Quantum feedback cooling of two trapped ions*

Shuo Zhang(张硕)[1,3], Wei Wu(吴伟)[2,†], Chun-Wang Wu(吴春旺)[2,‡], Feng-Guang Li(李风光)[1], Tan Li(李坦)[1], Xiang Wang(汪翔)[1], and Wan-Su Bao(鲍皖苏)[1]

[1] *Zhengzhou Information Science and Technology Institute, Zhengzhou* 450004, *China*
[2] *College of Science, National University of Defense Technology, Changsha* 410073, *China*
[3] *Henan Key Laboratory of Quantum Information and Cryptography, Zhengzhou* 450004, *China*

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We present a sub-Doppler cooling scheme of a two-trapped-ion crystal by quantum feedback control method. In the scheme, we obtain the motional information by continuously measuring the spontaneous emission photons from one single ion of the crystal, and then apply a feedback force to cool the whole chain down.We derive the cooling dynamics of the cooling scheme using quantum feedback theory and quantum regression theorem. The result shows that with experimentally achievable parameters, our scheme can achieve lower temperature and faster cooling rate than Doppler cooling.

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1. Introduction

Laser cooling of trapped ions is a crucial technology for its various applications in the field of quantum information processing[1] and quantum metrology.[2–4] The present laser cooling is normally performed with two steps. In the first step, the ion is cooled into the Lamb–Dicke (LD) regime via Doppler cooling,[5,6] while the sideband cooling[7,8] or coherent cooling scheme[9–15] is used to prepare the ion to the ground state in the second step.

Apart from these approaches, combining the laser cooling with feedback control theory[16–18] is an alternative choice. The feedback cooling scheme of a single ion by continuous measurement was proposed by Steixner *et al.* in 2005.[19] In this scheme, the cooling process is carried out by continuously acquiring the motional information of a single ion, and then feeding back a "cold damping" force. They theoretically predicted that the optimal cooling result could achieve a lower temperature than Doppler cooling. Then they combined the feedback cooling with electromagnetically induced transparency (EIT) cooling,[20] and the results showed that the final mean phonon number can achieve zero in an ideal case. The feedback cooling method was experimentally realized by Bushev *et al.* in 2006.[21] They demonstrated the feedback cooling of a single ion, and the cooling results coincided with analytical predictions. Further studies show that cooling can be performed by feeding back to the cooling laser,[22] and can be achieved using an oscillating mirror.[23]

In this work, we extend the feedback cooling method to a two-ion crystal, a simple and representative example of a trapped ion chain. The motional state of the ion chain in a sufficiently cold temperature can be characterized as normal modes. Hence the quantum motion of each ion is the linear combination of two normal modes,[24] the center of mass (COM) mode and the breathing mode. For this reason, we can obtain the motional information of two normal modes by measuring a single ion, and then apply a feedback force to a single ion to simultaneously cool the two normal modes. The analytical and numerical results show that our feedback cooling scheme can also achieve a lower temperature than Doppler cooling in a two-ion case.

2. Model

2.1. Hamiltonian and master equation

As shown in Fig. 1, we consider two ions with the same species in front of a mirror. The ions of mass m and charge e are trapped in a harmonic potential with trap frequency ν. The ions are trapped along the z axis, and equilibrium position of the i-th ion is denoted by z_i and momentum p_i. A single ion has an excited state $|\mathrm{e}\rangle$ with linewidth γ_e and a ground state $|\mathrm{g}\rangle$, and their energy difference is $\hbar\omega_{\mathrm{eg}}$. One of the two ions (assuming the first ion) is coupled by an external laser field with frequency ω_L, wave vector $\boldsymbol{k}$, and coupling strength Ω.

The Hamiltonian of the system reads ($\hbar = 1$)

$$H_{\mathrm{sys}} = H_{\mathrm{ph}} + H_{\mathrm{at\text{-}L}}. \tag{1}$$

H_{ph} is the motional degrees of freedom (d.o.f) of the two ions, including their kinetic energy, the trap potential, and the

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†Corresponding author. E-mail: weiwu@nudt.edu.cn
‡Corresponding author. E-mail: cwwu@nudt.edu.cn

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Coulomb interaction:

$$H_{\rm ph}=\sum_{i=1,2}\left(\frac{p_i^2}{2m}+\frac{1}{2}mv^2z_i^2\right)-\frac{e^2}{4\pi\varepsilon_0}\frac{1}{|z_1-z_2|}. \tag{2}$$

