Gravitational quasi-normal modes of static R^2 Anti-de Sitter black holes*

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In this paper, we study the gravitational quasi-normal modes (QNMs) for a static R^2 black hole (BH) in Anti-de Sitter (AdS) spacetime. The corresponding master equation of odd parity is derived and the QNMs are evaluated by the Horowitz and Hubeny method. Meanwhile the stability of such BH is also discussed through the temporal evolution of the perturbation field. Here we mainly consider the coefficient λ, which is related to the radius of AdS black hole, on the QNMs of the R^2 AdS BH. The results show that the $\mathrm{Re}(\omega)$ and $|\mathrm{Im}(\omega)|$ of the QNMs increase together as $|\lambda|$ increases for a given angular momentum number l. That indicates with a larger value of $|\lambda|$ the corresponding R^2 AdS BH returns to stable much more quickly. The dynamic evolution of the perturbation field is consistent with the results derived by the Horowitz and Hubeny method. Since in the conformal field theory the QNMs can reflect its approach to equilibrium, so our related results could be referential to studies of the AdS/CFT conjecture. The relationship between λ and the properties of the static R^2 BH might be helpful for the development of R^2 gravitational theory.

Keywords: static R^2 AdS black hole, gravitational perturbation, quasi-normal modes

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1. Introduction

Currently, a large number of astronomical observations indicate that our universe is in an accelerated expansion phase.[1–8] In order to explain such a phenomenon, many gravity models were proposed, in which the type $R+R^n$ (R is the Ricci scalar and R^n denotes all possible combinations of the curvature tensor) was considered as one of the promising candidates.[9] Firstly, the type $R+R^n$ is equivalent to the standard Einstein gravity with a special potential[10] for an additional scalar degree of freedom ϕ.[11] Starobinsky found that the potential of ϕ can describe a slow transition from a de Sitter phase to a flat Minkowski phase leading to a viable realization of the early universe in the inflationary scenario,[12,13] and he considers the simplest case could be possibly R^2 term to the standard Einstein action. Secondly, Stelle found that the addition of the quadratic curvature terms with $n=2$ makes the theory of gravity renormalized.[14] Therefore, it is meaningful to investigate some black holes predicted by R^2 gravity.

The evolution of a black hole perturbed by external fields can be divided into three stages: the first stage is the initial wave burst, the second stage corresponds to damped oscillations with complex frequencies (the modes of such oscillations are called quasi-normal modes (QNMs)[15]), the third stage is the late-time tail. Therefore, the frequencies of QNMs reflect the intrinsic properties of the black hole, which are independent from initial perturbations. In other words, QNMs may give the interior region of black holes.[16] The frequencies of QNMs are complex, the real part corresponds to the oscillation rate and the magnitude of the imaginary part reflects the damping rate.[17] So in order to obtain more information about the R^2 AdS black hole, it is necessary to study the QNMs of such a black hole.

So far, many works of perturbation theory have studied typical ringdown signals (QNMs) of various black holes in detail. For gravitational perturbation Regge and Wheeler first studied the gravitational perturbations in Schwarzschild spacetime.[18] Through numerical simulations for the scattering of Gaussian wavepackets in Schwarzschild spacetimes, they derived the corresponding QNMs. Vishveshwara[19] noticed that the waveform after a period of evolution consists of a damped sinusoid, with ringing frequency almost independent of the Gaussian parameters. Until 1983, the perturbation theory was given in systematic form by Chandrasekhar.[20] Besides the black hole's geometry, the properties of the QNM spectrum also depend on the properties of the perturbation field (e.g., spin). Therefore, some work is based on the research on QNMs for Scalar, Electromagnetic, Dirac perturbations (i.e., fields with integers spins).[21–27] For instance Vitor Cardoso studied the QNM of electromagnetic and gravitational perturbations of a Schwarzschild black hole in an AdS spacetime, and Li *et al.* have calculated the QNMs of Dirac

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perturbations in the regular spacetimes.[28] Among various types of perturbation, gravitational perturbation can unfold not only the interior properties but also the stability of the black hole.[29,30] Moreover, gravitational perturbations to a black hole can generate relatively strong gravitational waves (GWs). Recently the gravitational wave signals from the merging of two black holes detected by the interferometric LIGO.[31,32] Later, it indicates that the current experiment precision might meet the requirement of testing alternative gravity theories.[33] Therefore, as an explanation for the accelerated expanding universe, R^2 gravity would be verified by the corresponding GWs observation, and it has very important practical significance.

