Fast generating W state of three superconducting qubits via Lewis–Riesenfeld invariants*

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We propose a scheme for a fast generating three-qubit W state in a superconducting system by using a technique of shortcuts to adiabaticity, Lewis–Riesenfeld invariants. Three identical superconducting qubits (SQs) are connected by two coplanar waveguide resonators (CPWRs) capacitively. Under a certain limit condition, we convert the complicated SQ system into a simple three-state system. By designing experimentally accessible harmonic pulses, a three-SQ W state is implemented with quite short operation time and high fidelity. Numerical simulations prove that the scheme is robust against the parameter deviation. In addition, we also give detailed discussion about the scheme robustness against decoherence.

Keywords: W state, superconducting qubits, shortcuts to adiabaticity

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1. Introduction

Entanglement plays a crucial role in quantum information processing.[1–4] As two kinds of three (multi)-qubit entangled states, W states[5] and Greenberger–Horne–Zeilinger (GHZ) states[6] have received a great deal of attention. Comparing them, W states receive more attention than GHZ states because the former may keep bipartite entanglement when one of three qubits is collapsed, which will have advantages in quantum teleportation.[7] Therefore, a number of schemes have been proposed to investigate W states in different systems via different techniques for different purposes.[8–21]

As one of the most promising candidates for achieving quantum computers, the superconducting system has many advantages over other systems.[22–25] It has a natural superiority of scalability for achieving quantum computers. The superconducting device used in a quantum computer can be created more easily for large integration and scale with the mature microelectronic technique. In addition, coupling strengths among SQs can be controlled better with desired values by modulating electromagnetic signals and designing level configurations. By using superconducting systems, many schemes have been brought forward to generate W states.[26–29]

Among these references mentioned above for generating W states, a technique, stimulated Raman adiabatic passage (STIRAP), is widely used because of its robustness in quantum state transfer.[30–34] However, STIRAP usually requires a very long operation time, which will increase the probabilities of decoherence and errors and lead to useless dynamics finally. Therefore, a set of techniques called "shortcuts to adiabaticity (STA)" arise for speeding up adiabatic evolution processes.[35–44] Besides, some experimental realizations have been completed.[45,46] By using STA, some schemes were presented for the fast generation of W states in cavity quantum electrodynamics.[47–49] Very recently, in superconducting systems, Kang *et al.* proposed two schemes for fast generating W states.[50,51]

In this work, we implement the fast generation of three-SQ W states in a superconducting system by using a technique of STA, Lewis–Riesenfeld invariants. Unlike the scheme in the Refs. [50] and [51], on the one hand, the scheme we propose does not need four qubits which include an auxiliary qubit but only three qubits for the generation of three-SQ W state. On the other hand, the scheme needs to neither interpolate the system Hamiltonian nor add counterdiabatic driving pulses, which will reduce experimental difficulties, but only design accessible harmonic pulses to construct a Hermitian invariant of the system.

2. Model and effective dynamics

Figure 1 gives the fundamental model for generating a three-SQ W state. Three SQs are connected by two single-mode CPWRs capacitively. All SQs have identical Λ-type level configuration possessing one upper level $|e\rangle$ and two lower levels $|L\rangle$ and $|R\rangle$. For the j-th SQ, transition $|e\rangle_j \leftrightarrow |L\rangle_j$ is resonantly driven by a time-dependent laser field with Rabi frequency $\Omega_j(t)$. In addition, for SQ1, $|e\rangle_1 \leftrightarrow |R\rangle_1$ is resonantly coupled to CPWR-A and CPWR-B, synchronously, with coupling strength g_1. For SQ2(3), $|e\rangle_{2(3)} \leftrightarrow |R\rangle_{2(3)}$ is coupled to CPWR-$A(B)$ with coupling strength $g_{2(3)}$. The interaction Hamiltonian of the superconducting system is ($\hbar =$

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1):

$$H(t) = \sum_{j=1}^{3} \Omega_j(t)|e\rangle_j\langle L| + g_1(a_A + a_B)|e\rangle_1\langle R| + g_2 a_A|e\rangle_2\langle R| + g_3 a_B|e\rangle_3\langle R| + \text{H.c.}, \quad (1)$$

where $a_{A(B)}$ is the annihilation operator of CPWR-$A(B)$.

