Enhancement of multiple four-wave mixing via cascaded fibers with discrete dispersion decreasing*

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Cascaded fiber geometry with the dispersion of each fiber decreasing is proposed to enhance the multiple four-wave mixing (FWM) generation. The first fiber with relatively large dispersion initiates and accelerates the expansion of multiple FWM, and the second fiber with small dispersion would allow the phase-matching process (thus the spectrum broadening) to keep going. Numerical and experimental results show that with this geometry not only multiple FWM expansion can be accelerated, but also the efficiency of multiple FWM products can be effectively improved with shorter fibers.

Keywords: four-wave mixing, optical fiber, cascaded process

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1. Introduction

Multiple four-wave-mixing (FWM), also named cascaded FWM, in optical fibers has been demonstrated to be a promising way to achieve boradband optical frequency comb (OFC).[1–5] Compared with the cavity-defined approaches, such as mode-locked fiber lasers[6,7] and microresonator,[8] OFC based on multiple FWM is convenient to easily tune the frequency spacing, and is free from frequency chirps and pulse jitter inherently.[3] It has been proposed to serve as high precision frequency markers, calibration of astrophysical spectrometers, broadband spectroscopy and metrology.[2,9] Multiple FWM can also find its applications in high repetetion-rate, short-pulse source,[10] and wavelength division multiplexing systems.[11,12]

Some techniques have been proposed to enhance the multiple FWM, e.g., using triple pumps with unequally spaced frequencies,[13] adding an optical feedback into the nonlinear fiber.[14] Discrete dispersion-increasing fibers were cascaded to extend the multiple FWM spectrum, and the efficiency of multiple FWM products at shorter wavelengths has been effectively improved.[1] In addition, multistage mixers with synthesized dispersion were demonstrated to generate wideband OFC.[3]

As is well known, FWM in long fiber will degrade the performance of FWM generation. In this case, fiber random birefringence and undesirable variation of the zero dispersion wavelengths (ZDWs) along the fiber will reduce the efficiency of multiple FWM.[15] Furthermore, sophisticated technologies have to be adopted to counteract the low-threshold stimulated Brillouin scattering (SBS)[16] in the long fibers. So short fibers with high nonlinearity, such as highly nonlinear fibers (HNLFs)[17] and photonic crystal fibers,[1] are ideal to generate broadband multiple FWM products.

Here, we propose a cascaded-fiber geometry where the dispersion of each fiber decreases on wave propagation. The first fiber with relatively large dispersion initiates and accelerates the expansion of multiple FWM, and the second fiber with small dispersion would allow the phase-matching process to keep going, thus broadening the output spectrum. The proposed scheme is demonstrated numerically and experimentally, which shows that with this geometry the multiple FWM expansion can be accelerated, and the efficiency of multiple FWM products can be effectively improved with shorter fibers.

2. Simulations

To demonstrate the proposed cascaded fiber geometry with discrete dispersion-decreasing fibers, numerical simulations of multiple FWM in two fibers with the same ZDW but different dispersion slopes are carried out first. FWM in optical fibers is conventionally modeled with coupled amplitude equations.[18] However, for multiple FWM generation with hundreds of FWM products to be considered, the nonlinear Schrödinger equation should be used[1,18]

$$\frac{\partial E(z,t)}{\partial z} = \sum_{k\geq 2}^{4} \frac{\mathrm{i}^{k+1}}{k!}\beta_k \frac{\partial^k E}{\partial t^k} + \mathrm{i}\gamma |E(z,t)|^2 E(z,t), \qquad (1)$$

where $E(z,t)$ is the electric field envelope, β_k is the k-th order dispersion, and γ is the Kerr nonlinearity coefficient of the fiber. As the used fiber is short here, linear attenuation is neglected. Split-step-Fourier-method is used to

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solve Eq. (1),[1,12,14] where up to 4-th order dispersions are taken into account. Even though simulations with different parameters have been done, a detailed example is discussed here to show the proposed scheme. The ZDWs of both fibers are both 1550 nm, and the dispersion slopes are $S_0 = 0.075$ (ps/nm^2)/km and 0.00075 (ps/nm^2)/km, respectively, by which the 2nd and 3rd order dispersions, β_2 and β_3, are determined. The Kerr nonlinearity coefficient and the 4th order dispersion of both fibers are $\gamma = 10$ W^{-1}·km^{-1} and $\beta_4 = 10^{-4}$ ps^4/km, respectively.

