Generating EPR-entangled mechanical state via feeding finite-bandwidth squeezed light*

Cheng-qian Yi(伊程前), Zhen Yi(伊珍)†, and Wen-ju Gu(谷文举)

Institute of Quantum Optics and Information Photonics, School of Physics and Optoelectronic Engineering, Yangtze University, Jingzhou 434023, *China*

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Einstein–Podolski–Rosen (EPR) entanglement state is achievable by combining two single-mode position and momentum squeezed states at a 50:50 beam-splitter (BS). We investigate the generation of the EPR entangled state of two vibrating membranes in a ring resonator, where clockwise (CW) and counter-clockwise (CCW) travelling-wave modes are driven by lasers and finite-bandwidth squeezed lights. Since the optomechanical coupling depends on the location of the membranes, CW and CCW can couple to the symmetric and antisymmetric combination of mechanical modes for a suitable arrangement, which corresponds to a 50:50 BS mixing. Moreover, by employing the red-detuned driving laser and tuning the central frequency of squeezing field blue detuned from the driving laser with a mechanical frequency, the squeezing property of squeezed light can be perfectly transferred to the mechanical motion in the weak coupling regime. Thus, the BS mixing modes can be position and momentum squeezed by feeding the appropriate squeezed lights respectively, and the EPR entangled mechanical state is obtained. Moreover, cavity-induced mechanical cooling can further suppress the influence of thermal noise on the entangled state.

Keywords: EPR-entangled mechanical state, finite-bandwith squeezed light, squeezing transfer

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1. Introduction

Rapid advancement of microfabrication techniques including ground-state cooling,[1,2] strong optomechanical coupling,[3,4] optomechanically induced transparency (OMIT)[5–7] in cavity optomechanics promotes the control and measurement of massive mechanical oscillators as well as further applications in ultrahigh-precision sensing[8–10] and quantum information processing (QIP),[11–15] and enables tests of fundamental quantum physics.[16] Of particular fundamental interest is to investigate the entanglement in macroscopic scale, which is critical for a deeper understanding of the boundary between the classical and quantum world. Since entanglement is a distinguishing feature of quantum mechanics, where measurements of one system appear to determine the state of the other for a bipartite entangled state, and the peculiar property becomes a resource for constructing hybrid QIP systems exceeding the classical information processing limits.[17] Therefore, the possibility of entangling two mechanical oscillators has been widely investigated, such as exploring the ponderomotive force,[18,19] injecting entangled fields,[20,21] modulating the optomechanical coupling,[22] and using an entanglement-swapping protocol,[23] etc.

In particular, continuous-variable (CV) Einstein–Podolsky–Rosen (EPR) entanglement represents a figure of merit for characterizing the efficacy of entanglement-based CV quantum information protocols, including quantum key distribution (QKD), quantum teleportation, and entanglement swapping.[24] Continuous-variable EPR entanglement can be directly generated via nondegenerate optical down conversion processes,[25] and an alternative approach is to combine two single-mode position and momentum squeezed states at a 50:50 beam splitter (BS),[26] which can be extended to generate the EPR entangled state of massive systems. Hence methods of generating mechanical squeezing can be exploited to generate mechanical EPR entanglement, such as reservoir engineering with the use of two-tone driving,[27] employing a parametric amplifier (PA) inside the optomechanical cavity,[28] ponderomotive squeezing by Coulomb interaction,[29] feeding the finite-bandwidth squeezed reservoir,[30] as well as with the help of a nitrogen-vacancy (NV) center or quantum dots (QDs).[31–33] Moreover, entanglement of macroscopic oscillators is fragile to the unavoidable decoherence and dissipation caused by the surrounding thermal noise. Luckily besides squeezing mechanical motion, squeezed light and PA can help to more efficiently cool the motion of a macroscopic mechanical object below the quantum limit.[28,34]

Here we propose to exploit the combined effects of optomechanical squeezing and mechanical cooling to generate a steady EPR-entangled state of mechanical oscillators in a

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†Corresponding author. E-mail: yizhen@yangtzeu.edu.cn

