewed the interest in the study of the spin-dependent transport properties in various magnetic nanostructures with strong spin–orbit interaction.[6–23]

There has recently been some attempt to study spintronics in hybrid semiconductor heterostructures with the presence of both strong spin–orbit interaction and superconductivity.[24–30] Such systems can be realized by combining conventional (or unconventional) superconductors and semiconductor two-dimensional electron gases (2DEGs) or semiconductor nanowires with strong spin–orbit interaction. It was found that the Andreev reflection and the conductance spectra of such hybrid heterostructures can be affected significantly by the presence of the strong spin–orbit interaction.[24–27] Such systems can also be realized by inducing the superconducting pair potential into a semiconductor 2DEG or a semiconductor nanowire with strong spin–orbit interaction through superconductor-proximity effect.[31–37] The interplay between the induced superconducting pair potential and the strong spin–orbit interaction can lead to the formation of helical superconducting states in a semiconductor 2DEG or a semiconductor nanowire. This possibility has been verified unambiguously by some recent experimental studies.[38–44] It was both theoretically predicted and experimentally demonstrated that, due to the formation of helical superconducting states in some hybrid semiconductor heterostructures with strong spin–orbit interaction, some exotic quantum transport phenomena may occur.[31–44] These advances also offer a new route for studying spintronics in hybrid semiconductor heterostructures, in which the spin-dependent charge transport may be effectively controlled by the interplay of the strong spin–orbit interaction and the induced superconducting pair potential through the superconductor-proximity effect. A unique system to study such interplay is a double-barrier junction in which a semiconductor nanowire with strong Rashba spin–orbit coupling is sandwiched between two ferromagnetic (FM) leads. By making the Rashba semiconductor nanowire side-coupled to a conventional superconductor, the superconducting pair potential can be induced into the nanowire through the proximity effect. As in the spin transistor geometry, in such a hybrid structure, the FM leads can act as generator and detector of spin-polarized electrons or holes. Due to the formation of helical superconducting states in the

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semiconductor nanowire, the spin-polarized coherent charge transport in such a hybrid structure will depend on the competition between the intrinsic Rashba spin–orbit coupling and the induced superconducting pair potential in the semiconductor nanowire. In this paper, we present a theoretical investigation on the influences of these competing factors on the spin-polarized coherent quantum charge transport in such a hybrid structure. The investigation carried out in this paper is based on the extended Blonder–Tinkham–Klapwijk (BTK) approach.[45] Based on the extended BTK formalism, we solve self-consistently the Bogoliubov–de Gennes equation and calculate the charge conductance of the structure as functions of a series of adjustable physical parameters. The results show that, due to the quasiparticle interference in the semiconductor nanowire, the charge conductance of the structure can exhibit strong oscillations as a function of the bias voltage or the thickness of the nanowire, and the characteristic features of such oscillations can be effectively modulated by varying the strength of the Rashba spin–orbit coupling or the thickness of the semiconductor nanowire, or by applying an external magnetic field on the semiconductor nanowire. The strong oscillations of the conductance with the period of the geometrical resonances could be used for reliable spectroscopy of quasiparticle excitations in such structures. It is also found that two different types of zero-bias conductance peaks may occur under some particular conditions in such structures, which have different characteristics and should be due to different mechanisms. This is different from some previous theoretical predictions.[31–35]

The paper is organized as follows. In Section 2, we introduce the model Hamiltonian and the extended BTK formalism for the calculation of the charge conductance of the structure considered. In Section 3, we study numerically the characteristics of the conductance spectra of the structure in the presence of the interplay of Rashba spin–orbit coupling, induced superconducting pair potential, and external magnetic field. Some concluding remarks will be given in the end of the paper.

