Drift vortices in inhomogeneous collisional dusty magnetoplasma*

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For the sake of investigating the drift coherent vortex structure in an inhomogeneous dense dusty magnetoplasma, using the quantum hydrodynamic model a nonlinear controlling equation is deduced when the collision effect is considered. New vortex solutions of the electrostatic potential are obtained by a special transformation method, and three evolutive cases of monopolar vortex chains with spatial and temporal distribution are analyzed by representative parameters. It is found that the collision frequency, particle density, drift velocity, dust charge number, electron Fermi wavelength, quantum correction, and quantum parameter are all influencing factors of the vortex evolution. Compared to the uniform dusty system, the vortex solutions of the inhomogeneous system present richer spatial evolution and physical meaning. These results may explain corresponding vortex phenomena and support beneficial references for the dense dusty plasma atmosphere.

Keywords: drift vortices, dusty magnetoplasma, quantum hydrodynamic model

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1. Introduction

Collective dust-plasma interaction in nonuniform dusty magneto-plasmas has become a new and fascinating research area. From lunar, planetary systems, interstellar spaces and laboratory plasmas, charged dust plasmas are ubiquitous comprising neutral gas molecules, electrons, ions, and micron-sized and nanometer-sized charged dust grains. The dust can be billions of times heavier than the ions and acquire several thousands of electron charges.[1,2]

The charged dust grains cause a lot of new interesting phenomena, revise the existing plasma wave spectra,[3,4] and bring new novel eigenmodes.[5] A lot of experimental observation and theoretical investigation have achieved great development about the properties of these particles such as dust-acoustic solitons[6,7] and shocks,[6,8] dust ion-acoustic solitons,[6,9,10] and shocks,[6,11] dust-electron-acoustic shock waves,[12] dust lattice solitons,[13] and vortices with electrostatic and electromagnetic waves.[14,15] It is worth mentioning that the magnetized dusty plasmas have coherent vortices, which possibly exist as monopolar,[16] dipolar,[16,17] tripolar and chain vortex.[18] The vortices are related with nonlinear dispersive waves that possess, at least, a two-dimensional character.[5]

It is known that the quantum hydrodynamic (QHD) model is extended from the magnetohydrodynamic model in the case of nonzero magnetic field for dense plasmas with a quantum correction term generally known as the Bohm potential.[19] Now the QHD model is a very effective method to study theoretically a quantum plasma system, and has achieved a number of valuable results.[19–27] Especially in nonuniform magneto-plasmas, a number of dynamical equations and solutions for nonlinear waves are gained successfully with the QHD Model. For example, in a nonuniform dissipative quantum plasma with sheared ion flow, it is found that electrostatic monopolar, dipolar, and vortex street-type solutions can appear, and that the inclusion of the quantum statistical and Bohm potential terms significantly modifies the scale lengths of these structures.[16] The positive and negative bell-shaped solitary pulses are found to become explosive pulses depending mainly upon the angles of propagation and dust polarity in a magnetized quantum dusty plasma.[25] The linear and nonlinear propagation characteristics of the dust acoustic waves are investigated in an inhomogeneous dense dusty magneto-plasma, and the variation of drift shock waves with parameters is discussed.[26] Three quantum mechanical behaviors for electrons, ions, and dusts are studied for nonlinear quantum dust acoustic waves in nonuniform complex quantum dusty plasma.[27]

This paper aims to study drift monopolar vortices in an inhomogeneous dense charged dusty magneto-plasma comprising inertia dust particles, inertialess electrons and ions with the background of neutrals. In the next section, the QHD model is employed to derive a two-dimensional nonlinear dynamical equation when the collision between dust particles and neu-

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trals is considered. Then three groups of coherent vortex structure solutions are obtained by a special transformation method, and drift monopolar vortices are analyzed in detail with three-dimensional graphs by the typical parameters of dense astrophysics. The summary and discussion are presented in the final section.

2. Derivation of nonlinear dynamical equations

For a multi-component inhomogeneous quantum dusty magneto-plasma containing inertialess electrons, inertialess ions, and negatively charged dusts (e-i-d) with neutrals in the background and having a strong collision effect between dust particles and neutrals, the quantum hydrodynamic (QHD) model will be utilized to derive a set of nonlinear dynamical equations. Suppose that the ambient uniform magnetic field $\boldsymbol{B}_0$ is in the z direction, the density and temperature gradients are in the x direction, and the final derived nonlinear equations will be in the y–z plane. The phase velocity of the wave is ω/k, the electron and ion Fermi velocities are $v_{\rm Fe}$ and $v_{\rm Fi}$, dust velocity is $v_{\rm d}$, and they are related with $v_{\rm d} \ll \omega/k \ll v_{\rm Fi}, v_{\rm Fe}$.