Furthermore, according to the AdS/CFT correspondence an AdS spacetime corresponds to a thermal state in the conformal field theory. The temporal evolution of the perturbed thermal state returning to thermal equilibrium can be equivalently calculated through perturbing the corresponding AdS black hole. Recently, Daniel[34] studied new features of gravitational collapse in AdS spacetimes, and Lin[35] studied QNMs in Horava–Lifshitz gravity with $U(1)$ symmetry, by thoroughly investigating scalar perturbations in AdS spacetime. In this paper we aim to study gravitational perturbation of the R^2 black holes in AdS spacetimes. From the R^2 gravitational field equation, the first order of the R^2 AdS black hole under the odd gravitational perturbation is derived. Then we use the Horowitz[36] and Hubeny[37] method to study the influence of the coefficient λ on the oscillation frequency and damping rate of QNMs in the static R^2 AdS spacetimes. In addition, we also use the finite difference method[38,39] to describe the dynamic evolution of static R^2 AdS black holes.

The structure of the paper is as follows. In Section 2, we study the odd parity perturbation of R^2 BHs in the asymptotically AdS spacetimes and the main properties of the master wave equations. In Section 3 the specific analysis of the QNMs is described, which includes two parts: in part A, quasinormal modes for R^2 AdS black hole spacetime are evaluated using the method proposed by Horowitz and Hubeny.[36,37] In part B, the finite difference method[38,39] is used to analyze the dynamical evolution of the odd parity perturbation field in the R^2 AdS spacetimes. Finally, we summarize the important results and make some remarks in Section 4.

2. The gravitational perturbation of the static R^2 AdS BH

Generally, the quadratic and scale invariant theory of gravity can be described by the Weyl–Eddington action,[40] which is

$$S=\int \mathrm{d}^4x\sqrt{-g}(\alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}+\beta R^2), \tag{1}$$

where R is the Ricci scalar, and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. The two coupling constants α and β are dimensionless. When $\alpha=0$, $\beta\neq 0$ the pure R^2 gravity[41] is simplified to be

$$S=\int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{16\mu^2}R^2+L_{\mathrm{matter}}\right), \tag{2}$$

where L_{matter} is matter Lagrangian. From the action Eq. (2), the field equation of the pure R^2 gravity in a vacuum field ($T_{\mu\nu}=0$) can be written as

$$\widehat{G}_{\mu\nu}=RR_{\mu\nu}-\frac{1}{4}R^2g_{\mu\nu}-\nabla_\mu\nabla_\nu R+g_{\mu\nu}\nabla^2R=0. \tag{3}$$

Considering the general static spherically metric

$$\mathrm{d}s^2=-f(r)\,\mathrm{d}t^2+\frac{\mathrm{d}r^2}{f(r)}+r^2\,\mathrm{d}\Omega_2^2. \tag{4}$$

Non-flat spaces satisfy:

$$R_{\mu\nu}=3\lambda g_{\mu\nu}. \tag{5}$$

In fact λ can be a function of the coordinates so that $\lambda_\mu=\partial_\mu\lambda$ is a covariantly constant vector. In this case, spacetime is necessarily decomposable and equation (5) leads to $\lambda=\mathrm{const.}$[42] Solving Eq. (3) with the R^2 theory, we obtain the solution to a metric function:[43]

$$f(r)=1-\frac{M}{r}-\lambda r^2, \tag{6}$$

when $\lambda<0$ equation (6) describes Anti-de Sitter spacetimes, while $\lambda>0$ describes asymptotically de Sitter spacetime, when $\lambda=-1$ it is an AdS Schwarzschild black hole and with $\lambda=0$ it returns to a classical Schwarzschild black hole. Here $|\lambda|=1/R_{\mathrm{BH}}^2$, where R_{BH} is the radius of R^2 AdS BH, so λ can be seen as the coefficient related to the radius of R^2 AdS BH. In this paper, we focus on the gravitational perturbations of R^2 black holes in AdS spacetime.

