Adjustable quantum coherence effects in a hybrid optomechanical system*

Wen-Qing Xia(夏文清), Ya-Fei Yu(於亚飞), and Zhi-Ming Zhang(张智明)†

Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials & Devices (SIPSE), and Guangdong Provincial Key Laboratory of Quantum Engineering & Quantum Materials, South China Normal University, Guangzhou 510006, *China*

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We propose a system for achieving some adjustable quantum coherence effects, including the normal-mode splitting (NMS), the optomechanically induced transparency (OMIT), and the optomechanically induced absorption (OMIA). In this system, two tunnel-coupled optomechanical cavities are each driven by a coupling field and coupled to an atomic ensemble. Besides, one of the cavities is also injected with a probe field. When the system works under different conditions, we can obtain the NMS, the OMIT, and the OMIA, respectively. These effects can be flexibly adjusted by the tunnel coupling between the two cavities, the power of the coupling lasers, and the coupling strength between the atomic ensembles and the cavity fields. Furthermore, we can realize the OMIT even if the oscillating mirrors have relatively larger decay rates.

Keywords: optomechanically induced transparency, optomechanically induced absorption, normal-mode splitting

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1. Introduction

Some quantum interference phenomena in the cavity optomechanical system (OMS), such as the optomechanically induced transparency (OMIT)[1,2] and the optomechanically induced absorption (OMIA),[3] have attracted a great deal of attention. These effects promote the developments of the subluminal (superluminal) propagation, optical storage, optical switches, and so on.[4–7] On the other hand, the appearance of the normal-mode splitting (NMS) as an unambiguous evidence for the strong coupling between quantum systems has been investigated. For instance, the optomechanical NMS for recognizing the strong coupling of a cavity mode to a mechanical mode was observed.[8] The optomechanical systems have developed from pure systems to hybrid systems,[9–11] and some remarkable phenomena were found in the hybrid optomechanical systems. For example, a new way for detecting entanglement was presented.[12] A tunable multi-channel inverse OMIT was investigated in the optomechanical system assisted with the Coulomb interaction.[13] A nonlinear optomechanical system with two movable mirrors, in which EIT can be controlled and manipulated, was theoretically studied.[14]

With the hybrid optomechanial systems, it is worth noting that a crowd of novel optical phenomena were found under the assistance of an atomic ensemble. For example, the bistable phenomenon in a Fabry–Perot (FP) cavity, assisted by a cold two-level atomic ensemble, was observed.[15] The properties of multipartite entanglement, the Einstein–Podolshy–Rosen (EPR) state, and the mechanical vibrator cooled to the ground state were demonstrated in the hybrid system containing an atomic ensemble.[16–18] It was proven that the radiation pressure between the cavity field and the movable mirror can be enhanced by adding atomic ensembles.[19] It was also shown that a large number of atoms can broaden the window of OMIT.[20,21] The above studies indicate that the atomic ensemble plays an important role in the researches of optomechanics.

We also know from the previous researches that the two-mode cavity system has a more flexible controllability and becomes a research hot spot.[22–25] For example, Agarwal and Huang studied the inverse EIT (IEIT) and showed that its effect is equivalent to the perfect coherent absorption.[25] In a double-cavity optomechanical system, the perfect coherent absorption, transmission, and synthesis were achieved.[26]

In this paper, we propose a system for achieving some adjustable quantum coherence effects, including the NMS, the OMIT, and the OMIA. The system consists of two tunnel-coupled optomechanical cavities, in which there are two atomic ensembles. The two cavities are also driven by some external fields. When the system works under different conditions, we can obtain the NMS, the OMIT, and the OMIA, respectively. These effects can be flexibly adjusted by the power of the coupling lasers and the coupling strength between the atomic ensembles and the cavity fields. This is different from the system proposed in Ref. [27], in which the system just achieves these coherent processes but cannot adjust them flexibly. Comparing with the system in Ref. [26], the high quality factor of the oscillator is not required in our system.

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†Corresponding author. E-mail: zmzhang@scnu.edu.cn

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The organization of this paper is as follows. In Section 2, we propose the model and solve the quantum Langevin equations. In Section 3, we study the NMS in this system without any coupling fields. In Section 4, we discuss how to adjust the phenomena of OMIT and OMIA in this system by inputting one coupling field. Section 5 presents a brief summary.

2. Model and general formalism

The system studied in this work is shown schematically in Fig. 1. The system consists of two single-cavity optomechanical systems and two two-level atomic ensembles with N_i $(i=1,2)$ atoms. Each single-cavity optomechanical system is constructed with one fixed end mirror and one partially transmissive oscillating mirror with the effective mass m, the frequency $\omega_{\rm m}$, and the decay rate $\gamma_{\rm m}$. Two identical optical cavities of length L, frequency ω_0, and decay rate κ are coupled via the photon tunneling and are driven by two coupling fields with the frequencies of $\omega_{\rm c}$ and $\omega_{\rm d}$, respectively. At the same time, one probe field with the frequency $\omega_{\rm p}$ is injected into the left optical cavity to detect the transparency properties of the overall system. We define $P_{\rm c}$, $P_{\rm d}$, and $P_{\rm p}$ as the input powers of the relevant fields and $\varepsilon_{\rm c}=\sqrt{2\kappa P_{\rm c}/\hbar\omega_{\rm c}}$, $\varepsilon_{\rm d}=\sqrt{2\kappa P_{\rm d}/\hbar\omega_{\rm d}}$, and $\varepsilon_{\rm p}=\sqrt{2\kappa P_{\rm p}/\hbar\omega_{\rm p}}$ as the real field amplitudes. The Hamiltonian of the system can be written as