According to the QHD model, the dynamical equations for electrons, ions and negatively charged dust particles are given by

$$m_{\rm e}n_{\rm e}(\partial_t + \boldsymbol{v}_{\rm e}\cdot\nabla)\boldsymbol{v}_{\rm e} = -en_{\rm e}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}_{\rm e}\times\boldsymbol{B}_0\right) - \nabla p_{\rm e} + \frac{\hbar^2 n_{\rm e}}{2m_{\rm e}}\nabla\left(\frac{\nabla^2\sqrt{n_{\rm e}}}{\sqrt{n_{\rm e}}}\right), \tag{1}$$

$$m_{\rm i}n_{\rm i}(\partial_t + \boldsymbol{v}_{\rm i}\cdot\nabla)\boldsymbol{v}_{\rm i} = en_{\rm i}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}_{\rm i}\times\boldsymbol{B}_0\right) - \nabla p_{\rm i}, \tag{2}$$

$$m_{\rm d}n_{\rm d}(\partial_t + \boldsymbol{v}_{\rm d}\cdot\nabla)\boldsymbol{v}_{\rm d} = -Z_{\rm d}en_{\rm d}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}_{\rm d}\times\boldsymbol{B}_0\right) - m_{\rm d}n_{\rm d}\nu_{\rm dn}\boldsymbol{v}_{\rm d}, \tag{3}$$

where $\nu_{\rm dn}$ is the collision frequency between dust particles and neutrals; m_l, n_l, $\boldsymbol{v}_l$ (l = e, i, d), e and $Z_{\rm d}$ are mass, density, velocity, electron charge, and dust charge number, respectively; the electrostatic field $\boldsymbol{E} = -\nabla\phi$ (ϕ is the electrostatic potential); the electron and ion Fermi pressure $p_j = \hbar^2(3\pi^2)^{2/3}n_j^{5/3}/5m_j$ (j = e, i).

For the inertialess electrons and inertialess ions, considering the boundary conditions $n_j = n_{j0}$ and $\phi = 0$ at $z \to \pm\infty$, and the Talor expansion in the parallel component of Eqs. (1) and (2), the following proportions are deirved

$$\frac{\tilde{n}_{\rm e}}{n_{\rm e0}} = \frac{3e\phi}{2k_{\rm B}T_{\rm Fe}} - \frac{3}{4}\left(\frac{e\phi}{k_{\rm B}T_{\rm Fe}}\right)^2 + \frac{9\hbar^2 e}{16m_{\rm e}k_{\rm B}^2T_{\rm Fe}^2}\nabla^2\phi,$$
$$\frac{\tilde{n}_{\rm i}}{n_{\rm i0}} = -\frac{3e\phi}{2k_{\rm B}T_{\rm Fi}} - \frac{3}{4}\left(\frac{e\phi}{k_{\rm B}T_{\rm Fi}}\right)^2. \tag{4}$$

Here $k_{\rm B}$ is the Boltzmann constant; n_{j0} is the equilibrium density, $\tilde{n}_j$ is the perturbed density, $\tilde{n}_j \ll n_{j0}$, and $n_j = n_{j0} + \tilde{n}_j$. The electron and ion Fermi temperature $T_{{\rm F}j}$ and the equilibrium density n_{j0} are related by

$$T_{{\rm F}j} = \frac{\hbar^2}{2m_jk_{\rm B}}(3\pi^2)^{2/3}n_{j0}^{2/3}.$$

From the Poisson equation and Eq. (4)

$$\nabla^2\phi = -4\pi e(n_{\rm i} - n_{\rm e} - Z_{\rm d}n_{\rm d}), \tag{5}$$

the ratio of the dust number density perturbation $\tilde{n}_{\rm d}$ to density equilibrium $n_{\rm d0}$ is given by

$$\frac{\tilde{n}_{\rm d}}{n_{\rm d0}} = -a_1\phi - a_2\phi^2 - a_3\nabla^2\phi + a_4\nabla^2\phi, \tag{6}$$