2. Model and formulation

We consider a one-dimensional double-barrier junction in which a ballistic semiconductor nanowire with strong Rashba spin–orbit coupling is sandwiched between two FM electrodes by thin and insulating interfaces. We assume that the semiconductor nanowire is along the y axis and the directions of the exchange field in the FM electrodes are also along the y axis and the Rashba field in the central semiconductor nanowire is along the z axis, similar to the case of the spin-transistor geometry. The two interfaces between the central semiconductor nanowire and the FM leads are located at $y=0$ and $y=L$ and have infinitely narrow insulating barriers described by the δ function: $U(y)=U_0\{\delta(y)+\delta(y-L)\}$, with the strength of the interface barrier U_0. We assume that an s-wave superconducting pair potential is induced in the central semiconductor nanowire through the superconductor-proximity effect. Then in spin and Nambu space, the effective Hamiltonian for the structure is given by

$$H=\begin{pmatrix} H_0(k,\sigma) & \mathrm{i}\sigma_y\Delta\theta(y)\theta(L-y) \\ -\mathrm{i}\sigma_y\Delta\theta(y)\theta(L-y) & -H_0(-k,\sigma^*) \end{pmatrix}, \quad (1)$$

with the step function $\theta(y)$, the Pauli matrices σ_i ($i=x,y,z$), the superconducting pair potential Δ, and

$$\begin{aligned} H_0(k,\sigma) \;=\; & \xi_k + h_0\sigma_y\{\theta(-y)+\theta(y-L)\} \\ & +\{\alpha_{\mathrm{R}}k\sigma_z+B_0\sigma_x\}\theta(y)\theta(L-y) \\ & +U_0\{\delta(y)+\delta(y-L)\}, \end{aligned} \quad (2)$$

where $\xi_k=\hbar^2k^2/2m-E_{\mathrm{F}}$ is the kinetic energy with respect to the Fermi level E_{F}, h_0 is the exchange field in the FM leads, and α_{R} is the Rashba spin–orbit coupling constant in the semiconductor nanowire. The properties of the induced superconducting state in a Rashba semiconductor nanowire can also be modulated significantly by applying an external magnetic field in the direction perpendicular to the Rashba field.[31–36] To take this effect into account, we have included a Zeeman term in the Hamiltonian H_0, with the external magnetic field B_0. For simplicity, we choose the same effective mass in the FM leads and in the central semiconductor nanowire and neglect the Fermi wave vector mismatch.

The quasiparticle propagation in the system is described by the Bogoliubov–de Gennes equation $H\Psi=E\Psi$, where Ψ are the quasiparticle eigenfunctions.[45] For an injection wave from the left FM lead, the scattering wave function in each region of the system can be represented as the linear combination of the quasiparticle eigenfunctions of the Hamiltonian H. For an injection wave with energy E and spin σ ($\sigma=+$ for spin-polarization along the $+y$ axis and $\sigma=-$ for spin-polarization along the $-y$ axis), the scattering wave function in the left FM lead ($y\le 0$) can be represented as

$$\begin{aligned} \Psi_\sigma(y\le 0) \;=\; & \mathrm{e}^{\mathrm{i}k_{\mathrm{e},\sigma}y}|\sigma\rangle_{\mathrm{e}} + a_{\sigma\bar{\sigma}}\,\mathrm{e}^{\mathrm{i}k_{\mathrm{h},\bar{\sigma}}y}|\bar{\sigma}\rangle_{\mathrm{h}} + a_{\sigma\sigma}\,\mathrm{e}^{\mathrm{i}k_{\mathrm{h},\sigma}y}|\sigma\rangle_{\mathrm{h}} \\ & + b_{\sigma\sigma}\,\mathrm{e}^{-\mathrm{i}k_{\mathrm{e},\sigma}y}|\sigma\rangle_{\mathrm{e}} + b_{\sigma\bar{\sigma}}\,\mathrm{e}^{-\mathrm{i}k_{\mathrm{e},\bar{\sigma}}y}|\bar{\sigma}\rangle_{\mathrm{e}}, \end{aligned} \quad (3)$$

