Cooperative impulsive formation control for networked uncertain Euler–Lagrange systems with communication delays*

Liang-ming Chen(陈亮名), Chuan-jiang Li(李传江), Yan-chao Sun(孙延超)†, and Guang-fu Ma(马广富)

School of Astronautics, Harbin Institute of Technology, Harbin 150001, *China*

(Received 22 December 2016; revised manuscript received 14 February 2017; published online 20 April 2017)

This paper investigates the cooperative formation problem via impulsive control for a class of networked Euler–Lagrange systems. To reduce the energy consumption and communication frequency, the impulsive control method and cooperative formation control approach are combined. With the consideration of system uncertainties and communication delays among agents, neural networks-based adaptive technique is used for the controller design. Firstly, under the constraint that each agent interacts with its neighbors only at some sampling moments, an adaptive neural-networks impulsive formation control algorithm is proposed for the networked uncertain Euler–Lagrange systems without communication delays. Using Lyapunov stability theory and Laplacian potential function in the graph theory, we conclude that the formation can be achieved by properly choosing the constant control gains. Further, when considering communication delays, a modified impulsive formation control algorithm is proposed, in which the extended Halanay differential inequality is used to analyze the stability of the impulsive delayed dynamical systems. Finally, numerical examples and performance comparisons with continuous algorithm are provided to illustrate the effectiveness of the proposed methods.

Keywords: formation control, multi-agent systems, impulsive control, Euler–Lagrange system

PACS: 87.19.lr, 02.30.Yy, 45.80.+r **DOI:** 10.1088/1674-1056/26/6/068703

1. Introduction

With the development of the research community of system control, cooperative control of multi-agent systems (MAS) has gained increasing attention for the advantages of completing more complex assignments and increasing system reliabilities.[1] With these advantages, cooperative control of MAS has been used in many fields, such as the consensus of wireless networks,[2] synchronization of robots,[3] and formation of spacecrafts.[4] Thus, the cooperative control of MAS is a significant research issue.

One of the basic research problems in cooperative control of MAS is the distributed consensus problem, in which the distributed consensus control law is designed for each agent such that all the agents converge to a common state.[5] To solve the distributed consensus problem, an average-consensus control algorithm was investigated in Ref. [6] for the first-order linear systems under a connected undirected graph. The authors in Ref. [7] extended the results in Ref. [6] to the double integrator systems under a directed graph with a directed spanning tree.

Different from the distributed consensus control for the networked linear systems,[5–7] Liu *et al.*[8] designed a distributed consensus algorithm for the networked Euler–Lagrange systems with parametric uncertainties under a jointly connected undirected graph. The Euler–Lagrange system is a special second-order nonlinear system, which can be used to represent a large class of practical mechanical systems, such as underwater vehicles, robots, and satellites.[9] The distributed adaptive coordination for multiple Euler–Lagrange systems under a directed graph was developed in Refs. [10] and [11]. In Ref. [12], the synchronization problem of networked uncertain Euler–Lagrange systems on directed communication topologies was investigated.

For the distributed control problem of networked systems, model uncertainties, stochastic disturbances, and communication constraints are mainly considered. Unknown parameters were considered for the distributed consensus control for networked Euler–Lagrange systems in Ref. [13]. To handle the uncertainties, the authors in Ref. [14] designed a distributed consensus algorithm for the first-order systems with unknown bounded disturbances under an undirected graph. Due to the long distance between agents and the time for data-processing in each agent, communication delays always exist in practice. Considering the input and communication delays, Tian *et al.*[15] designed the decentralized consensus algorithms for the linear systems under undirected and directed graphs, in which the condition of system consensus is dependent on input delays but independent of communication delays. The distributed consensus problem under the undirected graph with communication delays was investigated in Ref. [16] for the networked linear systems.

Summarizing the above results, we can conclude that the consensus control algorithms proposed in Refs. [5]–[10], [12]–[16] are all continuous, which means that the energy for the control system should be provided all the time. However, the energy carried in each system is usually limited.[17,18] Be-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61304005, 61403103, 61673135, and 61603114).

†Corresponding author. E-mail: sunyanchao@hit.edu.cn

© 2017 Chinese Physical Society and IOP Publishing Ltd

sides, in the studies of Refs. [5]– [10], [12]– [16], the information interaction among agents is always required, which increases the energy consumption and communication burden.

To lower the energy consumption and communication burden in the distributed control of MAS, some researchers proposed the impulsive control for the distributed control problem. In Ref. [19], an impulsive consensus control algorithm was proposed for the networked first-order linear MAS under a directed graph. The authors in Ref. [20] extended the results in Ref. [19] to the networked second-order linear MAS. In Ref. [21], the heterogeneous communication delays among agents were further considered for the distributed consensus problem for the networked second-order linear MAS via impulsive control method. However, in the above references,[19–21] the system model is linear. Tang *et al.*[22] researched the leader-following impulsive consensus control problem for the networked nonlinear MAS with communication delays. In Ref. [23], the distributed impulsive consensus control algorithm was designed for the networked Euler–Lagrange systems. The authors in Ref. [24] extended the results in Ref. [23] to the case with probabilistic communication delays. However, the model parameters of the Euler–Lagrange systems need to be known for the distributed consensus controller designed in Refs. [23] and [24].

The consensus theory in multi-agent systems can be used as a powerful tool to solve the cooperative formation problem where the cooperative formation control algorithm for each agent is designed such that all agents keep relative distance or attitude. Based on the overview of the above references,[1–22] this paper studies the cooperative impulsive formation control problem for the networked Euler–Lagrange systems under a directed graph. The networked Euler–Lagrange systems are subjected to uncertain dynamics, external disturbances, and communication delays. Moreover, the information interactions of each agent with its neighbors are only at some discrete moments. The proposed cooperative formation algorithms are independent of the system model.

Compared with the previous results on cooperative continuous consensus control[5–10,12–16] and distributed impulsive consensus control,[19–24] this study mainly has three advantages.

(i) The cooperative impulsive formation control is investigated for the networked MAS. In comparison to the distributed continuous consensus control algorithm,[5–10,12,13,15,16,16] the energy consumption and the communication burden are reduced.

(ii) Each agent is modeled by nonlinear Euler–Lagrange equation, which can be used to represent a large class of practical mechanical systems. In comparison to the linear systems,[5,6,14,15] it is more challenging to design a cooperative formation control algorithm for a nonlinear Euler–Lagrange system. Furthermore, the uncertain dynamics and external disturbances are considered. Compared with Refs. [23] and [24], the proposed cooperative formation algorithms are independent on the system model.

(iii) The information interactions are more constrained. The communication delays between agents are considered, while in Refs. [5]–[10] and [12]–[14], the proposed algorithm cannot handle the case with communication delays. The information interactions among each agent and its neighbors are only at some sampling moments, while in Refs. [20]–[23], the information interactions of each agent with its neighbors are continuously required.

Notation 1 The subscripts "T" and "-1" stand for matrix transposition and inverse, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The notation $P > 0$ ($P \geq 0$) means that P is symmetric and positive- (semi-positive)-definite. diag$\{\cdots\}$ stands for a block-diagonal matrix. $\|\cdot\|$ is used to refer to the Euclidean vector norm. $\|\cdot\|_2$ stands for the usual $l_2[0,\infty)$ norm, and $\|\cdot\|_\infty$ represents the l_2-induced norm. $\otimes$ represents the Knonecker product. λ_i stands for the i-th eigenvalue of a square matrix. I_n represents the $n \times n$ identity matrix. $\mathbb{N}$ is the set of positive integer numbers. Given a positive constant τ, let $PC([-\tau,0],\mathbb{R}^n) = \{\varphi : [-\tau,0] \to \mathbb{R}^n\}$, $\varphi(t)$ is continuous everywhere except a finite number of points $\hat{t}_k$ at which $\varphi(\hat{t}_k^+)$ and $\varphi(\hat{t}_k^-)$ exist and $\varphi(\hat{t}_k^+) = \varphi(\hat{t}_k)$.

2. Problem formulation and preliminaries

In this section, the Euler–Lagrange system, the graph theory and the problem description are introduced.