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ring resonator. We consider the system consisted of two membranes located inside a ring resonator, and both the clockwise (CW) and counter-clockwise (CCW) travelling-wave modes are pumped by lasers and finite-bandwidth squeezed lights. Via carefully tuning the equilibrium position of two membranes, CW and CCW modes can couple to the center-of-mass (COM) motion and relative motion of oscillators respectively, which corresponds to a 50:50 BS mixing of mechanical modes. Furthermore, via employing the red-detuned driving laser and tuning the central frequency of squeezing field blue detuned from the driving laser with a mechanical frequency, the squeezing property of squeezed light can be perfectly transferred to the mechanical motion in the rotating-wave approximation (RWA). Therefore, the BS mixing modes can be position and momentum squeezed by feeding the appropriate squeezed lights respectively, and the EPR-entangled mechanical state is achievable. We focus on the weak optomechanical coupling regime and adiabatically eliminate the cavity field to achieve the explicit expression for EPR entanglement. We find that the entanglement can be enhanced with the increasing bandwith of squeezed lights, and maximum squeezing is obtained in the squeezed white noise limit. Moreover, the mechanical thermal noise can decrease the entanglement, and the critical thermal occupation where the entanglement disappears is also discussed.

The paper is organized as follows. In Section 2, the physical system is introduced. In Section 3, the generation of mechanical EPR-entangled state is derived, and the influence of the bandwidth of squeezed light and mechanical thermal noise on the entanglement is analyzed, and finally the conclusion is drawn.

2. Description of physical model

We consider the system comprising of two vibrating membranes located inside a ring cavity, where the CW and CCW travelling-wave modes are pumped by two independent laser fields accompanied with two finite-bandwidth squeezed lights through two separated partially transmitting mirrors, as shown in Fig. 1.

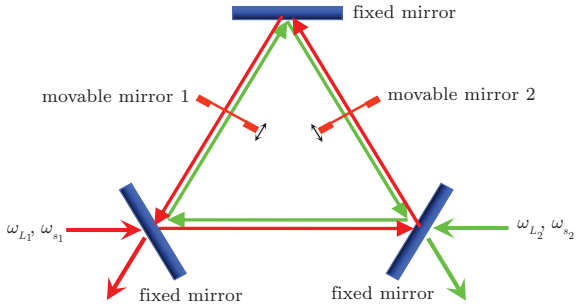


Fig. 1. (color online) Sketch of two mechanical membranes inside a ring resonator. Two laser fields accompanied with two finite-bandwidth squeezed fields drive the CW and CCW modes of the resonator respectively through two separated partially transmitting mirrors.

The membranes vibrate around the equilibrium positions $\bar{x}_j$ $(j=1,2)$ with the identical frequency ω_m and effective mass m, and mechanical modes are characterized by annihilation operators $\hat{b}_j$. The CW and CCW optical modes described by annihilation operators $\hat{a}_i$ $(i=1,2)$ with frequencies ω_i couple to the vibrating membranes via optomechanical interactions, and the Hamiltonian of the system is given by

$$\hat{H}=\sum_{i,j=1}^{2}\left[\omega_i\hat{a}_i^{\dagger}\hat{a}_i+\omega_m\hat{b}_j^{\dagger}\hat{b}_j-g_{ij}\hat{a}_i^{\dagger}\hat{a}_i(\hat{b}_j^{\dagger}+\hat{b}_j)+\mathrm{i}(E_i\,\mathrm{e}^{-\mathrm{i}\omega_{l_i}t}\hat{a}_i^{\dagger}-E_i^{*}\,\mathrm{e}^{\mathrm{i}\omega_{l_i}t}\hat{a}_i)\right], \tag{1}$$

where E_i is the laser driving strength at frequency ω_{l_i}. Optomechanical coupling strength between $\hat{a}_i$ and $\hat{b}_j$ is similar to the "membrane-in-the-middle" arrangement and characterized by[35]

$$g_{ij}=\frac{2\hbar\omega_i}{L}\sqrt{\frac{\hbar}{m\omega_m}}\Lambda_{ij}, \tag{2}$$

where L is the length of resonator, and

$$\Lambda_{ij}=\sin(2k_i\bar{x}_j)\sqrt{\frac{R_j(k_i)}{1-R_j(k_i)\cos^2(2k_i\bar{x}_j)}} \tag{3}$$

with $k_i=\omega_i/c$ the wave number of i-th cavity mode, $R_j(k_i)$ the j-th membrane intensity reflection coefficient for the i-th cavity mode. With a suitable choice of system parameters, such as the equilibrium position of membranes, it is possible to assume that the optomechanical coupling rates are equal in strength with different phases,[36,37] i.e.,

$$g_{11}=g_{12}=g,\quad g_{21}=-g_{22}=g. \tag{4}$$

Therefore, cavity mode $\hat{a}_1$ couples to the COM motion of membranes and $\hat{a}_2$ couples to the relative motion of membranes, as discussed in Ref. [38].