where $\bar{\sigma}\equiv-\sigma$ and $|\sigma\rangle_{\mathrm{e,h}}$ denotes the spinor eigenfunctions of electrons (e) or holes (h) with spin σ, with $|+\rangle_{\mathrm{e}}=[1,\mathrm{i},0,0]^{\mathrm{T}}$, $|-\rangle_{\mathrm{e}}=[1,-\mathrm{i},0,0]^{\mathrm{T}}$, $|+\rangle_{\mathrm{h}}=[0,0,1,-\mathrm{i}]^{\mathrm{T}}$, $|-\rangle_{\mathrm{h}}=[0,0,1,\mathrm{i}]^{\mathrm{T}}$. $a_{\sigma\sigma'}$ are the Andreev reflection coefficients and $b_{\sigma\sigma'}$ are the normal reflection coefficients. $k_{\mathrm{e},\sigma}$ and $k_{\mathrm{h},\sigma}$ are the wave numbers of electrons (e) and holes (h) with spin σ, which are determined by the electron and hole energy dispersion relations in the FM leads, respectively,

$$E=\hbar^2k_{\mathrm{e},\sigma}^2/2m-\sigma h_0-E_{\mathrm{F}}=E_{\mathrm{F}}-\hbar^2k_{\mathrm{h},\sigma}^2/2m+\sigma h_0. \quad (4)$$

Similarly, the scattering wave function in the right FM lead ($y \geq L$) can be represented as

$$\Psi_\sigma(y \geq L) = \sum_{\sigma'} \{ t^{(\mathrm{e})}_{\sigma\sigma'} \mathrm{e}^{\mathrm{i}k_{\mathrm{e},\sigma'}y} |\sigma'\rangle_{\mathrm{e}} + t^{(\mathrm{h})}_{\sigma\sigma'} \mathrm{e}^{\mathrm{i}k_{\mathrm{h},\sigma'}y} |\sigma'\rangle_{\mathrm{h}} \}, \quad (5)$$

where $t^{(\mathrm{e})}_{\sigma\sigma'}$ are the transmission coefficients of an incident electron with spin σ to the right FM lead as an electron with spin σ', and $t^{(\mathrm{h})}_{\sigma\sigma'}$ are the transmission coefficients of an incident electron with spin σ to the right FM lead as a hole with spin σ'.

The wave function in the central semiconductor nanowire can also be represented using the eigenfunctions of the Hamiltonian H. By solving the Bogoliubov–de Gennes equation, one finds that the quasiparticle energy dispersion relation in the central semiconductor nanowire is given by

$$E^2 = B_0^2 + \Delta^2 + \xi_k^2 + (\alpha_{\mathrm{R}} k)^2 \pm 2\sqrt{B_0^2\Delta^2 + B_0^2\xi_k^2 + (\alpha_{\mathrm{R}}k)^2\xi_k^2}. \quad (6)$$

From this dispersion relation, for an injection wave with energy E, one can obtain eight solutions for the quasiparticle wave number and will be denoted as k_n ($n = 1, 2, \ldots, 8$). The wave function in the central semiconductor nanowire can then be represented as

$$\psi(0 < y < L) = \sum_{n=1,2,\ldots,8} c_n \mathrm{e}^{\mathrm{i}k_n y} \psi_n(k_n), \quad (7)$$

where $\psi_n(k_n) = [u_\uparrow(k_n), u_\downarrow(k_n), v_\uparrow(k_n), v_\downarrow(k_n)]^{\mathrm{T}}$ denotes the spinor eigenfunction of the quasiparticles with wave number k_n, and $u_\sigma(k_n)$ and $v_\sigma(k_n)$ are given by

$$u_\uparrow(k_n) = \frac{2B_0\alpha_{\mathrm{R}}k_n + 2B_0E}{\Delta}, \quad (8)$$

$$u_\downarrow(k_n) = \frac{E^2 + B_0^2 - \Delta^2 - (\hbar^2k_n^2/2m + \alpha_{\mathrm{R}}k_n - E_{\mathrm{F}})^2}{\Delta}, \quad (9)$$

$$v_\downarrow(k_n) = \frac{E + E_{\mathrm{F}} - \hbar^2k_n^2/2m - \alpha_{\mathrm{R}}k_n}{\Delta} u_\uparrow(k_n) - \frac{B_0}{\Delta} u_\downarrow(k_n), \quad (10)$$

$$v_\uparrow(k_n) = \frac{\hbar^2k_n^2/2m - \alpha_{\mathrm{R}}k_n - E - E_{\mathrm{F}}}{\Delta} u_\downarrow(k_n) + \frac{B_0}{\Delta} u_\uparrow(k_n). \quad (11)$$

The amplitudes of the electron-like and hole-like quasiparticles, propagating in the central semiconductor nanowire, are given by the coefficients c_1–c_8.