where

$$a_1 = 3e/2k_{\rm B}T_{\rm Fe}[(n_{\rm e0}/Z_{\rm d}n_{\rm d0}) + (n_{\rm i0}/Z_{\rm d}n_{\rm d0})(T_{\rm Fe}/T_{\rm Fi})],$$
$$a_2 = 3e^2/4k_{\rm B}^2T_{\rm Fe}^2[(n_{\rm i0}/Z_{\rm d}n_{\rm d0})(T_{\rm Fe}/T_{\rm Fi})^2 - (n_{\rm e0}/Z_{\rm d}n_{\rm d0})],$$
$$a_3 = 9e\hbar^2n_{\rm e0}/16m_{\rm e}k_{\rm B}^2T_{\rm Fe}^2Z_{\rm d}n_{\rm d0},$$
$$a_4 = 1/4\pi eZ_{\rm d}n_{\rm d0},$$

respectively.

From Eq. (3), the perpendicular and parallel components of dust velocity can be obtained. Substituting them and Eq. (6) into the following dust continuity equation

$$\partial_t n_{\rm d} + \nabla\cdot(n_{\rm d}\boldsymbol{v}_{\rm d}) = 0, \tag{7}$$

and making the assumption $\partial_x \ll \partial_z \ll \partial_y$, this nonlinear equation can be deduced

$$\hat{D}\left\{-\frac{3a}{2}\partial_t\phi - b\partial_t\phi^2 + (\chi\lambda_{\rm Fe}^2 + Z_{\rm d}\rho_{\rm sd}^2 - \chi H^2)\partial_t\partial_y^2\phi + Z_{\rm d}\nu_{\rm dn}\rho_{\rm sd}^2\partial_y^2\phi + \frac{3}{2}v_*\partial_y\phi - D_1\partial_y\phi^2\right\} + Z_{\rm d}c_{\rm sd}^2\partial_z^2\phi = 0, \tag{8}$$

where the operator $\hat{D}$ is defined as $\hat{D} = d_t + \nu_{\rm dn} = \partial_t + v_{\rm dz}\partial_z + \nu_{\rm dn}$; $a = \chi + p\sigma$, $b = 3e/4k_{\rm B}T_{\rm Fe}(p\sigma^2 - \chi)$, $\chi = n_{\rm e0}/Z_{\rm d}n_{\rm d0}$, $p = n_{\rm i0}/Z_{\rm d}n_{\rm d0}$, $\sigma = T_{\rm Fe}/T_{\rm Fi}$, $T_{\rm Fe}$ and $T_{\rm Fi}$ are the Fermi temperature of the electron and ion, respectively; $\lambda_{\rm Fe} = \sqrt{k_{\rm B}T_{\rm Fe}/4\pi e^2n_{\rm e0}}$ is the electron Fermi wavelength; $\rho_{\rm sd} = \sqrt{k_{\rm B}T_{\rm Fe}/m_{\rm d}\Omega_{\rm cd}^2}$ is the dust Larmor radius at the electron temperature, $\Omega_{\rm cd} = 2\pi f_{\rm cd} = Z_{\rm d}eB_0/cm_{\rm d}$ is the dust cyclotron frequency, and $\partial_t \ll \Omega_{\rm cd}$ is used for the low frequency waves; $H = \sqrt{9\hbar^2/16m_{\rm e}k_{\rm B}T_{\rm Fe}}$ is the quantum parameter; $c_{\rm sd} = \sqrt{k_{\rm B}T_{\rm Fe}/m_{\rm d}}$ is the dust acoustic speed; $v_* = (-2ck_{\rm B}T_{\rm Fe}/3eB_0)k_{\rm n}$ is the drift velocity, $k_{nj} = |d_x\ln n_0|$ is the dense gradient; $D_1 = D_2(k_{\rm n} - k_{T_{\rm Fe}})$, $D_2 = 3c/4B_0$; $k_{T_{{\rm F}j}} =$

$|d_x \ln T_{Fe}|$ is the temperature gradient.