$$\begin{aligned}H &= \hbar\omega_0 c_1^+c_1+\hbar\omega_0 c_2^+c_2+\hbar\omega_{\rm m}b_1^+b_1+\hbar\omega_{\rm m}b_2^+b_2\\&+\frac{\hbar\omega_{\rm a}}{2}\sum_{\alpha=1}^{N_1}\sigma_z^\alpha+\frac{\hbar\omega_{\rm a}}{2}\sum_{\beta=1}^{N_2}\sigma_z^\beta\\&-\hbar g_{\rm m1}c_1^+c_1(b_1^++b_1)-\hbar g_{\rm m2}c_2^+c_2(b_2^++b_2)\\&-\hbar\lambda(c_1^+c_2+c_2^+c_1)\\&+\hbar g_{\rm ac}\sum_{\alpha=1}^{N_1}(c_1\sigma_+^\alpha+c_1^+\sigma_-^\alpha)+\hbar g_{\rm ac}\sum_{\beta=1}^{N_2}(c_2\sigma_+^\beta+c_2^+\sigma_-^\beta)\\&+{\rm i}\hbar\varepsilon_{\rm c}(c_1^+{\rm e}^{-{\rm i}\omega_{\rm c}t}-c_1{\rm e}^{{\rm i}\omega_{\rm c}t})\\&+{\rm i}\hbar\varepsilon_{\rm p}\left(c_1^+{\rm e}^{-{\rm i}\omega_{\rm p}t}-c_1{\rm e}^{{\rm i}\omega_{\rm p}t}\right)\\&+{\rm i}\hbar\varepsilon_{\rm d}(c_2^+{\rm e}^{-{\rm i}\omega_{\rm d}t}-c_2{\rm e}^{{\rm i}\omega_{\rm d}t}).\end{aligned}\tag{1}$$

Here the first four terms respectively describe the free Hamiltonian of the two cavities and the two oscillating mirrors, in which c_i (c_i^+) and b_i (b_i^+) $(i=1,2)$ are the annihilation (creation) operators of the cavity fields and the mechanical modes, respectively. The fifth and the sixth terms are the free Hamiltonian of the two atom ensembles. $\sigma_z=|e\rangle\langle e|-|g\rangle\langle g|$, $\sigma_+=|e\rangle\langle g|$, and $\sigma_-=|g\rangle\langle e|$ are the Pauli matrices for each atom. N_i is the number of atoms in the i-th atomic ensemble. We assume that all the atoms have the same excited (ground) state $|e\rangle$ $(|g\rangle)$. As a result, they have the same transition frequency $\omega_{\rm a}$. The seventh and the eighth terms are the interaction Hamiltonian showing the radiation pressure interaction between the cavities and the oscillating mirrors with optomechanical coupling rate $g_{\rm mi}$. For simplicity, we assume $g_{\rm m1}=g_{\rm m2}=g_{\rm m}=\frac{\omega_0}{L}\sqrt{\hbar/2m\omega_{\rm m}}$. The ninth term expresses the interaction between the two cavities, where λ is the tunnel coupling rate, which is proportional to the transmittance of the two oscillating mirrors in the two cavities. The tenth and the eleventh terms give the atom–field interaction Hamiltonian describing the coupling between each cavity and the atomic ensemble in the cavity. Here, $g_{\rm ac}=-\mu\sqrt{\omega_{\rm a}/2V\varepsilon_0}$ is the atom–field coupling rate, where μ is the electric dipole between the two levels, V describes the cavity mode volume, and ε_0 is the permittivity of the vacuum. We restrain each atomic ensemble in a thin layer whose size is much smaller than the wavelength of the cavity field in the cavity axial direction, so all the atoms have the same coupling strength $g_{\rm ac}$ with the cavity field. The last three terms describe the interaction of the two coupling fields with the cavity fields and that of the probe field with the left cavity field. It is necessary to point out that there is no direct interaction between the atoms and the mirrors, the interaction between them indirectly relies on the cavity fields.

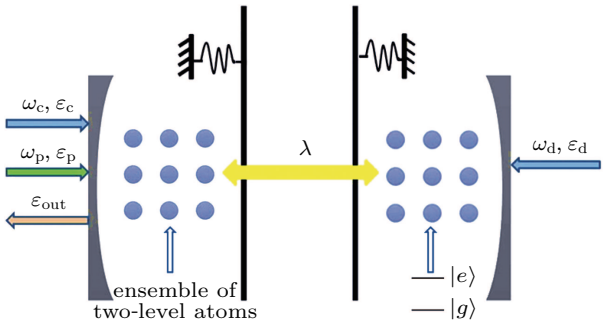


Fig. 1. (color online) Schematic diagram of the system.

As all the atoms have the same transition frequency $\omega_{\rm a}$, we can define the collective quasi-spin operators of the two atomic ensembles $A_1=\frac{1}{\sqrt{N_1}}\sum_{\alpha=1}^{N_1}\sigma_-^\alpha$ and $A_2=\frac{1}{\sqrt{N_2}}\sum_{\beta=1}^{N_2}\sigma_-^\beta$ to simplify the form of the Hamiltonian. In the large-N_i and low-excitation conditions, the collective quasi-spin operators satisfy the bosonic commutation relation[21] $[A_i,A_i^+]\approx 1$. Then we have $\sum_{\alpha=1}^{N_1}\sigma_z^\alpha=2{A_1}^+A_1-N_1$ and $\sum_{\beta=1}^{N_2}\sigma_z^\beta=2{A_2}^+A_2-N_2$, and the Hamiltonian can be simplified as