Considering a gravitational perturbation, the metric becomes

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}, \tag{7}$$

where $h_{\mu\nu}$ is a small perturbation, and the metric $\bar{g}_{\mu\nu}$ is the background metric of the R^2 BH (i.e., Eqs. (4) and (6)). Correspondingly,

$$g^{\mu\nu}=\bar{g}^{\mu\nu}-h^{\mu\nu}. \tag{8}$$

About the form of perturbation $h_{\mu\nu}$, we adopt the Regge–Wheeler gauge to deduce the master equation for the gravitational perturbation.[18] Here we take the odd parity into account, which is in the form as:

$$h_{\mu\nu}=\begin{pmatrix}0&0&0&h_0(r)\\0&0&0&h_1(r)\\0&0&0&0\\\mathrm{sym}&\mathrm{sym}&0&0\end{pmatrix}\exp(-\mathrm{i}kt)Q_{\mathrm{P}}(\theta), \tag{9}$$

it should be noticed that ϕ can be cancelled from $h_{\mu\nu}$ by choosing the magnetic quantum $M=0$. The symbol "sym" indicates that the missing components of $h_{\mu\nu}$ are to be found from the symmetry $h_{\mu\nu}=h_{\nu\mu}$.[18] Where

$$Q_{\rm p}(\theta)=\sin\theta\,{\rm d}{\rm P}_l(\cos\theta)/{\rm d}\theta, \tag{10}$$

$\mathrm{P}_l(\cos\theta)$ is the Legendre polynomial with angular quantum number l, which satisfies

$$Q''_{\rm p}-\cot(\theta)Q'_{\rm p}(\theta)=-l(l+1)Q_{\rm p}(\theta). \tag{11}$$

Then the perturbation equations with the first order are derived as follows:

$$\begin{aligned}\delta\widehat{G}_{03}=&\,h'_1-\frac{{\rm i}h''_0}{\omega}-\frac{{\rm i}h'_0(4-4f-2rf'+4r^2f''+r^3f''')}{r\omega(-2+2f+4rf'+r^2f'')}+\frac{h_1(6f'+6rf''+r^2f''')}{-2+2f+4rf'+r^2f''}\\&+\frac{{\rm i}h_0(20f^2-8r^2f'^2+(-2+r^2f'')(-2+2l+2l^2+r^2f'')+4rf'(2+2l+2l^2+5r^2f''+r^3f'''))}{2r^2\omega f(-2+2f+4rf'+r^2f'')}\\&+\frac{4f{\rm i}h_0(-6+l+l^2-6rf'+3r^2f''+6r^3f'''+r^4f^{(4)})}{2r^2\omega f(-2+2f+4rf'+r^2f'')},\end{aligned} \tag{12}$$

$$\begin{aligned}\delta\widehat{G}_{13}=&-\frac{2h_0}{r}+h'_0-\frac{28{\rm i}h_1f^3-2r^2\omega^2{\rm i}h_1(-2+4rf'+r^2f'')}{2r^2\omega(-2+2f+4rf'+r^2f'')}\\&+\frac{f{\rm i}h_1(4-4l-4l^2-4r^2\omega^2-8r^2f'^2-4r^2f''+2lr^2f''+2l^2r^2f''+r^4f''^4)}{2r^2\omega(-2+2f+4rf'+r^2f'')}\\&+\frac{4rf'f{\rm i}h_1(2+2l+2l^2+5r^2f''+r^3f''')}{2r^2\omega(-2+2f+4rf'+r^2f'')}\\&+\frac{4{\rm i}h_1f^2(-8+l+l^2-8rf'-2r^2f''+5r^3f'''+r^4f^{(4)})}{2r^2\omega(-2+2f+4rf'+r^2f'')},\end{aligned} \tag{13}$$