It is worth mentioning that a similar nonuniform plasma model[26] was obtained with the error coefficient b, which is corrected here, and the electrostatic drift shock waves were studied with the Korteweg–de-Vries–Burgers (KdVB) and Kadomtsev–Petviashvili–Burgers (KPB) equations. Besides, a similar uniform dense model was studied for the vortex street by the reductive perturbation method.[23]

In this paper, for the nonuniform dense dusty magnetoplasma we study the drift coherent dust vortices taking into account a strong collision effect between dust particles and neutrals. When $\nu_{dn} \gg d_t$ in the operation, and the terms $\partial_z^2\phi$ and $\partial_t\phi^2$ are kept, equation (8) is simplified as a two-dimensional nonlinear dynamic equation

$$-\frac{3a}{2}\partial_t\phi - b\partial_t\phi^2 + (\chi\lambda_{Fe}^2 + Z_d\rho_{sd}^2 - \chi H^2)\partial_t\partial_y^2\phi + Z_d\nu_{dn}\rho_{sd}^2\partial_y^2\phi + \frac{3}{2}v_*\partial_y\phi - D_1\partial_y\phi^2 + \frac{Z_d c_{sd}^2}{\nu_{dn}}\partial_z^2\phi = 0. \tag{9}$$

For the convenience of calculation, the following rescaling is defined as

$$\Phi = \frac{e\phi}{k_B T_{Fe}},\ H_e = \frac{H}{\rho_{sd}},\ \Lambda = \frac{\lambda_{Fe}}{\rho_{sd}},\ \nu = \frac{\nu_{dn}}{\Omega_{cd}},$$
$$\bar{y} = \frac{y}{\rho_{sd}},\ \bar{z} = \frac{z}{\rho_{sd}},\ \bar{t} = \Omega_{cd}t,\ u = \frac{v_*}{c_{sd}}. \tag{10}$$

Then equation (9) is normalized as

$$\partial_t\Phi + g_1\partial_t\Phi^2 + g_2\partial_t\partial_y^2\Phi - g_3\partial_y^2\Phi - g_4\partial_z^2\Phi + g_5\partial_y\Phi - g_6\partial_y\Phi^2 = 0, \tag{11}$$

where the bars have been dropped, and the dimensionless coefficients are

$$g_1 = \frac{1}{2a}(p\sigma^2 - \chi),\ g_2 = \frac{2}{3a}(\chi H_e^2 - \chi\Lambda^2 - Z_d),$$
$$g_3 = \frac{2Z_d}{3a}\nu,\ g_4 = \frac{2Z_d}{3a\nu},\ g_5 = -\frac{u}{a},$$
$$g_6 = \frac{3u}{4ak_{nd}}[\chi(k_{ne} - k_{T_{Fe}}) + p\sigma(k_{ni} - k_{T_{Fi}})]. \tag{12}$$

Under the dense astrophysical circumstances,[28–31] the following physical quantities can be taken as

$$n_{i0} = 1.5\times10^{21}\ \mathrm{cm}^{-3},\quad B_0 = 10^8\ \mathrm{Gs},$$
$$z_d = 10^3,\quad m_i = 3.35\times10^{-24}\ \mathrm{g},\ m_d = 10^{-15}\ \mathrm{g}, \tag{13}$$

the unit $1\ \mathrm{Gs} = 10^{-4}$ T. Besides, taking $n_{e0} = 6\times10^{18}\ \mathrm{cm}^{-3}$, $n_{d0} = 1.494\times10^{18}\ \mathrm{cm}^{-3}$, $\nu_{dn} = 10^6$ Hz, and $v_* = -c_{sd}$, the corresponding normalized parameters are

$$\Lambda = 1.21\times10^{-2},\ H_e = 6.89\times10^{-2},\ \nu = 0.624,$$
$$u = -1, \tag{14}$$
$$g_1 = 46.279,\ g_2 = -7.173,\ g_3 = 4.475,$$
$$g_4 = 11.5,\ g_5 = 0.011,\ g_6 = -0.25. \tag{15}$$

3. Vortex solutions and evolutive plots

For the sake of obtaining a coherent vortex solution of Eq. (12), we employ the following special transformation method. The solution of Eq. (11) is supposed to be

$$\Phi = f_1F(\xi) + f_2\frac{\partial F(\xi)}{\partial\xi}\frac{\partial\xi}{\partial z}, \tag{16}$$

where

$$\frac{\partial^2F(\xi)}{\partial\xi^2} = -\alpha^2F(\xi),$$
$$\frac{\partial^3F(\xi)}{\partial\xi^3} = -\alpha^2\frac{\partial F(\xi)}{\partial\xi},$$
$$\frac{\partial^4F(\xi)}{\partial\xi^4} = \alpha^4F(\xi). \tag{17}$$

The solutions of Eqs. (17) read as

$$F(\xi) = \eta_1\sin(\alpha\xi) + \eta_2\cos(\alpha\xi),$$
$$\frac{\partial F(\xi)}{\partial\xi} = \alpha[\eta_1\cos(\alpha\xi) - \eta_2\sin(\alpha\xi)], \tag{18}$$

where f_1, f_2, and ξ are the undetermined functions of (y,z,t); α is an undetermined constant; η_1 and η_2 are arbitrary constants.