Two continuous-wave (CW) seeds at 1555 nm and 1563 nm with the same power of 10 W are injected into the fiber. In order to model the parametric amplification of quantum noise, a white-Gaussian noise,[1,14] with optical signal-to-noise ratio (SNR) of 120 dB at 12.5 GHz, is added to the input seeds. Even though the noise can be set to be zero in the numerical simulations, the bandwidth of multiple FWM will be limited by the phase mismatch. The detailed theory of FWM can be found in Ref. [1] and Chapter 10 of Ref. [18].

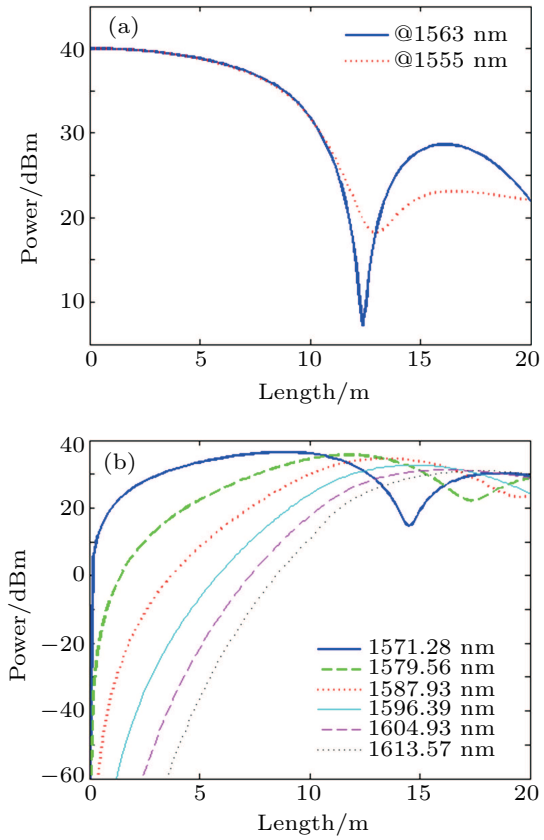


Fig. 1. (color online) Power evolutions along the fiber for (a) pumps, and (b) generated multiple FWM products at the first six idler wavelengths.

Two sections of 10 m fibers with $S_0 = 0.075$ (ps/nm^2)/km and 0.00075 (ps/nm^2)/km are cascaded successively. The power dynamical evolutions along the cascaded fibers are shown in Fig. 1. It can be seen that with the pump power decreasing along the fiber, higher order sidebands are generated successively. But after propagation of 12.39 m, the power reverts back to the pump and quasi-periodical evolutions with damping [Fig. 1(a)] begin to exhibit.[19] The power evolutions of the first six idler wavelengths are also periodic [Fig. 1(b)]. Figure 1(a) shows that the powers of the two pumps evolve differently. And it is found that the power dynamical evolutions of signal and idler with the same order are different[19] (data not shown).

To show the accelerated expansion of multiple FWM products in the cascaded fibers with discrete dispersion decreasing, spectrum evolutions in different fiber geometries are shown in Fig. 2. From Fig. 2(a), it can be seen that the comb expands quickly in the first 10-m fiber with large dispersion slope $S_0 = 0.075$ (ps/nm^2)/km. When the generated comb propagates in the following 10-m fiber with small dispersion slope $S_0 = 0.00075$ (ps/nm^2)/km, the comb expands further apparently till it arrives at $\sim$ 12 m. This is consistent with Fig. 1 on the power flows among FWM products. Further propagation almost brings in power flows among the generated products.

However, if the two fibers are interchanged, i.e., fibers are cascaded with dispersion increasing, the final output spectrum is not so broad, which is shown in Fig. 2(b). The comb in Fig. 2(b) is not so wide as that in Fig. 2(a) at 10 m (the interface of the two fibers), which shows that the comb expands slower in the fiber with smaller dispersion. Once it propagates in the second section with larger dispersion slope, the comb expands quickly till it arrives at $\sim$ 16 m. Figures 2(c) and 2(d) also show the spectrum evolutions in 20 m single-stage fiber with $S_0 = 0.075$ (ps/nm^2)/km and $S_0 = 0.00075$ (ps/nm^2)/km, respectively, and the final numbers of FWM products are less than that of Fig. 2(a).