$$\delta\widehat{G}_{23}=\frac{{\rm i}\omega h_0}{f^2}+h'+\frac{h_1(-4f^2+rf'(-2+4rf'+r^2f'')+f(4+4r^2f''+r^3f'''))}{rf(-2+2f+4rf'+r^2f'')}. \tag{14}$$

After decoupling the above equations, the Schrödinger wave equation for this case can be expressed as

$$\mathrm{d}^2\Phi/\mathrm{d}r_*^2+(\omega^2-V(r))\Phi=0, \tag{15}$$

where

$$\Phi(r)=\left(1-M/r-\lambda r^2\right)h_1/r, \tag{16}$$

r_* is a tortoise coordinate as

$$\frac{{\rm d}r_*}{{\rm d}r}=\left(1-\frac{M}{r}-\lambda r^2\right)^{-1}, \tag{17}$$

and the effective potential function $V(r)$ in odd parity is written as follows:

$$\begin{aligned}V(r)=&\frac{1}{4r^2(-2-2f+4rf'+r^2f'')^2}f(112f^3+2(-2+4rf'+r^2f'')(-14r^2f'^2+(-2+r^2f'')(2(-1+l+l^2)+r^2f'')\\&+rf'(8(1+l+l^2)+14r^2f''+3r^3f'''))+4f^2(-60+4l+4l^2+24rf'-6r^2f''+12r^3f'''+3r^4f^{(4)})\\&+f(144-32l-32l^2-204r^2f'^2+72r^4f''^2-48r^3f'''+3r^6f'''^2-12r^4f^{(4)}+8rf'(-12+8l\\&+8l^2+6r^2f''+18r^3f'''+3r^4f^{(4)})+2r^2f''(8l(1+l)+30r^3f'''+3r^4f^{(4)}))).\end{aligned} \tag{18}$$

According to Eq. (18), we can describe the effective potential functions of odd type perturbation $V(r)$ in R^2 AdS spacetime, and find how the potential relies on the parameters, such as the coefficient λ, angular quantum number l.

Figure 1 illuminates that the potential functions vary with r outside the event horizon with different values of λ and l. From Fig. 1(a), we can see obviously that as l increases, the potential value will increase. Figure 1(b) shows that the peak value of the potential barrier grows as $|\lambda|$ increases. Figure 1 indicates that the potential function $V(r)$ is divergent as $r\to\infty$, which behaves very differently from the potential function in asymptotically flat and dS spacetime. Due to its positive definiteness beyond the event horizon, the corresponding quasinormal modes perturbation should be stable.

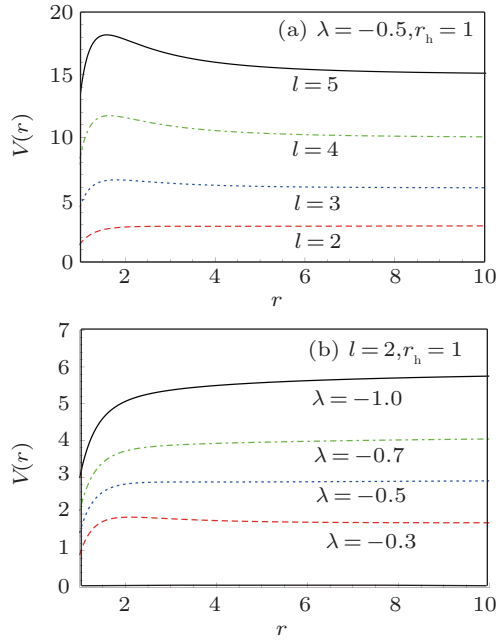


Fig. 1. (color online) The effective potential for odd parity gravitational perturbation in static R^2 AdS spacetime for (a) $\lambda=-0.5$, $r_{\rm h}=1$; (b) $l=2$, $r_{\rm h}=1$.

