Bifurcated overtones of one-way localized Fabry–Pérot resonances in parity-time symmetric optical lattices*

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Since the first observation of parity-time (PT) symmetry in optics, varied interesting phenomena have been discovered in both theories and experiments, such as PT phase transition and unidirectional invisibility, which turns PT-symmetric optics into a hotspot in research. Here, we report on the one-way localized Fabry–Pérot (FP) resonance, where a well-designed PT optical resonator may operate at exceptional points with bidirectional transparency but unidirectional field localization. Overtones of such one-way localized FP resonance can be classified into a blue shifted branch and a red shifted branch. Therefore, the fundamental resonant frequency is not the lowest one. We find that the spatial field distributions of the overtones at the same absolute order are almost the same, even though their frequencies are quite different.

Keywords: optical lattices, parity-time symmetry, Fabry–Pérot resonances, unidirectional field localization

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1. Introduction

The subject of parity-time (PT) symmetry is more alive than ever after the implementation of PT-related notions in optics, owing to the similarity between the paraxial wave equation and the Schrödinger equation.[1,2] The PT symmetry is a notion initially in quantum mechanics, which describes the non-Hermitian Hamiltonians ($\hat{H}^{\dagger} \neq \hat{H}$) that respect $PT\hat{H} = \hat{H}PT$. Since PT-symmetric potentials can be effectively realized in optics through delicately arranging gain/loss regions, tremendous theoretical and experimental studies have been conducted in such complex systems, enriching our understanding of wave dynamics in PT optical potentials.[3–12] For example, spontaneous PT symmetry breaking was firstly reported in optical coupled systems.[2] Unidirectional invisibility was proposed and verified at exceptional points in PT lattices.[6–8,13] PT lasing and absorption were observed in the PT symmetry breaking phase.[4,9,10] Those interesting phenomena were demonstrated to be closely related to singularity points in eigenvalue spectra of the system scattering matrix or Hamiltonian. Recently, researchers found out that singularity dynamics in finite PT systems can be much affected by boundary effects. A representative example is that unidirectional invisibility in PT lattices will break down as the number of period increases.[14,15] Therefore, it shows the very possibility to tailor singularity effects in the confined PT systems through boundaries.

In a previous work,[15] we focused only on the Bragg condition in PT-symmetric lattices, and found out that the light field is one-way localized in the lattices with certain periods at the Bragg condition. In this paper, we explore the whole spectrum and obtain varied overtones of one-way localized Fabry–Pérot (FP) resonances. As a very unique thing, we observe, for the first time, that overtones in PT cavities can be classified into a blue shifted branch and a red shifted branch with respect to the fundamental frequency. For the blue shifted and the red shifted overtones of the same absolute order, despite very different wavelengths, they have almost the same interference patterns. Since the FP resonances in PT cavities are featured with unidirectional field localizations, distinctive interference patterns emerge only for the light propagating in one direction but not for the reversed direction. Our finding may provide a unique route towards achieving directional responses to photons.

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2. Result and discussion

A PT optical structure has a complex refractive index distribution respecting the form $n(z) = n^*(-z)$.[2] In this section, we design a grating structure with a period L as shown in Fig. 1. The periodic unit-cell has four same-size sub-sections (length $L/4$) with refractive indices being $n_1 = n_0 + \Delta n + \mathrm{i}\Delta n$, $n_2 = n_0 - \Delta n + \mathrm{i}\Delta n$, $n_3 = n_0 - \Delta n - \mathrm{i}\Delta n$, and $n_4 = n_0 + \Delta n - \mathrm{i}\Delta n$, respectively. The incident light is TE polarized. The previous work of Ref. [15] has detailedly demonstrated that when $0 \le \Delta n/n_0 \le 0.15$ and $n_0/(3\Delta n)$ is a positive integer, the PT lattice having $N = 11[n_0/(3\Delta n)]^2$ unit-cells is always completely reflectionless for light propagating in opposite directions under the Bragg condition $\lambda = 2n_0 L$, with the light field one-way localized. From the expression $N = 11[n_0/(3\Delta n)]^2$, we conclude that a weak index modulation will lead to a large number of unit-cells in the desired PT lattice. In the following, we choose $n_0 = 3$ and $\Delta n = 0.2$ as an example, where $\Delta n = 0.2$ is a big number only for the sake of avoiding a very large finite element model.[15] We emphasize that Δn can be chosen with other more practical values (< 0.002) and the results to be demonstrated in the following will remain the same.