In the approximation that the displacement of each ion from its equilibrium position is much smaller than its distance, the motion of the ion chain can be described by the uncoupled normal modes. In the case of two ions, one can apply the transformation $Z_1=\frac{1}{\sqrt{2}}(z_1+z_2)$, $Z_2=\frac{1}{\sqrt{2}}(z_1-z_2)$, and get

$$H_{\rm ph}=\nu_1 b_1^\dagger b_1+\nu_2 b_2^\dagger b_2, \tag{3}$$

where $Z_1=\sqrt{\frac{1}{2m\nu}}\left(b_1+b_1^\dagger\right)$ and $Z_2=\sqrt{\frac{1}{2\sqrt{3}m\nu}}\left(b_2+b_2^\dagger\right)$ are known as the COM mode and the breathing mode with frequencies $\nu_1=\nu$ and $\nu_2=\sqrt{3}\nu$. b_1 and b_2 are their annihilation operators.

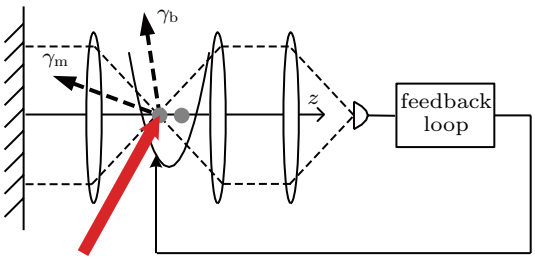


Fig. 1. (color online) Two ions are trapped in a harmonic potential. One of the two ions is driven by a laser field, and the spontaneously emitted photons to the mirror mode are collected by the photon detector. Then photon current goes through a feedback loop, and then feed back the electrode to cool the ion chain.

$H_{\text{at-L}}$ describes the interaction between the first ion and the laser field:

$$\begin{aligned}H_{\text{at-L}} &= -\Delta\left|\mathrm{e}\right\rangle_{11}\left\langle\mathrm{e}\right|+\frac{\Omega}{2}\Big(\left|\mathrm{e}\right\rangle_{11}\left\langle\mathrm{g}\right|\mathrm{e}^{-\mathrm{i}k_z z_1}\\ &\quad+\left|\mathrm{g}\right\rangle_{11}\left\langle\mathrm{e}\right|\mathrm{e}^{\mathrm{i}k_z z_1}\Big),\end{aligned} \tag{4}$$

where k_z is z-component of wave vector, $|\bullet\rangle_1$ denotes the state of the first ion, and the detuning $\Delta=\omega_{\rm L}-\omega_{\rm eg}$. In LD regime, $H_{\text{at-L}}$ can be expanded to the lowest order of $k_z z_1$, and rewrite Hamiltonian (4) in normal modes, yielding

$$\begin{aligned}H_{\text{at-L}} &= -\Delta\left|\mathrm{e}\right\rangle_{11}\left\langle\mathrm{e}\right|+\frac{\Omega}{2}\left(\left|\mathrm{e}\right\rangle_{11}\left\langle\mathrm{g}\right|+\left|\mathrm{g}\right\rangle_{11}\left\langle\mathrm{e}\right|\right)\\ &\quad-\mathrm{i}\frac{\Omega}{2}\left|\mathrm{e}\right\rangle_{11}\left\langle\mathrm{g}\right|\left[\eta_{1,z}\left(b_1+b_1^\dagger\right)+\eta_{2,z}\left(b_2+b_2^\dagger\right)\right]\\ &\quad+\text{H.c.},\end{aligned} \tag{5}$$

where the LD parameters are defined by

$$\eta_{1,z}=\frac{k_z}{2}\sqrt{\frac{1}{m\nu}},\quad \eta_{2,z}=\frac{k_z}{2}\sqrt{\frac{1}{\sqrt{3}m\nu}}. \tag{6}$$

The trapped ion can spontaneously emit photons from the atomic excited state into the radiation field, including the mirror mode and the remaining background modes, i.e., $\gamma_{\rm e}=\gamma_{\rm m}+\gamma_{\rm b}$, where $\gamma_{\rm m}$ and $\gamma_{\rm b}$ are spontaneous emission rates to the mirror mode and the background modes, respectively. Then we can define $\varepsilon=\frac{\gamma_{\rm m}}{\gamma_{\rm e}}$ the ratio between the spontaneously emission rate into the mirror mode and the total spontaneous emission rate. As is shown in Ref. [19], ε refers to the fraction of the solid angle covered by the lens that is the emission rate into mirror mode, and therefore $\gamma_{\rm b}=(1-\varepsilon)\gamma_{\rm e}$.