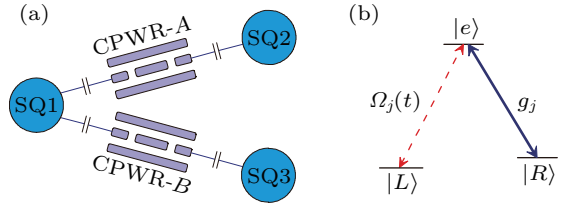


Fig. 1. (color online) (a) Diagrammatic sketch of the superconducting system. (b) Level configurations and relevant transitions of three SQs.

We suppose that the superconducting system is initially in state $|\phi_1\rangle = |L\rangle_1|R\rangle_2|R\rangle_3|0\rangle_A|0\rangle_B$ which represents three SQs in states $|L\rangle$, $|R\rangle$, and $|R\rangle$, respectively, and two CPWRs both are in the vacuum state. Then Hamiltonian (1) can be rewritten as

$$H(t) = \Omega_1(t)|\phi_1\rangle\langle\phi_2| + g_1(|\phi_2\rangle\langle\phi_3| + |\phi_2\rangle\langle\phi_4|) + g_2|\phi_3\rangle\langle\phi_5| + g_3|\phi_4\rangle\langle\phi_6| + \Omega_2(t)|\phi_5\rangle\langle\phi_7| + \Omega_3(t)|\phi_6\rangle\langle\phi_8| + \text{H.c.}, \quad (2)$$

with the orthogonal complete basis states

$$\begin{aligned}
|\phi_1\rangle &= |L\rangle_1|R\rangle_2|R\rangle_3|0\rangle_A|0\rangle_B,\\
|\phi_2\rangle &= |e\rangle_1|R\rangle_2|R\rangle_3|0\rangle_A|0\rangle_B,\\
|\phi_3\rangle &= |R\rangle_1|R\rangle_2|R\rangle_3|1\rangle_A|0\rangle_B,\\
|\phi_4\rangle &= |R\rangle_1|R\rangle_2|R\rangle_3|0\rangle_A|1\rangle_B,\\
|\phi_5\rangle &= |R\rangle_1|e\rangle_2|R\rangle_3|0\rangle_A|0\rangle_B,\\
|\phi_6\rangle &= |R\rangle_1|R\rangle_2|e\rangle_3|0\rangle_A|0\rangle_B,\\
|\phi_7\rangle &= |R\rangle_1|L\rangle_2|R\rangle_3|0\rangle_A|0\rangle_B,\\
|\phi_8\rangle &= |R\rangle_1|R\rangle_2|L\rangle_3|0\rangle_A|0\rangle_B,
\end{aligned} \quad (3)$$

where $|1\rangle_{A(B)}$ denotes a single photon in CPWR-$A(B)$. When we introduce a set of orthogonal states

$$|\psi_j^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\phi_{2j+1}\rangle \pm |\phi_{2j+2}\rangle), \quad (j = 1,2,3). \quad (4)$$

Hamiltonian (2) is written as

$$H(t) = \Omega_1(t)|\phi_1\rangle\langle\phi_2| + \sqrt{2}g|\phi_2\rangle\langle\psi_1^+| + g(|\psi_1^+\rangle\langle\psi_2^+| + |\psi_1^-\rangle\langle\psi_2^-|) + \Omega(t)(|\psi_2^+\rangle\langle\psi_3^+| + |\psi_2^-\rangle\langle\psi_3^-|) + \text{H.c.}, \quad (5)$$

in which we have set $g_1 = g_2 = g_3 = g$ and $\Omega_2(t) = \Omega_3(t) = \Omega(t)$ for convenience. Then using Eq. (5), we can easily find that $|\psi_j^-\rangle$ will not be involved in the whole evolution when the initial state is $|\phi_1\rangle$. Thus Hamiltonian (5) can be written by

$$H(t) = \Omega_1(t)|\phi_1\rangle\langle\phi_2| + \sqrt{2}g|\phi_2\rangle\langle\psi_1^+| + g|\psi_1^+\rangle\langle\psi_2^+| + \Omega(t)|\psi_2^+\rangle\langle\psi_3^+| + \text{H.c.} \quad (6)$$