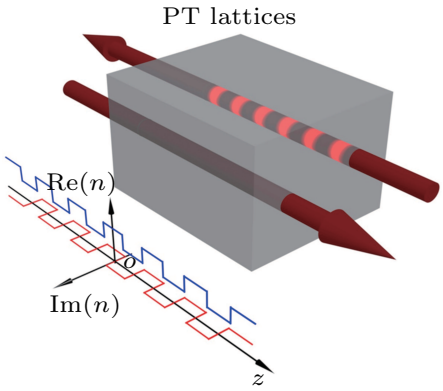


Fig. 1. (color online) Unidirectional field localization may occur at Fabry–Pérot resonances in a well-designed PT optical resonator.

We start from the transfer matrix method, where the characteristic matrix $\boldsymbol{m}$ for one unit-cell is given by[16]

$$\boldsymbol{m} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \prod_{j=4}^{1} \begin{pmatrix} \cos\left(\dfrac{\pi n_j L}{2\lambda}\right) & -\dfrac{\mathrm{i}}{n_j}\sin\left(\dfrac{\pi n_j L}{2\lambda}\right) \\ -\mathrm{i}n_j \sin\left(\dfrac{\pi n_j L}{2\lambda}\right) & \cos\left(\dfrac{\pi n_j L}{2\lambda}\right) \end{pmatrix}. \quad (1)$$

Due to the unimodular property, the characteristic matrix $\boldsymbol{M}$ for N unit-cells is

$$\boldsymbol{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = (\boldsymbol{m})^N = \begin{pmatrix} m_{11}u_{N-1}(a) - u_{N-2}(a) & m_{12}u_{N-1}(a) \\ m_{21}u_{N-1}(a) & m_{22}u_{N-1}(a) - u_{N-2}(a) \end{pmatrix}, \quad (2)$$

where

$$u_N(a) = \frac{\sin((N+1)\cos^{-1}a)}{\sqrt{1-a^2}}, \quad a = \frac{m_{11}+m_{22}}{2}.$$

Under the constraint of PT symmetry, the left-side reflection coefficient, the right-side reflection coefficient, and the transmission coefficient are respectively

$$\begin{aligned} r_{\mathrm{L}} &= \frac{(M_{11}+M_{12}n_0)n_0 - (M_{21}+M_{22}n_0)}{(M_{11}+M_{12}n_0)n_0 + (M_{21}+M_{22}n_0)}, \\ r_{\mathrm{R}} &= \frac{(M_{11}+M_{12}^*n_0)n_0 - (M_{21}^*+M_{22}n_0)}{(M_{11}+M_{12}^*n_0)n_0 + (M_{21}^*+M_{22}n_0)}, \\ t &= \frac{2n_0}{(M_{11}+M_{12}n_0)n_0 + (M_{21}+M_{22}n_0)}. \end{aligned} \quad (3)$$

To study the scattering process in the PT resonator, we employ $\boldsymbol{S}$-matrix $\boldsymbol{S} = [t \;\; r_{\mathrm{L}};\; r_{\mathrm{R}} \;\; t]$ for the two-port system, where the eigenvalues of the $\boldsymbol{S}$-matrix are determined by[17]

$$\lambda_{1,2} = t \pm \mathrm{i}\sqrt{r_{\mathrm{L}} r_{\mathrm{R}}}. \quad (4)$$

In Eq. (4), the EP, as the coalescence of eigenvalues ($\lambda_1 = \lambda_2$), is located at $r_{\mathrm{L}} = 0$ or $r_{\mathrm{R}} = 0$, at which we will obtain the transmission $T = |t|^2 = 1$ according to the generalized energy conservation $|r_{\mathrm{L}} r_{\mathrm{R}}| = t^2(1 - 1/|t|^2)$.[18] Therefore, the EP indicates that the PT resonator also operates at the FP resonance. As shown in Ref. [18], it is not common to achieve bidirectional transparency ($r_{\mathrm{L}} = 0$ and $r_{\mathrm{R}} = 0$) in PT systems due to the asymmetric refractive index modulation. So in the previous works, the transmission resonances in the PT systems are always unidirectional or anisotropic ($r_{\mathrm{L}} = 0$, $r_{\mathrm{R}} \neq 0$ or $r_{\mathrm{L}} \neq 0$, $r_{\mathrm{R}} = 0$).[6–8] However, from Eq. (3), we obtain one sufficient condition for both $r_{\mathrm{L}} = 0$ and $r_{\mathrm{R}} = 0$, namely,