As shown in Refs. [19] and [25], one can obtain the master equation for the d.o.f by adiabatic elimination of the atomic d.o.f and radiation field

$$\begin{aligned}\mathrm{d}\rho(t) &= \mathcal{L}_0\rho(t)\,\mathrm{d}t\\ &= -\mathrm{i}\left[H_{\rm ph},\rho(t)\right]+\mathcal{L}_{\rm m}\rho(t)+\mathcal{L}_{\rm b}\rho(t).\end{aligned} \tag{7}$$

The first term of Eq. (7) describes the free evolution of the ion motion. The second accounts for the dissipative dynamics of the motional d.o.f due to the mirror mode:

$$\mathcal{L}_{\rm m}\rho(t)=\gamma\mathcal{D}\left[c_{\rm m}(z_1)\right]\rho(t),$$

where the superoperator $\mathcal{D}[o]\bullet=o\bullet o^\dagger-\frac{1}{2}\left(o^\dagger o\bullet+\bullet o^\dagger o\right)$, $\gamma=\varepsilon\gamma_{\rm e}\frac{\Omega^2}{4}\frac{1}{\Delta^2+\left(\frac{\gamma_{\rm e}}{2}\right)^2}$ is the effective photon emission rate into the mirror mode, and $c_{\rm m}(z_1)$ is defined by

$$c_{\rm m}(z_1)\approx\sin\phi+(\cos\phi)\,k_{\rm eg}z_1-\frac{1}{2}(\sin\phi)\left(k_{\rm eg}z_1\right)^2. \tag{8}$$

Here, $k_{\rm eg}=\frac{\omega_{\rm eg}}{c}$ ($k_{\rm eg}\approx|\boldsymbol{k}|$ in optical frequency), $\phi=k_{\rm eg}L$ with L the distance from the mirror to the equilibrium position of the first ion.

The third term is the dissipative dynamics of the motional d.o.f due to the background mode:

$$\begin{aligned}\mathcal{L}_{\rm b}\rho(t) &= \left(A_{1,-}\mathcal{D}[b_1]+A_{1,+}\mathcal{D}\left[b_1^\dagger\right]\right)\rho(t)\\ &\quad+\left(A_{2,-}\mathcal{D}[b_2]+A_{2,+}\mathcal{D}\left[b_2^\dagger\right]\right)\rho(t).\end{aligned} \tag{9}$$

Here, $A_{j,\pm}$ are the heating/cooling coefficients of j-th normal mode, which are

$$A_{j,\pm}=\gamma_{\rm b}\eta_j^2\frac{\Omega^2}{4}\left(\frac{\eta_{j,z}^2}{\eta_j^2}\frac{1}{(\Delta\mp\nu_j)^2+\frac{\gamma_{\rm e}^2}{4}}+\frac{\alpha}{\Delta^2+\frac{\gamma_{\rm e}^2}{4}}\right), \tag{10}$$

with

$$\eta_j=\frac{k_{\rm eg}}{2}\sqrt{\frac{1}{m\nu_j}}=\frac{k_{\rm eg}}{k_z}\eta_{j,z},$$

and the geometry parameter $\alpha=\frac{2}{5}$ for dipole transition.[5]

2.2. Posteriori master equation

In the experimental setup, the photons emitted into the mirror modes are measured by a photon detector. One can convert the master equation (7) to the Ito form posteriori master equation according to the number of photon counts $\mathrm{d}N_{\rm c}(t)$ during time interval $(t,t+\mathrm{d}t]$:[19,26]

$$\mathrm{d}\rho_{\rm c}(t)=\mathcal{L}_0\rho_{\rm c}(t)\,\mathrm{d}t+\left(\frac{\mathcal{J}[c_{\rm m}]\rho_{\rm c}(t)}{\mathrm{Tr}_{\rm sys}\{\mathcal{J}[c_{\rm m}]\rho_{\rm c}(t)\}}-\rho_{\rm c}(t)\right)$$

