Tracking consensus for nonlinear heterogeneous multi-agent systems subject to unknown disturbances via sliding mode control*

Xiang Zhang(张翔)[1], Jin-Huan Wang(王金环)[1,†], De-Dong Yang(杨德东)[2], and Yong Xu(徐勇)[1]

[1] School of Sciences, Hebei Province Key Laboratory of Big Data Calculation, Hebei University of Technology, Tianjin 300401, China
[2] School of Control Science and Engineering, Hebei University of Technology, Tianjin 300130, China

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We investigate the tracking control for a class of nonlinear heterogeneous leader–follower multi-agent systems (MAS) with unknown external disturbances. Firstly, the neighbor-based distributed finite-time observers are proposed for the followers to estimate the position and velocity of the leader. Then, two novel distributed adaptive control laws are designed by means of linear sliding mode (LSM) as well as nonsingular terminal sliding mode (NTSM), respectively. One can prove that the tracking consensus can be achieved asymptotically under LSM and the tracking error can converge to a quite small neighborhood of the origin in finite time by NTSM in spite of uncertainties and disturbances. Finally, a simulation example is given to verify the effectiveness of the obtained theoretical results.

Keywords: multi-agent systems, tracking consensus, distributed adaptive control, sliding mode

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1. Introduction

With the development of communication networks and powerful embedded systems, distributed cooperative control for multi-agent systems (MAS) has received compelling attention from various research fields, such as robotic teams,[1] sensor networks,[2] and underwater vehicles.[3] The uncertainty and unpredictability of multi-agent systems often appear in practical applications. In the stability analysis of MAS with uncertainties, it is extremely challenging to construct the Lyapunov function properly.

Among collective behaviors of multiple agents, consensus is one of the most important behaviors, which means that all agents will reach a common state eventually. Recently, the consensus problem of multi-agent systems has been intensively studied.[4–10] In Refs. [6]– [8], the studies of MAS focus on linear systems. The consensus of nonlinear MAS is more challenging than that of linear MAS. Nonlinearity is ubiquitous in physical phenomena, however only a few works have been done to research the consensus of nonlinear MAS.[9–11] For example, Ren *et al*. studied the tracking control of second-order nonlinear MAS via two different kinds of sliding mode structures, whereas the considered model is without disturbance.[11] In practical implementations, the external disturbance may not be ignored and should be considered. In Refs. [12]–[21], the consensus problem of MAS with disturbances was discussed. Yang *et al*.[12] proposed the asymptotic convergence protocol of MAS with disturbances and some similar results can be found in Refs. [13] and [14] and the references therein. So far, most of the existing control algorithms for MAS are asymptotically convergent.[12–14,22–24] Compared to the asymptotic consensus, the finite-time consensus control is an effective approach with high performance, good robustness to uncertainties, and better disturbances rejection properties. Zhang *et al*.[15] investigated the finite-time tracking control for nonlinear MAS subject to external disturbances. Chen *et al*.[16] solved the finite-time cooperative tracking problem for a class of networked Euler–Lagrange systems. Continuous distributed finite-time control algorithms were designed for multiple agents with double integrators in Ref. [17]. Yu *et al*.[18] and He *et al*.[19] investigated the finite-time tracking control of second-order multi-agent systems with bounded external disturbances. The former is for linear systems and the latter is for nonlinear systems. Nevertheless, the bounds of the external disturbances in Refs. [12]–[20] are assumed to be known, which are actually difficult to obtain in practice. To estimate the unknown bounds, the adaptive control is a good method.[25,26] Zhao *et al*.[25,26] studied the attitude synchronization control problem for a group of spacecrafts and the containment control problem for second-order multi-agent systems using the sliding mode associated with adaptive methodology. At present, however, there are few results focusing on the finite-time distributed adaptive control problem for heterogeneous nonlinear multi-agent systems with unknown external disturbances.

From the review of the existing works on consensus of MAS, it can be observed that there are still some challenging issues to be addressed. One of the issues is to consider more general nonlinear MAS with heterogeneous dynamics

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†Corresponding author. E-mail: wjhuan228@163.com

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and external disturbances. The bounds of the external disturbances may be unknown, which is more practical and challenging. Another issue is to develop more efficient control algorithms, which have the following features: 1) distributed, which means each agent can only use the local information of its neighbors; 2) finite time, which makes the tracking error converge in finite time, 3) adaptive, which can estimate the unknown bounds of the external disturbances. However, the control mechanisms in most existing results only have some but not all of these features.

Motivated by the above discussion, in this paper, we investigate the distributed adaptive sliding mode control for tracking consensus of heterogeneous second-order nonlinear multi-agent systems with unknown external disturbances. The contributions of this paper are three-fold. 1) On account of the fact that the information of the position and velocity of the leader is unmeasurable, the neighbor-based distributed finite-time observers are proposed for the followers to estimate the information of the leader. 2) Two novel distributed adaptive sliding mode control strategies are designed. The asymptotic tracking is based on linear sliding mode (LSM) and the finite-time tracking is based on nonsingular terminal sliding mode (NTSM). 3) Our rigorous proof shows that the tracking error can converge to zero asymptotically by LSM or to a quite small neighborhood of the origin in finite time by NTSM in spite of uncertainties and disturbances. Therefore, the proposed control algorithms are insensitive and robust to the uncertainties and unknown external disturbances.

The rest of the paper is organized as follows. Section 2 introduces some necessary preliminaries and problem formulation. Section 3 solves the asymptotic tracking and finite-time tracking problem of the leader–follower MAS via LSM and NTSM, respectively. In Section 4, a numerical example is provided to demonstrate the effectiveness of the proposed control schemes. Finally, Section 5 concludes the paper.

2. Preliminaries and problem formulation

For the sake of better understanding, we recall some preliminaries on graph theory, which can be referred to Ref. [27] for more details, and then formulate our problem.

2.1. Graph theory

In this paper, we consider a class of leader–follower multi-agent systems consisting of one leader and N followers. The topology relationship of the followers and the leader can be described by a graph $\bar{\mathcal{G}}$. We assume that the information transfer between the followers is undirected (denoted by an undirected graph $\mathcal{G}$), while the agents in graph $\mathcal{G}$ are connected to the leader by directed edges, that is, the information can only be transferred from the leader to the followers.

The undirected graph $\mathcal{G} = \{V, E, \boldsymbol{A}\}$ consists of a vertex set $V = \{1, 2, \ldots, N\}$ denoting N agents, a set of undirected edges $E \subseteq V \times V$, and a weighted adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, where $a_{ii} = 0$ and $a_{ij} = a_{ji} \geq 0$. $a_{ij} > 0$ if and only if $(i, j) \in E$. Two nodes i and j are adjacent or neighbors if $(i, j) \in E$. The neighbors set of agent i is denoted by $\mathcal{N}_i = \{j \in V | (i, j) \in E\}$. The graph $\mathcal{G}$ is called connected if there is a path between any two nodes in $\mathcal{G}$. The Laplacian matrix $\boldsymbol{L}$ with respect to graph $\mathcal{G}$ is defined as

$$\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A},$$

where $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \ldots, d_N)$ is the degree matrix with the diagonal elements $d_i = \sum\limits_{j\in\mathcal{N}_i} a_{ij}, i = 1, 2, \ldots, N$.

The leader adjacency matrix associated with graph $\bar{\mathcal{G}}$ is defined as $\boldsymbol{B} = \mathrm{diag}(b_1, b_2, \cdots, b_N) \in \mathbb{R}^{N\times N}$, where $b_i > 0$ if the leader is a neighbor of agent i (that is, there is a directed edge from agent i to the leader) and otherwise $b_i = 0$. The graph $\bar{\mathcal{G}}$ is said to be connected if at least one agent in each component of graph $\mathcal{G}$ is connected to the leader by a directed edge. Note that graph $\bar{\mathcal{G}}$ is connected does not mean graph $\mathcal{G}$ is also connected. For convenience, we denote $\boldsymbol{H} = \boldsymbol{L} + \boldsymbol{B}$.

Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional real vector space and $\mathbb{R}^{n\times m}$ denotes the $n \times m$ real matrix space. $\mathbf{1}_n$ represents the $n \times 1$ all-one column vector and $\mathbf{0}$ represents the all-zero column vector. $\boldsymbol{I}_n$ denotes the n-dimensional identity matrix. Given a vector $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ and a constant α, we define $\mathrm{diag}(\boldsymbol{x}^\alpha) = \mathrm{diag}(x_1^\alpha, x_2^\alpha, \ldots, x_n^\alpha)$, $\mathrm{sig}^\alpha(\boldsymbol{x}) = [\mathrm{sig}^\alpha(x_1), \mathrm{sig}^\alpha(x_2), \ldots, \mathrm{sig}^\alpha(x_n)]^{\mathrm{T}}$, and $\mathrm{sig}^\alpha(x_i) = |x_i|^\alpha \mathrm{sgn}(x_i)$, $i = 1, 2, \ldots, n$, where $\mathrm{sgn}(\cdot)$ is the signum function. $\|\boldsymbol{x}\|_1 = \sum\limits_{i=1}^{n} |x_i|$, $\|\boldsymbol{x}\|_2 = \|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}$, and $\|\boldsymbol{x}\|_\infty = \max_{i=1,2,\ldots,n}\{|x_i|\}$ denote 1-norm, 2-norm, and infinity norm of the vector $\boldsymbol{x}$, respectively. A basic relationship among these norms is $\|\boldsymbol{x}\|_\infty \leq \|\boldsymbol{x}\|_2 = \|\boldsymbol{x}\| \leq \|\boldsymbol{x}\|_1$. Given matrices $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{m\times n}$ and $\boldsymbol{B} \in \mathbb{R}^{p\times q}$, their Kronecker product is defined as

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{bmatrix} a_{11}\boldsymbol{B} & a_{12}\boldsymbol{B} & \cdots & a_{1n}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & a_{22}\boldsymbol{B} & \ldots & a_{2n}\boldsymbol{B} \\ \vdots & \vdots & & \vdots \\ a_{m1}\boldsymbol{B} & a_{m2}\boldsymbol{B} & \ldots & a_{mn}\boldsymbol{B} \end{bmatrix} \in \mathbb{R}^{mp\times nq}.$$

2.2. Problem formulation

In this paper, the dynamics of each follower is described by the following second-order heterogeneous nonlinear MAS

$$\begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{v}_i(t), \\ \dot{\boldsymbol{v}}_i(t) = \boldsymbol{f}_i(\boldsymbol{x}_i, \boldsymbol{v}_i, t) + \boldsymbol{u}_i(t) + \boldsymbol{w}_i(t), \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $\boldsymbol{x}_i \in \mathbb{R}^n, \boldsymbol{v}_i \in \mathbb{R}^n$, and $\boldsymbol{u}_i \in \mathbb{R}^n$ are the position, velocity, and control input of the i-th follower, respectively. The unknown vector function $\boldsymbol{f}_i(\boldsymbol{x}_i, \boldsymbol{v}_i, t)$ is intrinsic nonlinear dynamics and $\boldsymbol{w}_i$ is the unknown external disturbance of the i-th

follower. We assume that $\|\boldsymbol{w}_i(t)\|_\infty \le l_i$, and l_i is an unknown constant.

The leader's dynamics is described as

$$\begin{cases} \dot{\boldsymbol{x}}_0(t) = \boldsymbol{v}_0(t), \\ \dot{\boldsymbol{v}}_0(t) = \boldsymbol{f}_0(\boldsymbol{x}_0, \boldsymbol{v}_0, t), \end{cases} \tag{2}$$

where $\boldsymbol{x}_0,\ \boldsymbol{v}_0 \in \mathbb{R}^n$ and $\boldsymbol{f}_0(\boldsymbol{x}_0, \boldsymbol{v}_0, t)$ is a nonlinear function.

To begin with, we have the following assumptions.

Assumption 1 In system (1), assume that $\hat{\boldsymbol{f}}_i(\boldsymbol{x}_i, \boldsymbol{v}_i, t)$ is the estimate of $\boldsymbol{f}_i(\boldsymbol{x}_i, \boldsymbol{v}_i, t)$ and satisfies $\|\boldsymbol{f}_i(\boldsymbol{x}_i, \boldsymbol{v}_i, t) - \hat{\boldsymbol{f}}_i(\boldsymbol{x}_i, \boldsymbol{v}_i, t)\| \le F_i(\boldsymbol{x}_i, \boldsymbol{v}_i, t),\ i = 1, 2, \ldots, N$, where $F_i(\cdot)$ are bounded continuous nonlinear functions.

Assumption 2 In system (2), there exists a non-negative constant c such that $\|\boldsymbol{f}_0(\boldsymbol{x}_0, \boldsymbol{v}_0, t)\| \le c$.

Lemma 1[27] Let $\boldsymbol{L}$ be the Laplacian of an undirected graph $\mathcal{G}$ with N vertices. Then, $\boldsymbol{L}$ is a symmetric and positive semi-definite matrix. Since each row sum of $\boldsymbol{L}$ is zero, $\boldsymbol{L}$ has a zero eigenvalue associated with the eigenvector $\boldsymbol{1}_N$, i.e., $\boldsymbol{L}\boldsymbol{1}_N = \boldsymbol{0}$. Moreover, if the graph is connected, $\boldsymbol{L}$ has only one zero eigenvalue and all the other eigenvalues are positive.

Lemma 2[28] If the graph $\bar{\mathcal{G}}$ is connected, then the symmetric matrix $\boldsymbol{H} = \boldsymbol{L} + \boldsymbol{B}$ associated with $\bar{\mathcal{G}}$ is positive definite.

Lemma 3[26] Suppose that there exists a continuous positive definite function $V(t)$ satisfying

$$\dot{V}(t) + \lambda_1 V(t) + \lambda_2 V(t)^\gamma \le 0,\ \forall\, t > t_0,$$

where $0 < \gamma < 1, \lambda_1 > 0, \lambda_2 > 0$, then $V(t)$ will converge to the equilibrium point in finite time T given by

$$T \le t_0 + \frac{1}{\lambda_1(1-\gamma)} \ln \frac{\lambda_1 V(t_0)^{(1-\gamma)} + \lambda_2}{\lambda_2}.$$

Lemma 4[26] For any positive scalar $\nu > 1/2$, $\tilde{a}\hat{a} \le -\frac{(2\nu-1)}{2\nu}\tilde{a}^2 + \frac{\nu}{2}a^2$ with $\tilde{a} = a - \hat{a}$.

3. Main results

In this section, to track the leader, the control protocol $\boldsymbol{u}_i$ will be designed based only on the relative information between agent i and its neighbors. In our problem, the position and velocity of the leader are not known to any follower even if it is connected to the leader during the tracking process. In this case, a second-order distributed observer is designed for each follower to estimate the state of the leader.

Similar to Ref. [4], the distributed finite-time estimator is proposed as

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}_i(t) = \hat{\boldsymbol{v}}_i(t) - \gamma \mathrm{sgn}\Big(\sum\limits_{j=1}^{N} a_{ij}(\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t)) \\ \qquad\qquad + b_i(\hat{\boldsymbol{x}}_i(t) - \boldsymbol{x}_0(t)) \Big), \\ \dot{\hat{\boldsymbol{v}}}_i(t) = -\beta \mathrm{sgn}\Big(\sum\limits_{j=1}^{N} a_{ij}(\hat{\boldsymbol{v}}_i(t) - \hat{\boldsymbol{v}}_j(t)) \\ \qquad\qquad + b_i(\hat{\boldsymbol{v}}_i(t) - \boldsymbol{v}_0(t)) \Big), \end{cases} \tag{3}$$

where $\hat{\boldsymbol{x}}_i$ and $\hat{\boldsymbol{v}}_i$ are the estimated position and velocity of the i-th follower to the leader, respectively. For convenience, we denote $\boldsymbol{\xi}_1 = [\xi_{11}^{\mathrm{T}}, \xi_{12}^{\mathrm{T}}, \ldots, \xi_{1N}^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{\xi}_2 = [\xi_{21}^{\mathrm{T}}, \xi_{22}^{\mathrm{T}}, \ldots, \xi_{2N}^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{\xi}_{1i} = \hat{\boldsymbol{x}}_i - x_0$ and $\boldsymbol{\xi}_{2i} = \hat{\boldsymbol{v}}_i - \boldsymbol{v}_0$, $i = 1, 2, \ldots, N$.

According to Eqs. (2) and (3), we have

$$\begin{cases} \dot{\boldsymbol{\xi}}_1 = \boldsymbol{\xi}_2 - \gamma \mathrm{sgn}((\boldsymbol{H} \otimes \boldsymbol{I}_n)\boldsymbol{\xi}_1), \\ \dot{\boldsymbol{\xi}}_2 = -\beta \mathrm{sgn}((\boldsymbol{H} \otimes \boldsymbol{I}_n)\boldsymbol{\xi}_2) - \boldsymbol{1}_N \otimes \boldsymbol{f}_0. \end{cases} \tag{4}$$

With some mild modification, we give Theorem 4.1 of Ref. [4] as follows.

Proposition 1[4] Consider the leader–follower multi-agent systems (1) and (2). Assume that the adjacency graph $\bar{\mathcal{G}}$ is connected and Assumptions 1 and 2 hold. Using the distributed estimator protocol (3) with $\gamma > 0$, $\beta > c$, the followers can obtain the estimation of the position and velocity of the leader in finite time T, namely, $\hat{\boldsymbol{x}}_i \to \boldsymbol{x}_0$, $\hat{\boldsymbol{v}}_i \to \boldsymbol{v}_0$ as $t \to T$, and $\hat{\boldsymbol{x}}_i(t) = \boldsymbol{x}_0(t)$, $\hat{\boldsymbol{v}}_i(t) = \boldsymbol{v}_0(t)$ for any $t \ge T$, $i = 1, 2, \ldots, N$.