2.1. Euler–Lagrange system

The dynamic system of each agent in this study is modeled by the following nonlinear second-order Euler–Lagrange equation

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + \omega_i, \tag{1}$$

where i denotes the index of the agents, $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates with p degrees of freedom. $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive-definite inertia matrix. $C_i(q_i,\dot{q}_i) \in \mathbb{R}^{p\times p}$ is the vector of Coriolis and centrifugal matrix. $g_i(q_i) \in \mathbb{R}^p$ denotes the vector of gravitational forces. $\tau_i \in \mathbb{R}^p$ is the vector of the generalized control forces. $\omega_i \in \mathbb{R}^p$ represents the external disturbance. For the existence of uncertain dynamics in practice, $M_i(q_i)$, $C_i(q_i,\dot{q}_i)$, and $g_i(q_i)$ are all considered as unknown for the controller design in this study. For simplicity, $M_i(q_i)$, $C_i(q_i,\dot{q}_i)$, and $g_i(q_i)$ are denoted as M_i, C_i, and g_i in the following statements.

Many mechanical systems can be modeled by the Euler–Lagrange equation and the following common properties are satisfied in this study:

Property 1 $\dot{M}_i - 2C_i$ is skew symmetric, i.e., $x^{\mathrm{T}}(\dot{M}_i - 2C_i)x = 0$, for all $x \in \mathbb{R}^p$.

Property 2 For any i, there exist positive constants $k_{\underline{m}i}$ and $k_{\overline{m}i}$, such that $0 < k_{\underline{m}i} I_p \leq M_i \leq k_{\overline{m}i} I_p$.

Property 3 The external disturbance ω_i is bounded by $\|\omega_i\| \leq \omega_{Mi}$, where ω_{Mi} is a positive constant.

2.2. Graph theory

A directed graph $G = (V, \varepsilon, A)$ is used to describe the information interaction among agents. $V = \{v_1, \ldots, v_n\}$ is the set of nodes and n represents the number of nodes. $\varepsilon \subseteq V \times V$ represents the set of edges, in which $(v_i, v_j) \in \varepsilon$ represents that agent j can receive the information from agent i. $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix of the directed graph G, where $a_{ij} \neq 0$ if and only if $(v_j, v_i) \in \varepsilon$, and $a_{ij} = 0$ if $(v_j, v_i) \notin \varepsilon$ or $i = j$. A directed path from node v_{i1} to v_{ik} is a sequence of ordered edges $v_{i1}, \ldots, v_{is}, \ldots, v_{ik}$, which satisfies $(v_{is}, v_{is+1}) \in \varepsilon$. A graph is said to be strongly connected if for any node v_i and v_j, there exists at least one directed path from v_i to v_j. The Laplacian matrix L of the directed graph G is defined as $L = D - A$, where $D = \mathrm{diag}(d_1, \ldots, d_n)$ is a diagonal matrix and $d_i = \sum_{j=1}^{n} a_{ij}$. The following Lemma 1 shows a useful property between the Laplacian matrix and strongly connected graph.

Definition 1[25] Suppose a directed graph G is strongly connected. Let $p = [p_1, \ldots, p_n]^{\mathrm{T}}$ be the left-eigenvector of its Laplacian matrix L associated with eigenvalue $\lambda_1 = 0$. The generalized graph Laplacian potential is defined as

$$V_L = \sum_{i,j=1}^{n} p_i a_{ij} (x_j - x_i)^2, \tag{2}$$

where x_i is the system state variable.

Lemma 1[26] Suppose the directed graph G is strongly connected with the Laplacian matrix L. Let $P = \mathrm{diag}\{p_i\} \in \mathbb{R}^{n\times n}$ and

$$Q = PL + L^{\mathrm{T}} P, \tag{3}$$

where p_i is defined as in Definition 1. Then the following identity holds

$$V_L = \sum_{i,j=1}^{n} p_i a_{ij} (x_j - x_i)^2 = x^{\mathrm{T}} Q x. \tag{4}$$

Moreover, $P > 0$ and $Q \geq 0$.

2.3. Problem description

The networked MAS are composed of N agents (denoted as $I = \{1, \ldots, N\}$), which are modeled by EL equations (1). The information interaction among N agents is a directed graph G_A with the Laplacian matrix L. The control objective of this study is to design a cooperative impulsive control algorithm for each agent such that all agents reach a formation. The formation is said to be achieved if the following condition holds

$$\lim_{t\to\infty} (q_i(t) - q_j(t) - \delta_{ij}) = 0, \quad \forall i, j \in I, \tag{5}$$

where δ_{ij} is the desired relative distance or attitude between agent i and j in formation problem. If the directed graph among all agents is strongly connected, δ_{ij} can be decomposed as $\delta_{ij} = \delta_i - \delta_j$. Then, the following Lemma in multi-agent systems is given to build a relationship among the graph, Laplacian potential and system formation.

Lemma 2[25] Let the directed graph be strongly connected and its Laplacian potential V_L be defined by Eq. (2). Then $V_L = 0$ if and only if $x_i = x_j$, $\forall i, j = 1, \ldots, N$.

3. Main results

In this section, the cooperative impulsive formation control algorithms for the networked Euler–Lagrange systems without and with communication delays are investigated respectively. The neural-networks approximation and adaptive technique are used to handle the system uncertain dynamics and external disturbances. The Lyapunov stability theory and Laplacian potential function in graph theory are used to prove the convergence of the system formation.

3.1. Cooperative impulsive formation control without communication delays

In this subsection, a cooperative impulsive formation control algorithm is designed for the networked Euler–Lagrange systems without communication delays. Firstly, the directed information interaction graph G_A satisfies the following assumption.

Assumption 1 The directed graph G_A that characterizes the information interaction among N agents is strongly connected.

First, the auxiliary variables are designed

$$\dot{q}_{ri} = -\rho(q_i + \delta_i), \tag{6}$$

$$s_i = \dot{q}_i - \dot{q}_{ri} = \dot{q}_i + \rho(q_i + \delta_i), \tag{7}$$

where ρ is a positive constant. Applying Eq. (7) into Eq. (1), we can obtain

$$M_i \dot{s}_i + C_i s_i = f_i(q_i, \dot{q}_i) + \tau_i + \omega_i, \tag{8}$$

$$f_i(q_i, \dot{q}_i) = -M_i \ddot{q}_{ri} - C_i \dot{q}_{ri} - g_i, \tag{9}$$

where $f_i(q_i, \dot{q}_i)$ is the uncertain nonlinear dynamics. The neural-networks are used to approximate for the uncertain dynamics in the following form

$$f_i(q_i, \dot{q}_i) = W_i^{\mathrm{T}} \theta_i(q_i, \dot{q}_i) + \varepsilon_i, \tag{10}$$

where W_i is the ideal constant approximation weight matrix, $\theta_i(\cdot)$ is the suitable basis set of functions, and ε_i is the approximation error of neural-networks which is assumed to be

bounded by $\|\varepsilon_i\| \le \varepsilon_{Mi}$, where ε_{Mi} is a positive constant. However, the ideal constant weight matrix W_i are usually unknown for the controller design and the uncertain dynamics cannot be approximated exactly by the neural-networks. Thus, the estimate of $f_i(q_i,\dot{q}_i)$ can be written as

$$\hat{f}_i(q_i,\dot{q}_i) = \hat{W}_i^{\mathrm{T}}\theta_i(q_i,\dot{q}_i), \tag{11}$$

where $\hat{W}_i$ is the estimate of W_i and the adaptive law for $\hat{W}_i$ will be given later. In the following statements, $\theta_i(q_i,\dot{q}_i)$ is simplified as θ_i.

Then, we propose the following cooperative impulsive formation control algorithm

$$\tau_i = -\hat{W}_i^{\mathrm{T}}\theta_i - k_i s_i - (\sigma_{1i}+\sigma_{2i})\frac{s_i}{\|s_i\|} + u_i, \tag{12}$$

$$u_i = -\alpha\sum_{k=1}^{\infty}\sum_{j=1}^{N} a_{ij}(s_i - s_j)\delta(t-t_k), \tag{13}$$

$$\dot{\hat{W}}_i = \gamma_i\theta_i s_i^{\mathrm{T}}, \tag{14}$$

where k_i, σ_{1i}, σ_{2i}, α, γ_i are positive constants, a_{ij} is the (i,j)-th entry of the adjacency matrix A of the directed graph G_A, and $\delta(t)$ denotes the impulsive effects at the moment $t = t_k$. In this study, we assume that the time moments t_k are strictly increasing in the time interval $[0,\infty)$, i.e.,

$$0 = t_0 < t_1 < \cdots < t_k < \cdots \lim_{k\to\infty} t_k = \infty.$$

Then, the following theorem is given to solve the impulsive formation control problem for the networked Euler–Lagrange systems.