Next, we introduce a second set of orthogonal states as follows:

$$\begin{aligned}
|\Psi_D\rangle &= \frac{1}{\sqrt{3}}(|\phi_2\rangle - \sqrt{2}|\psi_2^+\rangle),\\
|\Psi_\pm\rangle &= \frac{1}{\sqrt{6}}(\sqrt{2}|\phi_2\rangle \pm \sqrt{3}|\psi_1^+\rangle + |\psi_2^+\rangle),
\end{aligned} \quad (7)$$

then Hamiltonian (6) becomes

$$\begin{aligned}
H(t) &= H_0 + V(t),\\
H_0 &= \sqrt{3}g(|\Psi_+\rangle\langle\Psi_+| - |\Psi_-\rangle\langle\Psi_-|),\\
V(t) &= \frac{\Omega_1(t)}{\sqrt{3}}|\phi_1\rangle(\langle\Psi_D| + \langle\Psi_+| + \langle\Psi_-|) - \frac{\Omega(t)}{\sqrt{3}}[\sqrt{2}|\Psi_D\rangle - \frac{1}{\sqrt{2}}(|\Psi_+\rangle + |\Psi_-\rangle)]\langle\psi_3^+| + \text{H.c.}
\end{aligned} \quad (8)$$

By performing the unitary transformation $U = \exp(-iH_0t)$ and neglecting high oscillating terms under the limit condition $\Omega_1(t), \sqrt{2}\Omega(t) \ll 3g$, Hamiltonian (8) can be simplified into an effective Hamiltonian

$$H_e(t) = \frac{1}{\sqrt{3}}[\Omega_1(t)|\phi_1\rangle\langle\Psi_D| - \sqrt{2}\Omega(t)|\Psi_D\rangle\langle\psi_3^+|] + \text{H.c.}, \quad (9)$$

which can take the place of the whole Hamiltonian (1) to approximatively govern the system evolution under the limit condition $\Omega_1(t), \sqrt{2}\Omega(t) \ll 3g$.

3. Fast generating three-SQ *W* state via Lewis–Riesenfeld invariants

In this section, according to effective Hamiltonian (9), we implement the fast generation of three-SQ W state via Lewis–Riesenfeld invariants. First of all, it is necessary to give a brief introduction of Lewis–Riesenfeld invariants theory.[52] A quantum system is governed by a time-dependent Hamiltonian $H(t)$, and its time-dependent Hermitian invariant $I(t)$ satisfies

$$i\hbar\frac{\partial I(t)}{\partial t} = [H(t), I(t)]. \quad (10)$$

The solution of the time-dependent Schrödinger equation $i\partial_t|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$ can be written as a superposition of $I(t)$ eigenvectors $|\Phi_n(t)\rangle$

$$|\Psi(t)\rangle = \sum_n C_n e^{i\alpha_n}|\Phi_n(t)\rangle, \quad (11)$$

for which $C_n = \langle\Phi_n(0)|\phi_1\rangle$ is time-independent amplitude and α_n is the Lewis–Riesenfeld phase. The Lewis–Riesenfeld phase is defined by

$$\alpha_n(t) = \int_0^t dt'\langle\Phi_n(t')\Big|i\frac{\partial}{\partial t'} - H(t')|\Phi_n(t')\Big\rangle. \quad (12)$$

We choose one Hermitian invariant of the effective Hamiltonian (9) as

$$I(t) = \sin\nu\sin\beta|\phi_1\rangle\langle\Psi_D| + \sin\nu\cos\beta|\Psi_D\rangle\langle\psi_3^+| + \mathrm{i}\cos\nu|\psi_3^+\rangle\langle\phi_1| + \mathrm{H.c.}, \quad (13)$$

which has the dimension of energy. ν and β are two parameters which will be determined later. Eigenvectors of $I(t)$ with eigenvalues $\lambda_0 = 0$ and $\lambda_\pm = \pm 1$, respectively, are

$$\begin{aligned}|\Phi_0(t)\rangle &= \cos\beta\sin\nu|\phi_1\rangle - \mathrm{i}\cos\nu|\Psi_D\rangle - \sin\beta\sin\nu|\psi_3^+\rangle,\\ |\Phi_\pm(t)\rangle &= \frac{1}{\sqrt{2}}[(\cos\beta\cos\nu \pm \mathrm{i}\sin\beta)|\phi_1\rangle + \mathrm{i}\sin\nu|\Psi_D\rangle \\ &\quad - (\sin\beta\cos\nu \mp \mathrm{i}\cos\beta)|\psi_3^+\rangle].\end{aligned} \quad (14)$$

Inserting Eqs. (9) and (13) into Eq. (10), we obtain two Rabi frequencies as

$$\begin{aligned}\Omega_1(t) &= \sqrt{3}(\dot\beta\sin\beta\tan\nu - \cos\beta\dot\nu),\\ \Omega(t) &= -\frac{3}{\sqrt{6}}(\dot\beta\cos\beta\tan\nu + \sin\beta\dot\nu),\end{aligned} \quad (15)$$

in which the dot represents a time derivative.