After Substituting Eqs. (16) and (17) into Eq. (11) and setting the coefficients of $F(\xi)$ and $\partial F(\xi)/\partial\xi$ terms as zero, we can obtain an over-determined set of equations for the unknowns. Then there are the following solutions from the determinant equations

$$f_1 = c_3\exp\left(\frac{c_4zg_6 + c_5tg_6 + c_5g_1y}{g_6}\right),$$
$$f_2 = c_6\exp\left(\frac{c_4zg_6 + c_5tg_6 + c_5g_1y}{g_6}\right),$$
$$\xi = \frac{c_1zg_6 + c_2tg_6 + c_2g_1y}{g_6}, \tag{19}$$

and two equations for limiting the unknown coefficients

$$\left[\left(c_6{c_5}^3 + 3c_3{c_5}^2 - 3{c_2}^2c_6c_5\alpha^2 - {c_2}^2c_3\alpha^2\right)g_2 + \left(c_6\alpha^2{c_2}^2 - c_6{c_5}^2 - 2c_3c_5\right)g_3\right]{c_2}{g_1}^2$$
$$+\left(c_6\alpha^2{c_1}^2c_2 - 2c_3c_4c_1 - c_6{c_4}^2c_2\right){g_6}^2g_4 + c_2g_6(c_3 + c_6c_5)(g_6 + g_5g_1) = 0, \tag{20}$$

$$\left[\left(-3c_3c_5\alpha^2{c_2}^2+c_6\alpha^4{c_2}^4-3c_6{c_5}^2\alpha^2{c_2}^2+c_3{c_5}^3\right)g_2\right]{g_1}^2+g_6\left(c_3c_5-c_6\alpha^2{c_2}^2\right)\left(g_6+g_5g_1\right)$$
$$+\left(2c_6c_4\alpha^2c_1c_2+c_3\alpha^2{c_1}^2-c_3{c_4}^2\right){g_6}^2g_4+\left(-c_3{c_5}^2+2{c_2}^2c_6c_5\alpha^2+{c_2}^2c_3\alpha^2\right)g_3{g_1}^2=0, \quad (21)$$

where c_3 and c_6 are arbitrary constants. According to the above results, the constants c_1, c_2, c_4, c_5, and α will be determined. Here, three cases are discussed below.

3.1. Vortex chain with periodic distribution along z direction

Taking the constants

$$c_2=0,\ \ c_4=0, \quad (22)$$

the solutions in Eq. (19) become

$$\xi=c_1z,\ \ f_1=c_3\exp\left(\frac{c_5\left(tg_6+g_1y\right)}{g_6}\right),$$

$$f_2=c_6\exp\left(\frac{c_5\left(tg_6+g_1y\right)}{g_6}\right). \quad (23)$$

Substituting Eq. (22) into Eqs. (20) and (21) results in

$$\alpha=\pm\frac{\sqrt{-g_4c_5\left({c_5}^2g_2{g_1}^2-c_5g_3{g_1}^2+g_6g_5g_1+{g_6}^2\right)}}{g_6c_1g_4}, \quad (24)$$

where c_1, c_3, c_6 are arbitrary constants, c_5 is a constant limiting by

$$-g_4c_5\left({c_5}^2g_2{g_1}^2-c_5g_3{g_1}^2+g_6g_5g_1+{g_6}^2\right)\geq 0. \quad (25)$$

Substituting the above solutions into Eq. (16) results in the first group of vortex solution for Eq. (11)

$$\Phi_1=c_3\exp\left[\frac{c_5\left(tg_6+g_1y\right)}{g_6}\right]\times\left[\eta_1\sin\left(\frac{\sqrt{-g_4c_5\left({c_5}^2g_2{g_1}^2-c_5g_3{g_1}^2+g_6g_5g_1+{g_6}^2\right)}z}{g_6g_4}\right)\right.$$
$$\left.+\eta_2\cos\left(\frac{\sqrt{-g_4c_5\left({c_5}^2g_2{g_1}^2-c_5g_3{g_1}^2+g_6g_5g_1+{g_6}^2\right)}z}{g_6g_4}\right)\right]. \quad (26)$$

From Eq. (26), the coefficient of y variable directly relies on two nonlinear coefficients g_1 and g_6, in which g_6 is related with the drift velocity. The coefficient of z matters in all six coefficients of Eq. (11), which include the particle density, collision frequency, drift velocity, dust charge number, electron Fermi wavelength, quantum correction, and quantum parameter.