3.1. Linear sliding mode control design

In this subsection, a new sliding mode controller protocol is proposed to guarantee that each agent can achieve the consensus as time goes to infinity.

We design the LSM vector as

$$\begin{aligned} \boldsymbol{s}_i &= \sum_{j=1}^{N} h_{ij}(\boldsymbol{x}_j - \hat{\boldsymbol{x}}_j) + \mu \sum_{j=1}^{N} h_{ij}(\boldsymbol{v}_j - \hat{\boldsymbol{v}}_j), \\ &\quad i = 1, 2, \ldots, N,\ t \ge T, \end{aligned} \tag{5}$$

where h_{ij} is the element of $\boldsymbol{H}$ and $\mu > 0$.

Denote $\boldsymbol{\xi}_3 = [\xi_{31}^{\mathrm{T}}, \xi_{32}^{\mathrm{T}}, \ldots, \xi_{3N}^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{\xi}_4 = [\xi_{41}^{\mathrm{T}}, \xi_{42}^{\mathrm{T}}, \ldots, \xi_{4N}^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{\xi}_{3i} = \boldsymbol{x}_i - \hat{\boldsymbol{x}}_i$, $\boldsymbol{\xi}_{4i} = \boldsymbol{v}_i - \hat{\boldsymbol{v}}_i$, $i = 1, 2, \ldots, N$. It follows from Eqs. (1) and (3) that

$$\begin{cases} \dot{\boldsymbol{\xi}}_3 = \boldsymbol{\xi}_4 + \gamma \mathrm{sgn}((\boldsymbol{H} \otimes \boldsymbol{I}_n)\boldsymbol{\xi}_1), \\ \dot{\boldsymbol{\xi}}_4 = \boldsymbol{f} + \boldsymbol{u} + \boldsymbol{w} + \beta \mathrm{sgn}((\boldsymbol{H} \otimes \boldsymbol{I}_n)\boldsymbol{\xi}_2), \end{cases} \tag{6}$$

where $\boldsymbol{f} = [f_1^{\mathrm{T}}, f_2^{\mathrm{T}}, \ldots, f_N^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{u} = [u_1^{\mathrm{T}}, u_2^{\mathrm{T}}, \ldots, u_N^{\mathrm{T}}]^{\mathrm{T}}$, and $\boldsymbol{w} = [w_1^{\mathrm{T}}, w_2^{\mathrm{T}}, \ldots, w_N^{\mathrm{T}}]^{\mathrm{T}}$.

Then the LSM vector can be rewritten as

$$\boldsymbol{S} = (\boldsymbol{H} \otimes \boldsymbol{I}_n)\boldsymbol{\xi}_3 + \mu(\boldsymbol{H} \otimes \boldsymbol{I}_n)\boldsymbol{\xi}_4, \tag{7}$$

where $\boldsymbol{S}=[s_1^{\mathrm{T}},s_2^{\mathrm{T}},\ldots,s_N^{\mathrm{T}}]^{\mathrm{T}}$.

Let $\hat{l}_i$ be the estimate of the unknown constant l_i. The adaptation update law for the upper bound of $\|\boldsymbol{\omega}_i\|_\infty$ is designed as

$$\dot{\hat{l}}_i=\mu\rho_i\|\boldsymbol{s}_i\|_1,\quad i=1,2,\ldots,N \tag{8}$$

where $\mu>0$ and $\rho_i>0$.

The control law based on the linear sliding mode is given as

$$\begin{cases}\boldsymbol{u}_i=-\hat{\boldsymbol{f}}_i-\rho_1\boldsymbol{x}_i-\rho_2\boldsymbol{v}_i, & t<T,\\ \boldsymbol{u}_i=-\dfrac{1}{\mu}(\boldsymbol{v}_i-\hat{\boldsymbol{v}}_i)-\hat{\boldsymbol{f}}_i(\boldsymbol{x}_i,\boldsymbol{v}_i,t)\\ \quad -\alpha\Big(\sum\limits_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{x}_i-\boldsymbol{x}_j)+b_i(\boldsymbol{x}_i-\hat{\boldsymbol{x}}_i)\\ \quad +\mu\Big(\sum\limits_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{v}_i-\boldsymbol{v}_j)+b_i(\boldsymbol{v}_i-\hat{\boldsymbol{v}}_i)\Big)\Big)\\ \quad -(F_i+\hat{l}_i)\mathrm{sgn}(\boldsymbol{s}_i), & t\geq T,\quad i=1,2,\ldots,N,\end{cases}$$

where $0<\rho_2<1<\rho_1$, $\mu>0$, and $\alpha>0$.

Theorem 1 For the MAS (1) and (2), assume that the adjacency graph $\bar{\mathcal{G}}$ is connected and Assumptions 1 and 2 hold. Then, under the distributed observer (3), the adaptive law (8) and the control protocol (9) based on the sliding mode (5), the tracking consensus of the MAS can be reached asymptotically, namely, $\lim\limits_{t\to\infty}\|\boldsymbol{x}_i-\boldsymbol{x}_0\|=0$, $\lim\limits_{t\to\infty}\|\boldsymbol{v}_i-\boldsymbol{v}_0\|=0$, $i=1,2,\ldots,N$.

Proof Firstly, we show that $\boldsymbol{x}_i$ and $\boldsymbol{v}_i$ of system (1) are bounded during the time $0\leq t<T$.

Consider the following Lyapunov function

$$V_3(t)=\frac{1}{2}\sum_{i=1}^{N}\rho_1\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{x}_i+\frac{1}{2}\sum_{i=1}^{N}\rho_2\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{v}_i.$$

Taking the derivative of $V_3(t)$ yields

$$\begin{aligned}\dot{V}_3(t)&=\sum_{i=1}^{N}\rho_1\boldsymbol{x}_i^{\mathrm{T}}\dot{\boldsymbol{x}}_i+\sum_{i=1}^{N}\rho_2\boldsymbol{v}_i^{\mathrm{T}}\dot{\boldsymbol{v}}_i &(9)\\ &=\sum_{i=1}^{N}\rho_1\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{v}_i+\sum_{i=1}^{N}\rho_2\boldsymbol{v}_i^{\mathrm{T}}(\boldsymbol{f}_i+\boldsymbol{u}_i+\boldsymbol{\omega}_i). &(10)\end{aligned}$$

Owing to $\|\boldsymbol{w}_i(t)\|_\infty\leq l_i$ and Assumption 1, substituting (9) into the above equation yields

$$\begin{aligned}\dot{V}_3&=\sum_{i=1}^{N}\rho_1\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{v}_i+\sum_{i=1}^{N}\rho_2\boldsymbol{v}_i^{\mathrm{T}}(\boldsymbol{f}_i-\hat{\boldsymbol{f}}_i-\rho_1\boldsymbol{x}_i-\rho_2\boldsymbol{v}_i+\boldsymbol{\omega}_i)\\ &\leq\rho_1(1-\rho_2)\sum_{i=1}^{N}\|\boldsymbol{x}_i\|\|\boldsymbol{v}_i\|+\rho_2\sum_{i=1}^{N}\|\boldsymbol{v}_i\|F_i\\ &\quad-\rho_2^2\sum_{i=1}^{N}\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{v}_i+\rho_2\sum_{i=1}^{N}\|\boldsymbol{v}_i\|_1l_i\\ &\leq\rho_1(1-\rho_2)\sum_{i=1}^{N}\|\boldsymbol{x}_i\|\|\boldsymbol{v}_i\|+\rho_2\sum_{i=1}^{N}\|\boldsymbol{v}_i\|F_i+\rho_2\sum_{i=1}^{N}\|\boldsymbol{v}_i\|_1l_i\\ &\leq\frac{\rho_1(1-\rho_2)}{2}\sum_{i=1}^{N}\|\boldsymbol{x}_i\|^2+\frac{\rho_1(1-\rho_2)}{2}\sum_{i=1}^{N}\|\boldsymbol{v}_i\|^2+\frac{\rho_2}{2}\sum_{i=1}^{N}\|\boldsymbol{v}_i\|^2\\ &\quad+\frac{\rho_2}{2}\sum_{i=1}^{N}F_i^2+\frac{\rho_2}{2}\sum_{i=1}^{N}\|\boldsymbol{v}_i\|^2+\frac{\rho_2}{2}\sum_{i=1}^{N}l_i^2\\ &\leq\bar{k}V_3+\frac{\rho_2}{2}\sum_{i=1}^{N}F_i^2+\frac{\rho_2}{2}\sum_{i=1}^{N}l_i^2,\end{aligned}$$

where $\bar{k}=\max\{\frac{1-\rho_2}{2},2+\frac{\rho_1(1-\rho_2)}{\rho_2}\}$, and in the above inequality the Hölder inequality $|\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b}|\leq\|\boldsymbol{a}\|_1\|\boldsymbol{b}\|_\infty$ has been used.