$$\begin{aligned}H &= \hbar\omega_0 c_1^+c_1+\hbar\omega_0 c_2^+c_2+\hbar\omega_{\rm m}b_1^+b_1\\&+\hbar\omega_{\rm m}b_2^+b_2+\hbar\omega_{\rm a}A_1^+A_1+\hbar\omega_{\rm a}A_2^+A_2\\&-\hbar g_{\rm m}c_1^+c_1(b_1^++b_1)-\hbar g_{\rm m}c_2^+c_2(b_2^++b_2)\\&-\hbar\lambda(c_1^+c_2+c_2^+c_1)+\hbar J_1(c_1A_1^++c_1^+A_1)\\&+\hbar J_2(c_2A_2^++c_2^+A_2)\\&+{\rm i}\hbar\varepsilon_{\rm c}(c_1^+{\rm e}^{-{\rm i}\omega_{\rm c}t}-c_1{\rm e}^{{\rm i}\omega_{\rm c}t})\\&+{\rm i}\hbar\varepsilon_{\rm p}\left(c_1^+{\rm e}^{-{\rm i}\omega_{\rm p}t}-c_1{\rm e}^{{\rm i}\omega_{\rm p}t}\right)\\&+{\rm i}\hbar\varepsilon_{\rm d}(c_2^+{\rm e}^{-{\rm i}\omega_{\rm d}t}-c_2{\rm e}^{{\rm i}\omega_{\rm d}t}),\end{aligned}\tag{2}$$

where $J_1=\sqrt{N_1}g_{\rm ac}$ and $J_2=\sqrt{N_2}g_{\rm ac}$ are the effective coupling strengths between the atomic ensembles and the cavity fields. We can see that the coupling coefficient J_i can be enhanced by increasing the atomic number N_i in each ensemble. In the interaction picture with respect to $H_0=\hbar\omega_{\rm c}(c_1^+c_1+A_1^+A_1)+\hbar\omega_{\rm d}(c_2^+c_2+A_2^+A_2)$, the Hamiltonian can be rewritten as

$$\begin{aligned}H &= \hbar\Delta_{\rm c}c_1^+c_1+\hbar\Delta_{\rm d}c_2^+c_2\\ &\quad+\hbar\omega_{\rm m}b_1^+b_1+\hbar\omega_{\rm m}b_2^+b_2+\hbar\Delta_{\rm a1}A_1^+A_1+\hbar\Delta_{\rm a2}A_2^+A_2\\ &\quad-\hbar g_{\rm m}c_1^+c_1(b_1^++b_1)\\ &\quad-\hbar g_{\rm m}c_2^+c_2(b_2^++b_2)-\hbar\lambda\,{\rm e}^{-{\rm i}\Delta_{\rm L}t}(c_1^+c_2+c_2^+c_1)\\ &\quad+\hbar J_1(c_1A_1^++c_1^+A_1)+\hbar J_2(c_2A_2^++c_2^+A_2)\\ &\quad+{\rm i}\hbar\varepsilon_{\rm c}(c_1^+-c_1)+{\rm i}\hbar\varepsilon_{\rm p}\left(c_1^+\,{\rm e}^{-{\rm i}\delta t}-c_1\,{\rm e}^{{\rm i}\delta t}\right)\\ &\quad+{\rm i}\hbar\varepsilon_{\rm d}(c_2^+-c_2),\end{aligned}\tag{3}$$

where $\delta=\omega_{\rm p}-\omega_{\rm c}$, $\Delta_{\rm c}=\omega_0-\omega_{\rm c}$, $\Delta_{\rm d}=\omega_0-\omega_{\rm d}$, $\Delta_{\rm a1}=\omega_{\rm a}-\omega_{\rm c}$, $\Delta_{\rm a2}=\omega_{\rm a}-\omega_{\rm d}$, and $\Delta_{\rm L}=\omega_{\rm c}-\omega_{\rm d}$ are the detunings. Without loss of generality, we assume that the frequencies of the two coupling fields are the same, i.e., $\omega_{\rm c}=\omega_{\rm d}$, then $\Delta_{\rm c}=\Delta_{\rm d}=\Delta$, $\Delta_{\rm a1}=\Delta_{\rm a2}=\Delta_{\rm a}$, and $\Delta_{\rm L}=0$. By using Hamiltonian (3) and these conditions, we obtain the following Heisenberg–Langevin equations:

$$\begin{aligned}\dot b_1 &= -\left(\frac{\gamma_{\rm m}}{2}+{\rm i}\omega_{\rm m}\right)b_1+{\rm i}g_{\rm m}c_1^+c_1+\sqrt{\gamma_{\rm m}}b_1^{\rm in}(t),\\ \dot b_2 &= -\left(\frac{\gamma_{\rm m}}{2}+{\rm i}\omega_{\rm m}\right)b_2+{\rm i}g_{\rm m}c_2^+c_2+\sqrt{\gamma_{\rm m}}b_2^{\rm in}(t),\\ \dot c_1 &= -(\kappa+{\rm i}\Delta)c_1+{\rm i}g_{\rm m}c_1(b_1^++b_1)\\ &\quad-{\rm i}J_1A_1+{\rm i}\lambda c_2+\varepsilon_{\rm c}+\varepsilon_{\rm p}\,{\rm e}^{-{\rm i}\delta t}+\sqrt{2\kappa}c_1^{\rm in}(t),\\ \dot c_2 &= -(\kappa+{\rm i}\Delta)c_2+{\rm i}g_{\rm m}c_2(b_2^++b_2)\\ &\quad-{\rm i}J_2A_2+{\rm i}\lambda c_1+\varepsilon_{\rm d}+\sqrt{2\kappa}c_2^{\rm in}(t),\\ \dot A_1 &= -(\gamma_{\rm a}+{\rm i}\Delta_{\rm a})A_1-{\rm i}J_1c_1+\sqrt{2\gamma_{\rm a}}A_1^{\rm in}(t),\\ \dot A_2 &= -(\gamma_{\rm a}+{\rm i}\Delta_{\rm a})A_2-{\rm i}J_2c_2+\sqrt{2\gamma_{\rm a}}A_2^{\rm in}(t),\end{aligned}\tag{4}$$