In order to structure a stable vortex solution, assume $t+(g_1/g_6)y=Y$, and $Y\leq 0$ when $c_5>0$, otherwise $Y>0$ when $c_5<0$. After choosing typical parameters in Eq. (15) and substituting them into Eq. (25), the limitation is $c_5>0$ or $-6.7\times10^{-6}>c_5>-0.62$. Then taking c_5 as 0.05 ($Y\leq 0$), and -0.054 ($Y>0$), respectively, the evolution of the electrostatic potential Φ_1 with Y and z is displayed in Fig. 1.

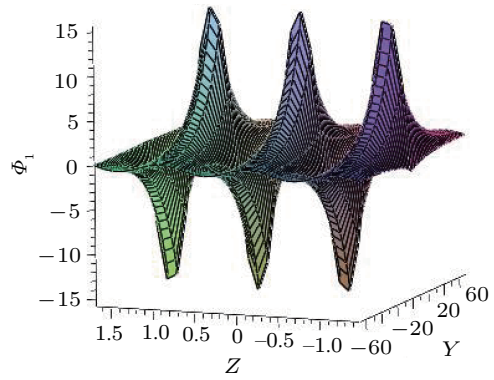


Fig. 1. (color online) Vortex distribution of the electrostatic potential Φ_1 of Eq. (26) with $c_3=3$, $\eta_1=4$, $\eta_2=3$, $g_1=46.279$, $g_2=-7.173$, $g_3=4.475$, $g_4=11.5$, $g_5=0.011$, $g_6=-0.25$, $c_5=0.05$ ($Y\leq 0$), and $c_5=-0.054$ ($Y>0$).

It is shown that the potential Φ_1 is a periodic change along the z direction, but in the Y direction it decays exponentially to zero, and finally presents a stable monopoalr vortex structure.[23] So a stable periodic vortex chain along the direction of the magnetic field is formed in this inhomogeneous dusty magneto-plasma.

3.2. Vortex chain with periodic distribution along y direction

When the constants

$$c_1=0,\ \ c_5=0, \quad (27)$$

the solutions in Eq. (19) are

$$\xi=\frac{c_2\left(tg_6+g_1y\right)}{g_6},\ \ f_1=c_3\,\mathrm{e}^{c_4z},\ \ f_2=c_6\,\mathrm{e}^{c_4z}. \quad (28)$$

Substituting Eq. (27) into Eqs. (20) and (21) results in

$$\alpha=\pm\frac{\sqrt{g_2g_6\left(g_6+g_5g_1\right)}}{g_1c_2g_2}, \quad (29)$$

$$c_4=\pm\frac{\sqrt{g_6g_4g_2g_3\left(g_6+g_5g_1\right)}}{g_6g_4g_2}, \quad (30)$$

where c_2, c_3, c_6 are arbitrary constants, and the parameters are limited by

$$g_2g_3g_4g_6\left(g_6+g_5g_1\right)\geq 0,\ \ g_2g_6\left(g_6+g_5g_1\right)\geq 0. \quad (31)$$

Substituting the above solutions into Eq. (16) results in the second group of vortex solutions for Eq. (11)

$$\Phi_2 = (c_3\eta_2 + c_2c_6\eta_1\alpha)\,\mathrm{e}^{c_4 z}\cos\left[\frac{\alpha c_2\,(tg_6 + g_1 y)}{g_6}\right] + (c_3\eta_1 - c_2c_6\eta_2\alpha)\,\mathrm{e}^{c_4 z}\sin\left[\frac{\alpha c_2\,(tg_6 + g_1 y)}{g_6}\right]. \quad (32)$$

In Eq. (32), the coefficient of the z variable is related to all dimensionless coefficients of Eq. (11), and the coefficient of y is related to g_1, g_2, g_5, and g_6 that are different from those in Eq. (26). Besides, the wave circle frequency

$$\omega_{\mathrm{w}} = |c_2\alpha| = |\sqrt{g_2 g_6\,(g_6 + g_5 g_1)}/g_1 g_2|,$$

and the wave velocity $v_{\mathrm{w}} = g_6/g_1$. They are relevant only with the dimensionless coefficients of Eq. (11), which show the natural character of wave in this plasma system.

