The interaction between a screw dislocation and a wedge-shaped crack in one-dimensional hexagonal piezoelectric quasicrystals*

Li-Juan Jiang(姜丽娟)† and Guan-Ting Liu(刘官厅)‡

College of Mathematics Science, Inner Mongolia Normal University, Huhhot 010022, *China*

(Received 12 September 2016; revised manuscript received 7 November 2016; published online 20 February 2017)

Based on the fundamental equations of piezoelasticity of quasicrystal material, we investigated the interaction between a screw dislocation and a wedge-shaped crack in the piezoelectricity of one-dimensional hexagonal quasicrystals. Explicit analytical solutions are obtained for stress and electric displacement intensity factors of the crack, as well as the force on dislocation. The derivation is based on the conformal mapping method and the perturbation technique. The influences of the wedge angle and dislocation location on the image force are also discussed. The results obtained in this paper can be fully reduced to some special cases already available or deriving new ones.

Keywords: one-dimensional hexagonal piezoelectric quasicrystals, dislocation, wedge-shaped crack, interaction

PACS: 46.05.+b, 46.50.+a, 61.72.Lk **DOI:** 10.1088/1674-1056/26/4/044601

1. Introduction

The interaction of dislocations with defects (cracks, interfaces, inhomogeneities, etc.) is of considerable importance for understanding the physical behavior of materials. In the open literature,[1–5] a lot of work on dislocations and cracks in pure elastic materials, piezoelectric solids, and quasicrystals have been studied in order to build a fracture theory that describes plastic deformation of dislocations near the crack tip. Many methods and techniques, such as the Green function method,[6] integral transformation,[7] and the complex variable method[8–14] can be used to solve these problems. Head[15] first investigated the interaction between a screw dislocation and a bi-material interface. Ohr *et al.*[16] derived the elastic field of a semi-infinite wedge crack and its interaction with a screw dislocation. The above-mentioned studies are based on the pure elastic materials. For the piezoelectric materials, various types of defects can adversely influence the performance of such piezoelectric devices. Pak[17] obtained closed form solutions for a screw dislocation in a piezoelectric solid and derived the generalized Peach–Koehler forces acting on the screw dislocation subjected to external loads. Zeng *et al.*[18] studied the interaction between screw dislocations and two asymmetrical interfacial cracks emanating from an elliptic hole under loads at infinity. Lee *et al.*[19] performed the interaction between a semi-infinite crack and a screw dislocation in a piezoelectric material. Chen *et al.*[20] investigated the electro–elastic stress on the interaction problem of a screw dislocation near the tip of a semi-infinite wedge-shaped crack in piezoelectric material. Liu *et al.*[21] derived closed-form solutions of the elastic and electrical fields induced by the screw dislocation using the conformal mapping method in conjunction with image principle. Recently, one-dimensional hexagonal quasicrystals have been paid a great deal of attention. Li *et al.*[22] performed an elastic field induced by a straight dislocation in a one-dimensional hexagonal quasicrystals with its line parallel to the quasiperiodic axis by superposition of the elastic fields of a pure edge part and a pure screw part. Liu *et al.*[23] investigated the interaction of defects in one-dimensional (1D) hexagonal quasicrystals using the complex variable function method. Li *et al.*[24] studied the interaction between the screw dislocation and wedge-shaped crack in 1D hexagonal quasicrystals. These explicit and exact solutions can provide a theoretical analysis for fracture problems.

However, for piezoelectricity of quasicrystals, this problem becomes more complicated due to the introduction of the electric field. Up to now, there has not been any research work on the interaction between a screw dislocation and a wedge-shaped crack in piezoelectricity of one-dimensional hexagonal quasicrystals. In this paper, explicit analytical solutions are obtained for the stress and electric displacement intensity factors of the crack, as well as the force on dislocation.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11262017, 11262012, and 11462020), the Natural Science Foundation of Inner Mongolia Autonomous Region, China (Grant No. 2015MS0129), the Programme of Higher-level Talents of Inner Mongolia Normal University (Grant No. RCPY-2-2012-K-035), and the Key Project of Inner Mongolia Normal University (Grant No. 2014ZD03).