$$\times \left(\mathrm{d}N_{\mathrm{c}}(t) - \gamma \mathrm{Tr}_{\mathrm{sys}}\left\{\mathcal{J}\left[c_{\mathrm{m}}\right]\rho_{\mathrm{c}}(t)\right\}\mathrm{d}t\right), \quad (11)$$

where the superoperator $\mathcal{L}_0$ has been defined in Eq. (7), ρ_{c} is the conditional density matrix, and $\mathcal{J}[o] = o \bullet o^{\dagger}$ is associated with a quantum jump process. The mean number of counts $\langle \mathrm{d}N_{\mathrm{c}}(t)\rangle$ is

$$\langle \mathrm{d}N_{\mathrm{c}}(t)\rangle \overset{\phi=\frac{\pi}{4}}{=} \frac{1}{2}\gamma \mathrm{d}t + \gamma k_{\mathrm{eg}} \langle z_1\rangle(t)\,\mathrm{d}t + O\left(\eta^2\right). \quad (12)$$

Here we have chosen $\phi = \frac{\pi}{4}$. Consequently, the stochastic variable $\mathrm{d}N_{\mathrm{c}}(t)$ consists of a deterministic term $\langle \mathrm{d}N_{\mathrm{c}}(t)\rangle$ and a Wiener increment $\mathrm{d}W(t)$ satisfying $\mathrm{d}^2W(t) = \mathrm{d}t$:

$$\mathrm{d}N_{\mathrm{c}}(t) = \frac{\gamma}{2}\mathrm{d}t + \gamma k_{\mathrm{eg}} \langle z_1\rangle(t)\,\mathrm{d}t + \sqrt{\frac{\gamma}{2}}\mathrm{d}W(t). \quad (13)$$

Then the photon current will be[19] (we have subtracted the direct current)

$$I_{\mathrm{c}}(t) = \frac{\mathrm{d}N_{\mathrm{c}}(t)}{\mathrm{d}t} = \gamma k_{\mathrm{eg}} \langle z_1\rangle_{\mathrm{c}}(t) + \sqrt{\frac{\gamma}{2}}\xi(t), \quad (14)$$

where $\langle \bullet \rangle_{\mathrm{c}} = \mathrm{Tr}_{\mathrm{sys}}\{\bullet \rho_{\mathrm{c}}(t)\}$, and the Gaussian white noise $\xi(t) \equiv \frac{\mathrm{d}}{\mathrm{d}t}W(t)$ satisfies $\langle \xi(t)\,\xi(t')\rangle = \delta(t-t')$.

The Ito form posteriori master equation (11) can be expanded to the first order in LD parameter, and we can get

$$\begin{aligned}\mathrm{d}\rho_{\mathrm{c}}(t) \;=\; & \mathcal{L}_0\rho_{\mathrm{c}}(t)\,\mathrm{d}t \\ & + 2\sqrt{\frac{\gamma}{2}}\mathrm{d}W(t)\sum_{i=1,2}\eta_i\left[\tilde{Z}_i\rho_{\mathrm{c}}(t)\right. \\ & \left. + \rho_{\mathrm{c}}(t)\tilde{Z}_i - 2\left\langle \tilde{Z}_i\right\rangle_{\mathrm{c}}(t)\rho_{\mathrm{c}}(t)\right], \end{aligned} \quad (15)$$

with $\tilde{Z}_j = b_j^{\dagger} + b_j$.

3. Feedback dynamics

The feedback loop is designed as shown in Fig. 2. We equally divide the measured photoncurrent into two parts:

$$\begin{aligned} I_{\mathrm{c1}}(t) \;&=\; I_{\mathrm{c2}}(t) = \frac{\gamma}{2}k_{\mathrm{eg}}\langle z_1\rangle_{\mathrm{c}}(t) + \frac{1}{2}\sqrt{\frac{\gamma}{2}}\xi(t) \\ &=\; \frac{\gamma}{2}\left(\eta_1\left\langle \tilde{Z}_1\right\rangle_{\mathrm{c}} + \eta_2\left\langle \tilde{Z}_2\right\rangle_{\mathrm{c}}\right)(t) + \frac{1}{2}\sqrt{\frac{\gamma}{2}}\xi(t). \end{aligned} \quad (16)$$