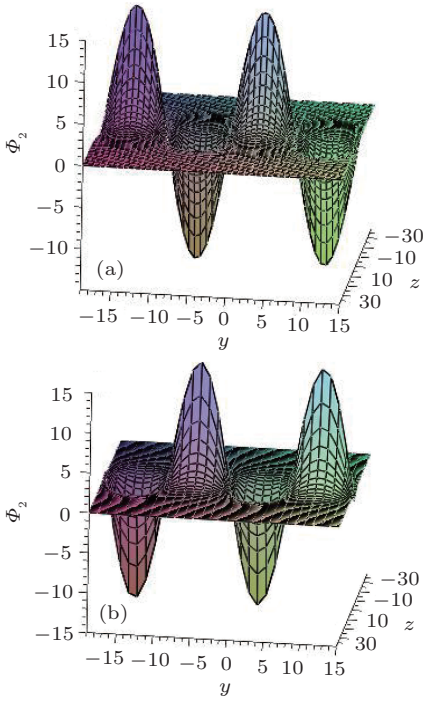


Fig. 2. (color online) Vortex distribution of the electrostatic potential Φ_2 of Eq. (32) with $c_3 = 3$, $c_6 = 2$, $\eta_1 = 4$, $\eta_2 = 3$, $g_1 = 46.279$, $g_2 = -7.173$, $g_3 = 4.475$, $g_4 = 11.5$, $g_5 = 0.011$, $g_6 = -0.25$. Panel (a) is for $t = 1$ and panel (b) for $t = 1500$.

Under the chosen parameters in Eq. (15), the vortex potential Φ_2 evolves in the y–z plane as shown in Fig. 2. It shows that along the y direction the drift vortices move periodically with time; in the z direction of the magnetic field, the spatial distribution presents the attenuation trend, and the local area of vortices is largely decided by

$$1/c_4 = g_6 g_4 g_2/\sqrt{g_6 g_4 g_2 g_3 (g_6 + g_5 g_1)},$$

which matters for six dimensionless coefficients.

3.3. Vortex chain with periodic distribution along oblique angle to z direction

If the constants are chosen as

$$c_5 = 0, \quad (33)$$

then

$$\xi = \frac{c_1 z g_6 + c_2 t g_6 + c_2 g_1 y}{g_6},$$
$$f_1 = c_3\,\mathrm{e}^{c_4 z},\quad f_2 = c_6\,\mathrm{e}^{c_4 z}. \quad (34)$$

Substituting Eq. (33) into Eqs. (20) and (21) results in the following expressions

$$c_1 = \frac{c_2\left(c_2{}^2 g_2 \alpha^2 g_1{}^2 - g_5 g_1 g_6 - g_6{}^2\right)}{\sqrt{-2 g_4 c_2 \alpha\left(-g_3 g_1{}^2 \alpha c_2 + \sqrt{A}\right) g_6}},$$
$$c_4 = -\frac{\sqrt{-2 g_4 c_2 \alpha\left(-g_3 g_1{}^2 \alpha c_2 + \sqrt{A}\right)}}{2 g_4 g_6},$$
$$A = \left(g_2{}^2 c_2{}^2 \alpha^2 + g_3{}^2\right) g_1{}^4 c_2{}^2 \alpha^2 - 2\alpha^2 c_2{}^2 g_2 g_1{}^2 g_6\,(g_6 + g_5 g_1) + g_1 g_5 g_6{}^2\,(g_5 g_1 + 2 g_6) + g_6{}^4, \quad (35)$$

where c_3, c_6 are arbitrary constants, c_2 and α are the constants that must ensure the values greater than zero in the above two radical signs.