According to Assumption 1, there exists a constant $\bar{m}>0$ such that $F_i\leq\bar{m}$. We have

$$\dot{V}_3(t)\leq\bar{k}V_3+\frac{\rho_2N\bar{m}^2}{2}+\frac{\rho_2}{2}\sum_{i=1}^{N}l_i^2.$$

Thus, for $t\in[0,T]$, we can obtain

$$\begin{aligned}V_3(t)\ \leq\ &\left[V_3(0)+\frac{1}{\bar{k}}\left(\frac{\rho_2N\bar{m}^2}{2}+\frac{\rho_2}{2}\sum_{i=1}^{N}l_i^2\right)\right]\mathrm{e}^{\bar{k}T}\\ &-\frac{1}{\bar{k}}\left(\frac{\rho_2N\bar{m}^2}{2}+\frac{\rho_2}{2}\sum_{i=1}^{N}l_i^2\right).\end{aligned}$$

On the other hand, since $\|\boldsymbol{x}_i\|=\sqrt{\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{x}_i}\leq\sqrt{\frac{2}{\rho_1}V_3(t)}$ and $\|\boldsymbol{v}_i\|=\sqrt{\boldsymbol{v}_i^{T}\boldsymbol{v}_i}\leq\sqrt{\frac{2}{\rho_2}V_3(t)}$, $i=1,2,\ldots,N$, it can be inferred that the states of system (1) cannot diverge to infinity during $t\in[0,T]$.

Next, when $t\geq T$, construct the Lyapunov function

$$V_4(t)=\frac{1}{2}\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{H}\otimes\boldsymbol{I}_n)^{-1}\boldsymbol{S}+\frac{1}{2}\sum_{i=1}^{N}\frac{1}{\rho_i}\tilde{l}_i^2,$$

where $\tilde{l}_i=l_i-\hat{l}_i$. Since the adjacency graph $\bar{\mathcal{G}}$ is connected, V_4 is positive definite by Lemma 2. Taking the derivative of V_4 yields

$$\begin{aligned}\dot{V}_4\ &=\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{H}\otimes\boldsymbol{I}_n)^{-1}\dot{\boldsymbol{S}}+\sum_{i=1}^{N}\frac{1}{\rho_i}\tilde{l}_i\dot{\tilde{l}}_i\\ &=\boldsymbol{S}^{\mathrm{T}}(\dot{\boldsymbol{\xi}}_3+\mu\dot{\boldsymbol{\xi}}_4)-\sum_{i=1}^{N}\mu\tilde{l}_i\|\boldsymbol{s}_i\|_1\\ &=\sum_{i=1}^{N}\boldsymbol{s}_i^{\mathrm{T}}\left[\boldsymbol{v}_i-\hat{\boldsymbol{v}}_i+\gamma\mathrm{sgn}\Big(\sum_{j=1}^{N}a_{ij}(\hat{\boldsymbol{x}}_i-\hat{\boldsymbol{x}}_j)+b_i(\hat{\boldsymbol{x}}_i-\boldsymbol{x}_0)\Big)\right]\\ &\quad+\mu\sum_{i=1}^{N}\boldsymbol{s}_i^{\mathrm{T}}\Big\{\boldsymbol{f}_i-\frac{1}{\mu}(\boldsymbol{v}_i-\hat{\boldsymbol{v}}_i)\\ &\quad-\alpha\Big[\sum_{j=1}^{N}a_{ij}(\boldsymbol{x}_i-\boldsymbol{x}_j)+b_i(\boldsymbol{x}_i-\hat{\boldsymbol{x}}_i)\\ &\quad+\mu\Big(\sum_{j=1}^{N}a_{ij}(\boldsymbol{v}_i-\boldsymbol{v}_j)+b_i(\boldsymbol{v}_i-\hat{\boldsymbol{v}}_i)\Big)\Big]\\ &\quad-\hat{\boldsymbol{f}}_i+\boldsymbol{\omega}_i-(F_i+\hat{l}_i)\mathrm{sgn}(\boldsymbol{s}_i)\\ &\quad+\beta\mathrm{sgn}\Big(\sum_{j=1}^{N}a_{ij}(\hat{\boldsymbol{v}}_i-\hat{\boldsymbol{v}}_j)+b_i(\hat{\boldsymbol{v}}_i-\boldsymbol{v}_0)\Big)\Big\}-\mu\sum_{i=1}^{N}\tilde{l}_i\|\boldsymbol{s}_i\|_1.\end{aligned}$$

From Proposition 1, we have $\hat{\boldsymbol{x}}_i\to\boldsymbol{x}_0$, $\hat{\boldsymbol{v}}_i\to\boldsymbol{v}_0$, $i=1,2,\ldots,N$ as $t\to T$. Thus,

$$\dot{V}_4\ =\mu\sum_{i=1}^{N}\boldsymbol{s}_i^{\mathrm{T}}[\boldsymbol{f}_i-\hat{\boldsymbol{f}}_i-(F_i+\hat{l}_i)\mathrm{sgn}(\boldsymbol{s}_i)+\boldsymbol{\omega}_i-\alpha\boldsymbol{s}_i]$$

$$
\begin{aligned}
&-\mu\sum_{i=1}^{N}\tilde{l}_i\|\boldsymbol{s}_i\|_1\\
&\le \mu\sum_{i=1}^{N}\|\boldsymbol{s}_i\|_2 F_i-\mu\sum_{i=1}^{N}\|\boldsymbol{s}_i\|_1 F_i-\mu\sum_{i=1}^{N}\|\boldsymbol{s}_i\|_1\hat{l}_i\\
&\quad+\mu\sum_{i=1}^{N}\|\boldsymbol{s}_i\|_1 l_i-\mu\alpha\sum_{i=1}^{N}\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i-\mu\sum_{i=1}^{N}\tilde{l}_i\|\boldsymbol{s}_i\|_1\\
&\le -\mu\alpha\sum_{i=1}^{N}\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i.
\end{aligned}
$$

The above inequality implies that $\dot{V}_4<0$ for any $\boldsymbol{s}_i\neq\boldsymbol{0}$. By the LaSalle–Yoshizawa Theorem (Theorem 2.17 in Ref. [29]), we can conclude that $\boldsymbol{s}_i\to\boldsymbol{0}$ as $t\to\infty$. Then the sliding surface is reached.

On the sliding mode $\boldsymbol{S}=\boldsymbol{0}$, using Eq. (7), we obtain

$$\boldsymbol{\xi}_3+\mu\boldsymbol{\xi}_4=\boldsymbol{0},$$

which implies that

$$\boldsymbol{\xi}_4=-\frac{1}{\mu}\boldsymbol{\xi}_3.$$

Hence

$$\dot{\boldsymbol{\xi}}_3=-\frac{1}{\mu}\boldsymbol{\xi}_3.$$

Choose the Lyapunov function as $\bar{V}_4=\frac{1}{2}\boldsymbol{\xi}_3^{\mathrm{T}}\boldsymbol{\xi}_3$. One can conclude that

$$\dot{\bar{V}}_4=-\frac{1}{\mu}\boldsymbol{\xi}_3^{\mathrm{T}}\boldsymbol{\xi}_3\le-\frac{2}{\mu}\bar{V}_4.$$

As a consequence, $\boldsymbol{\xi}_3\to\boldsymbol{0}$ as $t\to\infty$. At the same time, $\boldsymbol{\xi}_4\to\boldsymbol{0}$ as $t\to\infty$.

This completes the proof.

3.2. Nonsingular terminal sliding mode control design

In this subsection, we come up with a novel controller based on NTSM to make the tracking error of each agent converge to a quite small neighborhood of the origin in finite time.

We design the NTSM vector as

$$
\begin{aligned}
\boldsymbol{s}_i &= \sum_{j=1}^{N}h_{ij}(\boldsymbol{x}_j-\hat{\boldsymbol{x}}_j)+\mu\sum_{j=1}^{N}h_{ij}\mathrm{sig}(\boldsymbol{v}_j-\hat{\boldsymbol{v}}_j)^{\alpha},\\
&\quad i=1,2,\ldots,N,\ \ t\ge T,
\end{aligned}
\tag{11}
$$

where h_{ij} is the element of $\boldsymbol{H}$ and $\mu>0$, $1<\alpha<2$. Then the NTSM vector can be written as

$$\boldsymbol{S}=(\boldsymbol{H}\otimes\boldsymbol{I}_n)\boldsymbol{\xi}_3+\mu(\boldsymbol{H}\otimes\boldsymbol{I}_n)\mathrm{sig}(\boldsymbol{\xi}_4)^{\alpha},\quad t\ge T,\tag{12}$$

where $\boldsymbol{S}=[\boldsymbol{s}_1^{\mathrm{T}},\boldsymbol{s}_2^{\mathrm{T}},\ldots,\boldsymbol{s}_N^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{s}_i^{\mathrm{T}}=[s_{i1},s_{i2},\ldots,s_{in}]$, $i=1,2,\ldots,N$.

