Macroscopic resonant tunneling in an rf-SQUID flux qubit under a single-cycle sinusoidal driving*

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We experimentally demonstrate the observation of macroscopic resonant tunneling (MRT) phenomenon of the macroscopic distinct flux states in a radio frequency superconducting quantum interference device (rf-SQUID) under a single-cycle sinusoidal driving. The population of the qubit exhibits interference patterns corresponding to resonant tunneling peaks between states in the adjacent potential wells. The dynamics of the qubit depends significantly on the amplitude, frequency, and initial phase of the driving signal. We do the numerical simulations considering the intra-well and inter-well relaxation mechanism, which agree well with the experimental results. This approach provides an effective way to manipulate the qubit population by adjusting the parameters of the external driving field.

Keywords: superconducting quantum interference device (SQUID), macroscopic resonant tunneling (MRT), Landau–Zener transition

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1. Introduction

Macroscopic resonant tunneling (MRT) has attracted much more research interest in recent years. It has been shown that MRT is one of the most representative macroscopic quantum behaviors based on superconducting Josephson junctions and rf-SQUID system. Such MRT has been demonstrated in current-biased or flux-biased Josephson junctions.[1–7] Also resonant tunneling between energy levels in two adjacent potential wells in a superconducting quantum interference device (SQUID) has been obtained.[8–18] It has also been shown that Landau–Zener transition can enhance the resonant tunneling rate.[19–21] MRT has been used to prove that the low-frequency flux noise in both Josephson junctions and rf-SQUID system is the crucial source of quantum noise.[22–26]

In this article, we experimentally demonstrate the observation of MRT between macroscopic distinct states in an rf-SQUID system, which is driven by an external single-cycle sinusoidal driving signal. The symmetric double-well potential would be detuned under an external driving force. Resonant tunneling between two macroscopic fluxoid wells of the rf-SQUID has been observed when the energy levels in the adjacent wells are aligned. MRT depends significantly on the amplitude, frequency, and initial phase of the external driving field, which agrees with our numerical calculation. The paper is organized as follows. In Section 2, we give the measurement setup and experimental data. In Section 3, we model the system and calculate the energy-level of the rf-SQUID flux qubit with parameters measured in the experiment. Using the dissipative Landau–Zener transition mechanism, we numerically calculate the population distribution properties of the rf-SQUID under external sinusoidal driving. This is followed by our conclusion in the last section.

2. Experimental results

Our rf-SQUID sample is made of an $Nb/AlO_X/Nb$ Josephson junction and measured in a dilution refrigerator (< 20 mK, Oxford Triton400). The dynamics of an rf-SQUID can be analogous to a particle of mass Φ moving in a potential well.[27,28] The schematic of the measurement is shown in Fig. 1(a). Figures 1(b) and 1(c) show the measuring time profile with different initial phases 0 and π in single-cycle sinusoidal signal (generated by Agilent 33220), respectively. The initial state of the rf-SQUID is prepared in the left well of the symmetric double-well potential. The potential changes when the rf-SQUID is irradiated with the weak external sinusoidal driving. The energy levels change simultaneously. When the energy levels in the two adjacent wells are aligned, resonant tunneling between two wells will occur. A dc-SQUID is applied to read out the clockwise or anticlockwise states of the rf-SQUID, which corresponds to the particle being in the left or right potential well. The process is repeated 2000 times in order to calculate P_R, the probability when the qubit is in the

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right well.