By now we introduce the normal modes which are the symmetrical and antisymmetrical combinations of motion of two membranes

$$\hat{B}_1=(\hat{b}_1+\hat{b}_2)/\sqrt{2},\quad \hat{B}_2=(\hat{b}_1-\hat{b}_2)/\sqrt{2}, \tag{5}$$

which correspond to the BS mixing of two mechanical modes. In the rotating frame of the laser's frequency, the Hamiltonian becomes

$$\hat{H}=\sum_i\left[\Delta_i\hat{a}_i^{\dagger}\hat{a}_i+\omega_m\hat{B}_i^{\dagger}\hat{B}_i-\sqrt{2}g\hat{a}_i^{\dagger}\hat{a}_i(\hat{B}_i^{\dagger}+\hat{B}_i)\right.$$

$$+\mathrm{i}(E_i\hat{a}_i^{\dagger}-E_i^{*}\hat{a}_i)\Big], \tag{6}$$

where the detunings are $\Delta_i=\omega_i-\omega_{l_i}$. In parallel to the coherent driving, each cavity mode is subjected to an independent squeezed field with damping rate κ_i, which is the squeezed output field of an optical parametric oscillator (OPO) below the threshold. The correlation spectra are in the form of minus/sum of two Lorentzian functions and expressed as[39]

$$\begin{aligned}\langle\hat{a}_{\mathrm{in},i}(-\omega)^{\dagger}\hat{a}_{rmin,i}(\omega')\rangle&=\eta_i\frac{\gamma_i}{2}|\varepsilon_i|\left(\frac{1}{\lambda_i^2+\omega^2}-\frac{1}{\mu_i^2+\omega^2}\right)\delta(\omega+\omega'),\\ \langle\hat{a}_{\mathrm{in},i}(\omega)\hat{a}_{\mathrm{in},i}(\omega')\rangle&=\eta_i\frac{\gamma_i}{2}\varepsilon_i\left(\frac{1}{\lambda_i^2+\omega^2}+\frac{1}{\mu_i^2+\omega^2}\right)\delta(\omega+\omega'-2\Delta_s),\end{aligned} \tag{7}$$

where ε_i characterizes the nonlinearity of OPO, γ_i is the damping rate of OPO cavity, $\lambda_i=\gamma_i/2-|\varepsilon_i|$ and $\mu_i=\gamma_i/2+|\varepsilon_i|$ are the bandwidths of the squeezed light, η_i is the ratio between the transmitting rate at the output port and the damping rate, and $\Delta_s=\omega_s-\omega_{l_i}$ is the detuning between central frequency of squeezed light ω_s and the driving laser frequency.

We investigate the optomechanical interaction with the linearization procedure by employing a large laser driving to enhance the coupling, and the cavity and mechanical modes are written as a sum of coherent and quantum fluctuation parts, i.e., $\hat{a}_i=\alpha_i+\delta\hat{a}_i$, $\hat{B}_i=\beta_i+\delta\hat{B}_i$, where $\alpha_i=\langle\hat{a}_i\rangle$ and $\beta_i=\langle\hat{B}_i\rangle$ are the average amplitudes and $\delta\hat{a}_i$ and $\delta\hat{B}_i$ are the quantum fluctuation operators. The mean values obey the equations