As seen above, the comb expands quicker in the fibers with larger dispersion. To achieve a maximum parametric gain, the effective phase mismatch $\kappa = \Delta k + \gamma(P_1 + P_2)$ should be minimized,[18] where Δk is the wave-vector mismatch and P_i $(i = 1, 2)$ is the incident pump power. For countering the nonlinear phase $\gamma(P_1 + P_2)$, phase mismatch from dispersion (negative value) should be enough. So greater dispersion can accelerate comb expansion with better phase-match at given lengths. Similarly, increasing the spacing between comb components should have the same effect.[11]

Once broad FWM products have been achieved in fibers with large dispersion, further propagation in fibers with small dispersion will drive the energy flow to outer sidebands continuously. As shown in Fig. 2, cascading the fibers with discrete dispersion-decreasing can generate broadband comb with accelerated expansion.

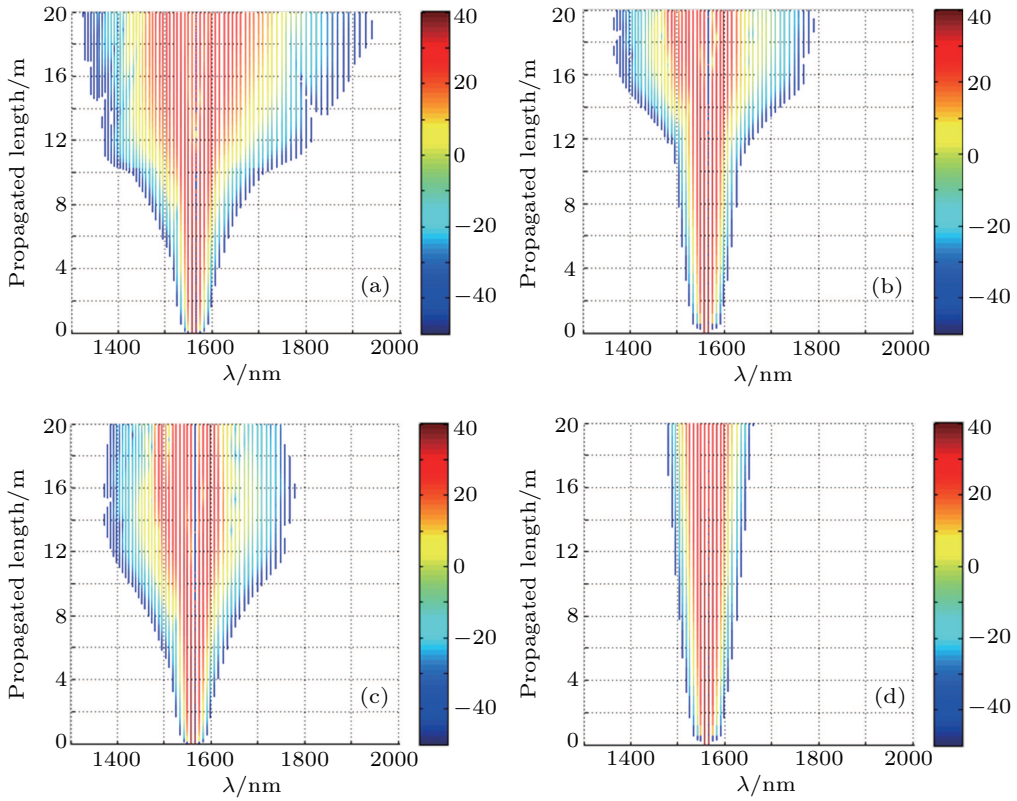


Fig. 2. (color online) Spectrum evolutions along the fiber for the multiple FWM. (a) Cascaded dispersion-decreased fibers with 10-m-long large-S_0 fiber and 10-m-long, small-S_0 fiber successively, (b) cascaded dispersion-increased fibers with 10-m-long small-S_0 fiber and 10-m-long large-S_0 fiber successively, (c) 20-m-long large-S_0 fiber, (d) 20-m-long small-S_0 fiber. Large $S_0 = 0.075$ (ps/nm^2)/km, small $S_0 = 0.00075$ (ps/nm^2)/km. Color bar shows the spectral power in unit dBm.