Let $\hat{l}_i$ be the estimate of the unknown constant l_i. The adaptation update law of $\hat{l}_i$ is given as

$$
\begin{aligned}
\dot{\hat{l}}_i &= -2k_io_i\hat{l}_i+\mu\alpha o_i\sum_{p=1}^{n}|s_{ip}||v_{ip}-\hat{v}_{ip}|^{\alpha-1},\\
&\quad i=1,2,\ldots,N,
\end{aligned}
\tag{13}
$$

where $k_i>0$, $o_i>0$, $\mu>0$, and $1<\alpha<2$ are the designable parameters, and s_{ip} and v_{ip} are the p-th components of $\boldsymbol{s}_i$ and $\boldsymbol{v}_i$, respectively.

The control law based on the nonsingular terminal sliding mode is designed as

$$
\begin{cases}
\boldsymbol{u}_i=-\hat{\boldsymbol{f}}_i-\rho_1\boldsymbol{x}_i-\rho_2\boldsymbol{v}_i, & t<T,\\
\boldsymbol{u}_i=\boldsymbol{u}_i^1+\boldsymbol{u}_i^2, & t\ge T,
\end{cases}
\tag{14}
$$

where

$$
\begin{aligned}
\boldsymbol{u}_i^1 &= -\frac{1}{\mu\alpha}\mathrm{sig}(\boldsymbol{v}_i-\hat{\boldsymbol{v}}_i)^{2-\alpha}-\hat{\boldsymbol{f}}_i(\boldsymbol{x}_i,\boldsymbol{v}_i,t)\\
&\quad -m_1\boldsymbol{s}_i-F_i\mathrm{sgn}(\boldsymbol{s}_i)-m_2\mathrm{sig}(\boldsymbol{s}_i)^{\eta},
\end{aligned}
\tag{15}
$$

$\boldsymbol{u}_i^2=[u_{i1}^2,u_{i2}^2,\ldots,u_{in}^2]^{\mathrm{T}}$ and

$$
\begin{cases}
u_{ip}^2=-\dfrac{s_{ip}}{|s_{ip}|}\hat{l}_i, & |s_{ip}|>\varepsilon_i\\
u_{ip}^2=-\dfrac{s_{ip}}{\varepsilon_i}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}\hat{l}_i^2, & |s_{ip}|\le\varepsilon_i,\\
i=1,2,\ldots,N,\quad p=1,2,\ldots,n,
\end{cases}
\tag{16}
$$

with $0<\rho_2<1<\rho_1$, $\mu>0$, $1<\alpha<2$, $0<\eta<1$, $m_1>0$, $m_2>0$, $\varepsilon_i>0$.

Denote $\boldsymbol{x}_i=[x_{i1},x_{i2},\ldots,x_{in}]^{\mathrm{T}}$, $\boldsymbol{v}_i=[v_{i1},v_{i2},\cdots,v_{in}]^{\mathrm{T}}$, $i=0,1,\ldots,N$.

Theorem 2 For the MAS (1) and (2), if the adjacency graph $\bar{\mathcal{G}}$ is connected and Assumptions 1 and 2 hold, then with the distributed observer (3), the adaptive law (13), and the control protocol designed in Eqs. (14)–(16) based on the sliding mode (11), the tracking error of MAS will converge to $|x_{ip}-x_{0p}|\le\Theta_{1ip}$ and $|v_{ip}-v_{0p}|\le\Theta_{2ip}$ in finite time, where

$$
\begin{aligned}
&\Theta_{1ip}=\max\{2\Psi_1,\Psi_2\},\quad \Theta_{2ip}=\left(\frac{\Psi_1}{\mu}\right)^{1/\alpha},\\
&\Psi_1=\min\left\{\sqrt{\frac{2\Phi_2}{\phi_2}},\sqrt{2}\left(\frac{\Phi_2}{\psi_2}\right)^{\frac{1}{\eta+1}}\right\},\\
&\Psi_2=\min\left\{\frac{\chi}{m_1},\left(\frac{\chi}{m_2}\right)^{1/\eta}\right\},\\
&\phi_2=\min\left\{2\mu\alpha m_1\bar{\nu}\lambda,\frac{k_io_i(2\nu_i-1)}{\nu_i}\right\},\\
&\psi_2=\min\left\{2^{\frac{\eta+1}{2}}\mu\alpha m_2\bar{\nu}\lambda^{\frac{\eta+1}{2}},\left(\frac{k_io_i(2\nu_i-1)}{\nu_i}\right)^{\frac{\eta+1}{2}}\right\},\\
&\Phi_1=\sum_{i=1}^{N}k_i\nu_i l_i^2,\quad \Phi_2=\Phi_1+\frac{\mu\alpha}{4}\sum_{i=1}^{N}\varepsilon_i,
\end{aligned}
$$

$\bar{\nu}=\min\{|v_{ip}-\hat{v}_{ip}|\}$, $\lambda=\lambda_{\min}(\boldsymbol{H})$, $\chi>0$, $\nu_i>\frac{1}{2}$, $i=1,2,\ldots,N$, $p=1,2,\ldots,n$.

Proof Firstly, by similar arguments as those in Theorem 2, one can obtain that $\boldsymbol{x}_i$ and $\boldsymbol{v}_i$ of system (1) are bounded during the time $0\le t<T$, which is omitted.

Next, when $t \geq T$, construct the Lyapunov function

$$V_5(t) = \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{H}\otimes \boldsymbol{I}_n)^{-1}\boldsymbol{S} + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{o_i}\tilde{l}_i^2,$$

where $\tilde{l}_i = l_i - \hat{l}_i, i = 1,2,\ldots,N$. Taking the derivative of $V_5(t)$ yields

$$\begin{aligned}\dot{V}_5 =\ & \boldsymbol{S}^{\mathrm{T}}(\boldsymbol{H}\otimes \boldsymbol{I}_n)^{-1}\dot{\boldsymbol{S}} + \sum_{i=1}^{N}\frac{1}{o_i}\tilde{l}_i\dot{\tilde{l}}_i \\ =\ & \boldsymbol{S}^{\mathrm{T}}\dot{\boldsymbol{\xi}}_3 + \mu\alpha\sum_{i=1}^{N}\boldsymbol{s}_i{}^{\mathrm{T}}\mathrm{diag}(|\boldsymbol{\xi}_4|^{\alpha-1})\dot{\boldsymbol{\xi}}_{4i} \\ & + 2\sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i - \mu\alpha\sum_{i=1}^{N}\tilde{l}_i\sum_{p=1}^{n}|s_{ip}||v_{ip}-\hat{v}_{ip}|^{\alpha-1}.\end{aligned}$$

(i) When $|s_{ip}| > \varepsilon_i$,

$$\begin{aligned}\dot{V}_5 =\ & \sum_{i=1}^{N}\boldsymbol{s}_i^{\mathrm{T}}\left[\boldsymbol{v}_i - \hat{\boldsymbol{v}}_i + \gamma\mathrm{sgn}\left(\sum_{j=1}^{N}a_{ij}(\hat{\boldsymbol{x}}_i - \hat{\boldsymbol{x}}_j) + b_i(\hat{\boldsymbol{x}}_i - \boldsymbol{x}_0)\right)\right] \\ & + \mu\alpha\sum_{i=1}^{N}\boldsymbol{s}_i^{\mathrm{T}}\mathrm{diag}(|\boldsymbol{\xi}_4|^{\alpha-1})\left\{\boldsymbol{f}_i - \frac{1}{\mu\alpha}\mathrm{sig}(\boldsymbol{v}_i - \hat{\boldsymbol{v}}_i)^{2-\alpha}\right. \\ & - m_1\boldsymbol{s}_i - \hat{\boldsymbol{f}}_i - F_i\mathrm{sgn}(\boldsymbol{s}_i) - m_2\mathrm{sig}(\boldsymbol{s}_i)^{\eta} - \frac{s_{ip}}{|s_{ip}|}\hat{l}_i + \boldsymbol{\omega}_i \\ & \left. + \beta\mathrm{sgn}\left(\sum_{j=1}^{N}a_{ij}(\hat{\boldsymbol{v}}_i - \hat{\boldsymbol{v}}_j) + b_i(\hat{\boldsymbol{v}}_i - \boldsymbol{v}_0)\right)\right\} \\ & + 2\sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i - \mu\alpha\sum_{i=1}^{N}\tilde{l}_i\sum_{p=1}^{n}|s_{ip}||v_{ip}-\hat{v}_{ip}|^{\alpha-1}.\end{aligned}$$

It follows from Proposition 1 that when $t \to T$, $\hat{\boldsymbol{x}}_i \to \boldsymbol{x}_0$, $\hat{\boldsymbol{v}}_i \to \boldsymbol{v}_0$, $i = 1,2,\ldots,N$. Thus

$$\begin{aligned}\dot{V}_5 \leq\ & -\mu\alpha m_1\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^2 \\ & - \mu\alpha m_2\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^{\eta+1} \\ & - \mu\alpha\sum_{i=1}^{N}\hat{l}_i\sum_{p=1}^{n}s_{ip}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}\frac{s_{ip}}{|s_{ip}|} \\ & + \mu\alpha\sum_{i=1}^{N}l_i\sum_{p=1}^{n}s_{ip}|v_{ip}-\hat{v}_{ip}|^{\alpha-1} + 2\sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i \\ & - \mu\alpha\sum_{i=1}^{N}\tilde{l}_i\sum_{p=1}^{n}s_{ip}|v_{ip}-\hat{v}_{ip}|^{\alpha-1} \\ \leq\ & -\mu\alpha m_1\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^2 \\ & - \mu\alpha m_2\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^{\eta+1} + 2\sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i.\end{aligned}$$