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\alpha_i&=-\left(\mathrm{i}\Delta_i+\frac{\kappa_i}{2}\right)\alpha_i+\mathrm{i}\sqrt{2}g\alpha_i(\beta_i^{*}+\beta_i)+E_i,\\ \frac{\mathrm{d}}{\mathrm{d}t}\beta_i&=-\left(\mathrm{i}\omega_m+\frac{\gamma_m}{2}\right)\beta_i+\mathrm{i}\sqrt{2}g|\alpha_i|^2,\end{aligned} \tag{8}$$

from which we can achieve the steady-state mean values

$$\alpha_i=\frac{E_i}{\mathrm{i}\Delta_i'+\kappa_i/2},\quad \beta_i=\frac{\mathrm{i}\sqrt{2}g|\alpha_i|^2}{\mathrm{i}\omega_m+\gamma_{\mathrm{m}}/2}, \tag{9}$$

with the redefined detuning $\Delta_i'=\Delta_i-\sqrt{2}g(\beta_i^{*}+\beta_i)$ and the mechanical damping rate γ_{m}. The linearized Hamiltonian becomes

$$\begin{aligned}\hat{H}=\sum_i\Big[&\Delta_i'\delta\hat{a}_i^{\dagger}\delta\hat{a}_i+\omega_m\delta\hat{B}_i^{\dagger}\delta\hat{B}_i\\ &-G_i(\delta\hat{a}_i^{\dagger}+\delta\hat{a}_i)(\delta\hat{B}_i^{\dagger}+\delta\hat{B}_i)\Big],\end{aligned} \tag{10}$$

where the effective optomechanical coupling strength is $G_i=g\alpha_i$, and α_i can be real by tuning the phase of the driving laser.

3. Preparation of EPR mechanical entanglement

Our motivation for generating an EPR-entangled mechanical state is to combine two single-mode position and momentum squeezed states at a 50:50 BS,[26] where the BS mixing is already obtained in Eq. (5). Therefore, we will focus on the discussion of squeezing properties of normal modes $\delta\hat{B}_i$, which is realizable by transferring the squeezing from the cavity field to mechanical motion. The efficient transfer of squeezing should fulfill two conditions: (i) realization of the BS-type optomechanical interaction via employing a red-detuned driving laser, i.e., $\Delta_i'=\omega_{\mathrm{m}}$, and (ii) the central frequency of squeezing field blue detuned from driving laser with a mechanical frequency,[40] i.e., $\Delta_{\mathrm{s}}=\omega_{\mathrm{m}}$. In the rotating frame of mechanical frequency and making use of RWA under the condition $G_i\ll\omega_{\mathrm{m}}$, the quantum Langevin equations can now be written in the form

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\delta\hat{a}_i&=-\frac{\kappa_i}{2}\delta\hat{a}_i+\mathrm{i}G_i\delta\hat{B}_i+\sqrt{\kappa_i}\hat{a}_{\mathrm{in},i},\\ \frac{\mathrm{d}}{\mathrm{d}t}\delta\hat{B}_i&=-\frac{\gamma_{\mathrm{m}}}{2}\delta\hat{B}_i+\mathrm{i}G_i\delta\hat{a}_i+\sqrt{\gamma_{\mathrm{m}}}\hat{B}_{\mathrm{in},i},\end{aligned} \tag{11}$$

where $\hat{B}_{\mathrm{in},i}$ is the input mechanical thermal noise and obeys the correlations $\langle\hat{B}_{\mathrm{in},i}(-\omega)^{\dagger}\hat{B}_{\mathrm{in},i}(\omega')\rangle=n_{\mathrm{th}}\delta(\omega+\omega')$, $\langle\hat{B}_{\mathrm{in},i}(\omega)\hat{B}_{\mathrm{in},i}(\omega')\rangle=0$. Here

$$n_{\mathrm{th}}=\{\exp[\hbar\omega_{\mathrm{m}}/K_{\mathrm{B}}T]-1\}^{-1}$$

is the average thermal occupation number of the mechanical mode, K_{B} is the Boltzmann constant, and T is the temperature of environment.