in which we have included the noise and damping terms. Here $b_i^{\rm in}(t)$, $c_i^{\rm in}(t)$, and $A_i^{\rm in}(t)$ are the thermal noise on the i-th oscillating mirror, the input quantum vacuum noise operator in the i-th cavity, and the quantum noise operator of the i-th atomic ensemble, respectively, and have zero mean value, i.e., $\langle b_i^{\rm in}(t)\rangle=\langle c_i^{\rm in}(t)\rangle=\langle A_i^{\rm in}(t)\rangle=0$. Moreover, the input quantum vacuum noise operators of all cavities and atomic ensembles satisfy the nonvanishing correlation functions $\left\langle c_i^{\rm in}(t)c_i^{\rm in^+}(t)\right\rangle=\delta(t-t')$ and $\left\langle A_i^{\rm in}(t)A_i^{\rm in^+}(t)\right\rangle=\delta(t-t')$.[29] The thermal noise operators of the oscillating mirrors obey the temperature-dependent correlation function[30]

$$\left\langle b_i^{\rm in}b_i^{\rm in^+}(t)\right\rangle=\int\omega\,{\rm e}^{-{\rm i}\omega(t-t')}N(\omega)\,{\rm d}\omega,$$

where

$$N(\omega)=\frac{\gamma_{\rm m}}{2\pi\omega_{\rm m}}\left[1+\coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)\right],$$

$k_{\rm B}$ is the Boltzmann constant, and T is the temperature of the oscillating mirrors. Each operator can be expressed as the sum of its steady value and its small fluctuation, i.e., $\hat h=h_{\rm s}+\delta\hat h$. By solving the equation of motion for the fluctuation $\delta\hat h$, we can understand the dynamical behaviors of the system.

First we can obtain the steady solutions of Eq. (4) as follows:

$$\begin{aligned}b_{1\rm s} &= \frac{{\rm i}g_{\rm m}|c_{1\rm s}|^2}{\gamma_{\rm m}/2+{\rm i}\omega_{\rm m}},\quad b_{2\rm s}=\frac{{\rm i}g_{\rm m}|c_{2\rm s}|^2}{\gamma_{\rm m}/2+{\rm i}\omega_{\rm m}},\\ c_{1\rm s} &= \frac{\left(\kappa+{\rm i}\Delta_2+\dfrac{J_2^2}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}}\right)\varepsilon_{\rm c}+{\rm i}\lambda\varepsilon_{\rm d}}{\left(\kappa+{\rm i}\Delta_1+\dfrac{J_1^2}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}}\right)\left(\kappa+{\rm i}\Delta_2+\dfrac{J_2^2}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}}\right)+\lambda^2},\\ c_{2\rm s} &= \frac{\left(\kappa+{\rm i}\Delta_1+\dfrac{J_1^2}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}}\right)\varepsilon_{\rm d}+{\rm i}\lambda\varepsilon_{\rm c}}{\left(\kappa+{\rm i}\Delta_1+\dfrac{J_1^2}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}}\right)\left(\kappa+{\rm i}\Delta_2+\dfrac{J_2^2}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}}\right)+\lambda^2},\\ A_{1\rm s} &= \frac{{\rm i}J_1c_{1\rm s}}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}},\quad A_{2\rm s}=\frac{{\rm i}J_2c_{2\rm s}}{\gamma_{\rm a}+{\rm i}\Delta_{\rm a}},\end{aligned}\tag{5}$$

where $\Delta_1=\Delta-g_{\rm m}(b_{1\rm s}^*+b_{1\rm s})$ and $\Delta_2=\Delta-g_{\rm m}(b_{2\rm s}^*+b_{2\rm s})$ give the effective detunings between the cavity modes and the coupling fields. Since $g_{\rm m}|b_{i\rm s}|$ $(i=1,2)$ are typically very small compared to $\omega_{\rm m}$, we can linearize this problem, and the linearized quantum Langevin equation can be obtained as

$$\begin{aligned}\delta\dot b_1 &= -\left(\frac{\gamma_{\rm m}}{2}+{\rm i}\omega_{\rm m}\right)\delta b_1\\ &\quad+{\rm i}g_{\rm m}(c_{1\rm s}^*\delta c_1+c_{1\rm s}\delta c_1^+)+\sqrt{\gamma_{\rm m}}b_1^{\rm in}(t),\\ \delta\dot b_2 &= -\left(\frac{\gamma_{\rm m}}{2}+{\rm i}\omega_{\rm m}\right)\delta b_2\\ &\quad+{\rm i}g_{\rm m}(c_{2\rm s}^*\delta c_2+c_{2\rm s}\delta c_2^+)+\sqrt{\gamma_{\rm m}}b_2^{\rm in}(t)),\\ \delta\dot c_1 &= -(\kappa+{\rm i}\Delta_1)\delta c_1+{\rm i}g_{\rm m}c_{1\rm s}(\delta b_1^++\delta b_1)\\ &\quad-{\rm i}J_1\delta A_1+{\rm i}\lambda\delta c_2+\varepsilon_{\rm p}\,{\rm e}^{-{\rm i}\delta t}+\sqrt{2\kappa}c_1^{\rm in}(t),\\ \delta\dot c_2 &= -(\kappa+{\rm i}\Delta_2)\delta c_2+{\rm i}g_{\rm m}c_{2\rm s}(\delta b_2^++\delta b_2)\\ &\quad-{\rm i}J_2\delta A_2+{\rm i}\lambda\delta c_1+\sqrt{2\kappa}c_2^{\rm in}(t),\\ \delta\dot A_1 &= -(\gamma_{\rm a}+{\rm i}\Delta_{\rm a})\delta A_1-{\rm i}J_1\delta c_1+\sqrt{2\gamma_{\rm a}}A_1^{\rm in}(t),\\ \delta\dot A_2 &= -(\gamma_{\rm a}+{\rm i}\Delta_{\rm a})\delta A_2-{\rm i}J_2\delta c_2+\sqrt{2\gamma_{\rm a}}A_2^{\rm in}(t).\end{aligned}\tag{6}$$