3. Numerical calculation of the QNM frequencies

In the study of the potential function in the static R^2 AdS spacetime, it was found that the potential function $V(r)\to\infty$ at infinity, so that some traditional numerical calculations are invalid, such as the WKB approximation method and the continuous fraction method, which are used to calculate QNMs frequency with $V(r)\longrightarrow 0$ at infinity. In order to solve this problem, Horowitz and Hubeny[36,37] proposed a way to calculate the QNMs frequency for perturbation in AdS spacetime. Therefore in this section, we will adopt the Horowitz Hobeny method to calculate the QNMs frequencies varying with λ, l, and overtone n. Then, we use the finite difference method[44,45] to figure out the dynamics evolution of the gravitational perturbation field.

3.1. The QNMs frequency calculated by Horowitz–Hobeny method

A static spherically symmetric metric Eq. (4) should be rewritten in the ingoing Eddington coordinates with $v=t+r_*$ as follows:

$$\mathrm{d}s^2=-f(r)\mathrm{d}^2v+2\mathrm{d}v\mathrm{d}r+r^2\mathrm{d}\theta^2+r^2\sin\theta\,\mathrm{d}\phi^2. \tag{19}$$

Applying such a method to our condition and defining a new wave function $\Psi(r)=\mathrm{e}^{\mathrm{i}\omega r}\Phi(r)$, the master equation Eq. (17) becomes

$$f(r)\frac{\partial^2\Psi}{\partial r^2}+(f'(r)-2\mathrm{i}\omega)\frac{\partial\Psi}{\partial r}-\frac{V(r)}{f(r)}\Psi=0. \tag{20}$$

It is worth noting that when $r\to r_h$, r_h is the event horizon, $V(r)/f(r)$ is a constant. Then changing coordinate variable r into $x=1/r$, equation (20) is expressed as

$$s(x)\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2}+\frac{t(x)}{x-x_+}\frac{\mathrm{d}\Psi}{\mathrm{d}x}+\frac{u(x)}{(x-x_+)^2}\Psi=0, \tag{21}$$

where $x_+=1/r_h$ and

$$\begin{aligned}s(x)&=-\frac{f(r)x^4}{x-x_+},\\ t(x)&=x^2[f(r)'-2f(r)x-2\mathrm{i}\omega],\\ u(x)&=(x-x_+)V(x).\end{aligned} \tag{22}$$

Around horizon $r_{\rm h}$, the functions $s(x)$, $t(x)$, and $u(x)$ can be expanded as

$$\begin{aligned}s(x)&=\sum_{m=0}^{4}s_m(x-x_+)^m,\\ t(x)&=\sum_{m=0}^{4}t_m(x-x_+)^m,\\ u(x)&=\sum_{m=0}^{4}u_m(x-x_+)^m,\end{aligned} \tag{23}$$

where $s_0=2x_+^2\kappa$, $t_0=2x_+^2(\kappa-\mathrm{i}\omega)$, and $u_0=0$ ($\kappa=f'(r_{\rm h})/2$ is the gravity on the surface of the black hole). Meanwhile supposing the wave function $\Psi\sim(x-x_+)^\alpha$ at the horizon $r_{\rm h}$, equation (21) yields

$$\alpha(\alpha-1)s_0+\alpha t_0=2x_+^2\alpha(\alpha\kappa-\mathrm{i}\omega)=0. \tag{24}$$

So $\alpha=0$ is corresponding to ingoing modes at the AdS horizon. Then the approximation solution of the wave function around the horizon should be $\Psi\sim(x-x_+)^0$. Therefore, we define

$$\Psi=\sum_{m=0}^{\infty}a_m(x-x_+)^m. \tag{25}$$

Substituting Eq. (23) and (25) into Eq. (24), the recurrence formula of a_m can be derived as

$$a_m=-\frac{1}{m(m-1)s_0+mt_0}\sum_{j=0}^{\infty}a_j[j(j-1)s_{m-j}+jt_{m-j}+u_{m-j}]. \tag{26}$$