In each part we apply a feedback loop. In the first channel we extract the sideband signal of COM mode with a bandpass filter, shifted by $-\frac{\pi}{2}$, and amplified. As a result, the feedback current is

$$I_{\mathrm{fb,c1}}(t) = \frac{G_1}{2}\left(\gamma\eta_1\left\langle \tilde{P}_1\right\rangle_{\mathrm{c}}(t) + \sqrt{\frac{\gamma}{2}}\Xi_1(t)\right)\cos\nu t. \quad (17)$$

Meanwhile, in the second channel we extract the sideband signal of breathing mode in the same way, and the corresponding feedback current is

$$I_{\mathrm{fb,c2}}(t) = \frac{G_2'}{2}\left(\gamma\eta_2\left\langle \tilde{P}_2\right\rangle_{\mathrm{c}}(t) + \sqrt{\frac{\gamma}{2}}\Xi_2(t)\right)\cos\sqrt{3}\nu t, \quad (18)$$

where G_1 and G_2' are the amplified factor of the first and second loop, respectively. $\tilde{P}_i = i\left(b_i^{\dagger} - b_i\right)$, and $\Xi_i(t)$ can be approximately treated as white noise, which is

$$\left\langle \Xi_i(t)\,\Xi_j(t')\right\rangle \approx \delta_{ij}\delta(t-t'). \quad (19)$$

Then the total feedback current I_{fb} which is the sum of $I_{\mathrm{fb,c1}}$ and $I_{\mathrm{fb,c2}}$, i.e., $I_{\mathrm{fb}} = I_{\mathrm{fb,c1}} + I_{\mathrm{fb,c2}}$.

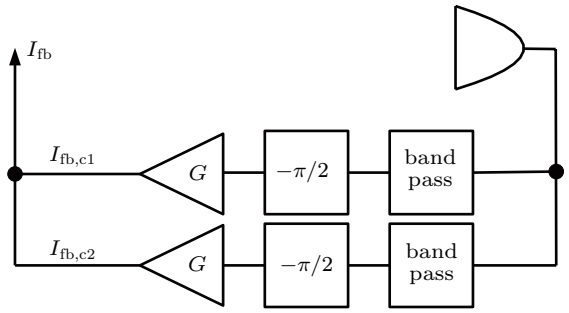


Fig. 2. The feedback loop contains two feedback channels. The photon current is divided into two parts. In the first channel, the sideband signal of COM mode is extracted with a bandpass filter, shifted by $-\pi/2$, and amplified, while in the second channel the sideband signal of breathing mode is extracted in the same way. Then the total feedback current I_{fb} is the sum of $I_{\mathrm{fb,c1}}$ and $I_{\mathrm{fb,c2}}$.

As the feedback current has been obtained, we can exert a feedback force on the trap electrode, which is proportional to z_1 to cool the first ion's motion. Because I_{fb} contains the information of the two normal modes, the two modes can be simultaneously cooled via feedback current. The feedback Hamiltonian in the interaction picture is

$$\begin{aligned} H_{\mathrm{fb}} \;&=\; I_{\mathrm{fb}}(t-\tau)\,U^{\dagger}\frac{z_1}{\eta_1}U \\ &\approx\; \sum_{i=1,2}\frac{G_j}{4}\left(\gamma\eta_i\langle P_i\rangle_{\mathrm{c}} + \sqrt{\frac{\gamma}{2}}\Xi_i\right)(t-\tau)\,\tilde{Z}_i. \end{aligned} \quad (20)$$

Here, τ is the time delay of the feedback loop, which is much smaller than $\frac{1}{\nu}$, and U is the unitary operator $U = \exp\left(-\mathrm{i}\nu b_1^{\dagger}b_1 t - \mathrm{i}\sqrt{3}\nu b_2^{\dagger}b_2 t\right)$, and $G_2 = G_2'\left(\frac{\eta_1}{\eta_2}\right) = (3)^{\frac{1}{4}}G_2'$. In the second line, we have made the rotating wave approximation and ignored the terms of $\exp(\pm\mathrm{i}\nu t \pm \mathrm{i}\sqrt{3}\nu t)$ and $\exp(\pm 2\mathrm{i}\nu t)$.