Theorem 1 The networked Euler–Lagrange systems with uncertain dynamics and external disturbances will converge to the desired formation with the cooperative impulsive control algorithm [Eqs. (12), (13), (14)] under the strongly connected directed graph G_A if the following conditions hold

$$\sigma_{1i} \ge \omega_{Mi} + \varepsilon_{Mi} \ge \|\omega_i + \varepsilon_i\|, \tag{15}$$

$$\sigma_{2i} \ge \left\|\tilde{W}_i^{\mathrm{T}}\right\|_{\mathrm{F}}\|\theta_i\|, \tag{16}$$

$$(\beta_1 - 1)k_{\bar{m}} + \alpha\lambda_{\min}(L+L^{\mathrm{T}}) - (\alpha^2/k_{\underline{m}})\lambda_{\max}(L^{\mathrm{T}}L) \ge 0, \tag{17}$$

$$\chi > \frac{\ln\beta_1}{\psi}, \tag{18}$$

where $\tilde{W}_i = W_i - \hat{W}_i$. $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm of the matrix. β_1, ψ are positive constants and will be designed later. $M = \mathrm{diag}[M_1,\ldots,M_N]$ and $0 < k_{\underline{m}} I_{Np} \le M \le k_{\bar{m}} I_{Np}$ where $k_{\underline{m}}$, $k_{\bar{m}}$ are positive constants.

$$\chi = \min\{2k_1/k_{\bar{m}1}, 2k_2/k_{\bar{m}2}, \ldots, 2k_N/k_{\bar{m}N}\}.$$

L is the Laplacian matrix of the directed graph G_A.

Proof Applying Eq. (12) into Eq. (8), we can obtain

$$M_i\dot{s}_i + C_i s_i = f_i(q_i,\dot{q}_i) - \hat{W}_i^{\mathrm{T}}\theta_i - k_i s_i - (\sigma_{1i} + \sigma_{2i})\frac{s_i}{\|s_i\|} + u_i + \omega_i. \tag{19}$$

Combining Eq. (10) and the definition of $\tilde{W}_i$, we can obtain

$$M_i\dot{s}_i + C_i s_i = \tilde{W}_i^{\mathrm{T}}\theta_i - k_i s_i - (\sigma_{1i}+\sigma_{2i})\frac{s_i}{\|s_i\|} + u_i + \omega_i + \varepsilon_i. \tag{20}$$

When $t \ne t_k$, $u_i = 0$, the closed-loop system (20) can be written as

$$M_i\dot{s}_i + C_i s_i = \tilde{W}_i^{\mathrm{T}}\theta_i - k_i s_i - (\sigma_{1i}+\sigma_{2i})\frac{s_i}{\|s_i\|} + \omega_i + \varepsilon_i. \tag{21}$$

When $t = t_k$, let $\lim_{t\to t_k^-} x(t_k) = x(t_k^-)$ and $\lim_{t\to t_k^+} x(t_k) = x(t_k^+)$. We assume that all the variables are right continuous at $t = t_k$, i.e., $x(t_k) = x(t_k^+)$. In the impulsive control algorithm (13), the Dirac function $\delta(t-t_k)$ depicts the impulsive effects of sudden appearances of u_i at the time moment $t = t_k$. Then, we have

$$s_i(t_k+\pi) - s_i(t_k-\pi) = \int_{t_k-\pi}^{t_k+\pi} M_i^{-1}\left[-C_i s_i + \tilde{W}_i^{\mathrm{T}}\theta_i - k_i s_i - (\sigma_{1i}+\sigma_{2i})\frac{s_i}{\|s_i\|} + u_i + \omega_i + \varepsilon_i\right]\mathrm{d}\tau, \tag{22}$$

where π is sufficiently small. According to Refs. [16], [20], [24], and [26], when $\pi \to 0^+$, we gain

$$\Delta s_i(t_k) = s_i(t_k^+) - s_i(t_k^-) = M_i^{-1}\left[-\alpha\sum_{j=1}^{N} a_{ij}(s_{\mathrm{i}} - s_j)\right]. \tag{23}$$

To further analyze the stability of the proposed cooperative impulsive formation control algorithm for the networked Euler–Lagrange systems, the following four proof steps are given. Steps 1 and 2 are given to prove the asymptotic convergence of the networked systems when $t \ne t_k$. Step 3 is to analyze the relationship of system variables when $t = t_k^-$ and $t = t_k^+$. Step 4 is to synthesize the results when $t \ne t_k$ and $t = t_k$ and conclude the global stability.

Step 1 Prove the boundness of the variables s_i, $\tilde{W}_i$ when $t \ne t_k$.

Lyapunov stability theory is used to analyze the system stability. Consider the Lyapunov function candidate for the i-th agent

$$V_{1i} = \frac{1}{2}\left[\gamma^{-1}\mathrm{tr}(\tilde{W}_i^{\mathrm{T}}\tilde{W}_i) + s_i^{\mathrm{T}}M_i s_i\right]. \tag{24}$$

For $t \in [t_k, t_{k+1}), k \ge 0$, the derivative of V_{1i} can be written as

$$\dot{V}_{1i} = \gamma^{-1}\mathrm{tr}(\tilde{W}_i^{\mathrm{T}}\dot{\tilde{W}}_i) + 1/2 s_i^{\mathrm{T}}\dot{M}_i s_i + s_i^{\mathrm{T}}M_i\dot{s}_i. \tag{25}$$

Applying Eqs. (21) and (14) into Eq. (25), we can obtain

$$\dot{V}_{1i} = -\operatorname{tr}(\tilde{W}_i^{\mathrm{T}}\theta_i s_i^{\mathrm{T}}) + 1/2 s_i^{\mathrm{T}}\dot{M}_i s_i + s_i^{\mathrm{T}}\left[-C_i(q_i,\dot{q}_i)s_i + \tilde{W}_i^{\mathrm{T}}\theta_i - k_i s_i - (\sigma_{1i}+\sigma_{2i})\frac{s_i}{\|s_i\|} + \omega_i + \varepsilon_i\right]$$
$$= -k_i s_i^{\mathrm{T}} s_i - \operatorname{tr}(\tilde{W}_i^{\mathrm{T}}\theta_i s_i^{\mathrm{T}}) + s_i^{\mathrm{T}}\tilde{W}_i^{\mathrm{T}}\theta_i + 1/2 s_i^{\mathrm{T}}\left[\dot{M}_i - 2C_i(q_i,\dot{q}_i)\right]s_i - s_i^{\mathrm{T}}\left[(\sigma_{1i}+\sigma_{2i})\frac{s_i}{\|s_i\|} + \omega_i + \varepsilon_i\right]. \quad (26)$$

As $s_i^{\mathrm{T}}\tilde{W}_i^{\mathrm{T}}\theta_i$ is a scalar, it follows that $\operatorname{tr}(\tilde{W}_i^{\mathrm{T}}\theta_i s_i^{\mathrm{T}}) = \operatorname{tr}(s_i^{\mathrm{T}}\tilde{W}_i^{\mathrm{T}}\theta_i) = s_i^{\mathrm{T}}\tilde{W}_i^{\mathrm{T}}\theta_i$. By using the Property 1 in Euler–Lagrange system, we have

$$\dot{V}_{1i} = -k_i s_i^{\mathrm{T}} s_i - (\sigma_{1i}+\sigma_{2i})\|s_i\| + s_i^{\mathrm{T}}(\omega_i+\varepsilon_i)$$
$$\le -k_i s_i^{\mathrm{T}} s_i - (\sigma_{1i}+\sigma_{2i})\|s_i\| + \|s_i\|\,\|\omega_i+\varepsilon_i\|. \quad (27)$$

According to Eq. (15), it follows that

$$\dot{V}_{1i} \le -k_i s_i^{\mathrm{T}} s_i - \sigma_{2i}\|s_i\| \le 0. \quad (28)$$

According to the Lyapunov stability theory, equations (24) and (28) imply that both s_i and $\tilde{W}_i$ are bounded.

Step 2 Prove the asymptotic convergence of the variable s_i when $t \ne t_k$.