From Lemma 5, one has $k_i\tilde{l}_i\hat{l}_i \leq -\frac{k_i(2\nu_i-1)}{2\nu_i}\tilde{l}_i^2 + \frac{k_i\nu_i}{2}l_i^2, \nu_i > \frac{1}{2}$, and substituting it into the above inequality yields

$$\begin{aligned}\dot{V}_5 \leq\ & -\mu\alpha m_1\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^2 \\ & - \mu\alpha m_2\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^{\eta+1} \\ & - \sum_{i=1}^{N}\left(\frac{k_i(2\nu_i-1)}{2\nu_i}\tilde{l}_i^2\right)^{\frac{\eta+1}{2}} \\ & + \sum_{i=1}^{N}\left(\frac{k_i(2\nu_i-1)}{2\nu_i}\tilde{l}_i^2\right)^{\frac{\eta+1}{2}} - \sum_{i=1}^{N}\frac{k_i(2\nu_i-1)}{2\nu_i}\tilde{l}_i^2 \\ & + \sum_{i=1}^{N}\frac{k_i\nu_i}{2}l_i^2 + \sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i.\end{aligned}$$

Thus, we obtain

$$\dot{V}_5 \leq -\phi_2 V_5 - \psi_2 V_5^{\frac{\eta+1}{2}} + \Phi_1, \tag{17}$$

where

$$\begin{aligned}& \phi_2 = \min\left\{2\mu\alpha m_1\bar{\nu}\lambda, \frac{k_i o_i(2\nu_i-1)}{\nu_i}\right\}, \\ & \psi_2 = \min\left\{2^{\frac{\eta+1}{2}}\mu\alpha m_2\bar{\nu}\lambda^{\frac{\eta+1}{2}}, \left(\frac{k_i o_i(2\nu_i-1)}{\nu_i}\right)^{\frac{\eta+1}{2}}\right\}, \\ & \bar{\nu} = \min\{|v_{ip}-\hat{v}_{ip}|\}, \quad \lambda = \frac{1}{\lambda_{\max}(\boldsymbol{H}\otimes\boldsymbol{I}_n)^{-1}} = \lambda_{\min}(\boldsymbol{H}), \\ & \Phi_1 = \sum_{i=1}^{N}\left(\frac{k_i(2\nu_i-1)}{2\nu_i}\tilde{l}_i^2\right)^{\frac{\eta+1}{2}} + \sum_{i=1}^{N}\frac{k_i\nu_i}{2}l_i^2 + \sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i.\end{aligned}$$

By $0 < \eta < 1$ and Lemma 5, we have

$$\begin{aligned}& \left(\frac{k_i(2\nu_i-1)}{2\nu_i}\tilde{l}_i^2\right)^{\frac{\eta+1}{2}} + k_i\tilde{l}_i\hat{l}_i \\ \leq\ & \frac{k_i(2\nu_i-1)}{2\nu_i}\tilde{l}_i^2 + k_i\tilde{l}_i\hat{l}_i, \leq \frac{k_i\nu_i}{2}l_i^2,\end{aligned}$$

and then $\Phi_1 = \sum_{i=1}^{N}k_i\nu_i l_i^2$.

(ii) When $|s_{ip}| \leq \varepsilon_i$,

$$\begin{aligned}\dot{V}_5 \leq\ & -\mu\alpha m_1\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^2 \\ & - \mu\alpha m_2\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^{\eta+1} \\ & - \mu\alpha\sum_{i=1}^{N}\tilde{l}_i^2\sum_{p=1}^{n}\frac{|s_{ip}|^2|v_{ip}-\hat{v}_{ip}|^{2\alpha-2}}{\varepsilon_i} \\ & + \mu\alpha\sum_{i=1}^{N}l_i\sum_{p=1}^{n}|s_{ip}||v_{ip}-\hat{v}_{ip}|^{\alpha-1} \\ & + 2\sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i - \mu\alpha\sum_{i=1}^{N}\tilde{l}_i\sum_{p=1}^{n}s_{ip}|v_{ip}-\hat{v}_{ip}|^{\alpha-1} \\ \leq\ & -\mu\alpha m_1\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^2 \\ & - \mu\alpha m_2\sum_{i=1}^{N}\sum_{p=1}^{n}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}|s_{ip}|^{\eta+1} \\ & - \mu\alpha\sum_{i=1}^{N}\sum_{p=1}^{n}\left(\frac{s_{ip}|v_{ip}-\hat{v}_{ip}|^{\alpha-1}}{\sqrt{\varepsilon_i}} - \frac{\sqrt{\varepsilon_i}}{2}\right)^2\end{aligned}$$

$$+\frac{\mu\alpha}{4}\sum_{i=1}^{N}\varepsilon_i+2\sum_{i=1}^{N}k_i\tilde{l}_i\hat{l}_i$$
$$\leq -\phi_2 V_5-\psi_2 V_5^{\frac{\eta+1}{2}}+\Phi_2,$$

where $\varepsilon_i>0$ and $\Phi_2=\Phi_1+\frac{\mu\alpha}{4}\sum_{i=1}^{N}\varepsilon_i$.

Define $\Omega=\max\{\Psi_1,\Psi_2\}$, $\Psi_1=\min\{\sqrt{\frac{2\Phi_2}{\phi_2}},\sqrt{2}(\frac{\Phi_2}{\psi_2})^{\frac{1}{\eta+1}}\}$, $\Psi_2=\min\{\frac{\chi}{m_1},(\frac{\chi}{m_2})^{\frac{1}{\eta}}\}$, $\chi>0$.

In the following, we will show that once $|s_{ip}|\leq\Omega$, $\forall\, p=1,2,\ldots,n$, s_{ip} will stay therein forever. Similar to the proof in Ref. [26], the analysis will be divided into two cases.

(I) For the case $v_{ip}-\hat{v}_{ip}\neq 0$, when $s_{ip}\notin(|s_{ip}|\leq\Psi_1)$, the following inequality holds:

$$\dot{V}_5\leq -\phi_2 V_5-\psi_2 V_5^{\frac{\eta+1}{2}},$$

which means that all the sliding modes s_{ip} can converge to the region $|s_{ip}|\leq\Psi_1$.

(II) For the case $v_{ip}-\hat{v}_{ip}=0$, we also need to illustrate that $v_{ip}-\hat{v}_{ip}=0$, $|s_{ip}|>\Psi_2$ is not an attractor, and the reachability of NTSM (11) can be guaranteed. By substituting the controller (14) into Eq. (16), it is easy to obtain

$$\dot{\boldsymbol{\xi}}_4=-\bar{\boldsymbol{F}}\mathrm{sgn}(\boldsymbol{S})-m_1\boldsymbol{S}-m_2\mathrm{sig}(\boldsymbol{S})^{\eta}+\boldsymbol{Q},$$

where $\bar{\boldsymbol{F}}=\mathrm{diag}(\boldsymbol{F}\otimes\boldsymbol{1}_n)$, $\boldsymbol{F}=[F_1,F_2,\ldots,F_N]^{\mathrm{T}}$, $\boldsymbol{Q}=[q_1^{\mathrm{T}},q_2^{\mathrm{T}},\ldots,q_N^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{q}_i=\boldsymbol{f}_i-\hat{\boldsymbol{f}}_i+\boldsymbol{\omega}_i+\boldsymbol{u}_i^2$, $i=1,2,\ldots,N$, $\boldsymbol{q}_i=(q_{ip})_{n\times 1}$. The upper bound of q_{ip} is assumed as χ, $|q_{ip}|\leq\chi$. Since $s_{ip}\notin(|s_{ip}|\leq\Psi_2)$, then $\dot{v}_{ip}-\dot{\hat{v}}_{ip}\neq 0$. Thus, there exists at least one i such that $\dot{\boldsymbol{\xi}}_{4i}\neq 0$ for $|s_{ip}|>\Psi_2$. We get a contradiction.

In cases (I) and (II), once $s_{ip}\notin(|s_{ip}|\leq\Omega=\max\{\Psi_1,\Psi_2\})$, one has

$$\dot{V}_5\leq -\phi_2 V_5-\psi_2 V_5^{\frac{\eta+1}{2}}.$$

Hence, the sliding variables s_{ip} can reach the region $|s_{ip}|\leq\Omega$ in finite time.

We next prove that the tracking error of MAS will converge to the region $|x_{ip}-x_{0p}|\leq\Theta_{1ip}$ and $|v_{ip}-v_{0p}|\leq\Theta_{2ip}$ by two cases.