The Ito form of the conditional master equation including feedback control is

$$\begin{aligned}\mathrm{d}\tilde{\rho}_{\mathrm{c}}(t) \;=\; & \left(\mathcal{L}_{\mathrm{m}} + \mathcal{L}_{\mathrm{b}}\right)\tilde{\rho}_{\mathrm{c}}(t)\,\mathrm{d}t \\ & + \sum_{i=1,2}\left\{2\eta_i\sqrt{\frac{\gamma}{2}}\mathrm{d}W(t)\left[\tilde{Z}_i\tilde{\rho}_{\mathrm{c}}(t)\right.\right. \\ & \left. + \tilde{\rho}_{\mathrm{c}}(t)\tilde{Z}_i - 2\left\langle \tilde{Z}_i\right\rangle_{\mathrm{c}}(t)\tilde{\rho}_{\mathrm{c}}(t)\right] \\ & + \left(\frac{G_i}{4}\gamma\eta_i\langle P_1\rangle_{\mathrm{c}}(t-\tau)\,\mathrm{d}t + \frac{G_i}{4}\sqrt{\frac{\gamma}{2}}\mathrm{d}W_{\Xi_i}(t-\tau)\right. \\ & \left.\left. + \frac{G_i^2}{64}\gamma\mathcal{K}_i\,\mathrm{d}t\right)\mathcal{K}_i\tilde{\rho}_{\mathrm{c}}(t)\right\}, \end{aligned} \quad (21)$$

where the Wiener noise increment $\mathrm{d}W_{\Xi_i}(t) = \Xi_i(t)\,\mathrm{d}t$ satisfies $\mathrm{d}W_{\Xi_i}(t)\,\mathrm{d}W_{\Xi_j}^{\dagger}(t) = \delta_{ij}\mathrm{d}t$, and $\mathcal{K}_i\tilde{\rho}_{\mathrm{c}}(t) = -\mathrm{i}\left[\tilde{Z}_i, \tilde{\rho}_{\mathrm{c}}\right]$.

Redefine $\mu(t) = E[\tilde{\rho}_c(t)]$, where $E[\bullet]$ denotes the ensemble average over the trajectories during the time interval $(t-\tau, t]$, and Eq. (21) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu = \sum_{i=1,2}\left(A_{i,-}\mathcal{D}[b_i]\mu + A_{i,+}\mathcal{D}\left[b_i^\dagger\right]\mu - i\frac{G_j}{8}\gamma\eta_i\left[\tilde{Z}_i, \tilde{P}_i\mu + \mu\tilde{P}_i\right] - \frac{G_i^2}{64}\gamma\left[\tilde{Z}_i, \left[\tilde{Z}_i, \mu\right]\right]\right). \quad (22)$$

Then we can obtain the equations for the expectation values from Eq. (22) using quantum regression theorem:[25,26]

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left\langle b_1^\dagger b_1\right\rangle_\mu &= \left(-W_1 - \frac{G_1}{2}\gamma\eta_1\right)\left\langle b_1^\dagger b_1\right\rangle_\mu + \frac{G_1\gamma\eta_1}{4}\langle b_1 b_1\rangle_\mu \\
&\quad + \frac{G_1\gamma\eta_1}{4}\langle b_1 b_1\rangle_\mu^* - \frac{G_1\gamma\eta_1}{4} + \frac{G_1^2+G_2^2}{32}\gamma + A_{1,+}, \\
\frac{\mathrm{d}}{\mathrm{d}t}\langle b_1 b_1\rangle_\mu &= \frac{G_1}{2}\gamma\eta_1\left\langle b_1^\dagger b_1\right\rangle_\mu + \left(-W_1 - \frac{G_1}{2}\gamma\eta_1\right)\langle b_1 b_1\rangle_\mu \\
&\quad + \frac{G_1\gamma\eta_1}{4} - \frac{G_1^2+G_2^2}{32}\gamma, \\
\frac{\mathrm{d}}{\mathrm{d}t}\left\langle b_2^\dagger b_2\right\rangle_\mu &= \left(-W_2 - \frac{G_2}{2}\gamma\eta_2\right)\left\langle b_2^\dagger b_2\right\rangle_\mu + \frac{G_2\gamma\eta_2}{4}\langle b_2 b_2\rangle_\mu \\
&\quad + \frac{G_2\gamma\eta_2}{4}\langle b_2 b_2\rangle_\mu^* - \frac{G_2\gamma\eta_2}{4} + \frac{G_1^2+G_2^2}{32}\gamma + A_{2,+}, \\
\frac{\mathrm{d}}{\mathrm{d}t}\langle b_2 b_2\rangle_\mu &= \frac{G_2}{2}\gamma\eta_2\left\langle b_2^\dagger b_2\right\rangle_\mu + \left(-W_2 - \frac{G_2}{2}\gamma\eta_2\right)\langle b_2 b_2\rangle_\mu \\
&\quad + \frac{G_2\gamma\eta_2}{4} - \frac{G_1^2+G_2^2}{32}\gamma, \qquad (23)
\end{aligned}$$

with $\langle\bullet\rangle_\mu = \mathrm{Tr}(\bullet\mu)$ and $W_i = A_{i,-} - A_{i,+}$.