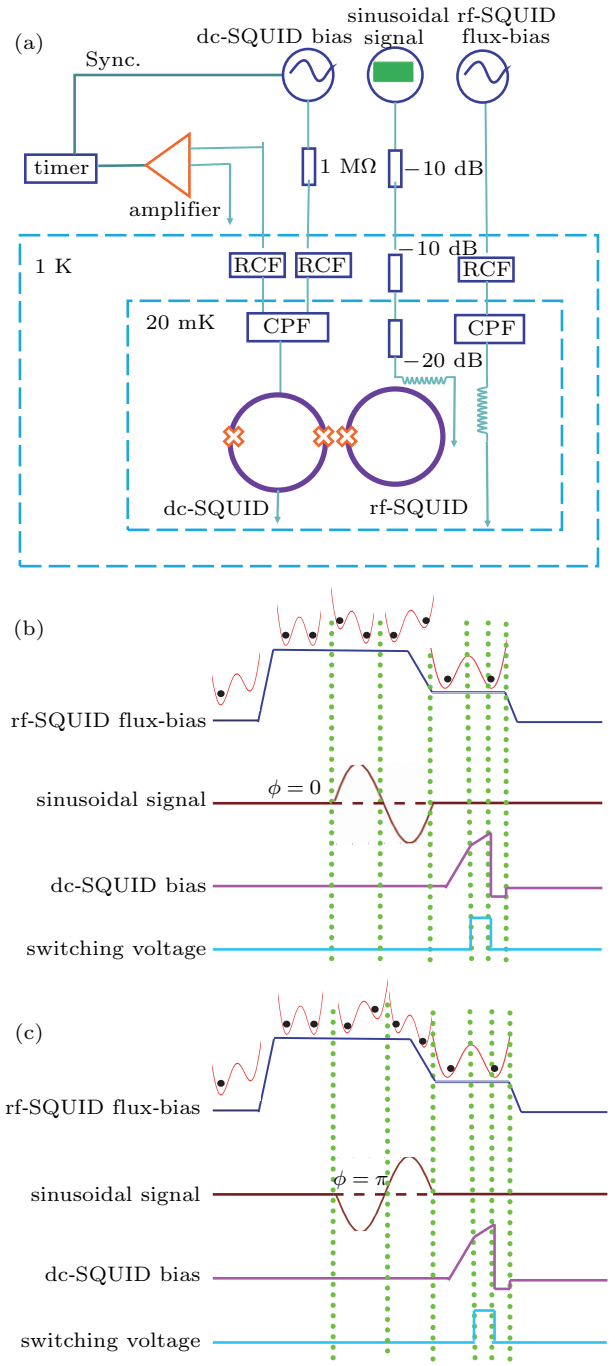


Fig. 1. (color online) (a) Schematic diagram of the measurement setup. (b) Time profile of manipulation and measurement with initial phase 0. (c) Time profile of manipulation and measurement with initial phase π.

The probability P_R as a function of the frequency and amplitude of the signal is shown in Fig. 2(a) and Fig. 2(b). As we can see, along the amplitude axis P_R shows several peaks at certain values. At these values, P_R has a non-monotonous dependence on frequency. From the parameter measured independently, we convert the amplitude to the flux unit and find that the spacing of adjacent peaks is equal to that of the anti-crossings in the energy-level structure in Fig. 3(b), which means the behavior of P_R is caused by the resonant tunneling between the quantized states in adjacent potential wells.

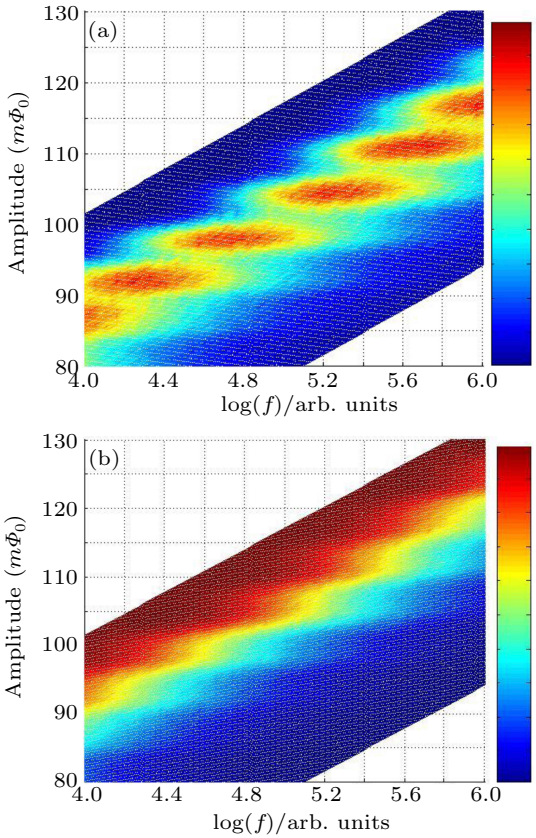


Fig. 2. (color online) (a) Relationship between P_R and signal frequency with different amplitude, the initial phase of the signal is 0. (b) Relationship between P_R and signal frequency with different amplitude, the initial phase of the signal is π.