Here we focus on the weak optomechanical coupling regime $G_i\ll\kappa_i$, where the cavity mode would reach fast to the steady state on the time scale of optomechanical coupling. Thus the cavity mode can be adiabatically eliminated which equals to

$$\delta\hat{a}_i=\frac{\mathrm{i}G_i}{\kappa_i/2}\delta\hat{B}_i+\sqrt{\kappa_i}\int_{-\infty}^{t}\exp\left[-\frac{\kappa_i}{2}(t-\tau)\right]\hat{a}_{\mathrm{in},i}(\tau)\,\mathrm{d}\tau. \tag{12}$$

Via substituting the expression into Eq. (11), the effective equation for the motion of the oscillator becomes

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\delta\hat{B}_i=&-\left(\frac{\gamma_{\mathrm{m}}}{2}+\frac{\gamma_{\mathrm{opt},i}}{2}\right)\delta\hat{B}_i+\sqrt{\gamma_{\mathrm{opt},i}}\hat{B}_{\mathrm{sq},i}\\ &+\sqrt{\gamma_{\mathrm{m}}}\hat{B}_{\mathrm{in},i},\end{aligned} \tag{13}$$

where $\gamma_{\mathrm{opt},i}=4G_i^2/\kappa_i$ is the cavity-induced damping rate in the resolved-sideband limit,[41,42] and the total damping rate

is $\Gamma_i = \gamma_m + \gamma_{opt,i}$. The mechanical motion is surrounded by two parts of noise, where

$$\hat{B}_{sq,i} = \frac{i\kappa_i}{2}\int_{-\infty}^{t} \exp\left[-\frac{\kappa_i}{2}(t-\tau)\right]\hat{a}_{in,i}(\tau)\,d\tau \tag{14}$$

is the accompanying finite-bandwidth squeezed noise transferred from the cavity field, which corresponds to the non-Markovian Langevin force on mechanical motion.

In order to solve Eq. (13), it is convenient to perform the Fourier transform[43]

$$\delta\hat{B}_i(\omega) = \frac{1}{\sqrt{2\pi}}\int e^{i\omega t}\delta\hat{B}_i(t)\,dt,$$
$$\delta\hat{B}_i(-\omega)^\dagger = \frac{1}{\sqrt{2\pi}}\int e^{i\omega t}\delta\hat{B}_i^\dagger(t)\,dt,$$

and the solution in the frequency domain can be obtained as

$$\delta\hat{B}_i(\omega) = \frac{iG_i\sqrt{\kappa_i}}{(i\omega-\kappa_i/2)(i\omega-\Gamma_i/2)}\hat{a}_{in,i}(\omega) - \frac{\sqrt{\gamma_m}}{i\omega-\Gamma_i/2}\hat{B}_{in,i}(\omega),$$

$$\delta\hat{B}_i(-\omega)^\dagger = \frac{-iG_i\sqrt{\kappa_i}}{(i\omega-\kappa_i/2)(i\omega-\Gamma_i/2)}\hat{a}_{in,i}(-\omega)^\dagger - \frac{\sqrt{\gamma_m}}{i\omega-\Gamma_i/2}\hat{B}_{in,i}(-\omega)^\dagger. \tag{15}$$

In addition, the squeezing of mechanical motion is characterized by the variances of the generalized quadrature operator, which is defined in the frequency domain as

$$\hat{X}_{i,\theta}(\omega) = \frac{1}{\sqrt{2}}[e^{i\theta}\delta\hat{B}_i(\omega) + e^{-i\theta}\delta\hat{B}_i(-\omega)^\dagger]. \tag{16}$$

It represents the position operator $\hat{X}_i$ for $\theta = 0$ and the momentum operator $\hat{P}_i$ for $\theta = -\pi/2$. By the aid of the correlations of the noise source in Eq. (7), the spectrum of quadrature operator defined by

$$S_{X_{i,\theta}}(\omega) = \langle\hat{X}_{i,\theta}(\omega)\hat{X}_{i,\theta}(\omega')\rangle\delta(\omega+\omega')$$

becomes

$$S_{X_{i,\theta}}(\omega) = \frac{G_i^2\kappa_i\eta_i\gamma_i/2}{(\omega^2+\kappa_i^2/4)(\omega^2+\Gamma_i^2/4)}\left(\frac{|\varepsilon_i|-\cos 2\theta\varepsilon_i}{\lambda_i^2+\omega^2} - \frac{|\varepsilon_i|+\cos 2\theta\varepsilon_i}{\mu_i^2+\omega^2}\right) + \frac{G_i^2\kappa_i/2}{\left(\omega^2+\frac{\kappa_i^2}{4}\right)\left(\omega^2+\frac{\Gamma_i^2}{4}\right)} + \frac{\gamma_m(n_{th}+1/2)}{\omega^2+\frac{\Gamma_i^2}{4}}, \tag{17}$$

and the steady-state mean square of quadrature fluctuations in the time domain is determined by the relation[43]

$$\langle\hat{X}_{i,\theta}^2\rangle = \frac{1}{2\pi}\int S_{X_{i,\theta}}(\omega)\,d\omega.$$