The steady solutions for the mean phonon number can be obtained by solving Eq. (23):

$$\begin{aligned}
N_{1,\mathrm{fb}} &= \frac{N_1 + \frac{G_1\tilde{\gamma}_1\eta_1}{4}(2N_1-1) + \frac{G_1^2+G_2^2}{32}\tilde{\gamma}_1}{1+G_1\tilde{\gamma}_1\eta_1}, \\
N_{2,\mathrm{fb}} &= \frac{N_2 + \frac{G_2\tilde{\gamma}_2\eta_2}{4}(2N_2-1) + \frac{G_1^2+G_2^2}{32}\tilde{\gamma}_2}{1+G_2\tilde{\gamma}_2\eta_2}, \qquad (24)
\end{aligned}$$

where the steady solutions $N_{1,\mathrm{fb}} = \left\langle b_1^\dagger b_1\right\rangle_\mu$ and $N_{2,\mathrm{fb}} = \left\langle b_2^\dagger b_2\right\rangle_\mu$ are the feedback cooling results of COM mode and breathing mode, respectively. $N_1 = \frac{A_{1,+}}{W_1}$ and $N_2 = \frac{A_{2,+}}{W_2}$ are the corresponding pure laser cooling results of COM mode and breathing mode, respectively. The dimensionless parameter

$$\tilde{\gamma}_i = \frac{\gamma}{W_i} = \frac{\varepsilon N_i}{(1-\varepsilon)(1+\alpha)\eta_i^2} \quad (i=1,2). \qquad (25)$$

As seen in Ref. [19], the last terms of $N_{i,\mathrm{fb}}$ in Eq. (24) originate from the feedback noise term. We can see that the noise of the two channels can influence each other, giving rise to external noise in comparison with single-ion feedback cooling case.

4. Results

To evaluate the cooling result, we define R as the ratio between the feedback cooling result and pure Doppler cooling result

$$R = \frac{\left(N_{1,\mathrm{fb}} + \frac{1}{2}\right)\hbar\nu_1 + \left(N_{2,\mathrm{fb}} + \frac{1}{2}\right)\hbar\nu_2}{\left(N_1 + \frac{1}{2}\right)\hbar\nu_1 + \left(N_2 + \frac{1}{2}\right)\hbar\nu_2}. \qquad (26)$$

As shown in the definition of $\tilde{\gamma}_i$ $(i=1,2)$ of Eq. (25), when $\varepsilon = 0$, $R = 1$, which indicates that no feedback current is applied, and the final phonon numbers are the same as pure laser cooling. On the other hand, in an ideal case $\varepsilon = 1$ (i.e. $\tilde{\gamma}_i \longrightarrow \infty$), which means that the photons are all spontaneously emitted into the mirror mode. Hence from Eq. (24), we can find that $N_{i,\mathrm{fb}} \approx \frac{1}{2}N_i$ for sufficiently small G_1 and G_2, and the corresponding optimal ratio $R \approx \frac{1}{2}$, which is the same as that of single ion case.[19]

In Fig. 3, we plot R as a function of amplified factors G_1 and G_2 for different ε. As ε increases, more motional information can be obtained, and the ion chain can be cooled to a lower temperature. For $\varepsilon = 0.05$, $G_1 = G_2 \sim 1.2$, $R < 0.65$, which is advantageous over pure Doppler cooling.

In Fig. 4, we compare the cooling dynamics of the feedback cooling with Doppler cooling. $\langle n_1(t)\rangle$ and $\langle n_2(t)\rangle$ are the mean phonon numbers of COM mode and breathing mode at time t, respectively. We find that the feedback assisted cooling is more than 30% below Doppler limit, and the cooling rate is also much faster than Doppler cooling.