Substituting the above solutions into Eq. (16) results in the third group of vortex solutions for Eq. (11)

$$\Phi_3 = (c_3\eta_2 + c_6c_2\eta_1\alpha)\,\mathrm{e}^{c_4 z}\cos\left[\frac{\alpha\,(c_1 z g_6 + c_2 t g_6 + c_2 g_1 y)}{g_6}\right] + (c_3\eta_1 - c_6c_2\eta_2\alpha)\,\mathrm{e}^{c_4 z}\sin\left[\frac{\alpha\,(c_1 z g_6 + c_2 t g_6 + c_2 g_1 y)}{g_6}\right]. \quad (36)$$

By employing the parameters chosen in Fig. 2, the vortex potential Φ_3 evolves with time in the y–z plane as shown in Fig. 3. The wave circle frequency ω_{w} is the same as that for Φ_2. The spatial distribution presents an attenuation trend along the z direction of the magnetic field. The drift vortices move with time at $\theta = \arctan(c_2 g_1/c_1 g_6)$ to the z axis, which include the constants c_2 and α. So this case is more representative and may illustrate general evolutive phenomena.

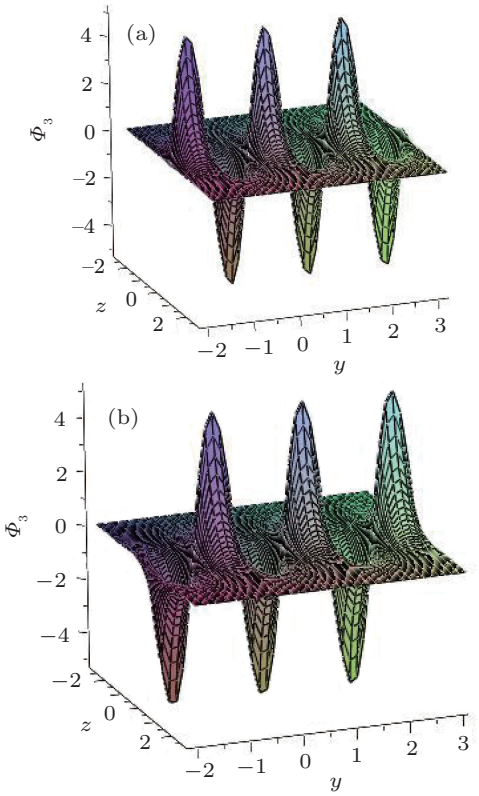


Fig. 3. (color online) Vortex distribution of the electrostatic potential Φ_3 of Eq. (36) with $c_2 = -0.2$, $c_3 = 1$, $c_6 = 2$, $\alpha = 0.1$, $\eta_1 = 4$, $\eta_2 = 3$, $g_1 = 46.279$, $g_2 = -7.173$, $g_3 = 4.475$, $g_4 = 11.5$, $g_5 = 0.011$, $g_6 = -0.25$. Panel (a) for $t = 1$ and panel (b) for $t = 136$.

4. Summary and discussion

In order to study the drift coherent vortex structure for an inhomogeneous dense dusty magneto-plasma, using the QHD model a two-dimensional nonlinear equation is derived when the collision between dust particles and neutrals is dominant. Then employing the special transformation method a new vortex solution of the electrostatic potential is obtained successfully. Three evolutive cases of the vortex chain are analyzed in detail with time in the y–z plane. In the vortex expression of Eqs. (26), (32), and (36), the coefficients of y and z variables are related to all six coefficients of Eq. (11). Furthermore, the wave circle frequency and the wave velocity in Eq. (32) only matter to the dimensionless coefficients of Eq. (11) and present the natural character of the system. So the particle density, collision frequency, drift velocity, dust charge number, electron Fermi wavelength, quantum correction, and quantum parameter are all influencing factors to the vortex evolution.

Compared with the uniform dusty system,[23] the similarity is that the solutions are all constituted by the exponential function and trigonometric function, which form the monopolar vortex chains. The difference is the analytic expressions because the constructing ways of solutions are different. The vortex chain of the uniform system is periodic along the z direction of the magnetic field and is decaying exponentially in the y direction, which presents one case with space. But in the inhomogeneous system, three groups of vortex solutions present richer physical spatial forms. They may periodically move along the y direction, the z direction or a certain oblique angle to the z direction, and show attenuation in another direction.

These results may explain corresponding vortex phenomena and support beneficial references for the inhomogeneous dense dusty magneto-plasmas, which manifest coherent nonlinear local structures in dense astrophysical circumstances like the atmosphere of planetary and interstellar spaces where collision and drift phenomena are remarkable.

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