In order to achieve the entanglement between the mechanical modes $\hat{b}_1$ and $\hat{b}_2$, we should make the BS-type mixing of modes $\delta\hat{B}_1$ and $\delta\hat{B}_2$ position and momentum squeezed respectively, and the measurement of entanglement is indicated by the degree of squeezing. The explicit criterion for EPR entanglement of $\hat{b}_1$ and $\hat{b}_2$ is characterized by the correlation of observables of quadrature operators $\hat{u}_i = (\hat{b}_i^\dagger + \hat{b}_i)/\sqrt{2}$ and $\hat{v}_i = i(\hat{b}_i^\dagger - \hat{b}_i)/\sqrt{2}$, which is the two-mode squeezing condition and fulfills[44]

$$E = \frac{1}{2}\left[\langle(\hat{u}_1+\hat{u}_2)^2\rangle + \langle(\hat{v}_1-\hat{v}_2)^2\rangle\right] = \langle\hat{X}_1^2\rangle + \langle\hat{P}_2^2\rangle < 1. \tag{18}$$

Therefore, to achieve the optimal entanglement we should obtain the maximum squeezing indicated by Eq. (17), where ε_1 should be real and positive for the position operator $\hat{X}_1$ of $\delta\hat{B}_1$, and ε_2 should be real and negative for the momentum operator $\hat{P}_2$ of $\delta\hat{B}_2$. Moreover, it is feasible to assume the other parameters of two mechanical oscillators are identical via employing the symmetric driving, i.e., $G_i = G$, $\kappa_i = \kappa$, $\Gamma_i = \Gamma$, $\gamma_i = \gamma$, $|\varepsilon_i| = \varepsilon$, $\mu_i = \mu$, and we have the analytical results

$$\langle\hat{X}_1^2\rangle = \langle\hat{P}_2^2\rangle = -\frac{G^2\eta\gamma\varepsilon 8(2\mu+\kappa+\Gamma)}{\mu\Gamma(\kappa+\Gamma)(2\mu+\kappa)(2\mu+\Gamma)} + \frac{2G^2}{\Gamma(\kappa+\Gamma)} + \frac{\gamma_m}{\Gamma}(n_{th}+1/2). \tag{19}$$

The criterion for EPR entanglement becomes $E = 2\langle\hat{X}_1^2\rangle < 1$. Due to the weak optomechanical coupling compared to the cavity damping rate we have $\Gamma \ll \kappa$, and the strong cavity-induced mechanical cooling rate compared to high-Q mechanical oscillator $\gamma_m \ll \gamma_{opt}$, the variance can be further simplified as

$$\langle\hat{X}_1^2\rangle \approx -\frac{2\eta\gamma\varepsilon}{\mu(2\mu+\Gamma)} + \frac{1}{2} + \frac{\gamma_m}{\Gamma}(n_{th}+1/2), \tag{20}$$

which is contributed by the squeezed light, vacuum fluctuation and cavity-induced suppression of thermal noise, respectively.

The EPR entanglement of modes $\hat{b}_1$ and $\hat{b}_2$ is achievable when $\langle\hat{X}_1^2\rangle < 1/2$. For squeezed light produced from the ideal single-sided OPO cavity, $\eta \to 1$. In Fig. 2 we plot the EPR entanglement as a function of the damping rate of the OPO cavity with the parameters applied in Ref. [40]. The damping rate connects to the bandwidth of squeezed light, and figure 2 shows that the increase of bandwidth will improve the mechanical entanglement. Especially, in the squeezed white noise limit parameters

$$N = \frac{\varepsilon\gamma}{2}\left(\frac{1}{\lambda^2} - \frac{1}{\mu^2}\right), \quad M = \frac{\varepsilon\gamma}{2}\left(\frac{1}{\lambda^2} + \frac{1}{\mu^2}\right)$$

with squeezing bandwidth $\lambda \to \infty$, $\mu \to \infty$, the variance becomes

$$\langle \hat{X}_1^2 \rangle = N - M + \frac{1}{2} + \frac{\gamma_m}{\Gamma}(n_{th} + 1/2). \tag{21}$$

For example, with the parameters given in Fig. 2, $E = 0.184$ for $n_{th} = 10$, which is consistent with the numerical result.