As mentioned above, appropriate lengths of the cascaded fibers are critical for generating the comb efficiently. Once power reversion happens between generated products, the comb generation becomes complex. In Ref. [1], the number of FWM products with acceptable SNR is found to drop after the optimal length has been reached, which can be explained with the spectrum dynamics in Fig. 2(c). With further propagation, the parametric noise will deteriorate the comb generation, which may result in Fig. 8 of Ref. [3]. On the other hand, if the comb is not expanded sufficiently in the leading fiber with large dispersion, i.e., the fiber is not long enough, the broadest comb products may not be achieved with the proposed scheme.

3. Experiments

To further verify the proposed scheme, preliminary experiments of multiple FWM are carried out with the setup shown in Fig. 3. Due to the limitations of maximum pump power and HNLFs available in our laboratory, pumps with relatively low power and HNLFs with relatively long length are used. Two CWs at 1555 nm and 1556.3 nm as dual-pump are phase-modulated (by setting the "coherent control" inside the lasers) to suppress the SBS in HNLFs. In order to obtain pumps with higher peak powers, an electro-optic modulator is used to modulate the amplitudes of CWs, and then pulses with a duty cycle of 0.1 and repetition of 2 MHz are obtained. Then they are amplified by an EDFA and the total output average power is 29 dBm. Two HNLFs with the same dispersion slope ($S_0 = 0.017$ (ps/nm^2)/km) and Kerr nonlinearity coefficient $\gamma = 10.5$ W^{-1}·km^{-1} but different ZDWs (fiber A at 1451 nm and fiber B at 1545 nm) are employed. Firstly, numerical simulations of multiple FWM generation in fiber A are carried out with these parameters as shown in Fig. 4, where the number of multiple FWM products with SNR higher than 20 dB is calculated to describe the expansion of multiple FWM. As can be seen in Fig. 4, multiple FWM generation is initiated with a fast pace at the beginning of the fiber. While after a certain length (here it is 94 m, which is determined by the parameters used), the number of multiple FWM decreases. Then according to the simulations, an appropriate fiber length is adopted in the experiments. Four conditions are compared with each other to study the scheme, by employing 94-m-long fiber A plus 106-m-long fiber B, 106-m-long fiber B plus 94-m-long fiber A, 200-m-long fiber A, and 200-m-long fiber B, respectively. The output spectra measured under these cases are shown in Fig. 5. For a more convenient comparison, the number of multiple FWM products with SNR higher than 20 dB ($n_{20\ \mathrm{dB}}$), the bandwidth (calculated by components with SNR higher than 15 dB), and the conversion efficiency (total power of spectrum with bandwidth mentioned above/total pump power) of

the four cases are extracted in Table 1.

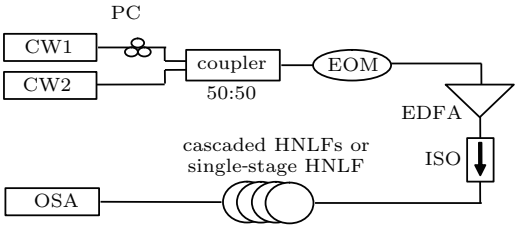


Fig. 3. Experimental setup of multiple FWM. CW: continues wave, PC: polarization controller, EOM: electro-optic modulator, EDFA: erbium-doped fiber amplifier, ISO: isolator, HNLF: highly nonlinear fiber, OSA: optical spectrum analyzer.

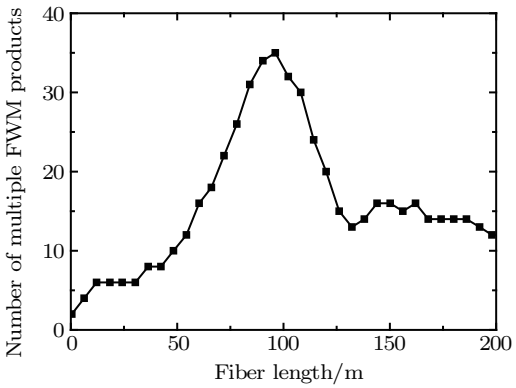


Fig. 4. Simulation of the multiple FWM generation along fiber A, with the same parameters of inputs as those in the experiments.