Consider the Lyapunov function candidate

$$V_{2i} = \frac{1}{2}s_i^{\mathrm{T}} M_i s_i. \quad (29)$$

For $t \in [t_k, t_{k+1})$, $k \ge 0$, the Dini derivative of V_{2i} can be written as

$$\dot{V}_{2i} = 1/2 s_i^{\mathrm{T}}\dot{M}_i s_i + s_i^{\mathrm{T}} M_i \dot{s}_i. \quad (30)$$

Applying the closed-loop system (21) into Eq. (30), we can obtain

$$\dot{V}_{2i} = 1/2 s_i^{\mathrm{T}}\dot{M}_i s_i + s_i^{\mathrm{T}}\left[-C_i(q_i,\dot{q}_i)s_i + \tilde{W}_i^{\mathrm{T}}\theta_i - k_i s_i - (\sigma_{1i}+\sigma_{2i})\frac{s_i}{\|s_i\|} + \omega_i + \varepsilon_i\right]. \quad (31)$$

According to the property of Frobenius norm, we can obtain

$$\left\|\tilde{W}_i^{\mathrm{T}}\theta_i\right\|_F \le \left\|\tilde{W}_i^{\mathrm{T}}\right\|_{\mathrm{F}}\|\theta_i\|. \quad (32)$$

Combining the conclusion in Step 1, Eq. (32) and Ref. [28], we can conclude that $\left\|\tilde{W}_i^{\mathrm{T}}\theta_i\right\|_F$ is bounded. Thus, we have

$$\dot{V}_{2i} = -k_i s_i^{\mathrm{T}} s_i - (\sigma_{1i}+\sigma_{2i})\|s_i\| + s_i^{\mathrm{T}}(\omega_i+\varepsilon_i+\tilde{W}_i^{\mathrm{T}}\theta_i)$$
$$\le -k_i s_i^{\mathrm{T}} s_i - (\sigma_{1i}+\sigma_{2i})\|s_i\| + \|\omega_i+\varepsilon_i\|\,\|s_i\| + \left\|\tilde{W}_i^{\mathrm{T}}\right\|_{\mathrm{F}}\|\theta_i\|\,\|s_i\|. \quad (33)$$

According to Eqs. (15) and (16), we can obtain

$$\dot{V}_{2i} \le -k_i s_i^{\mathrm{T}} s_i \le -(2k_i/k_{\bar{m}i})V_{2i}. \quad (34)$$

Therefore, we can conclude that

$$V_{2i}[s_i(t)] \le V_{2i}[s_i(t_k^+)]\,\mathrm{e}^{-(2k_i/k_{\bar{m}i})(t-t_k)},\ \ t \in [t_k, t_{k+1}),\ k \ge 0. \quad (35)$$

Define the general Lyapunov function V_2

$$V_2 = \sum_{i=1}^{N} V_{2i}. \quad (36)$$

Then, we have

$$V_2[s(t)] \le V_2[s(t_k^+)]\,\mathrm{e}^{-\chi(t-t_k)},\ \ t \in [t_k,\ t_{k+1}),\ k \ge 0, \quad (37)$$

where $s = [s_1^{\mathrm{T}}, \ldots, s_N^{\mathrm{T}}]^{\mathrm{T}}$.

Step 3 Analyze the relationship of s_i when $t = t_k^-$ and $t = t_k^+$.

To analyze the change of s_i when $t = t_k$, integrating two sides of Eq. (23), one can obtain

$$\Delta s = s(t_k^+) - s(t_k^-) = -\alpha M^{-1}(L \otimes I_p)s(t_k^-). \quad (38)$$

Then, we can obtain

$$s(t_k^+) = \left[I_{Np} - \alpha M^{-1}(L \otimes I_p)\right]s(t_k^-). \quad (39)$$

Then, we have

$$V_2(s(t_k^+)) = \sum_{i=1}^{N}\frac{1}{2}s_i^{\mathrm{T}}(t_k^+)M_i s_i(t_k^+) = \frac{1}{2}s^{\mathrm{T}}(t_k^+)Ms(t_k^+). \quad (40)$$

Applying Eq. (39) into Eq. (40), we obtain

$$V_2(s(t_k^+)) = \frac{1}{2}s^{\mathrm{T}}(t_k^-)[I_{Np} - \alpha M^{-1}(L\otimes I_p)]^{\mathrm{T}} M [I_{Np} - \alpha M^{-1}(L\otimes I_p)]s(t_k^-)$$
$$= -\frac{1}{2}s^{\mathrm{T}}(t_k^-)\left[(\beta_1 - 1)M + \alpha(L\otimes I_p) + \alpha(L^{\mathrm{T}}\otimes I_p) - \alpha^2(L^{\mathrm{T}}\otimes I_p)M^{-1}(L\otimes I_p)\right]s(t_k^-) + \frac{\beta_1}{2}s^{\mathrm{T}}(t_k^-)Ms(t_k^-), \quad (41)$$

where $0 < \beta_1 < 1$.

Since the information interaction topology among agents is a directed graph, L is not a symmetric matrix. But the components of Eq. (41), $\alpha(L\otimes I_p)+\alpha(L^{\mathrm{T}}\otimes I_p)$ and $\alpha^2(L^{\mathrm{T}}\otimes I_p)M^{-1}(L\otimes I_p)$ are symmetric. Thus, we can obtain

$$V_2(s(t_k^+)) \le -\frac{1}{2}\left[(\beta_1-1)k_{\bar{m}}+\alpha\lambda_{\min}(L+L^{\mathrm{T}})-(\alpha^2/k_{\underline{m}})\lambda_{\max}(L^{\mathrm{T}}L)\right]s^{\mathrm{T}}(t_k^-)s(t_k^-)+\frac{\beta_1}{2}s^{\mathrm{T}}(t_k^-)Ms(t_k^-).$$

Combining Eqs. (17) and (41), we can obtain

$$V_2(s(t_k^+)) \le \beta_1 V_2(s(t_k^-)). \tag{42}$$

Thus, equation (42) implies the abrupt change of the variable s_i when $t=t_k^-$ and $t=t_k^+$.

Step 4 Prove the global formation of the networked Euler–Lagrange systems when $t\to\infty$.

Combining Eqs. (34) and (42), we can obtain

$$V_2[s(t)] \le \beta_1^{k-1}V_2[s(t_0^+)]\,\mathrm{e}^{-\chi(t-t_0)}. \tag{43}$$

According to the average impulsive interval definition in Ref. [29], we have

$$N_\zeta \ge \frac{t-t_0}{\psi}-N_0, \tag{44}$$

where N_ζ denotes the number of the impulsive times of impulsive sequence $\zeta=\{t_1,t_2,\ldots\}$ in time interval (t_0,t), ψ is the minimum average impulsive interval, and N_0 is a positive integer. Thus, we can obtain

$$\begin{aligned} V_2[s(t)] &\le \beta_1^{N_\zeta}V_2[s(t_0^+)]\,\mathrm{e}^{-\chi(t-t_0)} \\ &\le V_2[s(t_0^+)]\beta_1^{-N_0}\,\mathrm{e}^{(\ln\beta_1/\psi-\chi)(t-t_0)}. \end{aligned} \tag{45}$$

Using Eq. (18), we conclude that

$$\lim_{t\to\infty}V_2[s(t)]=0. \tag{46}$$

To get the formation of the networked systems, we consider the Laplacian potential function

$$V_{\mathrm{L}}=\sum_{i,j=1}^{N}p_ia_{ij}(s_j-s_i)^2=s^{\mathrm{T}}Qs\ge 0, \tag{47}$$

where p_i, Q are defined in Definition 1.

Because the information interaction graph G_A among agents is strongly connected, we can obtain $Q\ge 0$ by Lemma 1. Therefore, we have a relationship between the Laplacian potential function V_{L} and the Lyapunov function V_2.

$$\begin{aligned} V_{\mathrm{L}} &= \sum_{i,j=1}^{N}p_ia_{ij}(s_j-s_i)^2 \\ &= s^{\mathrm{T}}Qs\le\lambda_{\max}(Q)s^{\mathrm{T}}s\le(2\lambda_{\max}(Q)/k_{\underline{m}})V_2. \end{aligned} \tag{48}$$

Combining Eqs. (46), (47), and (48), we conclude that

$$\lim_{t\to\infty}V_{\mathrm{L}}=0. \tag{49}$$

According to Lemma 2, equation (49) implies

$$\lim_{t\to\infty}(s_i-s_j)=0.$$

According to Eq. (7), we obtain

$$s_i-s_j=\dot{q}_i-\dot{q}_j+\rho(q_i-q_j-\delta_{ij}).$$

Because δ_{ij} is constant, we have

$$\begin{aligned} q_i-q_j-\delta_{ij} &= \mathrm{e}^{-\rho t}(q_i(0)-q_j(0)-\delta_{ij}) \\ &\quad+\mathrm{e}^{-\rho t}\int_0^t\mathrm{e}^{\rho\tau}(s_i-s_j)\,\mathrm{d}\tau. \end{aligned} \tag{50}$$

Because $\rho>0$, we can gain

$$\lim_{t\to\infty}\mathrm{e}^{-\rho t}(q_i(0)-q_j(0)-\delta_{ij})=0$$

and

$$\begin{aligned} \lim_{t\to\infty}\mathrm{e}^{-\rho t}\int_0^t\mathrm{e}^{\rho\tau}(s_i-s_j)\,\mathrm{d}\tau &= \lim_{t\to\infty}\frac{\mathrm{e}^{-\rho t}(s_i-s_j)}{\rho}(\mathrm{e}^{\rho t}-1) \\ &= 0. \end{aligned}$$

According to Eq. (50), we can conclude that

$$\lim_{t\to\infty}(q_i-q_j-\delta_{ij})=0,$$

which implies the formation of the networked Euler–Lagrange systems is achieved. This completes the proof.