Then we make transformations $\delta b_{1,2}\to\delta b_{1,2}\,{\rm e}^{-{\rm i}\omega_{\rm m}t}$, $\delta c_{1,2}\to\delta c_{1,2}\,{\rm e}^{-{\rm i}\Delta_{1,2}t}$, $\delta A_{1,2}\to\delta A_{1,2}\,{\rm e}^{-{\rm i}\Delta_{\rm a}t}$, and assume $\Delta=\Delta_{\rm a}$, i.e., the detuning between the cavity field and the coupling field is the same as that between the atomic ensemble and the coupling field. We drive the mechanical modes with red-detuned coupling fields, i.e., $\Delta_1\approx\Delta_2\approx\omega_{\rm m}$. Under these conditions, equation (6) can be simplified to

$$\begin{aligned}\delta\dot b_1 &= -\frac{\gamma_{\rm m}}{2}\delta b_1+{\rm i}g_{\rm m}c_{1\rm s}^*\delta c_1+\sqrt{\gamma_{\rm m}}b_1^{\rm in}(t),\\ \delta\dot b_2 &= -\frac{\gamma_{\rm m}}{2}\delta b_2+{\rm i}g_{\rm m}c_{2\rm s}^*\delta c_2+\sqrt{\gamma_{\rm m}}b_2^{\rm in}(t)),\\ \delta\dot c_1 &= -\kappa\delta c_1+{\rm i}g_{\rm m}c_{1\rm s}\delta b_1\\ &\quad-{\rm i}J_1\delta A_1+{\rm i}\lambda\delta c_2+\varepsilon_{\rm p}\,{\rm e}^{-{\rm i}\Omega t}+\sqrt{2\kappa}c_1^{\rm in}(t),\end{aligned}$$

$$\begin{aligned}
\delta\dot{c}_2 &= -\kappa\delta c_2 + \mathrm{i}g_{\mathrm{m}}c_{2\mathrm{s}}\delta b_2 \\
&\quad - \mathrm{i}J_2\delta A_2 + \mathrm{i}\lambda\delta c_1 + \sqrt{2\kappa}c_2^{\mathrm{in}}(t), \\
\delta\dot{A}_1 &= -\gamma_{\mathrm{a}}\delta A_1 - \mathrm{i}J_1\delta c_1 + \sqrt{2\gamma_{\mathrm{a}}}A_1^{\mathrm{in}}(t), \\
\delta\dot{A}_2 &= -\gamma_{\mathrm{a}}\delta A_2 - \mathrm{i}J_2\delta c_2 + \sqrt{2\gamma_{\mathrm{a}}}A_2^{\mathrm{in}}(t),
\end{aligned} \tag{7}$$

where $\Omega = \delta - \omega_{\mathrm{m}} \approx \omega_{\mathrm{p}} - \omega_0$ is the detuning of the probe field from the cavity resonance frequency. Now we begin to derive an expression for the transmission coefficient of the system. For this reason, we use the input–output relation[29]

$$\varepsilon_{\mathrm{out}} + \varepsilon_{\mathrm{p}}\,\mathrm{e}^{-\mathrm{i}\Omega t} = \kappa\langle\delta c_1\rangle, \tag{8}$$

and make use of the ansatz

$$\langle\delta h\rangle = h_-\,\mathrm{e}^{-\mathrm{i}\Omega t} + h_+\,\mathrm{e}^{\mathrm{i}\Omega t}(h = c_i, b_i, A_i \quad (i = 1,2)), \tag{9}$$

$$\varepsilon_{\mathrm{out}} = \varepsilon_{\mathrm{out}-}\,\mathrm{e}^{-\mathrm{i}\Omega t} + \varepsilon_{out+}\,\mathrm{e}^{\mathrm{i}\Omega t}, \tag{10}$$

where $\varepsilon_{\mathrm{out}-}$ and $\varepsilon_{\mathrm{out}+}$ are the components of $\varepsilon_{\mathrm{out}}$ oscillating at ω_{p} and $2\omega_{\mathrm{c}} - \omega_{\mathrm{p}}$ in the original frame. From Eqs. (8)–(10), we have

$$\varepsilon_{\mathrm{out}-} = \kappa c_{1-} - \varepsilon_{\mathrm{p}}. \tag{11}$$

By taking mean values in the set of equations (7) and using Eq. (9), we can obtain

$$c_{1-} = \frac{\kappa - \mathrm{i}\Omega + \dfrac{g_{\mathrm{m}}^2|c_{2\mathrm{s}}|^2}{\gamma_{\mathrm{m}}/2 - \mathrm{i}\Omega} + \dfrac{J_2^2}{\gamma_{\mathrm{a}} - \mathrm{i}\Omega}}{\left(\kappa - \mathrm{i}\Omega + \dfrac{g_{\mathrm{m}}^2|c_{1\mathrm{s}}|^2}{\gamma_{\mathrm{m}}/2 - \mathrm{i}\Omega} + \dfrac{J_1^2}{\gamma_{\mathrm{a}} - \mathrm{i}\Omega}\right)\left(\kappa - \mathrm{i}\Omega + \dfrac{g_{\mathrm{m}}^2|c_{2\mathrm{s}}|^2}{\gamma_{\mathrm{m}}/2 - \mathrm{i}\Omega} + \dfrac{J_2^2}{\gamma_{\mathrm{a}} - \mathrm{i}\Omega}\right) + \lambda^2}\varepsilon_{\mathrm{p}}. \tag{12}$$

By substituting Eq. (12) into Eq. (11), we obtain the expression for the transmission coefficient

$$T = \left|\frac{\varepsilon_{\mathrm{out}-}}{\varepsilon_{\mathrm{p}}}\right|^2 = \left|\frac{\kappa\left(\kappa - \mathrm{i}\Omega + \dfrac{g_{\mathrm{m}}^2|c_{2\mathrm{s}}|^2}{\gamma_{\mathrm{m}}/2 - \mathrm{i}\Omega} + \dfrac{J_2^2}{\gamma_{\mathrm{a}} - \mathrm{i}\Omega}\right)}{\left(\kappa - \mathrm{i}\Omega + \dfrac{g_{\mathrm{m}}^2|c_{1\mathrm{s}}|^2}{\gamma_{\mathrm{m}}/2 - \mathrm{i}\Omega} + \dfrac{J_1^2}{\gamma_{\mathrm{a}} - \mathrm{i}\Omega}\right)\left(\kappa - \mathrm{i}\Omega + \dfrac{g_{\mathrm{m}}^2|c_{2\mathrm{s}}|^2}{\gamma_{\mathrm{m}}/2 - \mathrm{i}\Omega} + \dfrac{J_2^2}{\gamma_{\mathrm{a}} - \mathrm{i}\Omega}\right) + \lambda^2} - 1\right|^2. \tag{13}$$