For generating the W state, the system should evolve along the invariant dark mode $|\Phi_0(t)\rangle$ from the initial state $|\phi_1\rangle$ to the target state

$$|\Psi_W\rangle = \frac{1}{\sqrt{3}}(|\phi_1\rangle + |\phi_7\rangle + |\phi_8\rangle),$$

which demands

$$\begin{aligned}&\nu(0) = \nu(t_\mathrm{f}) = \pi/2, \quad \beta(0) = 0,\\ &\beta(t_\mathrm{f}) = -\arctan\sqrt{2},\end{aligned} \quad (16)$$

where t_f is the final operation time, and the scheme starts at $t = 0$ and ends at $t = t_\mathrm{f}$. Besides, $H_\mathrm{e}(t)$ must have the same eigenvectors as $I(t)$ at two boundary times.[53] It requires $[H_\mathrm{e}(0), I(0)] = 0$ and $[H_\mathrm{e}(t_\mathrm{f}), I(t_\mathrm{f})] = 0$, which give two boundary conditions with respect to two Rabi frequencies

$$\Omega_1(0) = 0, \quad \Omega_1(t_\mathrm{f}) = 2\Omega(t_\mathrm{f}). \quad (17)$$

Substituting Eqs. (16) and (17) into Eq. (15), we find two additional boundary conditions

$$\dot\nu(0) = \dot\nu(t_\mathrm{f}) = 0. \quad (18)$$

Now all boundary conditions have been given. However, the conditions $\nu(0) = \nu(t_\mathrm{f}) = \pi/2$ imply infinite $\Omega_1(t)$ and $\Omega(t)$, and hence we must weaken the conditions and can set

$$\nu(0) = \nu(t_\mathrm{f}) = \pi/2 - \varepsilon, \quad (19)$$

with a time-independent small value ε.

According to the boundary conditions we discuss the above points, we choose ν and β as

$$\nu = \pi/2 - \varepsilon, \quad \beta(t) = -\arctan\sqrt{2}t/t_\mathrm{f}. \quad (20)$$

Correspondingly, two Rabi frequencies are

$$\begin{aligned}\Omega_1(t) &= \Omega_0\sin(\arctan\sqrt{2}t/t_\mathrm{f}),\\ \Omega(t) &= \Omega_0\cos(\arctan\sqrt{2}t/t_\mathrm{f})/\sqrt{2},\end{aligned} \quad (21)$$

which are experimentally accessible harmonic pulses with the amplitude

$$\Omega_0 = \sqrt{3}\arctan\sqrt{2}\cot\varepsilon/t_\mathrm{f}.$$

Now we can use Eq. (11) to calculate the final fidelity for generating the W state

$$\begin{aligned}F(t_\mathrm{f}) &= |\langle\Psi_W|\Psi(t_\mathrm{f})\rangle|^2\\ &= |\cos^2\varepsilon + \sin^2\varepsilon\cos(\arctan\sqrt{2}/\sin\varepsilon)|^2,\end{aligned} \quad (22)$$

with corresponding Lewis–Riesenfeld phases

$$\alpha_0 = 0, \quad \alpha_\pm = \pm\arctan\sqrt{2}/\sin\varepsilon. \quad (23)$$

Then, for appropriate Rabi frequencies and the final fidelity $F(t_\mathrm{f}) = 1$, we choose $\arctan\sqrt{2}/\sin\varepsilon = 2\pi$, i.e.,

$$\varepsilon = \arcsin(\arctan\sqrt{2}/2\pi) \approx 0.1526. \quad (24)$$

Up to now, we have implemented the construction of STA for a fast generating three-SQ W state via Lewis–Riesenfeld invariants.

4. Numerical simulations and discussion

In this section, we discuss the availability and robustness of the scheme by numerical simulations. First of all, we need to ensure that $\varepsilon = 0.1526$ is right and pick an appropriate operation time. In Fig. 2, we plot a contour image of the final fidelity versus ε and t_f. We can clearly see that $\varepsilon = 0.1526$ guarantees a high-fidelity generation of the three-SQ W state with a suitable final time. Ω_0 is inversely proportional to t_f, which requires a large enough t_f to meet the limit condition $\Omega_1(t), \sqrt{2}\Omega(t) \ll 3g$, so the final fidelity increases with the increase of t_f.