Remark 1 The Laplacian potential V_{L} defined in Eq. (47) is positive-semidefinite, but not positive-definite. Thus, The Laplacian potential V_{L} is not a Lyapunov function, but a function for the analysis of the system formation.

Remark 2 In Eq. (12), except the impulsive component, τ_i contains both the continuous control part and the adaptive control part. According to the definition of impulsive system and control in Ref. [27], we can call our proposed controller the impulsive formation controller. Moreover, we will try to design fully impulsive formation controller in our future work.

3.2. Cooperative impulsive formation control with communication delays

In this subsection, the communication delays among agents are considered in the networked Euler–Lagrange systems. Since the main procedures of this subsection are similar with the former subsection, the differences are mainly stated here.

To analyze the stability of the networked systems with delays, the following Lemma is given firstly.

Lemma 3[30] (Extended Halanay differential inequality)

Suppose $q \geq 0$, and $\mu(t)$ satisfies the scalar impulsive differential inequality

$$D^+\mu(t) \leq p\mu(t) + q[\mu(t)]_\tau, \quad t \neq t_k, t \geq t_0, \tag{51}$$

$$\mu(t_k^+) \leq b_k\mu(t_k^-) + d_k[\mu(t)]_{\tau^-},$$

$$0 < b_{k-1} + d_{k-1}\,\mathrm{e}^{\eta\tau} \leq 1, k \in \mathbb{N}, \tag{52}$$

$$\mu(t) = \phi(t), \quad t \in [t_0 - \tau, t_0], \tag{53}$$

where D^+ represents the distribution derivative, $b_0 = 1$, $d_0 = 0$, $\mu(t)$ is continuous at $t \neq t_k, t \geq t_0$, $\mu(t_k) = \mu(t_k^+)$ and $\mu(t_k^-)$ exists, $\phi \in PC([t_0 - \tau, t_0], \mathbb{R})$, $[\mu(t)]_{\tau^-} = \sup_{-\tau \leq \kappa < 0}[\mu(t+\kappa)]$. Assume that there exists a small enough number $\eta > 0$ such that for all $k \in \mathbb{N}$, the following condition is satisfied

$$\left(p + \frac{q}{b_{k-1} + d_{k-1}\,\mathrm{e}^{\eta\tau}}\right)(t_k - t_{k-1}) < -\ln(b_{k-1} + d_{k-1}\,\mathrm{e}^{\eta\tau}). \tag{54}$$

Then there exists a constant $\vartheta \geq 1$ such that

$$\mu(t) \leq \|\phi\|_\tau \vartheta\,\mathrm{e}^{-\eta(t-t_0)}, \quad t \geq t_0, \tag{55}$$

where $\|\phi\|_\tau = \sup_{-\tau \leq \kappa \leq 0}\|\phi(\kappa)\|$.

The cooperative impulsive formation control algorithm for the networked systems with communication delays is designed as Eqs. (12), (14), and

$$u_i = -\alpha \sum_{k=1}^{\infty}\sum_{j=1}^{N} a_{ij}(s_i - s_j(t-T))\delta(t - t_k), \tag{56}$$

where T represents the communication delays between agents.

Theorem 2 The networked Euler–Lagrange systems with uncertain dynamics and external disturbances can achieve the desired formation with the cooperative impulsive control algorithm (12), (14), and (56) under the strongly connected directed graph G_A with communication delays T if the conditions (15), (16), (68), (69) hold.

Proof With the comparison of the two cooperative impulsive formation control algorithms (13) and (56), the difference is only in the impulsive control effects. Thus, the stability analysis in this subsection when $t \neq t_k$ is the same with the former subsection.

When $t = t_k$, the closed-loop networked systems can be written as

$$\Delta s_i(t_k) = s_i(t_k^+) - s_i(t_k^-) = M_i^{-1}\left[-\alpha\sum_{j=1}^{N} a_{ij}(s_i - s_j(t-T))\right]. \tag{57}$$

Integrating both sides of Eq. (57), we have

$$\Delta s = s(t_k^+) - s(t_k^-) = -\alpha M^{-1}(L_{\mathrm{D}}s(t_k^-) - L_{\mathrm{F}}s(t_k^- - T)), \tag{58}$$

where $L_{\mathrm{D}} = D \otimes I_N$, $L_{\mathrm{F}} = A \otimes I_N$. $L_{\mathrm{D}}^{\mathrm{T}} = L_{\mathrm{D}}$ but $L_{\mathrm{F}}^{\mathrm{T}} \neq L_{\mathrm{F}}$. Then, we have

$$s(t_k^+) = (I_{Np} - \alpha M^{-1}L_{\mathrm{D}})s(t_k^-) + \alpha M^{-1}L_{\mathrm{F}}s(t_k^- - T). \tag{59}$$

The Lyapunov function $V_2(s(t_k^+))$ can be written as

$$\begin{aligned} V_2(s(t_k^+)) &= \sum_{i=1}^{N}\frac{1}{2}s_i^{\mathrm{T}}M_i s_i = \frac{1}{2}s^{\mathrm{T}}(t_k^+)Ms(t_k^+) \\ &= \frac{1}{2}[(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})s(t_k^-) + \alpha M^{-1}L_{\mathrm{F}}s(t_k^- - T)]^{\mathrm{T}}M[(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})s(t_k^-) + \alpha M^{-1}L_{\mathrm{F}}s(t_k^- - T)] \\ &= \frac{1}{2}s^{\mathrm{T}}(t_k^-)(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})^{\mathrm{T}}M(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})s(t_k^-) \\ &\quad + \frac{\alpha}{2}s^{\mathrm{T}}(t_k^-)(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})^{\mathrm{T}}L_{\mathrm{F}}s(t_k^- - T) + \frac{\alpha}{2}s^{\mathrm{T}}(t_k^- - T)L_{\mathrm{F}}^{\mathrm{T}}(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})s(t_k^-) \\ &\quad + \frac{\alpha^2}{2}s^{\mathrm{T}}(t_k^- - T)L_{\mathrm{F}}^{\mathrm{T}}M^{-1}L_{\mathrm{F}}s(t_k^- - T). \end{aligned} \tag{60}$$

According to the inequality $X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leq \varepsilon X^{\mathrm{T}}X + 1/\varepsilon Y^{\mathrm{T}}Y$, we obtain that there exists a constant $\varepsilon > 0$ such that

$$\begin{aligned} &\frac{\alpha}{2}s^{\mathrm{T}}(t_k^-)(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})^{\mathrm{T}}L_{\mathrm{F}}s(t_k^- - T) \\ &\quad + \frac{\alpha}{2}s^{\mathrm{T}}(t_k^- - T)L_{\mathrm{F}}^{\mathrm{T}}(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})s(t_k^-) \leq \frac{\alpha\varepsilon}{2}s^{\mathrm{T}}(t_k^-)(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})^{\mathrm{T}}(I_{Np} - \alpha M^{-1}L_{\mathrm{D}})s(t_k^-) \\ &\quad + \frac{\alpha}{2\varepsilon}s^{\mathrm{T}}(t_k^- - T)L_{\mathrm{F}}^{\mathrm{T}}L_{\mathrm{F}}s(t_k^- - T). \end{aligned} \tag{61}$$