$$M_{11} = M_{22} = \pm 1, \quad M_{12} = M_{21} = 0. \quad (5)$$

In Figs. 2(a)–2(d), we calculate the real part (solid blue lines) and the imaginary part (dashed red lines) of M_{11}, M_{12}, M_{21}, and M_{22}, respectively. In the calculation, we set the period $L = 200$ nm and the number of periods $N = 275$. The results show that besides the EP at the Bragg condition $\lambda = 1.2$ μm, there are several other EPs in the neighboring spectrum regions. They can be basically classified into two categories. To be specific, for the EPs at around 1.162 μm, 1.171 μm, 1.179 μm, 1.188 μm, 1.2 μm, 1.213 μm, 1.222 μm, and 1.231 μm, we find $M_{11} = M_{22} = 1$, $M_{12} = M_{21} = 0$, $r_{\mathrm{L(R)}} = 0$, and $t = 1$. For the EPs at around 1.166 μm, 1.175 μm, 1.183 μm, 1.193 μm, 1.208 μm, 1.217 μm, 1.226 μm, and 1.236 μm, we find $M_{11} = M_{22} = -1$, $M_{12} = M_{21} = 0$, $r_{\mathrm{L(R)}} = 0$, and $t = -1$.

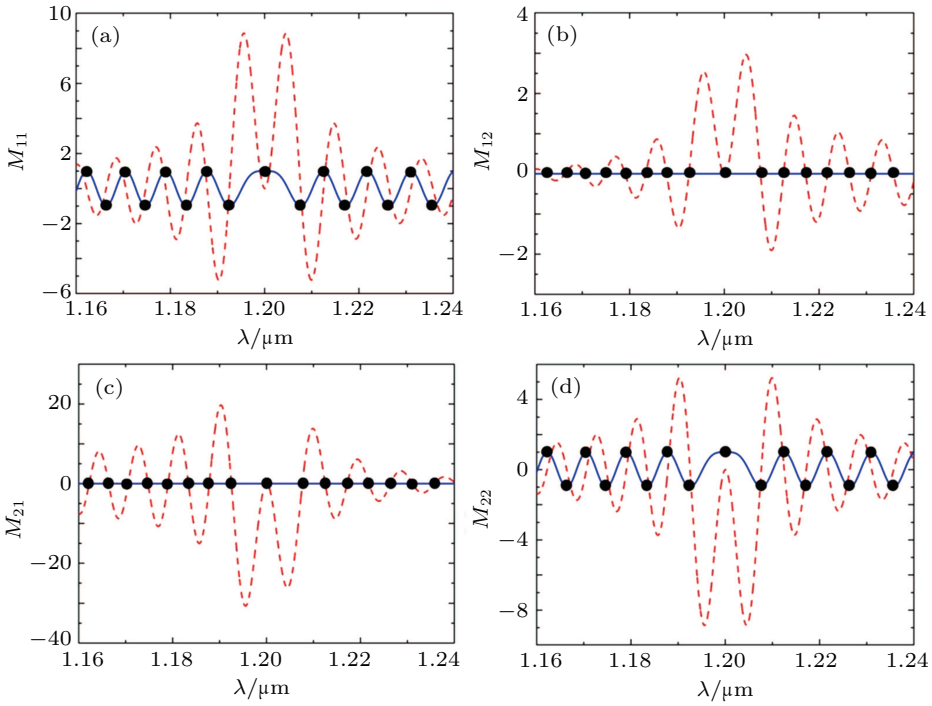


Fig. 2. (color online) Calculations of (a) M_{11}, (b) M_{12}, (c) M_{21}, and (d) M_{22}. Here the solid blue lines and dashed red lines show the real part and imaginary part, respectively. The black dots mark the positions at which $|M_{11(22)}| = 1$ and $|M_{12(21)}| = 0$.

The calculations of $T(=|t|^2)$, $R_{\rm L}(=|r_{\rm L}|^2)$, and $R_{\rm R}(=|r_{\rm R}|^2)$ in Fig. 3(a) further verify that the aforementioned EPs indeed correspond to FP resonances due to unitary transmission as well as bidirectional transparency, except for a special EP as marked by the dashed arrow in the vicinity of 1.2 μm. At such a peculiar point, the system still has a unitary transmission ($T = 1$), but the reflection is unidirectionally vanished ($R_{\rm L} = 0$, $R_{\rm R} \approx 0.3$). As inferred later from the FP resonance patterns in Fig. 4(a), we can find that the fundamental resonance (marked by the order $n = 0$ in Fig. 3(a) and the black dot in Fig. 3(b)) occurs at 1.2 μm with only one antinode in the PT resonator.