(III) In case (I), $|s_{ip}|\leq\Psi_1$, we can obtain $s_{ip}=\tau_{ip}$, $|\tau_{ip}|\leq\Psi_1$, which also means

$$\begin{aligned}\boldsymbol{S}&=(\boldsymbol{H}\otimes\boldsymbol{I}_n)\boldsymbol{\xi}_3+\mu(\boldsymbol{H}\otimes\boldsymbol{I}_n)\mathrm{sig}(\boldsymbol{\xi}_4)^{\alpha}\doteq\boldsymbol{\tau}\\&=[\tau_{11},\cdots,\tau_{1n},\cdots,\tau_{Nn}]^{\mathrm{T}}.\end{aligned}$$

Thus,

$$\begin{aligned}&\boldsymbol{\xi}_3+\mu\mathrm{sig}(\boldsymbol{\xi}_4)^{\alpha}=(\boldsymbol{H}^{-1}\otimes\boldsymbol{I}_n)\boldsymbol{\tau}\doteq\bar{\boldsymbol{\tau}}\\&=[\bar{\tau}_{11},\cdots,\bar{\tau}_{1n},\cdots,\bar{\tau}_{Nn}]^{\mathrm{T}}.\end{aligned}\tag{18}$$

Since $v_{ip}-\hat{v}_{ip}\neq 0$, the above equality can be rewritten as

$$(x_{ip}-\hat{x}_{ip})+\mu\mathrm{sig}(v_{ip}-\hat{v}_{ip})^{\alpha}=\bar{\tau}_{ip},$$

that is,

$$(x_{ip}-\hat{x}_{ip})+(\mu-\bar{\tau}_{ip}\mathrm{sig}(v_{ip}-\hat{v}_{ip})^{-\alpha})\mathrm{sig}(v_{ip}-\hat{v}_{ip})^{\alpha}=0.\tag{19}$$

Then when $\mu-\bar{\tau}_{ip}\mathrm{sig}(v_{ip}-\hat{v}_{ip})^{-\alpha}>0$, equation (19) is still kept in the form of TSM, which also means $|v_{ip}-\hat{v}_{ip}|\leq(\frac{\Psi_1}{\mu})^{1/\alpha}$ and $|x_{ip}-\hat{x}_{ip}|\leq\mu|v_{ip}-\hat{v}_{ip}|^{\alpha}+\bar{\tau}_{ip}\leq 2\Psi_1$.

(IV) In case (II), we have $|s_{ip}|\leq\Psi_2$. Similar to the above arguments, we have $s_{ip}=\varpi_{ip}$, $|\varpi_{ip}|\leq\Psi_2$, which can also lead to $|x_{ip}-\hat{x}_{ip}|\leq\Psi_2$.

In summary, the tracking error of MAS will converge to $|x_{ip}-x_{0p}|\leq\Theta_{1ip}=\max\{2\Psi_1,\Psi_2\}$ and $|v_{ip}-v_{0p}|\leq\Theta_{2ip}=(\frac{\Psi_1}{\mu})^{1/\alpha}$.

Corollary 1 For the MAS (1) and (2), if l_i, the upper bound of $\boldsymbol{\omega}_i$, is a known constant, then under the conditions of Theorem 2, the tracking error of MAS will converge to zero in finite time.

4. Simulation results

In this section, a numeral example is presented to illustrate the theoretical results. The example considers MAS (1) and (2) with one leader indexed by 0 and four followers indexed by 1–4. The communication topology graph $\bar{\mathcal{G}}$ is given in Fig. 1.

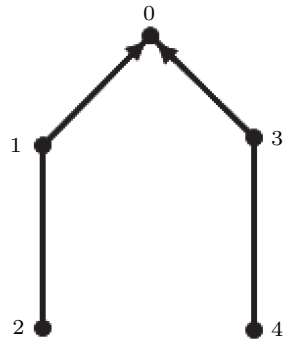


Fig. 1. The communication topology graph $\mathcal{G}$.

The dynamics of the agents described in Eqs. (1) and (2) are given by $\boldsymbol{f}_0=[-0.1\sin t,-0.2\sin t-0.3\cos t]^{\mathrm{T}}$ and $\boldsymbol{f}_i=1.1[x_{i1}^2,x_{i2}^2]^{\mathrm{T}}$. It is obvious that the nonlinear term $\boldsymbol{f}_i(\boldsymbol{x}_i,\boldsymbol{v}_i,t)$ and $\boldsymbol{f}_0(\boldsymbol{x}_0,\boldsymbol{v}_0,t)$ can satisfy Assumptions 1 and 2, where $\hat{\boldsymbol{f}}_i=[x_{i1}^2,x_{i2}^2]^{\mathrm{T}}$, $F_i=0.1\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{x}_i$, $\boldsymbol{x}_i=[x_{i1},x_{i2}]^{\mathrm{T}}$, $i=1,2,3,4$. The external disturbances are chosen as $\boldsymbol{w}(i)=0.1[\sin it,\ \cos it]^{\mathrm{T}}$, $i=1,2,3,4$. In all the simulations, $\gamma=0.5$, $\beta=0.8$, $\mu=0.2$, $\alpha=11/9$, $m_1=1.4$, $m_2=1.6$, $\eta=3/5$, $\varepsilon_i=0.2$, $k_i=0.001$, $\rho_i=0.45$, $o_i=0.9$, $i=1,2,3,4$. We choose the initial value of the MAS as $\boldsymbol{x}_0(t)=[0.1\sin t,0.2\sin t+0.3\cos t]^{\mathrm{T}}$, $\boldsymbol{v}_0(t)=[0.1\cos t,0.2\cos t-0.3\sin t]^{\mathrm{T}}$, $\boldsymbol{x}_1(0)=[-0.1,0.1]^{\mathrm{T}}$,

$\boldsymbol{x}_2(0) = [0.2,-0.2]^{\mathrm{T}}$, $\boldsymbol{x}_3(0) = [-0.5,-0.5]^{\mathrm{T}}$, $\boldsymbol{x}_4(0) = [0.6,0.6]^{\mathrm{T}}$, $\boldsymbol{v}_1(0) = [0.1,-0.1]^{\mathrm{T}}$, $\boldsymbol{v}_2(0) = [0.3,-0.3]^{\mathrm{T}}$, $\boldsymbol{v}_3(0) = [0.4,0.4]^{\mathrm{T}}$, $\boldsymbol{v}_4(0) = [-0.7,-0.7]^{\mathrm{T}}$.

Figures 2 and 3 show the response curves of the estimated parameters $\hat{\boldsymbol{x}}_i$ and $\hat{\boldsymbol{v}}_i$, $i = 1,2,3,4$, respectively, from which one observes that the state estimates generated from observer (3) converge to their real values fast and accurately in finite time. The position and velocity tracking trajectories of MAS based on linear sliding mode (5) are shown in Figs. 4 and 5, respectively. It can be seen that the trajectory of each agent is convergent. Figures 6 and 7 describe the position and velocity tracking trajectories of dynamics via nonsingular terminal sliding mode (11), which can guarantee the states of the followers to converge to a small neighborhood of the leader in finite time.

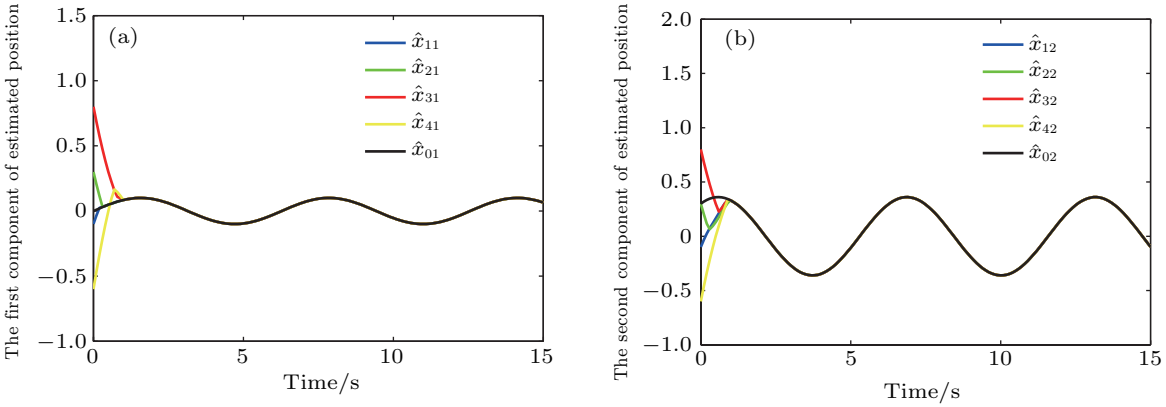


Fig. 2. (color online) The estimated position of the i-th follower to the leader ($i = 1,2,3,4$): (a) $\hat{x}_{i1}$, (b) $\hat{x}_{i2}$.