There are sixteen unknown coefficients in the wave functions given by Eqs. (3), (5), and (7). They can be determined by the following boundary conditions at the two interfaces at $y = 0$ and $y = L$:

$$\Psi_\sigma|_{y=0^-} = \Psi_\sigma|_{y=0^+}, \quad (12)$$

$$\hat{V}^{(\mathrm{F})}\Psi_\sigma|_{y=0^-} = \hat{V}^{(\mathrm{S})}\Psi_\sigma|_{y=0^+} - \frac{\hbar}{mi}\frac{2mU_0}{\hbar^2}(\sigma_0 \otimes \tau_z)\Psi_\sigma|_{y=0^+}, \quad (13)$$

$$\Psi_\sigma|_{y=L^-} = \Psi_\sigma|_{y=L^+}, \quad (14)$$

$$\hat{V}^{(\mathrm{F})}\Psi_\sigma|_{y=L^+} = \hat{V}^{(\mathrm{S})}\Psi_\sigma|_{y=L^-} + \frac{\hbar}{mi}\frac{2mU_0}{\hbar^2}(\sigma_0 \otimes \tau_z)\Psi_\sigma|_{y=L^-}, \quad (15)$$

where $\hat{V}^{(\mathrm{F})} = (-\frac{\mathrm{i}\hbar}{m}\frac{\mathrm{d}}{\mathrm{d}y})(\sigma_0 \otimes \tau_z)$ is the velocity operator in the FM leads and $\hat{V}^{(\mathrm{S})} = (-\frac{\mathrm{i}\hbar}{m}\frac{\mathrm{d}}{\mathrm{d}y})(\sigma_0 \otimes \tau_z) + \frac{\alpha_{\mathrm{R}}}{\hbar}(\sigma_z \otimes \tau_0)$ is the velocity operator in the central semiconductor nanowire, with σ_0 denoting the unit matrix in the spin space, τ_0 and τ_z the Pauli matrices in the Nambu space. After the sixteen coefficients in the wave functions (3), (5), and (7) have been determined from the above boundary conditions, following the derivation of Ref. [45], the differential charge conductance at zero temperature will be obtained as

$$G(V) = \frac{\mathrm{e}^2}{h} \sum_{\sigma,\sigma'} \left[1 + \mathrm{Re}\left(\frac{k_{\mathrm{h},\sigma'}}{k_{\mathrm{e},\sigma}}\right) |a_{\sigma\sigma'}(E)|^2 - \mathrm{Re}\left(\frac{k_{\mathrm{e},\sigma'}}{k_{\mathrm{e},\sigma}}\right) |b_{\sigma\sigma'}(E)|^2 \right], \quad (16)$$

where $E = eV$, with the bias voltage V.

3. Results and discussion

Based on the theoretical formulation introduced in Section 2, in this section we will study numerically the influence of the interplay of the Rashba spin–orbit coupling, the induced superconducting pair potential, and the external magnetic field on the conductance spectra of the structure considered. For convenience, below we will define the strength of the Rashba spin–orbit coupling by a dimensionless parameter $\alpha \equiv 2m\alpha_{\mathrm{R}}/\hbar^2k_{\mathrm{F}}$, with the Fermi wave vector k_{F}. The strength of the interface barrier is also defined by a dimensionless parameter $Z \equiv 2mU_0/\hbar^2k_{\mathrm{F}}$. The exchange field in the FM leads, the external magnetic field applied on the central semiconductor nanowire, and the superconducting pair potential induced in the nanowire will all be defined in units of Fermi energy E_{F}. The thickness of the central semiconductor nanowire will be defined in units of $1/k_{\mathrm{F}}$.