There are three kinds of direct interactions in the system: the coupling between the two cavities, the interaction between the cavities and the oscillators, and the interaction of the cavities with the atomic ensembles. In the following, we will discuss the impacts of these three kinds of interactions on the output spectrum. In this paper, we choose realistic parameters in a recent experiment:[17] the wavelength of the control fields $\lambda_{\mathrm{c,d}} = 2\pi c/\omega_{\mathrm{c,d}} = 1064$ nm, $L = 25$ mm, $m = 145$ ng, $\kappa = 2\pi \times 215 \times 10^3$ Hz, $\gamma_{\mathrm{a}} = 2\pi \times 400$ Hz, $\omega_{\mathrm{m}} = 2\pi \times 947 \times 10^3$ Hz, $\gamma_{\mathrm{m}} = 2\pi \times 141$ Hz (the quality factor $Q = \omega_{\mathrm{m}}/\gamma_{\mathrm{m}} = 6716$). In addition, we have considered the situation in which the atoms are resonant with the cavity, i.e., $\omega_{\mathrm{a}} = \omega_0$, this is corresponding to $\Delta = \Delta_{\mathrm{a}} \approx \omega_{\mathrm{m}}$. This assumption has also been used by the well-known work.[17]

3. Adjustable normal-mode splitting

In this section, we focus on the impact of the tunnel coupling between the two cavities on the output spectrum, i.e., we consider the situation in which $\varepsilon_{\mathrm{c}} = \varepsilon_{\mathrm{d}} = J_1 = J_2 = 0$. In this case, the transmission coefficient (13) can be simplified as

$$T = 1 - \frac{\kappa^2(\kappa^2 + \Omega^2 + 2\lambda^2)}{(\kappa^2 + \Omega^2 + \lambda^2)^2 - 4\lambda^2\Omega^2}. \tag{14}$$

Based on Eq. (14), we plot the transmission coefficient T as a function of the detuning Ω in Fig. 2(a). It can be seen that when the tunnel coupling between the two cavities is weak, there appears a single resonance around $\Omega = 0$ in the probe output spectrum. If the tunnel coupling λ is tuned into the strong regime, i.e., $\lambda \geq \kappa$, the resonance splits into two, i.e.,

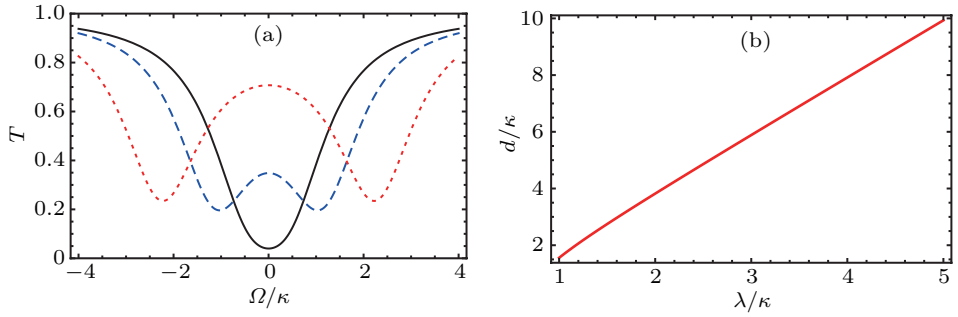


Fig. 2. (color online) (a) The normal-mode spiting with the tunnel coupling λ as the parameter: black-solid line: $\lambda = 0.5\kappa$; blue-dashed line: $\lambda = 1.2\kappa$; red-dotted line: $\lambda = 2.3\kappa$. (b) The normalized separation d/κ as a function of the normalized tunnel coupling λ/κ. Here $\kappa = 2\pi \times 215 \times 10^3$ Hz.[17]

there appears the normal-mode splitting, and the separation between the two resonances is increased with the tunnel coupling λ. From Eq. (14), we can obtain the frequencies of the two resonance peaks as

$$\Omega_{\pm} = \pm\sqrt{-(2\lambda^2+\kappa^2)+\lambda\sqrt{9\lambda^2+4\kappa^2}}, \tag{15}$$

and their separation

$$d = \Omega_{+} - \Omega_{-} = 2\sqrt{-(2\lambda^2+\kappa^2)+\lambda\sqrt{9\lambda^2+4\kappa^2}}. \tag{16}$$

We plot the normalized separation d/κ versus the normalized tunnel coupling λ/κ in Fig. 2(b). It can be seen that the separation of the split window is approximately linearly increased with increasing tunnel coupling when the tunnel coupling λ is switched into the strong regime, i.e., $\lambda \geq \kappa$. So we can control λ to achieve and adjust the behavior of the NMS.

4. Adjustable optomechanically induced transparency and optomechanically induced absorption

In what follows, we examine the nature of the output probe spectrum when we input the coupling field into the system. We find that the optomechanically induced transparency (OMIT) and the optomechanically induced absorption (OMIA) can be achieved in this system, and their windows can be adjusted by controlling the power of the coupling field and the interaction of the cavity field with the atomic ensemble. What is more, in the optomechanical system with a single cavity, the window of the OMIT is not perfect at a higher decay γ_m (low quality factor) of the oscillating mirror. However, we will show that it is still perfect in our system even if a larger mechanical decay γ_m exists. To see this clearly, we choose a large mechanical decay $\gamma_m = 2\pi \times 14.1 \times 10^3$ HZ and a low quality factor $Q = \omega_m/\gamma_m = 67$, which are based on Ref. [28], to complete the numerical simulation.