However, the unavoidable mechanical thermal noise can decrease the squeezing and thus EPR entanglement, as shown in Fig. 2. There exists a critical thermal number corresponding to $E = 1$, which is achieved from Eq. (20) as

$$n_{th}^c = \frac{2\eta\gamma\varepsilon\Gamma}{\mu(2\mu+\Gamma)\gamma_m} - \frac{1}{2}. \tag{22}$$

When the initial thermal number is below n_{th}^c for a given squeezed light, two mechanical modes are EPR entangled, and otherwise not entangled.

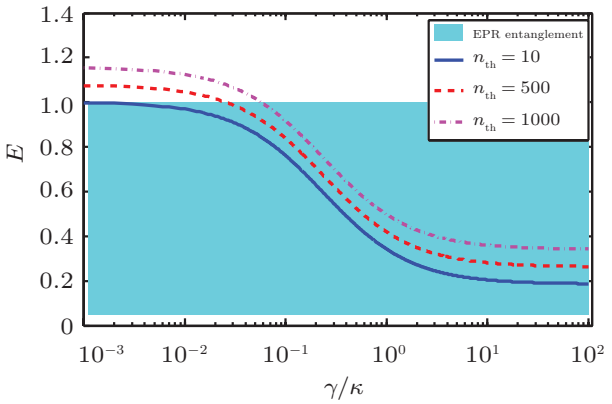


Fig. 2. (color online) EPR entanglement versus the damping rate γ of the OPO cavity with different mechanical thermal numbers $n_{th} = 10$ (blue solid), $n_{th} = 500$ (red dashed), $n_{th} = 1000$ (magenta dot-dashed). The other parameters are $\omega_m/2\pi = 1$ MHz, $\kappa/2\pi = 380$ kHz, $G/2\pi = 110$ kHz, $\gamma_m/2\pi = 10$ Hz, $\eta = 1$, and $\varepsilon = 0.2\gamma$.

Moreover, the maximum squeezing of the output field of the OPO cavity occurs at the threshold $\gamma/2 = \varepsilon$, and then equation (20) gives

$$\langle \hat{X}_1^2 \rangle = \frac{\Gamma/2}{4\varepsilon + \Gamma} + \frac{\gamma_m}{\Gamma}(n_{th} + 1/2), \tag{23}$$

which indicates the maximum squeezing

$$\langle \hat{X}_1^2 \rangle = \gamma_m(n_{th} + 1/2)/\Gamma$$

in the limit $\varepsilon \gg \Gamma$. The squeezing is mainly affected by the unavoidable mechanical thermal noise. With the strong cavity-induced mechanical cooling, $\langle \hat{X}_1^2 \rangle \to 0$, and mechanical modes approach an ideal EPR entanglement.

4. Conclusion

In conclusion, following the proposal of the generation of an EPR entanglement state by combining two single-mode position and momentum squeezed states at a 50:50 BS, we study the generation of the EPR-entangled mechanical state of two vibrating membranes in a ring resonator, where CW and CCW travelling-wave modes are driven by lasers and finite-bandwidth squeezed lights. Via tuning the location of membranes, CW and CCW couples to the symmetric and anti-symmetric combination of mechanical modes, corresponding to a 50:50 BS mixing. Moreover, for each cavity mode via employing a red-detuned driving laser and tuning the central frequency of squeezing field blue detuned from the driving laser with a mechanical frequency, the squeezing property of squeezed light can be perfectly transferred to the mechanical motion in the RWA. Thus, the BS mixing modes can be position and momentum squeezed by feeding the appropriate squeezed lights, and the EPR-entangled mechanical state is achievable. In addition, cavity-induced mechanical cooling can significantly suppress the influence of thermal noise to achieve an ideal entanglement.

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