First, we neglect the influence of the external magnetic field (setting $B_0 = 0$) and focus on the interplay of the Rashba spin–orbit coupling and the superconducting pair potential. In Figs. 1(a) and 1(b) we plot the charge conductance as a function of the bias voltage for $L = 1000/k_{\mathrm{F}}$ and $L = 2000/k_{\mathrm{F}}$, respectively. In both panels, the other parameters are set to be $h_0/E_{\mathrm{F}} = 0.5$, $\Delta/E_{\mathrm{F}} = 0.003$, $Z = 5$, $\alpha = 0.1$ (the dashed line) and 0.3 (the solid line). Such parameters are experimentally realizable in InSb-based or InAs-based semiconductor nanowires and hybrid structures.[31–44] From Fig. 1, one can see that the conductance is a monotonous function of the bias voltage in the voltage region of $eV < \Delta$ but exhibits strong oscillatory behaviors in the voltage region of $eV > \Delta$. From

the theoretical points of view, such oscillations arise from the quantum interference of incoming and outgoing quasiparticle waves in the central semiconductor nanowire of the structure. The quantum interference of incoming and outgoing quasiparticle waves in the central semiconductor nanowire can lead to the formation of quasi-stationary states that are localized mainly in the central semiconductor nanowire. Due to the formation of such quasi-stationary states, resonant transmission will occur if the energy of an incident quasiparticle from the leads coincides with the energy of a quasi-stationary state in the central semiconductor nanowire. Such resonant transmission can lead to the occurrence of a series of conductance peaks in the conductance spectra. Since the energies of these quasi-stationary states are spin–orbit split due to the Rashba spin–orbit coupling and hence can be modulated by varying the strength of the Rashba spin–orbit coupling, the conductance spectra of the structure can also be modulated significantly by varying the strength of the Rashba spin–orbit coupling. From Figs. 1(a) and 1(b) one can see clearly that, as the strength of the Rashba spin–orbit coupling is varied, both the peak positions, the peak values, and the oscillation amplitudes of the conductance can all be modulated significantly, and if the Rashba spin–orbit coupling is sufficiently strong, a conductance peak can be split explicitly into two subpeaks of approximately equal height.

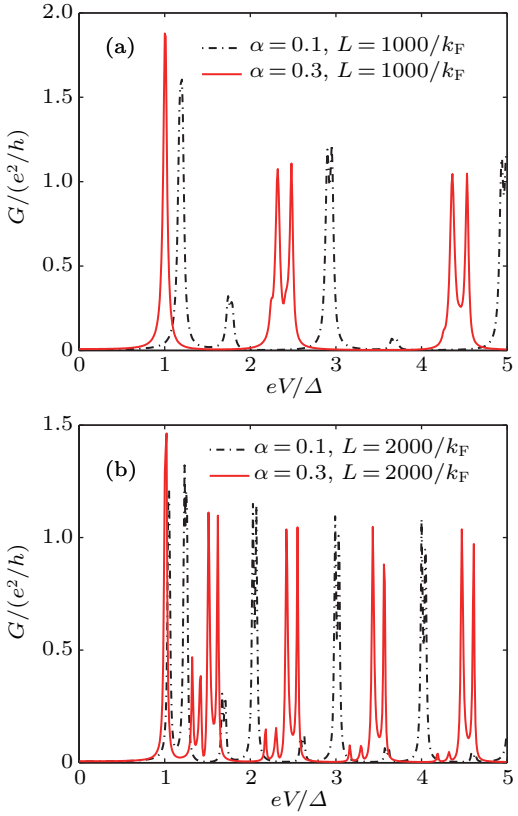


Fig. 1. (color online) The charge conductance as a function of the bias voltage for two different thicknesses of the semiconductor nanowire: (a) $L = 1000/k_F$, (b) $2000/k_F$. In both panels, the other parameters are taken to be $h_0/E_F = 0.5$, $\Delta/E_F = 0.003$, $Z = 5$, $\alpha = 0.1$ (the dashed line) and 0.3 (the solid line).