Here we set $a_0=1$. According to another boundary condition at $r\to\infty(x\to 0)$, $\Psi\to 0$, yielding

$$\sum_{m=0}^{\infty}a_m(-x_+)^m=0. \tag{27}$$

Taking Eqs. (25) and (26) into the above equation, through an iterative method to solve the algebraic equation of ω, we can obtain the frequencies of QNMs. We list some of them in Tables 1 and 2, where n is the overtone number, ω is the complex quasi-normal modes frequency, and l is the angular momentum. Here we mainly focus on the coefficient λ in the fundamental mode ($n=0$) and the first overtone mode ($n=1$).

Assuming $r_h = 1$, for $l = 2, 3$. When $\lambda = -1$, the static R^2 AdS BH returns to a Schwarzschild AdS black hole, and the results listed here are consistent with the ones in Refs. [46] and [47].

The results show that except for the case with $\lambda = -1$, $\mathrm{Re}(\omega)$ and $|\mathrm{Im}(\omega)|$ both increase as $|\lambda|$ increases with $n = 0$ and $n = 1$. That means the coefficient λ has a different effect on R^2 BH and Schwarzschild BH. We consider that in gravitational perturbations the frequency scales are relevant to the radius of the AdS BH. Since the temperature scale is also related to the radius in the AdS BH, the frequency scale depends on the temperature. In the dual CFT higher temperatures will approach to thermal equilibrium much faster. AdS spacetime is completely different from asymptotically flat spacetime and dS spacetime, in which the frequency scales increase with $|\lambda|$. Therefore negative λ is related to the description of the evolution of the gravitational field in such R^2 AdS spacetimes. In particular, all frequencies have negative imaginary parts, which indicates that the spacetime is stable for this kind of perturbation.

Table 1. QNM of gravitational perturbations for $r_h = 1$ and $l = 2$.

λ	$n = 0$		$n = 1$	
	$-\omega_\mathrm{I}$	ω_R	$-\omega_\mathrm{I}$	ω_R
−0.3	0.47117115i	1.64018421	1.31918111i	2.34960694
−0.4	0.71128652i	1.85685315	1.82988057i	2.76657727
−0.5	0.96914326i	2.05379903	2.35002326i	3.14629641
−0.6	1.24312387i	2.24258741	2.49874826i	3.42351683
−0.7	1.52932003i	2.43204137	2.92358923i	3.75261325
−0.8	1.82168081i	2.62705503	3.42568970i	4.09825431
−0.9	2.09626144i	2.80778111	3.91012895i	4.45689141
−1.0	2.40423438i	3.03311417	4.89817789i	4.96073481

Table 2. QNM of gravitational perturbations for $r_\mathrm{h} = 1$ and $l = 3$.

λ	$n = 0$		$n = 1$	
	$-\omega_\mathrm{I}$	ω_R	$-\omega_\mathrm{I}$	ω_R
−0.3	0.27016758i	2.25990103	0.96261205i	3.14885045
−0.4	0.44498975i	2.56157878	1.38830706i	3.32012356
−0.5	0.62580446i	2.82633664	1.82232134i	3.67866002
−0.6	0.81452861i	3.06470253	2.27066985i	4.02169877
−0.7	1.01032798i	3.28281121	2.72977539i	4.34474123
−0.8	1.21293039i	3.48453858	3.20122118i	4.65201002
−0.9	1.42237676i	3.67264841	3.68618807i	4.94813388
−1.0	1.63888098i	3.84925835	4.18548766i	5.23767394