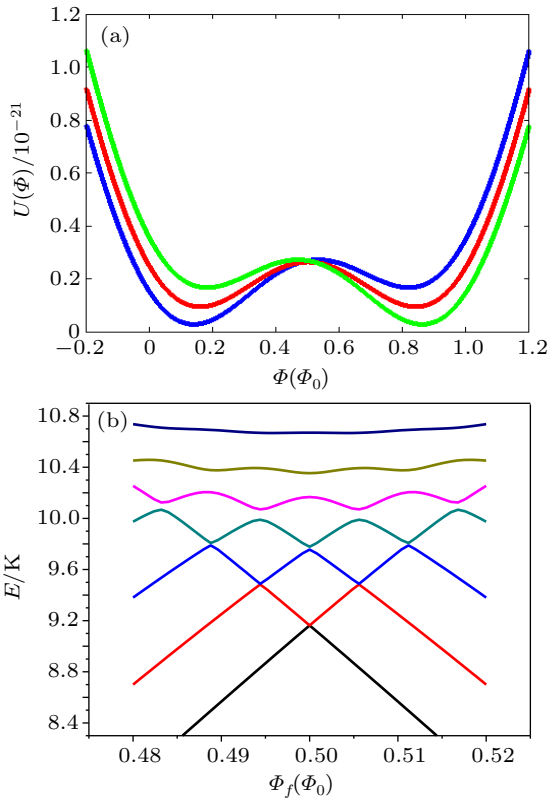


Fig. 3. (color online) (a) Schematic picture of double-well potential of the rf-SQUID flux qubit driven by a single-cycle sinusoidal signal. The initial state is a symmetric double-well potential (red line) in Fig. 2(a). (b) Energy-level structure of the rf-SQUID flux qubit with parameters measured in the experiment.

3. Theoretical analysis

The Hamiltonian of the system is $H = H_0 + H_1$, where H_0 is the Hamiltonian of the rf-SQUID system and H_1 is the Hamiltonian of external sinusoidal driving field

$$\begin{aligned} H_0 &= \frac{P_\Phi^2}{2m} + U(\Phi, \Phi_f^q) \\ &= \frac{P_\Phi^2}{2m} + U_0 \left\{ \frac{1}{2} \left[\frac{2\pi(\Phi - \Phi_f^q)}{\Phi_0} \right]^2 - \beta_L \cos\left(\frac{2\pi\Phi}{\Phi_0} \right) \right\}, \quad (1) \\ H_1 &= A\sin(\omega t) = A\sin(2\pi f t + \phi), \quad (2) \end{aligned}$$

where $P_\Phi = -\mathrm{i}\hbar\partial/\partial\Phi$, $\beta_L = 2\pi L I_c/\Phi_0$ and Φ_0 is the flux quantum. A, f, and ϕ are the amplitude, frequency, and initial phase of the sinusoidal signal, respectively. Figure 3 shows the double-well potential of the rf-SQUID flux qubit using parameters determined experimentally. The critical current $I_c = 0.8$ μA, the inductance of the superconducting loop $L = 1080$ pH, and the capacitance of the junction $C = 90$ fF, thus $\beta_L = 2.6$. The potential has a symmetric double well separated by a barrier ΔU_0, $\Delta U_0/k_B \approx 11$ K and k_B is the Boltzmann constant. The double-well potential will be tilted when the system is driven by a weak single-cycle electromagnetic signal, which causes the double-well potential to oscillate as $\Delta U_0 + A\sin(2\pi f t + \phi)$ and particle to tunnel between double-well potential. The driving frequency f increases from 10^4 Hz to 10^6 Hz. The driving field will also affect the corresponding energy-levels of the rf-SQUID flux qubit and cause anti-crossings at certain values of the flux bias as shown in Fig. 3(b).