By comparing the corresponding results shown in Figs. 1(a) and 1(b), one can see that the number of the conductance peaks increases with the increase of the thickness of the central semiconductor nanowire. From the theoretical point of view, this is due to the fact that, as the thickness of the central semiconductor nanowire is increased, more and more quasi-stationary states can be formed in the nanowire due to the quantum interference of incoming and outgoing quasiparticle waves. When more quasi-stationary states are formed in the nanowire, the number of the conductance peaks will increase correspondingly, as can be seen from the results shown in Figs. 1(a) and 1(b).

Next, we consider the influence of the external magnetic field applied on the central semiconductor nanowire on the coherent charge transport through the structure. In Fig. 2 we plot the charge conductance as a function of the bias voltage V in the presence of the external magnetic field, where the dashed line is for $B_0/E_F = 0.2$ and the solid line for $B_0/E_F = 0.4$, and the other parameters are set to be $L = 1000/k_F$, $h_0/E_F = 0.5$, $\Delta/E_F = 0.003$, $Z = 5$, $\alpha = 0.3$. From Fig. 2 one can see that the conductance spectra of the structure can be modulated significantly by the external magnetic field. As the strength of the external magnetic field is varied, the peak positions, the peak values, and the oscillation amplitudes of the conductance can all be modulated significantly. The spin–orbit splitting of the conductance peaks can also be suppressed by the external magnetic field.

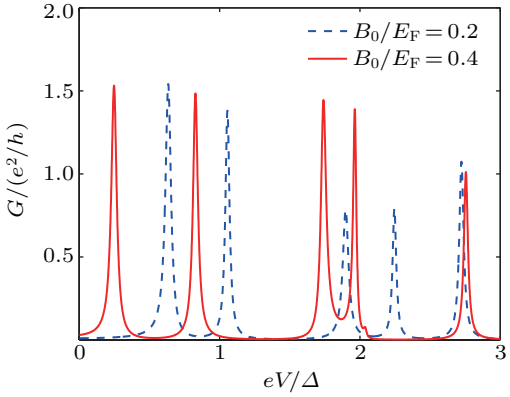


Fig. 2. (color online) The charge conductance as a function of the bias voltage V in the presence of the external magnetic field. The dashed line is for $B_0/E_F = 0.2$ and the solid line for $B_0/E_F = 0.4$. The other parameters are taken to be $L = 1000/k_F$, $h_0/E_F = 0.5$, $\Delta/E_F = 0.003$, $Z = 5$, $\alpha = 0.3$.

By comparing the corresponding results shown in Figs. 1 and 2, one can notice that a significant consequence of the external magnetic field is that the conductance peaks may also occur in the voltage region of $eV < \Delta$ when the strength of the applied external magnetic field is sufficiently strong, indicating that quasi-stationary states can be formed within the energy region of $E < \Delta$ in the presence of the external magnetic

field. From the theoretical point of view, this is due to the suppression of the superconducting gap by the external magnetic field.

It is also noticed that, due to the interplay of the Rashba spin–orbit coupling, the superconducting pair potential, and the external magnetic field in the semiconductor nanowire, the conductance peaks in the voltage region of $eV < \Delta$ can also occur at zero bias voltage under some particular conditions for the structure considered. This possibility is illustrated in Figs. 3(a) and 3(b), where we plot the charge conductance as a function of the bias voltage for $B_0/E_F = 0.5$ and $B_0/E_F = 0.8$, respectively. In both panels, the other parameters are set to be $h_0/E_F = 0.5$, $\Delta/E_F = 0.003$, $Z = 5$, $\alpha = 0.3$, $L = 1000/k_F$ (the solid line) and $1300/k_F$ (the dashed line). From Figs. 3(a) and 3(b) one can see that, for the case of $L = 1000/k_F$, zero-bias conductance peaks (ZBCPs) can occur both at $B_0/E_F = 0.5$ and at $B_0/E_F = 0.8$. Such ZBCPs are unstable with respect to the small variation of the thickness of the central semiconductor nanowire. For example, for the cases shown in Figs. 3(a) and 3(b), the ZBCPs disappear as the thickness of the central semiconductor nanowire is changed from $L = 1000/k_F$ to $1300/k_F$, while the other parameters remain the same.