4.1. Adjustable OMIT in the case of $\varepsilon_d = J_2 = 0$

In this subsection, we study the transparency spectra in the situation of $\varepsilon_d = J_2 = 0$, i.e., there is no right atomic ensemble and no right coupling field. The transparency spectra are plotted in Figs. 3(a) and 3(b), respectively. Figure 3(a) shows the transparency spectrum with the power P_c of the left coupling field as the parameter. It can be seen that there appears the normal-mode splitting when $P_c = 0$ and $\lambda = 1.2\kappa$ (the coupling between the two cavities is in the strong coupling regime), this is in agreement with the discussion in Section 3. When $P_c > 0$, i.e., when we input the left coupling field, there appears the optomechanically induced transparency, and the transparency window near $\Omega = 0$ becomes higher and wider with increasing P_c. Figure 3(b) shows the transparency spectrum with the coupling strength J_1 between the left atomic ensemble and the left cavity field as the parameter. We can see that the transparency window near $\Omega = 0$ becomes higher and wider with increasing J_1. It should be pointed out that we can obtain a perfect transparency $(T = 1)$ when J_1 is large enough. These features can be explained as follows.

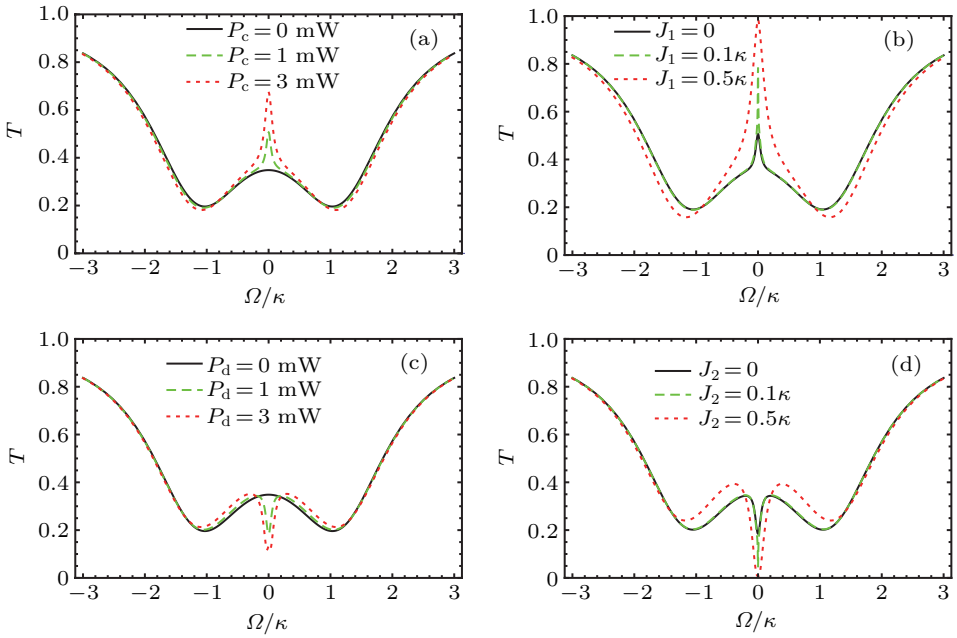


Fig. 3. (color online) (a) The transparency spectrum with the left coupling field power P_c as a parameter in the case of $P_d = 0$ and $J_2 = 0$. (b) The transparency spectrum with the coupling strength J_1 as a parameter in the case of $P_d = 0$ and $J_2 = 0$. (c) The transparency spectrum with the right coupling field power P_d as a parameter in the case of $P_c = 0$ and $J_1 = 0$. (d) The transparency spectrum with the coupling strength J_2 as a parameter in the case of $P_c = 0$ and $J_1 = 0$. The other parameters are the wavelength of the control fields $\lambda_{c,d} = 2\pi c/\omega_{c,d} = 1064$ nm, $L = 25$ mm, $m = 145$ ng, $\gamma_a = 2\pi \times 400$ Hz, $\omega_m = 2\pi \times 947 \times 10^3$ Hz, $\gamma_m = 2\pi \times 14.1$ kHz, $\lambda = 1.2\kappa$, and $\kappa = 2\pi \times 215 \times 10^3$ Hz.

According to the previous researches, the OMIT window can be adjusted via the effective optomechanical coupling strength $G_{\rm OM1}=g_{\rm m}^2|c_{1\rm s}|^2$ in the left cavity, which is equivalent to the Rabi frequency in the atomic electromagnetically induced transparency.[2,3] By using our previous assumption $\Delta_1\approx\Delta_2\approx\Delta_{\rm a}\approx\omega_{\rm m}$ in the present situation ($\varepsilon_{\rm d}=J_2=0$), we can obtain from Eq. (5) the effective optomechanical coupling strength between the left cavity and the mechanical mode, which is proportional to $|c_{1\rm s}|^2$, and

$$|c_{1\rm s}|^2=\frac{\varepsilon_{\rm c}^2}{(\kappa_1')^2+(\Delta_1')^2}, \tag{17}$$

with

$$\begin{aligned}\kappa_1'&=\kappa+\frac{J_1^2\gamma_{\rm a}}{\gamma_{\rm a}^2+\omega_{\rm m}^2}+\frac{\lambda^2\kappa}{\kappa^2+\omega_{\rm m}^2},\\ \Delta_1'&=\omega_{\rm m}-\frac{J_1^2\omega_{\rm m}}{\gamma_{\rm a}^2+\omega_{\rm m}^2}-\frac{\lambda^2\omega_{\rm m}}{\kappa^2+\omega_{\rm m}^2},\end{aligned} \tag{18}$$