The spectrum of multiple FWM expands faster in fiber with larger dispersion, but the power reverts back from sidebands to pumps after a certain fiber length, which leads to the reduction of conversion efficiency. While parametric gain bandwidth is broader in fibers with smaller dispersion,[17] which is consistent with the result of a comparison between Figs. 5(c) and 5(d) with single-stage fiber. As shown in Fig. 5(a) with 94-m-long fiber A and 106-m-long fiber B, the spectrum of multiple FWM is broader than that under non-cascaded conditions, and the conversion efficiency of the process is also increased. In Fig. 5(c) with 200-m-long fiber A, the output pump power is larger than that in Fig. 5(a), while the output bandwidth is narrower, which indicates that the following fiber with small dispersion makes the energy flow from pumps to the sidebands. However, if the two fibers in Fig. 5(a) are interchanged, i.e., the fibers are cascaded with dispersion increasing, the final output spectrum is not so broad, which are shown in Fig. 5(b), which emphasizes the appropriate design of fiber dispersion.

Table 1. Numbers of multiple FWM products with SNR higher than 20 dB, bandwidth and conversion efficiency in the four cases in Fig. 5.

Conditions	Fig. 5(a)	Fig. 5(b)	Fig. 5(c)	Fig. 5(d)
$n_{20\ \mathrm{dB}}$	52	15	19	27
Bandwidth/nm	92	29	37	58
Conversion efficiency	26.0%	9.3%	12.4%	14.8%

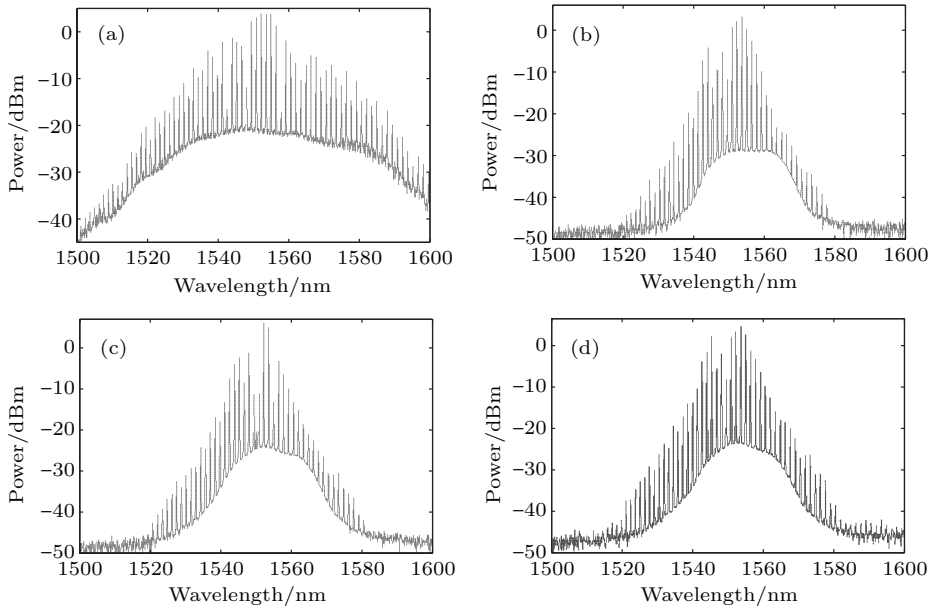


Fig. 5. Output spectra of multiple FWM generated in (a) 94-m-long fiber A plus 106-m-long fiber B, (b) 106-m-long fiber B plus 94-m-long fiber A, (c) 200-m-long fiber A, and (d) 200-m-long fiber B.

Therefore, in order to enhance the efficiency of multiple FWM, broaden the bandwidth and increase the power of the components, the principle of dipersion decreasing should be maintained. To optimize the fiber geometry of multiple FWM, the appropriate lengths of fiber under different dispersions and pumping powers should be estimated and employed to make sure the pump power keeps flowing to the sidebands.

4. Conclusions

Accelerating expansion of multiple FWM products is proposed and demonstrated in a cascaded fiber geometry with discrete dispersion-decreasing fibers. Simulations and experi-

ments demonstrate that this geometry can accelerate the multiple FWM expansion, thereby effectively improving the efficiency of multiple FWM products with shorter fibers.

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