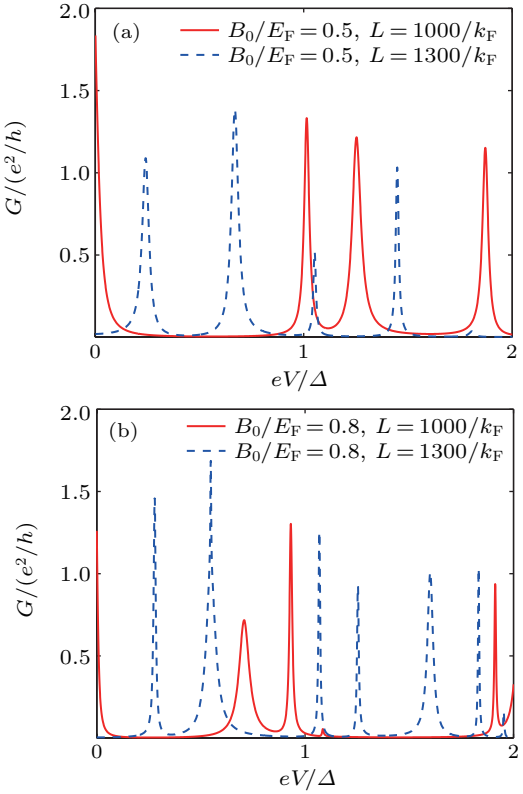


Fig. 3. (color online) Illustration of zero-bias conductance peak in the cases that the strength of the external magnetic field is smaller than the critical value B_c. Panel (a) shows the charge conductance as a function of the bias voltage V for $B_0/E_F = 0.5$, $L = 1000/k_F$ (the solid line) and $1300/k_F$ (the dashed line). Panel (b) shows the corresponding results for $B_0/E_F = 0.8$. In both panels, the other parameters are taken to be $h_0/E_F = 0.5$, $\Delta/E_F = 0.003$, $Z = 5$, $\alpha = 0.3$.

It should be stressed that the mechanism of the ZBCPs shown in Figs. 3(a) and 3(b) should be different from that proposed in some previous theoretical works.[31–35] The mechanism of the ZBCPs proposed in those theoretical works is due to the formation of zero-energy Majorana bound states (MBS) at the ends of a semiconductor nanowire. According to the analyses in those theoretical works, for a semiconductor nanowire with strong Rashba spin–orbit coupling and proximity-induced superconducting pair potential and subjected to an external magnetic field, if the strength of the external magnetic field exceeds a critical value $B_c \equiv \sqrt{\Delta^2 + E_F^2}$, the external magnetic field will drive the semiconductor nanowire through a topological quantum phase transition to a topologically nontrivial superconducting phase with localized MBS at the ends of the nanowire. When the strength of the external magnetic field is smaller than this critical value, the semiconductor nanowire is in a topologically trivial superconducting phase and no MBS can occur. For the cases shown in Figs. 3(a) and 3(b), the strength of the external magnetic field is substantially smaller than this critical value, so no MBS can occur in these cases. From the theoretical point of view, the ZBCPs shown in Figs. 3(a) and 3(b) should be due to the formation of zero-energy quasi-stationary states in the central semiconductor nanowire, arising from the combined effect of the suppression of the superconducting gap by the external magnetic field and the quantum interference of incoming and outgoing quasiparticle waves in the nanowire.

For the structure considered in the present paper, ZBCPs may also occur if the strength of the external magnetic field exceeds the critical value $B_c = \sqrt{\Delta^2 + E_F^2}$, as predicted in Refs. [31]–[35]. This is illustrated in Fig. 4. From the results shown in Fig. 4, one can see that the strength of the external magnetic field at which such ZBCPs shall occur also depends on the thickness of the semiconductor nanowire. For example, for the cases shown in Fig. 4, ZBCPs occur at $B_0/B_c = 1.19$ for $L = 1000/k_F$ but disappear as the thickness of the semiconductor nanowire is increased from $L = 1000/k_F$ to $1300/k_F$, while the other parameters remain the same.