3.2. Dynamics evolution in the finite difference method

Through the finite difference method[38,39] we evaluate the temporal evolution of the gravitational perturbation field in R^2 AdS spacetime. The results can reflect the stability of such R^2 AdS black holes under the odd parity gravitational perturbations. Applying the coordinate transformation $(t, r) \longrightarrow (\mu, \nu)$ to Eq. (17)

$$\mu = t - r_*, \nu = t + r_*, \tag{28}$$

yields

$$\frac{\partial^2 \Psi}{\partial u \partial v} + \frac{1}{4} V(r) \Psi = 0, \tag{29}$$

where $\Psi(u, v) \to \Psi_{ij}$ and

$$\frac{\partial \Psi}{\partial u} \to \frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{\triangle u},$$
$$\frac{\partial \Psi}{\partial v} \to \frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{\triangle v}. \tag{30}$$

Using central difference rule

$$\Psi_{i,j} = \frac{\Psi_\mathrm{W} + \Psi_\mathrm{E}}{2}, \tag{31}$$

finally, we obtain the discrete expression of Eq. (29)

$$\Psi_\mathrm{N} = \Psi_\mathrm{W} + \Psi_\mathrm{E} - \Psi_\mathrm{S} - \triangle u \triangle v V(r) \frac{\Psi_\mathrm{W} + \Psi_\mathrm{E}}{8}, \tag{32}$$

here we use the notation

$$\Psi_\mathrm{N} = \Psi_{i+1,j+1}, \Psi_\mathrm{S} = \Psi_{i-1,j-1},$$
$$\Psi_\mathrm{E} = \Psi_{i+1,j-1}, \Psi_\mathrm{W} = \Psi_{i-1,j+1}. \tag{33}$$

Following the original work of Refs. [48], [49], and [50], we now discuss the relationship between the effective potential and the angular quantum number l, and the coefficient λ.

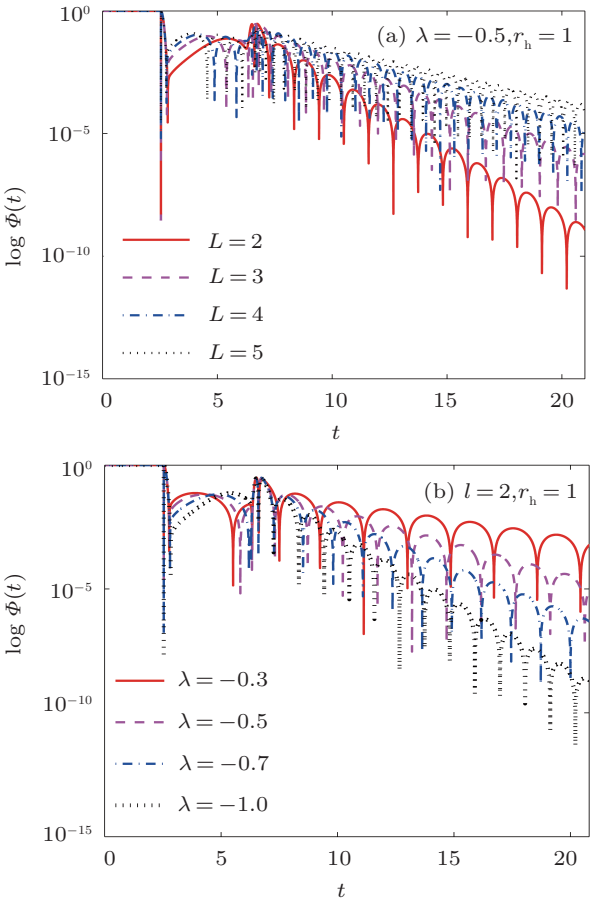


Fig. 2. (color online) The dynamical evolution of odd parity gravitational perturbation to the R^2 AdS BH. Here (a) the line, the dashed, the dot–dashed, and the dotted lines correspond to $l = 2$, 3, 4, and 5. (b) The dotted, the dot–dashed, the dashed, and the solid lines correspond to $\lambda = -1$, -0.7, -0.5, and -0.3.