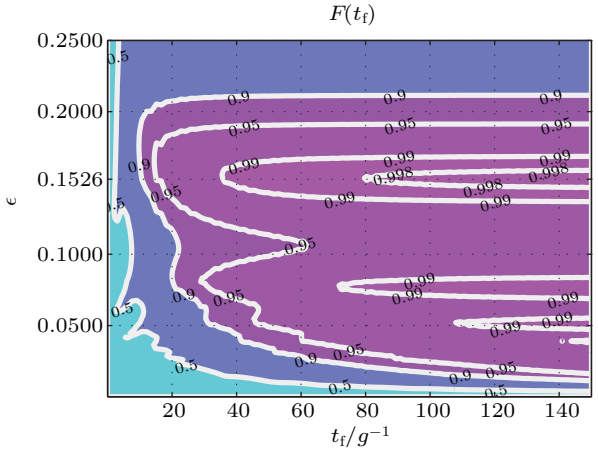


Fig. 2. (color online) Contour image for the final fidelity $F(t_\mathrm{f})$ versus ε and t_f.

Next, in order to show that our proposed STA scheme is fast, we contrast the STA scheme and the STIRAP scheme for generating three-SQ W state in Fig. 3. In the STIRAP scheme, we adopt the following Gaussian Rabi frequencies, respectively corresponding to $\Omega_1(t)$ and $\Omega(t)$:

$$\begin{aligned}\Omega_1'(t) &= \frac{2}{\sqrt{5}}\Omega_0' \exp[-(t-t_{\rm f}/2-t_0)^2/t_{\rm c}^2],\\ \Omega'(t) &= \frac{1}{\sqrt{5}}\Omega_0' \exp[-(t-t_{\rm f}/2-t_0)^2/t_{\rm c}^2]\\ &\quad -\Omega_0' \exp[-(t-t_{\rm f}/2+t_0)^2/t_{\rm c}^2],\end{aligned} \tag{25}$$

with two parameters $t_0 = 0.14t_{\rm f}$ and $t_{\rm c} = 0.19t_{\rm f}$. Since the STIRAP scheme needs either a very long operation time or very large differences between system eigen energies to weaken non-adiabatic couplings[54] as shown in Fig. 3, for the STIRAP scheme a larger Ω_0', which gives higher system eigen energies, will shorten the operation time; however, that will consume more resources. Besides, the limit condition $\Omega_1'(t), \sqrt{2}\Omega'(t) \ll 3g$ requires a smaller Ω_0' for high-fidelity generation of three-SQ W state. Therefore, for the STIRAP scheme in Fig. 3, when $\Omega_0' = 20g$ the final fidelity impossibly increases up to near unity even with an infinitely large operation time, which indicates that shortening the operation time by increasing the system energy is limited by the condition $\Omega_1'(t), \sqrt{2}\Omega'(t) \ll 3g$. For the STA scheme, when the operation time is $t_{\rm f} = 50/g$ the final fidelity is over 0.94 which gives the harmonic pulse amplitude $\Omega_0 \approx 0.2g$. Correspondingly, however, in the STIRAP scheme when $\Omega_0' = 0.2g$ it requires $t_{\rm f} = 200/g$, which is 4 times that for the STA scheme to reach the same final fidelity. Therefore, the STA scheme is fast for generating the three-SQ W state. In the following discussion, we pick $t_{\rm f} = 50/g$ as the final operation time.

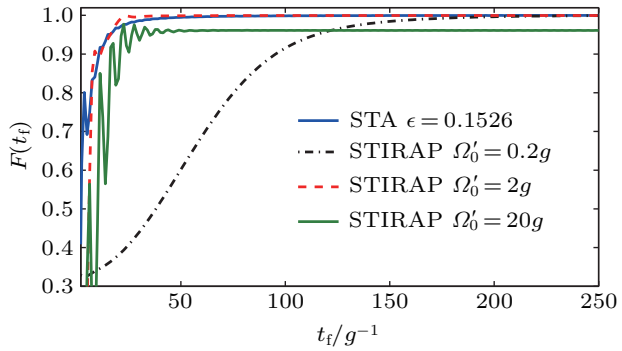


Fig. 3. (color online) Contrast between the STA scheme and the STIRAP scheme.