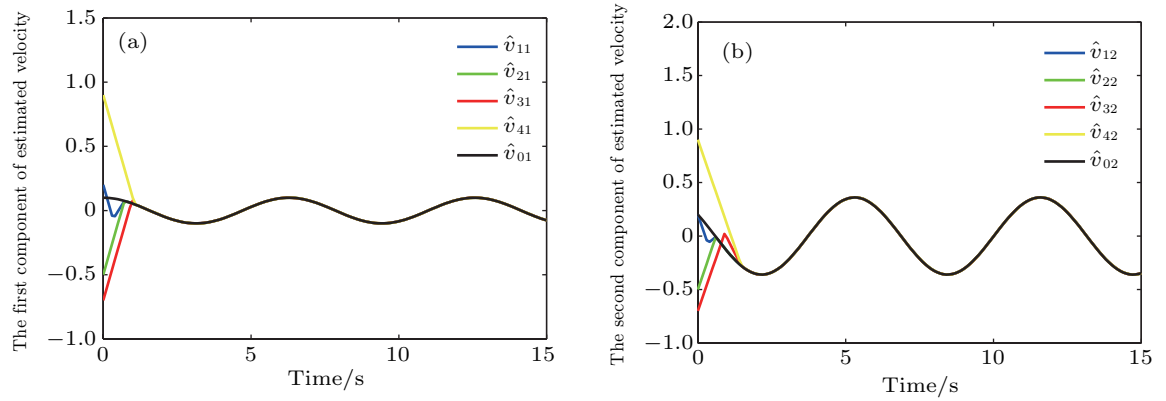


Fig. 3. (color online) The estimated velocity of the i-th follower to the leader ($i = 1,2,3,4$): (a) $\hat{v}_{i1}$, (b) $\hat{v}_{i2}$.

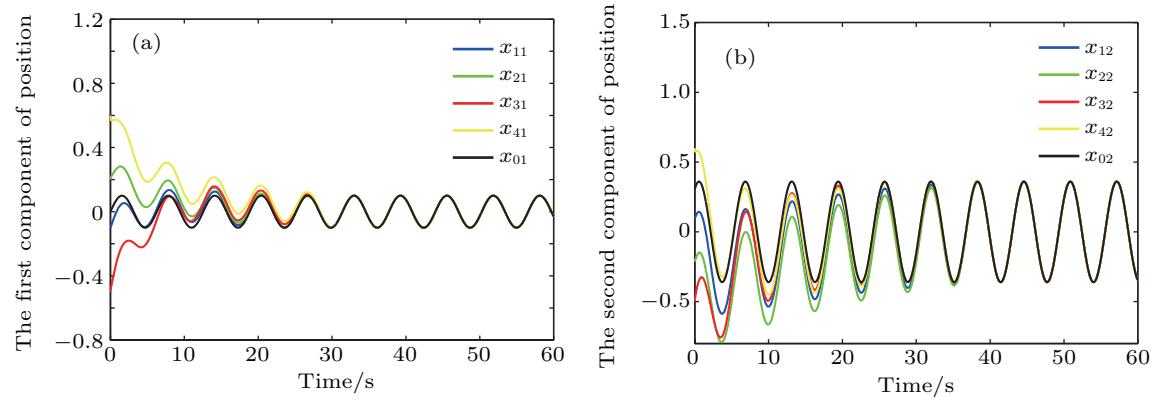


Fig. 4. (color online) The position tracking trajectories of MAS based on the LSM: (a) $\hat{x}_{i1}$, (b) $\hat{x}_{i2}$.

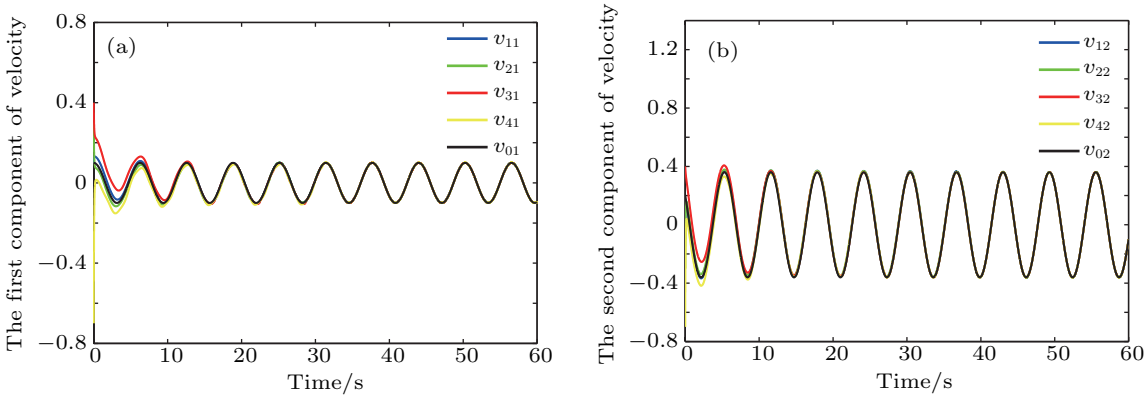


Fig. 5. (color online) The velocity tracking trajectories of MAS based on the LSM: (a) $\hat{v}_{i1}$, (b) $\hat{v}_{i2}$.

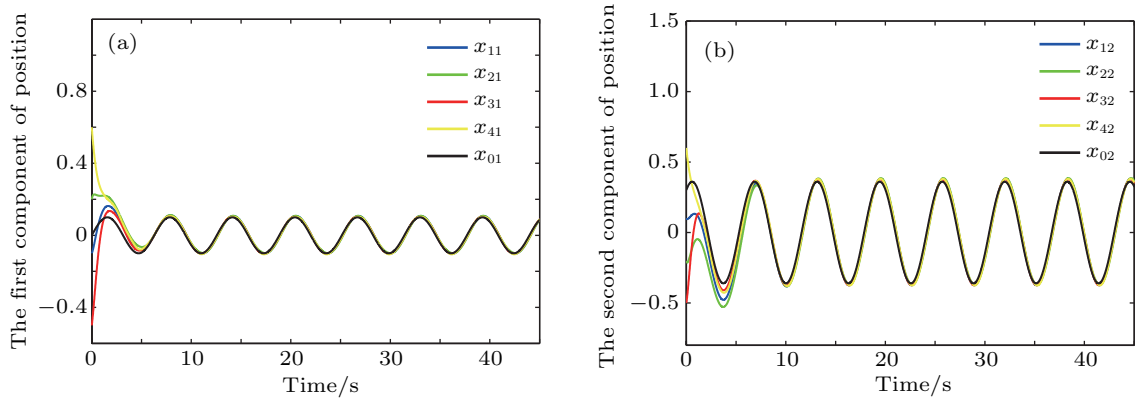


Fig. 6. (color online) The position tracking trajectories of MAS based on the NTSM: (a) $\hat{x}_{i1}$, (b) $\hat{x}_{i2}$.

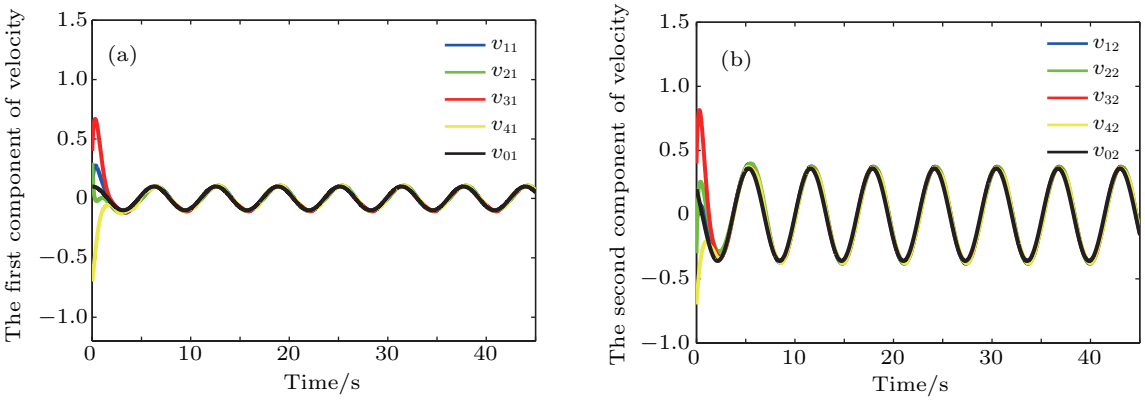


Fig. 7. (color online) The velocity tracking trajectories of MAS based on the NTSM: (a) $\hat{v}_{i1}$, (b) $\hat{v}_{i2}$.

5. Conclusion

We discuss the tracking control for a class of nonlinear heterogeneous leader–follower multi-agent system subject to unknown external disturbances. A second-order distributed observer is first proposed for each follower to estimate the position and velocity of the leader. Then, two new distributed adaptive control protocols based on linear sliding mode and nonsingular terminal sliding mode, respectively, are designed to guarantee the tracking errors to converge to zero asymptotically or to a quite small neighborhood of the origin in finite time. Simulations have been carried out to illustrate the validity of the main results. One of the challenging works for further research is to generalize the results to high-order multi-agent systems with uncertainties and unknown disturbances.

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