Thus, equation (60) can be written as

$$V_2(s(t_k^+)) \leq bV_2(s(t_k^-)) + dV_2(s(t_k^-))_T, \tag{62}$$

where

$$b = \lambda_{\max}^2(I_{Np} - \alpha M^{-1}L_{\mathrm{D}}) + \frac{\alpha\varepsilon}{\lambda_{\min}(M)}\lambda_{\max}^2(I_{Np} - \alpha M^{-1}L_{\mathrm{D}}), \tag{63}$$

$$d = \frac{\alpha^2}{\lambda_{\min}^2(M)}\lambda_{\max}(L_{\mathrm{F}}^{\mathrm{T}}L_{\mathrm{F}}) + \frac{\alpha}{\varepsilon\lambda_{\min}(M)}\lambda_{\max}(L_{\mathrm{F}}^{\mathrm{T}}L_{\mathrm{F}}). \tag{64}$$

Thus, we have

$$D^{+}V_2(s(t)) \le -\chi V_{2i}(s(t)),\ \ t \in [t_k, t_{k+1}), k \ge 0,$$
$$V_2(s(t_k^{+})) \le bV_2(s(t_k^{-})) + dV_2(s(t_k^{-}))_T,\ \ t = t_k.$$

Let $V(s(t)) = \phi(t) \in PC([-T,0],\mathbb{R}^n) = \{\phi : [-T,0] \to \mathbb{R}^n\}$, $\phi(t)$ is continuous for any $t \in [t_0 - T, t_0]$. Using Lemma 3, we can obtain if the following conditions hold

$$0 < b + d\,\mathrm{e}^{\eta T} \le 1, \tag{65}$$
$$(-\chi)(t_k - t_{k-1}) < -\ln(b + d\,\mathrm{e}^{\eta T}), \tag{66}$$

then, we have

$$V_2(s(t)) \le \|\phi\|_\tau \vartheta\, \mathrm{e}^{-\eta(t-t_0)},\ \ t \ge t_0, \tag{67}$$

where $\vartheta \ge 1$ and $\|\phi\|_\tau = \sup_{-\tau \le \kappa \le 0} \|\phi(\kappa)\|$.

By simplifying, inequalities (65) and (66) can be further written as

$$0 < \eta \le \frac{1}{T} \ln \frac{1-b}{d}, \tag{68}$$
$$\psi > \frac{\ln(b+d)}{\chi}. \tag{69}$$

By the similar proof procedures in Step 4 in Subsection 1 which is omitted here, we can conclude that the formation of the networked Euler–Lagrange systems with communication delays is achieved. This completes the proof.

Remark 3 For the proposed cooperative formation control law (12), there may exist singularity effect when s_i is small. Thus, one practical approach that can be used to avoid the singularity is by replacing $s_i/\|s_i\|$ with $s_i/\|s_i\| + \xi$, where ξ is a small positive constant.

Remark 4 In the impulsive formation system, the main control factors have influences on the dynamical behaviors of the system. In practice, we should balance our control parameters to meet the need of physical background and practical applications. If the number of impulsive time is more, the impulsive control effect and communication among agents will increase. When the amplitude of the pulses are larger, impulsive effect will become the main system control effect and the amplitude of the controller is larger. When the delay time becomes larger, the system convergent time becomes longer.

4. Illustrative example

In this section, a numerical simulation of satellite formation flying is presented to verify the effectiveness of the proposed cooperative impulsive formation control algorithms.

Consider a group of eight satellites ($I = \{1,\ldots,8\}$), the equations of relative orbit motion in the local-vertical, local-horizontal (LVLH) rotating frame can be described as[31]

$$\ddot{x}_i - 2\omega_0 \dot{y}_i - \omega_0^2 x_i - \dot{\omega}_0 y_i + \frac{\mu_e(R_0 + x_i)}{R_i^3} - \frac{\mu_e}{R_0^2} = \frac{\tau_{oix}}{m_i},$$
$$\ddot{y}_i^2 + 2\omega_0 \dot{x}_i - \omega_0^2 y_i + \dot{\omega}_0 x_i + \frac{\mu_e y_i}{R_i^3} = \frac{\tau_{oiy}}{m_i},$$
$$\ddot{z}_i + \frac{\mu_e z_i}{R_i^3} = \frac{\tau_{oiz}}{m_i},$$

where $[x_i, y_i, z_i]$ are the relative coordinate of the satellite i in the LVLH frame, R_0 is the orbit radius, m_i is the mass of the satellite, μ_e is the gravitation constant, $\omega_0 = \sqrt{\mu_e/R_0^3}$ is the orbit angular velocity, R_i is the distance between the satellite and the geocenter, $[\tau_{oix}, \tau_{oiy}, \tau_{oiz}]$ are the control forces. Note that the above relative orbit motion equations can be transformed into Euler–Lagrange equation $m_i\ddot{q}_i + C_i\dot{q}_i + g_i = \tau_{oi}$, $i = 1,\ldots,8$ by $q_i = [x_i, y_i, z_i]^{\mathrm{T}}$, $\tau_{oi} = [\tau_{oix}, \tau_{oiy}, \tau_{oiz}]^{\mathrm{T}}$,

$$C_i = 2m_i \begin{bmatrix} 0 & -\omega_0 & 0 \\ \omega_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$g_i = m_i \begin{bmatrix} -\omega_0 x_i - \dot{\omega}_0 y_i + \mu_e(R_0 + x_i)/R_i^3 - \mu_e/R_0^2 \\ -\omega_0 y_i + \mu_e y_i/R_i^3 + \dot{\omega}_0 x_i \\ \mu_e z_i/R_i^3 \end{bmatrix}.$$

This simulation's examples assume that the relative orbit reference of the satellite formation system follows a near-circular orbit with the initial orbit elements

$$[a\ e\ i\ \Delta\ \varpi\ f] = [7136.0 \quad 0.001 \quad 60^\circ \quad 10^\circ \quad 30^\circ \quad 0^\circ],$$

where a is the semi-major axis (in unit km) of the reference orbit, e is eccentricity, i is the inclination, Δ is the longitude of the ascending node, ϖ is argument of periapsis, and f is the true anomaly. For satellites $i = \{1,2,3\}$, $m_i = 35$ kg and $i = \{4,5,6,7,8\}$, $m_i = 40$ kg.

The connected directed communication topology G_A among eight satellites is described in Fig. 1.

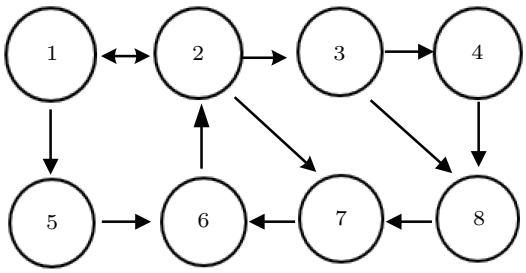


Fig. 1. The communication topology among satellites.

The Laplacian matrix of the directed graph G_A is

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

The initial states of eight satellites are $q_1(0) = [15, 20, 20]^{\mathrm{T}}$ m, $q_2(0) = [25, -10, 25]^{\mathrm{T}}$ m, $q_3(0) = [-20, -20, -5]^{\mathrm{T}}$ m, $q_4(0) = [-5, 10, 30]^{\mathrm{T}}$ m, $q_5(0) = [0, -15, -15]^{\mathrm{T}}$ m, $q_6(0) = [-25, 15, -25]^{\mathrm{T}}$ m, $q_7(0) = [10, -5, -10]^{\mathrm{T}}$ m, $q_8(0) = [-10, 0, -20]^{\mathrm{T}}$ m, $\dot{q}_i(0) = 0$ m/s, $\forall i = 1, \ldots, 8$. $\delta_1 = [150, 50, 150]^{\mathrm{T}}$ m, $\delta_2 = [150, 150, 150]^{\mathrm{T}}$ m, $\delta_3 = [50, 150, 150]^{\mathrm{T}}$ m, $\delta_4 = [50, 50, 150]^{\mathrm{T}}$ m, $\delta_5 = [150, 50, 50]^{\mathrm{T}}$ m, $\delta_7 = [50, 150, 50]^{\mathrm{T}}$ m, $\delta_8 = [50, 50, 50]^{\mathrm{T}}$ m.

The simulation parameters of neural-networks (NN) are given as: the activation function of NN for i-th satellite used in this simulation can be denoted as

$$\theta_i(z) = [\theta_{i1}(z), \ldots, \theta_{i6}(z)]^{\mathrm{T}},$$

where $\theta_{ij}(z)$ is a Gaussian function

$$\theta_{ij}(z) = \exp\left(-\frac{\|z - c_{ij}\|^2}{{\sigma_{ij}}^2}\right), \quad j = 1, \ldots, 6,$$

where $z = [q_i, \dot{q}_i]^{\mathrm{T}} \in \mathbb{R}^6$. It is assumed that all the activation function of neural-networks is the same, c_{ij} represents the center of the receptive field which is evenly distributed in $[-5, 5]^3 \times [-0.5, 0.5]^3$. $\sigma_{ij} = 2$ is the width of Gaussian function. The initial weight matrix are all zeros, which denoted $\hat{W}_i(0) = 0_{6\times 3}$.