To see the STA scheme availability, it is necessary to show the time dependence of the fidelity and populations for states investigated. So in Fig. 4, we plot the time dependence of the fidelity for generating a three-SQ W state and populations P_1 for $|\phi_1\rangle$, $P_{7(8)}$ for $|\phi_{7(8)}\rangle$, and $P_{\rm e} = \sum_{n=2}^{6} P_n$ for other states. Apparently, figure 4(a) shows that the fidelity is nearly unity at the final time $t_{\rm f} = 50/g$. Figure 4(b) further proves that the STA scheme implements three-SQ W nearly perfectly, because all populations of $P_{1,7,8}$ are about $1/3$ at the final time but the sum of populations for other states not involved in the W state is near zero.

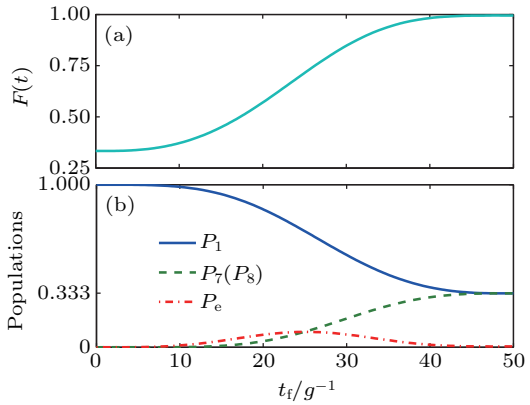


Fig. 4. (color online) (a) Time dependence of the fidelity; (b) time dependence of the populations for $|\phi_1\rangle$, $|\phi_{7(8)}\rangle$ and other states.

In a real experiment, perfect controls of parameters are scarcely possible. Hence it is necessary to investigate the sensitivities of the STA scheme to variations in control parameters. Here we define $\delta x = x' - x$ as the deviation of a parameter with x denoting the ideal value and x' denoting the actual value. In Fig. 5, we mainly consider the sensitivities of the STA scheme to variations in $t_{\rm f}$, Ω_0, and g. Through comparing Figs. 5(a)–5(c), we can find that the STA scheme is insensitive to variations in $t_{\rm f}$ and g but somewhat sensitive to those in Ω_0. Numerically speaking, however, the STA scheme is insensitive to variations in all three parameters because the final fidelity is always beyond 0.98 in each of the three figures.

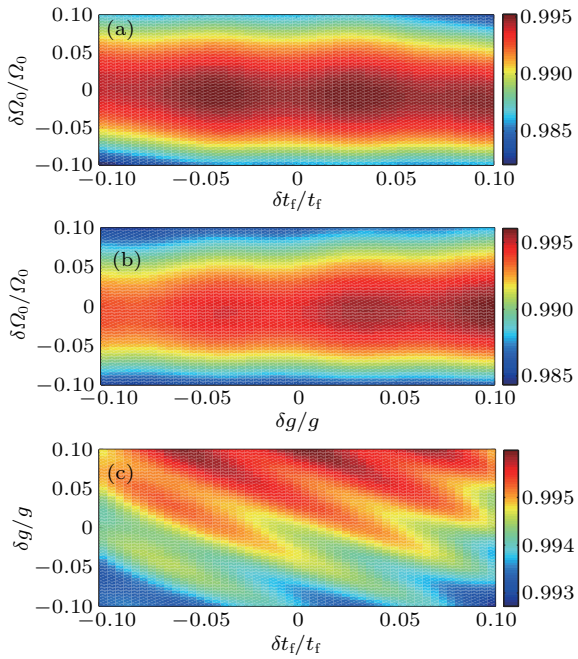


Fig. 5. (color online) Effects of the variations in the control parameters on final fidelity. (a) Final fidelity versus $\delta t_{\rm f}/t_{\rm f}$ and $\delta\Omega_0/\Omega_0$. (b) Final fidelity versus $\delta g/g$ and $\delta\Omega_0/\Omega_0$. (c) Final fidelity versus $\delta t_{\rm f}/t_{\rm f}$ and $\delta g/g$.