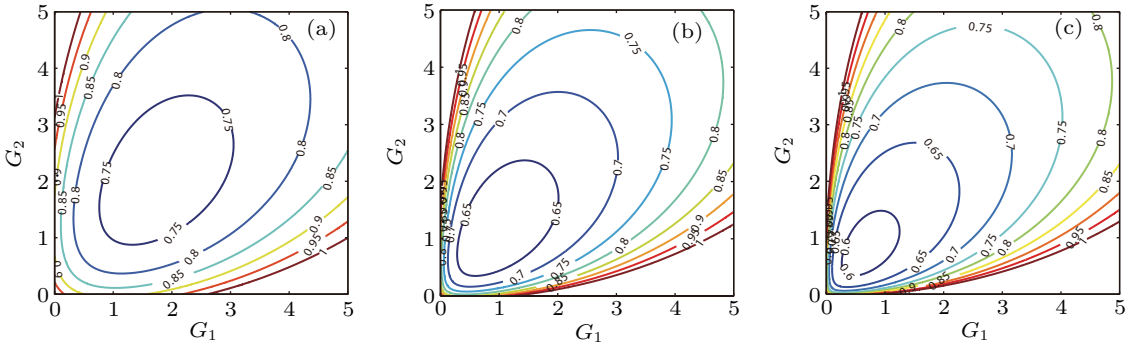


Fig. 3. (color online) R as a function of G_1 and G_2 for (a) $\varepsilon = 0.01$, (b) $\varepsilon = 0.05$, and (c) $\varepsilon = 0.1$. The other parameters are $\eta_1 = 0.1$, $\eta_2 = 0.075$, $N_1 = 15$, and $N_2 = 9.1$.

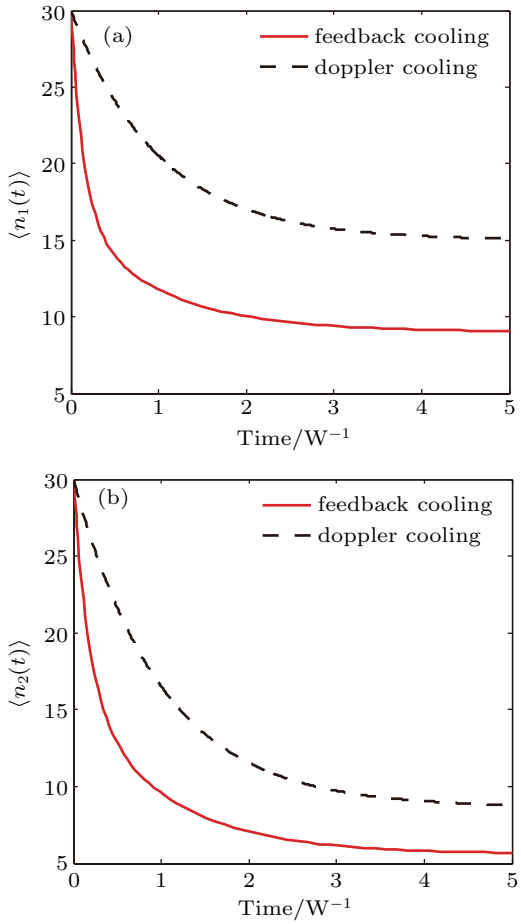


Fig. 4. (color online) The cooling dynamics of (a) the COM mode and (b) breathing mode. The parameters are $G_1 = 1.1$, $G_2 = 1.2$, $\eta_1 = 0.1$, $\eta_2 = 0.075$, $N_1 = 15$, $N_2 = 9.1$, and $\varepsilon = 0.05$. The solid line and dashed line correspond to the feedback cooling and Doppler cooling, respectively.

As seen from feedback cooling experiment of a single trapped ion,[21] the limitation to the feedback cooling result is the fluorescence collection efficiency ε, which is about 0.05 for a linear trap.[27] It is a primary challenge in our scheme as well. To further improve the cooling efficiency, one can use parabolic mirror[28,29] or spherical mirror[30] to collect more fluorescence. Additionally, because the motional information is obtained from the fluorescence of a single ion, our scheme requires that the ion chain should be spatially addressed by the laser field. It is also achievable with present techniques.[31]

5. Conclusion

We have shown that the feedback cooling is achievable in two-ion crystal. By collecting the spontaneous emission photons of a single ion, we can simultaneously obtain the motional information of the COM mode and the breathing mode. Via applying a feedback force related to the motional signal, the ion chain can be successfully cooled to the temperature below Doppler limit. The cooling scheme can also be extended to the ion string of more than two ions.

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