By comparing the corresponding results shown in Figs. 3 and 4, one can notice that there are some significant differences between the two types of ZBCPs shown therein. For example, for the cases shown in Fig. 4, most of the conductance peaks are suppressed by the strong external magnetic field, and the peak value at zero bias voltage is also substantially smaller than that shown in Fig. 3. From the theoretical point of view, such ZBCPs should be due to the formation of zero-energy bound states at the ends of the semiconductor nanowire.[31–35] But as was argued in some recent papers,[46–50] since it cannot be ruled out that some other mechanisms may also lead to the occurrence of ZBCPs with similar characteristic features,

whether such ZBCPs can be considered as the convincing evidence for the existence of zero-energy MBS in such systems is questionable from the theoretical point of view. The clarification of such subtle issues is beyond the scope of the present paper.

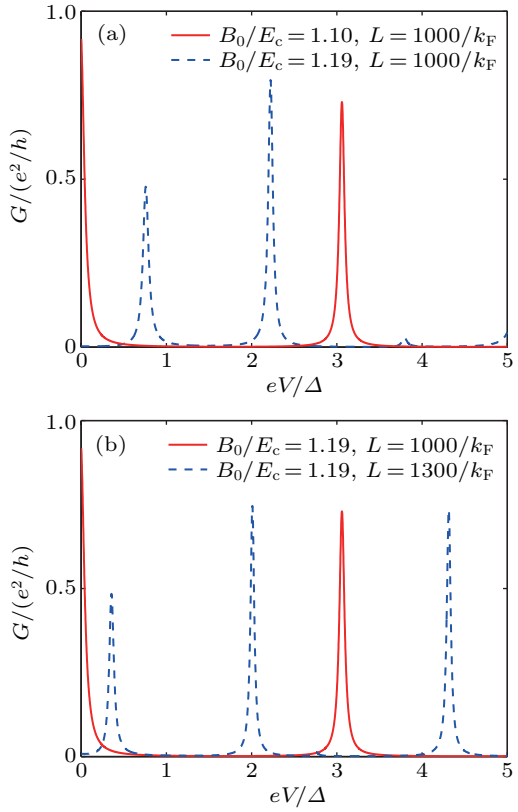


Fig. 4. (color online) Illustration of zero-bias conductance peak in the cases that the strength of the external magnetic field exceeds the critical value B_c. Panel (a) shows the charge conductance as a function of the bias voltage V for $L = 1000/k_F$, $B_0/B_c = 1.1$ (the dashed line) and 1.19 (the solid line). Panel (b) shows the corresponding results for $B_0/B_c = 1.19$, $L = 1000/k_F$ (the solid line) and $1300/k_F$ (the dashed line). In both panels, the other parameters are taken to be $h_0/E_F = 0.5$, $\Delta/E_F = 0.003$, $Z = 5$, $\alpha = 0.3$.

4. Summary

We have analyzed the influence of the interplay of Rashba spin–orbit coupling, induced superconducting pair potential, and external magnetic field on spin-polarized coherent charge transport in ferromagnet/semiconductor nanowire/ferromagnet double barrier junctions. The coherence effect is characterized by the strong oscillations of the charge conductance as a function of the bias voltage or the thickness of the nanowire. Such oscillations arise from the quantum interference of incoming and outgoing quasiparticles in the nanowire and can be effectively modulated by varying the strength of the Rashba spin–orbit coupling, the thickness of the nanowire, or the strength of the external magnetic field applied on the nanowire. Due to the formation of quasi-stationary states in the nanowire resulting from the quantum interference of incoming and outgoing quasiparticles, a series of conductance peaks may occur in the conductance spectra at the energies of the geometrical resonances. It is also shown that two different types of zero-bias conductance peaks may occur under some particular conditions, which have different characteristics and may be due to different mechanisms. The strong oscillations of the conductance with the period of the geometrical resonances could be used for reliable spectroscopy of quasiparticle excitations in such structures.

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