For more convenient simplification, we set $g_1 = g_2 = g_3$ and $\Omega_2(t) = \Omega_3(t)$. However, perfectly obtaining the two groups of equal parameters is scarcely possible in experiment either. Therefore, it is necessary to investigate the effects of the variations in the equal parameters on the final fidelity. In Fig. 6, we plot the effects of the variations in each pair of equal parameters on the final fidelity, in which $\Omega_{02} = \Omega_0$ and $\Omega_{03} = \Omega_0$ are for $\Omega_2(t)$ and $\Omega_3(t)$, respectively. Because of the symmetry of SQ2 and SQ3, figures 6(a) and 6(b) are identical. Generally speaking, the variations in each pair of equal parameters will destroy the final fidelity to a certain extent. However, as shown in Figs. 6(a)–6(d), the lowest final fidelity is still over 0.975, which indicates that the scheme is robust against the variations in the equal parameters.

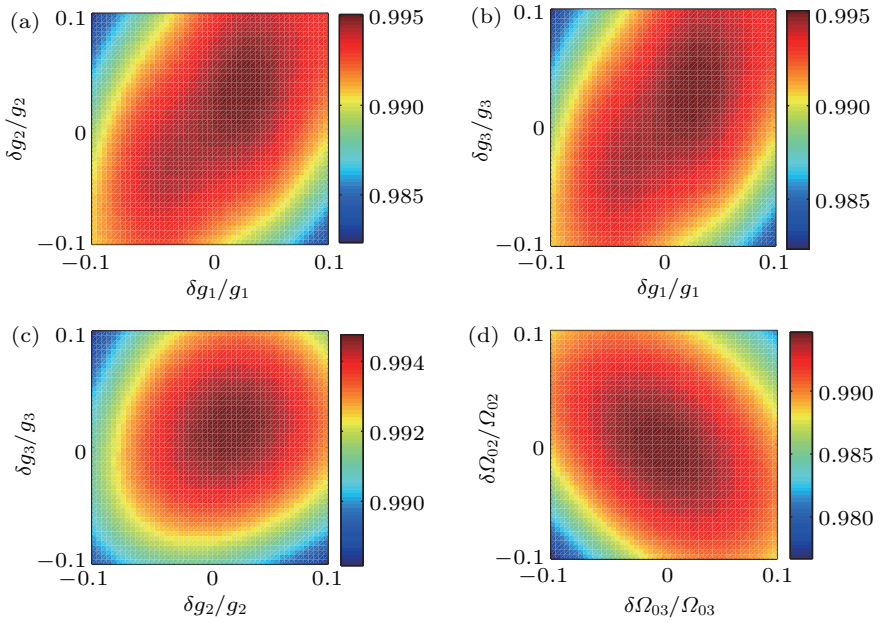


Fig. 6. (color online) Effects of the variations in the equal parameters on the final fidelity. (a) Final fidelity versus $\delta g_1/g_1$ and $\delta g_2/g_2$. (b) Final fidelity versus $\delta g_1/g_1$ and $\delta g_3/g_3$. (c) Final fidelity versus $\delta g_2/g_2$ and $\delta g_3/g_3$. (d) Final fidelity versus $\delta\Omega_{02}/\Omega_{02}$ and $\delta\Omega_{03}/\Omega_{03}$.

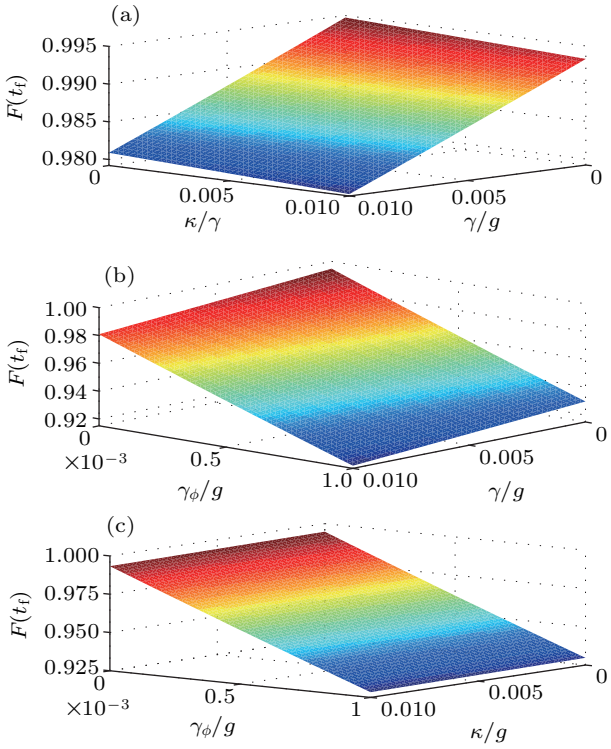


Fig. 7. (color online) (a) Final fidelity versus κ/g and γ/g with $\gamma_\phi = 0$. (b) Final fidelity versus γ_ϕ/g and γ/g with $\kappa = 0$. (c) Final fidelity versus γ_ϕ/g and κ/g with $\gamma = 0$.