The control parameters are $\rho = 0.01$, $\sigma_{1i} = 0.005$, $\sigma_{2i} = 0.005$, $k_i = 1$, $\alpha = 0.5$, $\gamma_i = 1$, $\forall i = 1, \ldots, 8$. Communication delays are set as $T = 1$ s. The impulsive intervals are 5 s, i.e., $t_{k+1} - t_k = 5$ s, $k \geq 0$. Note that the above chosen control parameters satisfy the conditions (15)–(18), (68) and (69).

The simulation results are shown below and the figures are drawn in Figs. 2–13.

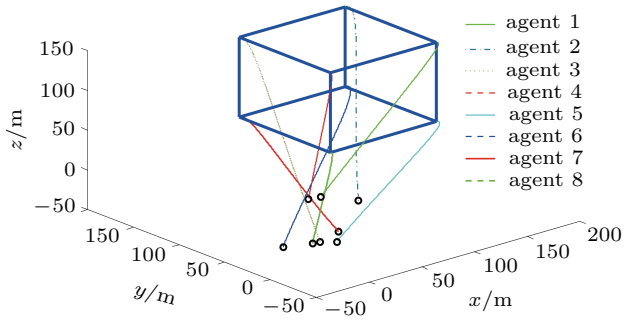


Fig. 2. (color online) Trajectories of eight satellites in Case 1.

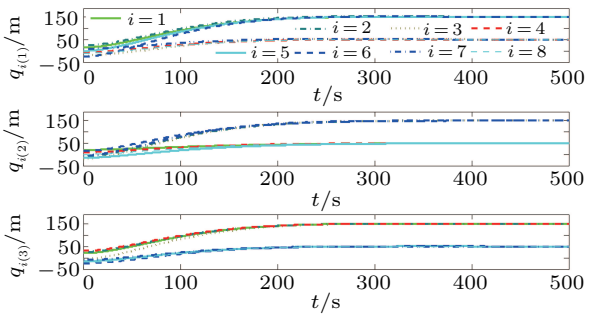


Fig. 3. (color online) Positions of eight formation satellites in Case 1.

Case 1 Cooperative impulsive formation control without communication delays;

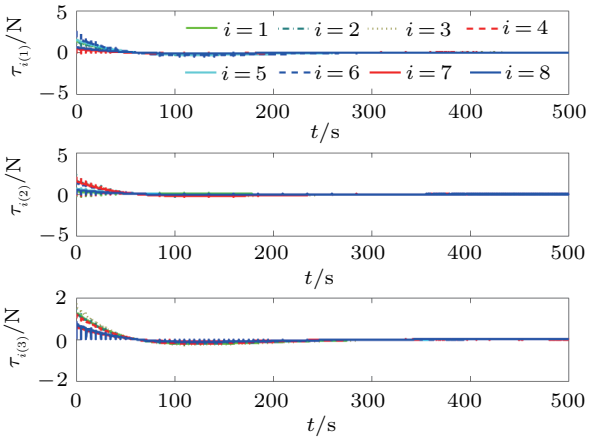


Fig. 4. (color online) Control forces of eight satellites in Case 1.

Case 2 Cooperative impulsive formation control with communication delays;

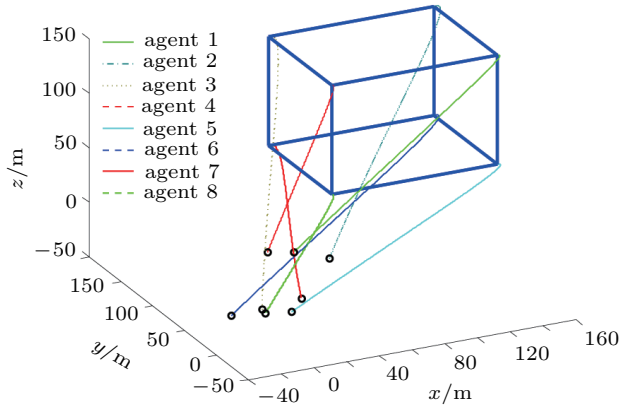


Fig. 5. (color online) Trajectories of eight satellites in Case 2.

Case 3 Continuous cooperative formation control algorithm with the same control parameters.

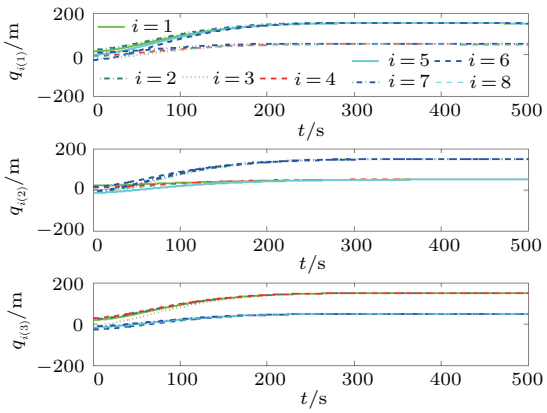


Fig. 6. (color online) Positions of eight formation satellites in Case 2.

To compare the control energy consumption among Case 1, Case 2, and Case 3, the control energy consumption for the networked systems is defined as

$$J = \sum_{i=1}^{8} \int_0^{500} \|\tau_i(t)\| \, \mathrm{d}t.$$

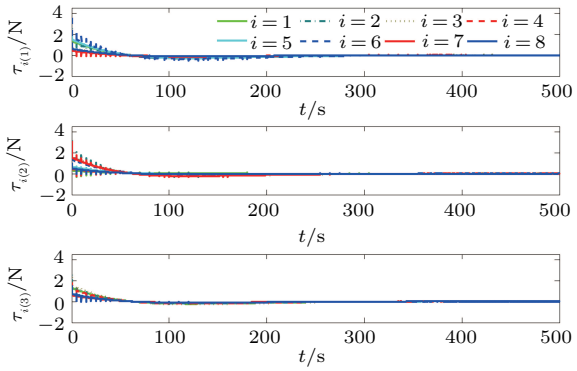


Fig. 7. (color online) Control forces of eight satellites in Case 2.

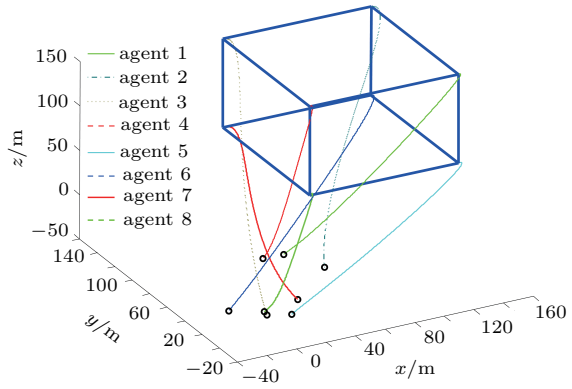


Fig. 8. (color online) Trajectories of eight satellites in Case 3.

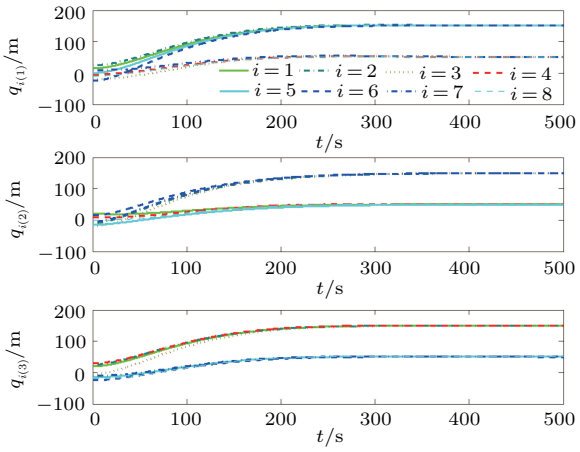


Fig. 9. (color online) Positions of eight formation satellites in Case 3.