In order to explore the dynamics of MRT in the double-well of rf-SQUID flux qubit with external driving, we start from a two-level approach developed by Amin and Averin.[22] It has been proved that this two-level model can also be applied to resonant tunneling problem in other double-well systems. In the presence of low-frequency flux noise, the resonant tunneling rate from the initial well to the target well will have a Gaussian line shape, which is given by

$$\Gamma_{0\to1}(\varepsilon) = \sqrt{\frac{\pi}{8}} \frac{\Delta^2}{W} \exp\left[-\frac{(\varepsilon - \varepsilon_p)^2}{2W^2} \right], \quad (3)$$

where $|0\rangle$ and $|1\rangle$ are the two lowest states in the left and right well. It is interesting to see that in the presence of low frequency flux noise, the resonant tunneling rate is shifted to $\pm\varepsilon_p$, so the first resonant tunneling peak will split into two peaks with a distance of $2\varepsilon_p$ for $\Gamma_{0\to1}$ and $\Gamma_{1\to0}$. The resonant peak agrees well with the Gaussian function (3). This result has been experimentally demonstrated by Harris, Johansson, *et al.*[23] However, the two-level approach only considers the tunneling between the two lowest states and the effect of other higher energy states in the double-well has been ignored. Here the two-level model should be generalized to the resonant tunneling between higher states in the wells. We also take intra-well and inter-well relaxation into consideration. If the ground state $|0\rangle$ in the initial well is aligned with the $|n\rangle$ excited state of the final well, the relaxation rate of the $|n\rangle$ excited state is $\Gamma = n\Gamma_1$,[29] where Γ_1 is the relaxation rate of the first excited state. The resonant tunneling rate is given by

$$\Gamma_{0\to n}(\varepsilon) = \sqrt{\frac{\pi}{8}} \frac{\Delta_n^2}{W} \mathrm{Re}\left[\omega\left(\frac{\varepsilon - \varepsilon_p + \mathrm{i}n\gamma_1}{\sqrt{2}W} \right) \right], \quad (4)$$

where $\gamma_1 = \Gamma_1/2$ and Δ_n is the tunnel splitting between the ground state $|0\rangle$ in the initial well and the $|n\rangle$ excited state of the final well. For quantum noise, $\varepsilon_p \neq 0$, the high resonant peaks are also shifted just like that in the two-level approach.

In our experiment, the low-frequency Gaussian-white noise has a bandwidth of about 9 MHz. The Landau–Zener transition rate between quantum states $|i,L\rangle$ (left well, $i = 0,1,2,\ldots$) and $|j,R\rangle$ (right well, $j = 0,1,2,\ldots$) can be written as

$$W_{ij}(\varepsilon_{ij}, A) = \frac{\Delta_{ij}^2}{2} \sum_n \frac{\Gamma_2 \mathrm{J}_n^2(x)}{(\varepsilon_{ij} - n\omega)^2 + \Gamma_2^2}, \quad (5)$$

where $\Gamma_2 = 1/T_2$ is the dephasing rate and $\mathrm{J}_n(x)$ are the first kind of Bessel functions with $x = A/\omega$. The time evolution of population for state $|j,R\rangle$ can be calculated by a rate equation

$$\begin{aligned} \frac{\mathrm{d}P_{j,R}}{\mathrm{d}t} &= -\sum_i W_{ji} P_{j,R} + \sum_i W_{ij} P_{i,L} \\ &\quad - \sum_{j'} \Gamma_{j\to j'} P_{j,R} + \sum_{j'} \Gamma_{j'\to j} P_{j',R} \\ &\quad - \sum_i \Gamma_{j\to i} P_{j,R} + \sum_i \Gamma_{i\to j} P_{i,L}. \quad (6) \end{aligned}$$

In order to calculate the population distribution of the flux qubit, we adopt two different decoherence effects. The first one $\Gamma_{j\to j'}$ is the intra-well relaxation rate from $|j,R\rangle$ to $|j',R\rangle$, which causes the dissipation within the same well. The other one $\Gamma_{j\to i}$ is the inter-well relaxation rate from $|j,R\rangle$ to $|i,L\rangle$. Then we calculate the rate equation in the stationary case.

Figures 4(a) and 4(b) show P_R as a function of frequency and amplitude of a single-cycle sinusoidal field with the experimental parameters, the initial phases of the signal are 0 and π, respectively. The probability of the qubit appears. Interference patterns apppear in P_R, which correspond to resonant tunneling peaks between states in the adjacent potential wells. The numerical results show good consistency with the experimental ones.