Finally, we consider the STA scheme robustness against decoherence caused by SQ energy relaxations, CPWR photon decay and SQ dephasing. Taking decoherence into account, the whole evolution of the superconducting system will be governed by the master equation

$$\begin{aligned}\dot{\rho} = &-\mathrm{i}[H(t),\rho(t)] + \sum_{l=A,B}\kappa_l L[a_l] \\ &+ \sum_{k=L,R}\Bigg\{\sum_{j=1}^{3}\frac{\gamma_{\phi_{k,j}}}{2}[S_{k,j}^{z}\rho(t)S_{k,j}^{z} - \rho(t)] \\ &+ \sum_{j=1}^{3}\gamma_{k,j}L[S_{k,j}^{-}]\Bigg\},\end{aligned} \tag{26}$$

in which $H(t)$ is Hamiltonian (expression (1)).

$$\begin{aligned}&L[\hat{a}_l] = \hat{a}_l\rho\hat{a}_l^{\dagger} - \hat{a}_l^{\dagger}\hat{a}_l\rho/2 - \rho\hat{a}_l^{\dagger}\hat{a}_l/2, \\ &L[S_{k,j}^{-}] = S_{k,j}^{-}\rho S_{k,j}^{\dagger} - S_{k,j}^{\dagger}S_{k,j}^{-}\rho/2 - \rho S_{k,j}^{\dagger}S_{k,j}^{-}/2, \\ &S_{k,j}^{-} = |k_j\rangle\langle e_j|, \quad S_{k,j}^{z} = |e_j\rangle\langle e_j| - |k_j\rangle\langle k_j|,\end{aligned}$$

κ_l is the photon decay rate of CPWR-l, and $\gamma_{\phi_{k,j}}$ ($\gamma_{k,j}$) is the dephasing rate (energy relaxation rate) of the j-th SQ for the decay path $|e\rangle \to |k\rangle$. For simplicity, we set $\kappa_l = \kappa$, $\gamma_{k,j} = \gamma/2$, and $\gamma_{\phi_{k,j}} = \gamma_\phi/2$.

By solving the master equation numerically, we can plot three-dimensional images of the final fidelities versus the two corresponding decoherence factors from among SQ energy relaxation, CPWR photon decay and SQ dephasing. Through

comparing Figs. 7(a)–7(c), one can clearly find that the STA scheme for generating the three-SQ W state is insensitive to SQ energy relaxation and CPWR photon decay, especially the latter. Figure 7(a) shows that the final fidelity can keep over 0.98 even when $\kappa = \gamma = 0.01g$ in the absence of SQ dephasing. However, in the presence of SQ dephasing even when $\gamma_\phi = 0.001g$, SQ dephasing will dominate the effects of decoherence on the final fidelities. But in Fig. 7(b) the final fidelity reaches to very high $F(t_f) = 0.92$ when $\gamma = 0.01g$ and $\gamma_\phi = 0.001g$, so the STA scheme is still robust against decoherence.

By using a set of recent experimental parameters $\{T_{\gamma_\phi} \sim 100\ \mu s,\ g > 2\pi \times 300\ \text{MHz},\ g/\kappa > 10^2,\ g/\gamma > 10^2\}$ with $T_{\gamma_\phi} = 1/\gamma_\phi$,[55] we can obtain the following parameter relations $\{\kappa/g < 10^{-2},\ \gamma/g < 10^{-2},\ \gamma_\phi/g < 10^{-5}\}$. Adopting the parameter relations above, we can achieve the generation of the three-SQ W state with the relatively high fidelity over $F(t_f) = 0.965$.

5. Conclusions

In this paper, we achieve the fast generation of the W state of three SQs via Lewis–Riesenfeld invariants. Based on the skilled simplification of the superconducting system, the STA scheme is used for shortening the operation time greatly. There are no additions of auxiliary driving pulses but experimentally accessible harmonic pulses adopted in the scheme. Adequate numerical simulations show that the STA scheme for the fast generation of the three-SQ W state is robust against variations in parameter and decoherence induced by SQ energy relaxation and CPWR photon decay. The scheme is somewhat sensitive to SQ dephasing, but the high-fidelity three-SQ W state may still be implemented under the current experimental parameters.

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