However, in stark contrast to the FP resonances in the lossless optical resonator (e.g., a high-refractive-index layer), the fundamental frequency ω_0 in the PT resonator is not the lowest one, since the overtones have a blue shifted branch (blue dots) as well as a red shifted branch (red dots), as shown in Fig. 3(b). Here we set the orders of the blue shifted overtones to be $n = 1, 2, 3, \ldots$, while the orders of the red shifted overtones are $n = -1, -2, -3, \ldots$. In Fig. 3(b), we find $\Delta\omega_{\pm 10} = 6.3 \times 10^{-3}$, $\Delta\omega_{\pm 20} = 10.28 \times 10^{-3}$, $\Delta\omega_{\pm 30} = 14.08 \times 10^{-3}$, $\Delta\omega_{\pm 40} = 17.81 \times 10^{-3}$, $\Delta\omega_{\pm 50} = 21.51 \times 10^{-3}$, $\Delta\omega_{\pm 60} = 25.19 \times 10^{-3}$, and $\Delta\omega_{\pm 70} = 28.86 \times 10^{-3}$. The calculations clearly show that the frequency deviations of the same absolute order overtones from the fundamental one ($\Delta\omega_{n0} = |\omega_n - \omega_0|/\omega_0$) are the same.

As exemplified in Fig. 4(a), we observe an interesting property that the localization of light only occurs for the backward propagation at the FP resonances in the PT resonator due to the designed anti-symmetric distribution of the imaginary part of the refractive index. In Figs. 4(b)–4(d), we also notice that for the blue shifted and red shifted overtones of the same absolute order ($n = \pm 1, \pm 2, \pm 3$), they share similar interference patterns, even though they have very different wavelengths. Here the blue curves in Figs. 4(b)–4(d) describe

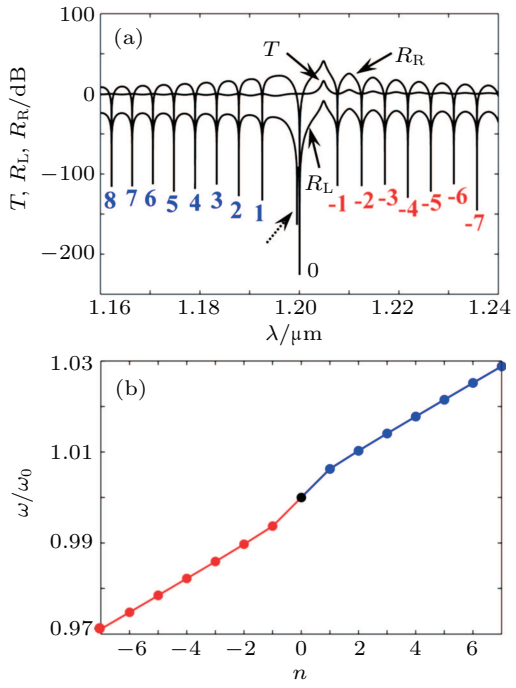


Fig. 3. (color online) Scattering properties of the PT resonator. (a) Calculations of T, $R_{\rm L}$, and $R_{\rm R}$. (b) Spectral positions of Fabry–Pérot resonances (marked by the numbers in panel (a)]. At the resonances ($n = 0$, ± 1, ± 2, ± 3, …), the PT resonator is bidirectionally transparent.