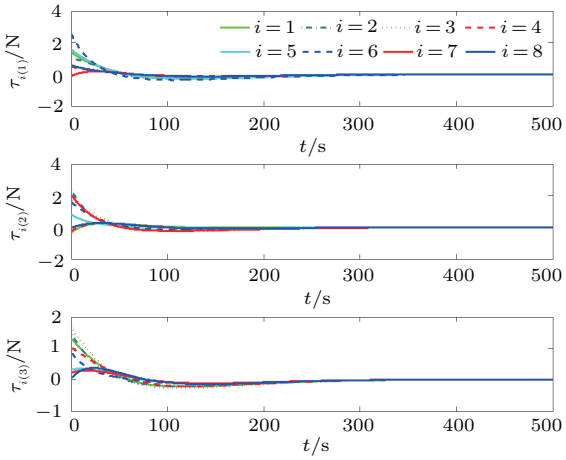


Fig. 10. (color online) Control forces of eight satellites in Case 3.

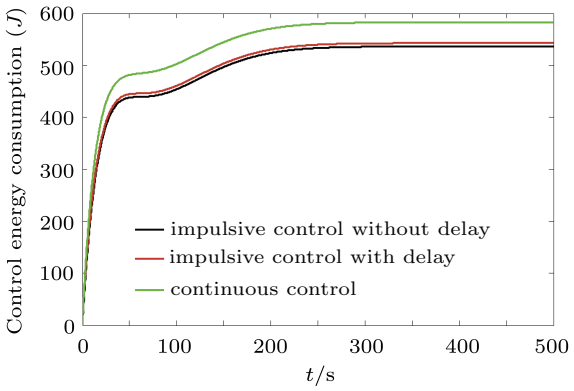


Fig. 11. (color online) Comparison of control energy consumption.

To analyze the impulsive effect on system performance, the figures of sample signal and impulsive effect in Case 1 are given below.

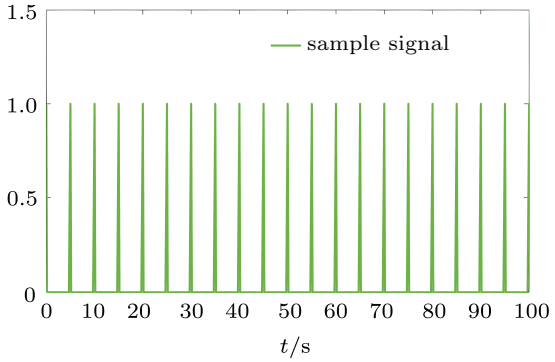


Fig. 12. (color online) Sample signal in Case 1 and Case 2.

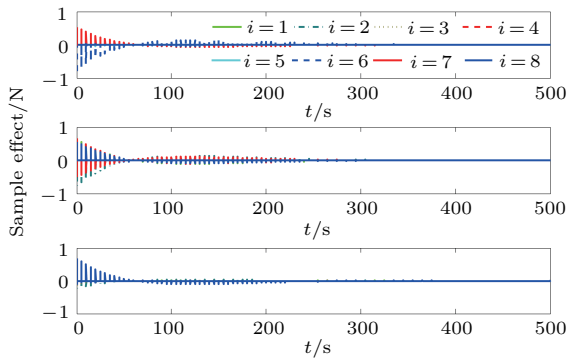


Fig. 13. (color online) Impulsive effect on the controller in Case 1.

We can see that the square formation of eight satellites is achieved in the above three simulation examples. Comparing Figs. 4, 7, and 10, we can obtain that the impulsive controller is discontinuous with some impulsive effect at the sample moments. From Fig. 11, we can conclude that the impulsive controller without communication delays has the least control energy consumption, the impulsive controller with communication delays is the second mainly because of the cost for the communication delays. The continuous controller has the most control energy consumption, which proves the effectiveness of reducing control energy consumption with the impulsive controller. Figure 12 gives the sample period moments

within 100 s. Figure 13 gives the impulsive effect in Eq. (13), which shows that the impulsive effect will converge to the origin finally.

To sum up, the simulation results in Figs. 12 and 13 illustrate the effectiveness of the proposed cooperative impulsive formation control algorithms. The comparison in Fig. 11 proves less energy consumption of impulsive formation algorithm than the continuous formation algorithm.

5. Conclusion

The paper focuses on the cooperative impulsive formation control problem for a class of networked Euler–Lagrange systems with uncertain dynamics and external disturbances under a connected directed graph. Furthermore, the communication delays among agents are considered. The main features of the proposed impulsive formation control algorithms are that the controllers are impulsive and the information interactions among agents are only at some discrete moments, which reduce the energy consumption and the communication frequency among agents. To handle the uncertain dynamics in the networked systems, neural-networks approximation and adaptive technique are used to compensate for the unknown nonlinear dynamics. With the proposed impulsive formation control algorithm, all agents converge to the desired formation. With the consideration of communication delays, the modified impulsive formation control algorithm is proposed. Finally, the simulation examples are presented to demonstrate the effectiveness of the proposed impulsive formation control laws and the performance improvements of impulsive formation control law upon the continuous formation control law. Future work will concentrate on the distributed impulsive leader-following formation problem with communication delays.

References

[1] Ren W and Beard R W 2008 *Distributed Consensus in Multi-vehicle Cooperative Control* (London: Springer-Verlag) pp. 25–40

[2] Argyriou A and Pandharipande A 2010 *IEEE Trans. Wireless Commun.* **9** 3112

[3] Bullo F, Cortes J and Martinez S 2009 *Distributed control of robotic networks* (Princeton: Princeton University Press) pp. 40–50

[4] Beard R W, Lawton J and Hadaegh F Y 2001 *IEEE Trans. Control Syst. Technol.* **9** 777

[5] Ren W and Beard R W 2005 *IEEE Trans. Autom. Control* **50** 655

[6] OlfatiSaber R, Fax A and Murray R M 2007 *Proc. IEEE* **2007** 215

[7] Ren W 2008 *IEEE Trans. Autom. Control* **53** 1503

[8] Liu Y, Min H, Wang S, Ma L and Liu Z 2014 *Journal of the Franklin Institute* **351** 3351

[9] Mei J, Ren W and Ma G F 2011 *IEEE Trans. Autom. Control* **56** 1415

[10] Mei J, Ren W, Chen J and Ma G F 2013 *Automatica* **49** 1723

[11] Chen L M, Lv Y Y, Li C J and Ma G F 2016 *Chin. Phys. B* **25** 128701

[12] Zhang W, Wang Z and Guo Y 2014 *Int. J. Syst. Sci.* **45** 145

[13] Min H, Sun F, Wang S and Li H 2011 *IET Control Theory and Appl.* **5** 145

[14] Ajorlou A, Asadi M M, Aghdam A G and Blouin S 2015 *Syst. Control Lett.* **82** 86

[15] Tian Y P and Liu C L 2008 *IEEE Trans. Autom. Control* **53** 2122

[16] Olfatisaber R and Murray R M 2004 *IEEE Trans. Autom. Control* **49** 1520

[17] Peng B D, Song Y, Sheng L, Wang P W, Hei D W, Zhao J, Li Y, Zhang M and Li K N 2016 *Acta Phys. Sin.* **65** 157801 (in Chinese)

[18] Zhou Z C and Joos B 2016 *Chin. Phys. B* **25** 088701

[19] Guan Z H, Wu Y and Feng G 2012 *IEEE Trans. Circ. Syst. I* **59** 170

[20] Guan Z H, Liu Z W, Feng G and Jian M 2012 *Automatica* **48** 1397

[21] Wang Y and Yi J 2015 *IET Control Theory Appl.* **9** 336

[22] Tang Y, Xing X, Karimi H and Kocarev L 2016 *IEEE Trans. Indus. Electron.* **63** 1299

[23] Wu X, Zhou J, Xiang L, Lin C and Zhang H 2013 *Multibody System Dyn.* **30** 37

[24] Ma C, Shi P, Zhao X and Zeng Q 2015 *IEEE Trans. Cybern.* **45** 1126

[25] Lewis F L, Zhang H, Hengster-Movric K and Das A 2009, *Cooperative Control of Multi-Agent Systems: Optimal and Adaptive Design Approaches* (Berlin: Springer) pp. 30–45

[26] Qu Z 2009 *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles* (Berlin: Springer) pp. 103–113

[27] Yang T 2001 *Impulsive Control Theory* (London: Springer-Verlag) pp. 83–97

[28] Wang L, Chai T and Zhai L 2009 *IEEE Trans. Indus. Electron.* **56** 3296

[29] Lu J, Ho D W C and Cao J 2010 *Automatica* **46** 1215

[30] Wu Q, Zhou J and Xiang L 2011 *Neurocomputing* **74** 3204

[31] Alfriend K T, Vadali S R, Gurfil P, How J and Breger L S 2009 *Spacecraft formation flying: Dynamics, control and navigation* (Oxford: Butterworth-Heinemann) pp. 104–111