When the initial phase of the signal is 0, in the first half cycle of the sinusoidal waveform, the left well is lifted and the right well is lowered. The particle tunnels mostly from the left well to the right one. It is noted that when the energy levels in the two adjacent wells are aligned, resonant tunneling, MRT, will occur. In the second half cycle, the right well is lifted and the left well is lowered. The dynamics is similar as that in the first half cycle. When the amplitude of the driving field is too

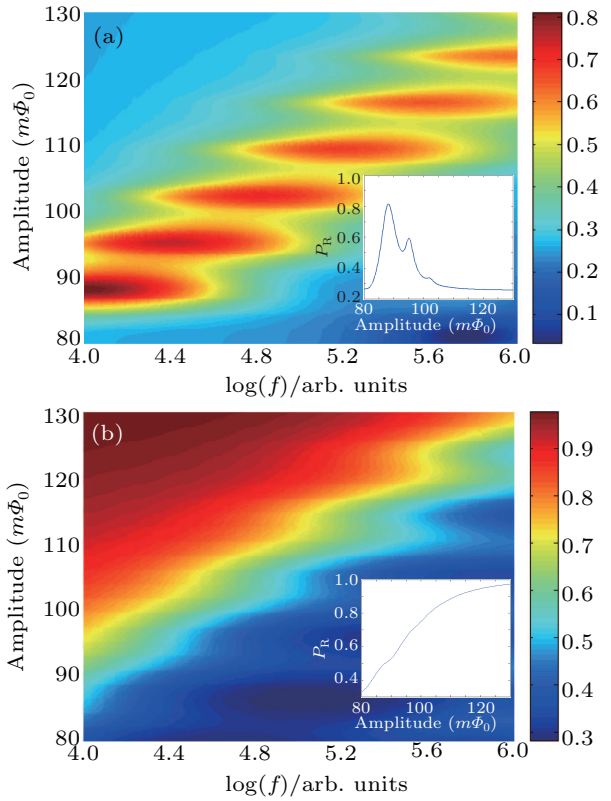


Fig. 4. (color online) Numerical simulation P_R as a function of frequency and amplitude when the qubit is driven by a single-cycle sinusoidal signal. The initial phases of the signal are 0 (Fig. 4(a)) and π (Fig. 4(b)), respectively. The inset shows P_R as a function of amplitude of the sinusoidal field when the frequency is 10 kHz.

small, the potential well is not tilted heavily and MRT is forbidden by the potential barrier. On the other hand, when the amplitude of the driving field is too large, in the second half cycle, the particle is totally kicked to the left well. Because the particle is initialized in the left well, the total effect manifests itself as peaks as shown in Fig. 2(a) and Fig. 4(a).

Due to the different tunneling rate when the ground state in one well is aligned to different excited states in the other well, the peaks are only visible in the corresponding range of frequency and amplitude of the driving field. The spacing of the peaks is equal to that of the anti-crossing in the spectrum (Fig. 3(b)) where the energy levels in the wells are aligned.

When the initial phase of the signal is π, because the particle is initiated in the left well, so the first half cycle of the driving field has no effect on the population. In the second half cycle, the same process happens as that in the first half cycle when the initial phase is 0. The difference is when the amplitude is large, the particle will be totally kicked to the right well, thus the peaks disappear as shown in Fig. 2(b) and Fig. 4(b).

Because the dynamics of P_R depends heavily on the initial phase of the driving field, we also numerically calculate P_R with different initial phase of the driving field, as shown in Fig. 5, and its dependence on the amplitude when the frequency is fixed to 10 kHz is also shown in the corresponding inset.