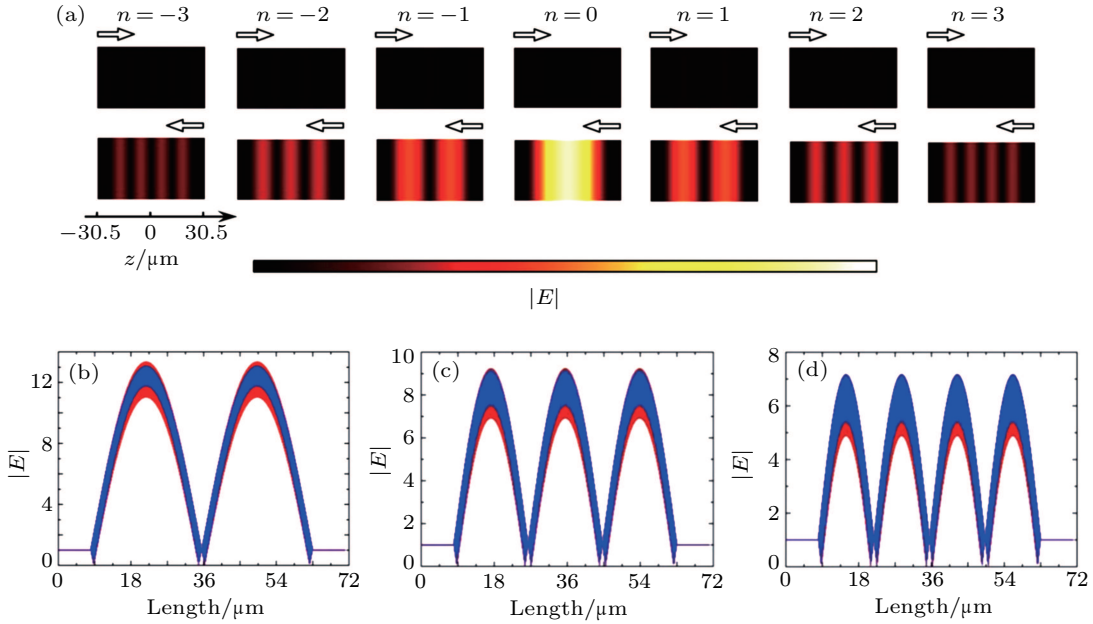


Fig. 4. (color online) Field distributions at FP resonances in the PT resonator. (a) Electric field distributions for the light propagating from the left and the right through the PT resonator at FP resonances ($n = 0, \pm1, \pm2, \pm3$), where the field is observed to be unidirectionally localized. The blue and red curves of field distributions in panels (b), (c), and (d) present the spatial modal degeneracy for $n = \pm1$, ±2, and ±3, respectively.

the electric field distributions of the blue-shifted overtones at $n = 1, 2, 3$, while the red curves show the electric field distributions of the red-shifted overtones at $n = -1, -2, -3$. Very different from the FP resonances in the lossless optical resonator, the FP resonances in the designed PT resonator exhibit beat-like electric field distributions, where the fast oscillation is caused by the oscillated PT modulation.

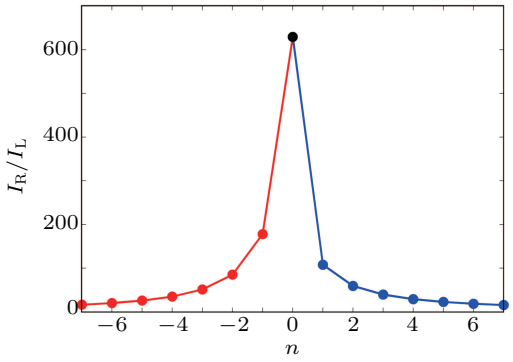


Fig. 5. (color online) Contrast ratio $\eta = I_R/I_L$ for different orders of resonances. I_R and I_L are the maximum intensities for the light propagating from the right and the left through the PT resonator at FP resonances ($n = 0, \pm1, \pm2, \pm3, \ldots$).

Unidirectional field localization could be useful in many aspects, such as directional sensing,[19,20] one-way nonlinearity, and non-blind invisibility. In order to evaluate the degree of unidirectional field localization, we calculate the contrast ratio $\eta = I_R/I_L$ for different orders of FP resonances in Fig. 5, where I_R and I_L denote the maximum intensities for the light propagating from the right and the left through the PT resonator, respectively. The results show that the contrast ratio reaches maximum ($\eta = 629.5$) at the fundamental resonance ($n = 0$), indicating a remarkable intensity difference up to three orders of magnitude. For the overtones of the same absolute order, the contrast ratio of the red shifted overtone is always larger than that of the blue shifted one. For example, at $n = -1$ and 1, η equals to 178 and 107.8, respectively. This can be explained by the existence of weak field enhancement for the forward propagation at the blue shifted overtones, enabling I_L of the blue shifted overtones to be larger than that of the red shifted ones.

Asymmetric field localization still exists when the PT symmetry is breaking. In this case, the system will not be transparent due to reflections and non-unitary transmission ($r_{L(R)} \neq 0$, $|t| \neq 1$). The reported FP resonances with bidirectional transparency, spatial modal degeneracy, and unidirectional field localization are unique in the PT systems. As aforementioned, for the FP resonances in lossless systems, the fundamental frequency should be the lowest one. Also under the restraint of T-reversal symmetry, the FP resonances in lossless systems must have strong field localization for the light propagating in both directions. In open systems not obeying the PT symmetry, the eigenvalues of S-matrix at EPs usually have imaginary parts, indicating that such non-Hermitian systems operate in either gained or lossy conditions and cannot be regarded as conservative.

3. Conclusion

We have studied an unusual phenomenon of one-way localized FP resonances with spatial modal degeneracy. The FP resonances have both blue shifted and red shifted overtones, and the spatial modal degeneracy is observed for the overtones

of the same absolute order. We reveal that one-way field localization occurs at the EP, where the PT system is bidirectionally transparent for the outside observer but inside the light field is localized only for the backward propagation.

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