where κ_1' and Δ_1' are the effective decay and the effective detuning, respectively. We can see from Eq. (17) that the effective optomechanical coupling strength $G_{\rm OM1}$ is increased with increasing power $P_{\rm c}$ of the left coupling field ($\varepsilon_{\rm c}=\sqrt{2\kappa P_{\rm c}/\hbar\omega_{\rm c}}$). We also find that our result is the same as that in Ref. [27] if there is no atomic ensemble, i.e., if $J_1=0$. Now because of the existence of the atomic ensemble, the effective decay κ_1' adds a term $J_1^2\gamma_{\rm a}/(\gamma_{\rm a}^2+\omega_{\rm m}^2)$ and the effective detuning Δ_1' reduces a term $J_1^2\omega_{\rm m}/(\gamma_{\rm a}^2+\omega_{\rm m}^2)$, their overall effect is that the effective optomechanical coupling is increased when the system is in the sideband resolved limit, i.e., $\omega_{\rm m}\gg\kappa$.[25] This broadens and heightens the window of OMIT.

It should be pointed out that, in our system, even if the oscillating mirror has a relatively larger decay rate ($\gamma_{\rm m}=2\pi\times14.1$ kHz), the perfect transparency window can also be obtained by reasonably controlling the power of the coupling field and the interaction strength between the left cavity and the atomic ensemble, as shown in Figs. 3(a) and 3(b). This is better than that in Ref. [27], in which the perfect transparency window only appears for a relatively smaller decay rate ($\gamma_{\rm m}=2\pi\times141$ Hz).

4.2. Adjustable OMIA in the case of $\varepsilon_{\rm c}=J_1=0$

In this subsection, we study the transparency spectra in the situation of $\varepsilon_{\rm c}=J_1=0$, i.e., there is no left atomic ensemble and no left coupling field. The transparency spectra are plotted in Figs. 3(c) and 3(d), respectively. Figure 3(c) shows the transparency spectrum with the power $P_{\rm d}$ of the right coupling field as the parameter. It can be seen that there appears the normal-mode splitting when $P_{\rm d}=0$ and $\lambda=1.2\kappa$ (the coupling between the two cavities is in the strong coupling regime), this is in agreement with the discussion in Section 3. When $P_{\rm d}>0$, i.e., when we input the right coupling field, instead of a resonance peak near $\Omega=0$ as in Fig. 3(a), there appears a resonance valley, this is the so-called optomechanically induced absorption. The absorption window near $\Omega=0$ becomes deeper and wider with increasing $P_{\rm d}$. Figure 3(d) shows the transparency spectrum with the coupling strength J_2 between the right atomic ensemble and the right cavity field as the parameter. We can see that the absorption window near $\Omega=0$ becomes deeper and wider with increasing J_2. It should be pointed out that one can obtain a full absorption ($T=0$) when J_2 is large enough. These features can be explained as follows.

It was shown that the width and the depth of the OMIA can be adjusted by changing the effective optomechanical coupling $G_{\rm OM2}=g_{\rm m}^2|c_{2\rm s}|^2$ in the right cavity.[3,27] For the present situation ($\varepsilon_{\rm c}=J_1=0$), we can obtain

$$|c_{2\rm s}|^2=\frac{\varepsilon_{\rm d}^2}{(\kappa_2')^2+(\Delta_2')^2}, \tag{19}$$

with

$$\begin{aligned}\kappa_2'&=\kappa+\frac{J_2^2\gamma_{\rm a}}{\gamma_{\rm a}^2+\omega_{\rm m}^2}+\frac{\lambda^2\kappa}{\kappa^2+\omega_{\rm m}^2},\\ \Delta_2'&=\omega_{\rm m}-\frac{J_2^2\omega_{\rm m}}{\gamma_{\rm a}^2+\omega_{\rm m}^2}-\frac{\lambda^2\omega_{\rm m}}{\kappa^2+\omega_{\rm m}^2}.\end{aligned} \tag{20}$$

It can be seen from Eq. (19) that $G_{\rm OM2}=g_{\rm m}^2|c_{2\rm s}|^2$ is proportional to the power $P_{\rm d}$ ($\varepsilon_{\rm d}=\sqrt{2\kappa P_{\rm d}/\hbar\omega_{\rm d}}$) of the right coupling field. Equation (20) tells us that when adding the right atomic ensemble, the effective detuning Δ_2' subtracts an extra term $J_2^2\omega_{\rm m}/(\gamma_{\rm a}^2+\omega_{\rm m}^2)$, the effective decay κ_2' adds an extra term $J_2^2\gamma_{\rm a}/(\gamma_{\rm a}^2+\omega_{\rm m}^2)$, and these lead to that the effective optomechanical coupling in the right cavity is enhanced in the sideband resolved limit. Thus we can broaden and deepen the windows of OMIA by increasing the power $P_{\rm d}$ of the right coupling field and the interaction strength J_2 between the right cavity and the right atomic ensemble.

We have also examined the situation in which both the two atomic ensembles and the two coupling fields exist, but not found any more interesting features, so we do not report the situation here.

5. Conclusion

We have theoretically investigated the normal-mode splitting, the optomechanically induced transparency, and the optomechanically induced absorption in an optomechanical system, which consists of two tunnel-coupled optomechanical cavities assisted by two atomic ensembles. Our analytical and numerical results show that the NMS appears when the tunnel coupling strength between the two cavities is strong, and the OMIT (OMIA) is influenced by the optomechanical interaction in the left (right) cavity. If only the optomechanical interaction in the left cavity exists, the OMIT is achieved and its window can be adjusted by controlling the power of the left coupling field and the coupling strength between the left cavity

and the left atomic ensemble. In this situation, we can obtain the perfect transparency window even if there is a large mechanical decay. On the other hand, when switching off the left coupling field and turning on the right coupling field to drive the right cavity, there appears an absorption dip. The depth and width of the absorption dip can be similarly adjusted by changing the power of the right coupling field and the interaction strength of the right cavity with the right atomic ensemble.

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