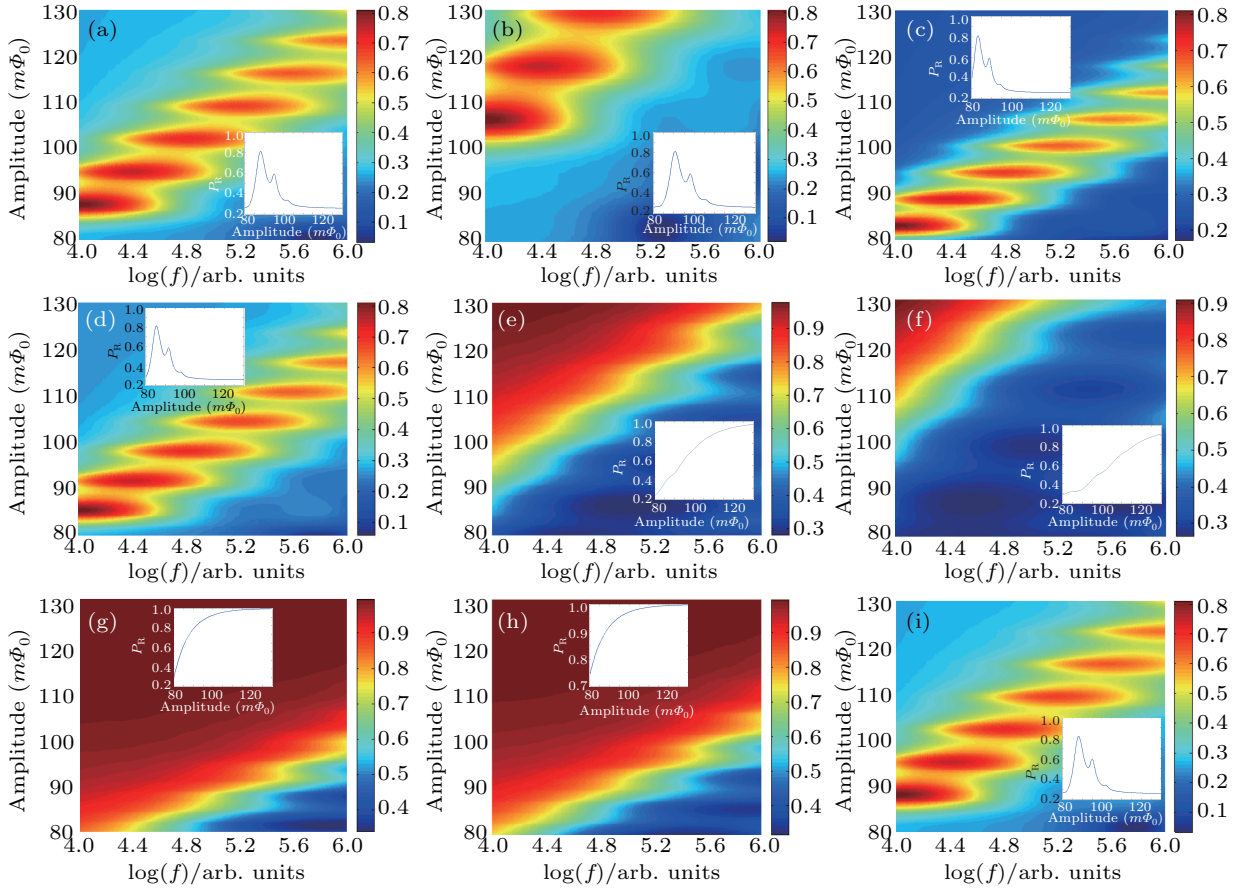


Fig. 5. (color online) Numerical calculations of P_R with different initial phase ϕ. The inset shows its dependence on the amplitude when the frequency is 10 kHz. (a) $\phi = 0$; (b) $\phi = \pi/4$; (c) $\phi = \pi/2$; (d) $\phi = 3\pi/4$; (e) $\phi = \pi$; (f) $\phi = 5\pi/4$; (g) $\phi = 3\pi/2$; (h) $\phi = 7\pi/4$; (i) $\phi = 2\pi$.

4. Conclusion

To summarize, using an rf-SQUID system, we have investigated the macroscopic resonant tunneling phenomenon of the macroscopic distinct flux states in a double-well potential when an external single-cycle sinusoidal driving is applied. Coherent resonant peaks due to the MRT have been observed and the dependence of MRT on the amplitude, frequency, and initial phase of the external driving signal is demonstrated. The significant dependence of the population distribution of the qubit on the parameters of the driving signal provides us with a new way to manipulate the quantum states.

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