Figure 2 illuminates that the corresponding R^2 AdS black holes can return to be stable at the final stage. The dynamics evolution of each mode in Fig. 2 corresponds to the case of Fig. 1. From Fig. 2(a), since $V(r)$ increases with the growth of l, the oscillation frequency $\mathrm{Re}(\omega)$ turns to be much more intense and decay rate $|\mathrm{Im}(\omega)|$ becomes much slower. From Fig. 2(b), the values of $V(r)$ are raised slightly with the increase of $|\lambda|$, the oscillation rate $\mathrm{Re}(\omega)$ becomes tighter and decay rate $|\mathrm{Im}(\omega)|$ becomes quicker. These results are consistent with the results of we obtained by the Horowitz and Hubeny method. Moreover, the dynamic evolution of the perturbation field indicates the stability of gravitational perturbation directly.

4. Conclusion

In this paper, we have studied the QNMs of odd gravitational perturbation for the R^2 black hole in asymptotically AdS spacetimes. We proceed directly from the calculation of the R^2 gravitational field equation, and obtained the first-order perturbation form of the R^2 gravitational field equation. The structure of AdS spacetime around $r \to \infty$ is very different from the conditions in asymptotically flat and de sitter spacetimes.[51–55] For the asymptotically AdS spacetime, the wave function Φ should vanish there when the potential diverges at infinity. It is noteworthy that the outgoing wave coming from the horizon will be scattered off the potential and turn to be an ingoing wave entering the horizon. The corresponding quasi-normal perturbations in AdS spacetime are considered to be modes with only ingoing waves near the horizon. Therefore, we used the Horowitz and Hubeny method to solve the QNMs of odd gravitational perturbation for R^2 black hole in asymptotically AdS spacetimes, and through the finite difference method we studied the dynamic evolution of R^2 AdS black holes.

The results obtained in this paper are summarized as follows: (i) From the Horowitz and Hubeny method, the static R^2 AdS BH reduces to a Schwarzschild AdS black hole when the coefficient $\lambda = -1$, and the results list are consistent with the ones in Refs. [46] and [47]. In addition to $\lambda = -1$, the oscillation frequency $\mathrm{Re}(\omega)$ and decay frequency $\mathrm{Im}(\omega)$ both increase with the increase of the coefficient λ. (ii) From the finite difference method, if we increase the value of the coefficient λ, the oscillation frequency $\mathrm{Re}(\omega)$ increases and decay rate $|\mathrm{Im}(\omega)|$ becomes quicker. This is consistent with the results obtained by the Horowitz and Hubeny method. Therefore, the coefficient λ (i.e., the radius of the R^2 AdS BH) plays an important role in the stability of the R^2 AdS BH. Furthermore, concerning the increase of the angular quantum number l, the quasi-normal modes of gravitational perturbations of R^2 AdS black holes oscillate more quickly and damp more slowly.

From the above results, we can draw the following conclusions: For odd gravitational perturbations in AdS spacetime, it is found that frequencies all have a negative imaginary part. The results also imply that the R^2 AdS black hole is stable against gravitational perturbations, these will decay exponentially with time. Moreover, the dynamic evolution of the perturbation field indicates the stability of gravitational perturbation directly. On the other hand, due to the effect of the coefficient λ on AdS spacetime, the decay rate of $|\mathrm{Im}(\omega)|$ becomes much faster with larger coefficient λ, which is related to the radius of the AdS R^2 black hole. In terms of the AdS/CFT correspondence, this implies that with smaller radius, the R^2 AdS BH approaches to equilibrium faster. Therefore, the results we obtained in this work may not only contribute the coefficient λ for QNMs of R^2 AdS black holes, but also bring referential information for studies of AdS/CFT correspondence.

There are some issues to be further studied in our future work. Firstly, the AdS space time we discussed is spherically symmetric. However, in order to reveal more AdS/CFT dual properties, the study of non-spherically symmetric in more general cases would give us much more useful information. Secondly, we will discuss the QNMs of even gravitational perturbation in AdS separately though the expression of the master equation is rather lengthy. Finally, how to combine the knowledge of quantum gravity to QNMs frequencies is still unknown.

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