†E-mail: 1530284866@qq.com

‡Corresponding author. E-mail: guantingliu@imnu.edu.cn

© 2017 Chinese Physical Society and IOP Publishing Ltd

2. Statement of the problem

The physical problem considered in this paper is shown in Fig. 1. The wedge-shape crack is assumed to infinitely extend in the negative x axis, and the wedge angle is denoted as α_0. A charged screw dislocation located at the point $z_{\rm d}(r_{\rm d},\theta_{\rm d})$, with the Burgers vectors $(b_1^{\parallel},b_2^{\parallel},b_3^{\parallel},b_3^{\perp},b_3^{\phi})$ is assumed to be straight and infinitely long in the z direction.

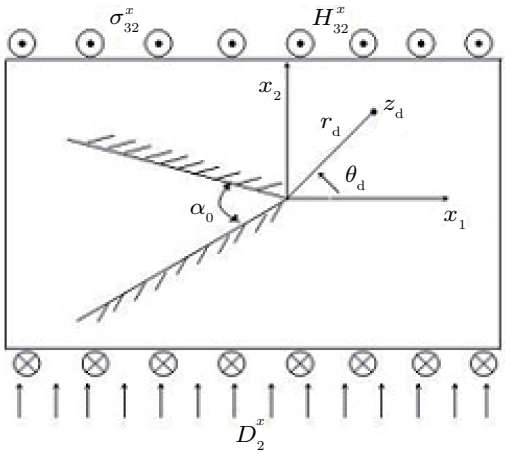


Fig. 1. A 1D hexagonal quasicrystal containing screw dislocation near a wedge crack.

As shown in Fig. 1, we have the following boundary conditions:

$$\begin{aligned}&\sqrt{x^2+y^2}\to\infty:\ \ \sigma_{zy}=\tau_1,\ H_{zy}=\tau_2,\ D_y=D_0,\\&(x,y)\in\theta=\pm(\pi-\alpha_0/2):\ \ \sigma_{zy}=0,\ H_{zy}=0,\ D_y=0,\end{aligned}\tag{1}$$

where τ_1, τ_2, and D_0 are the stress of the phonon field, the stress of the phason field, and the electric displacement, respectively, at the infinite region.

The stress–strain relations for 1D hexagonal piezoelectric quasicrystals, referred to the Cartesian coordinate (x,y,z) with xoy coincident with the periodic plane and the z axis identical to the quasi-periodic direction, read as follows:[25]

$$\begin{aligned}\sigma_{xx}&=C_{11}\varepsilon_{xx}+C_{12}\varepsilon_{yy}+C_{13}\varepsilon_{zz}+R_1\omega_{zz}-e_{31}E_z,\\\sigma_{yy}&=C_{12}\varepsilon_{xx}+C_{11}\varepsilon_{yy}+C_{13}\varepsilon_{zz}+R_1\omega_{zz}-e_{31}E_z,\\\sigma_{zz}&=C_{13}\varepsilon_{xx}+C_{13}\varepsilon_{yy}+C_{33}\varepsilon_{zz}+R_2\omega_{zz}-e_{33}E_z,\\\sigma_{yz}&=\sigma_{zy}=2C_{44}\varepsilon_{zy}+R_3\omega_{zy}-e_{15}E_y,\\\sigma_{zx}&=\sigma_{xz}=2C_{44}\varepsilon_{zx}+R_3\omega_{zx}-e_{15}E_x,\\\sigma_{xy}&=\sigma_{yx}=2C_{66}\varepsilon_{xy},\end{aligned}\tag{2}$$

$$\begin{aligned}H_{zz}&=R_1\varepsilon_{xx}+R_1\varepsilon_{yy}+R_2\varepsilon_{zz}+K_1\omega_{zz}-d_{33}E_z,\\H_{zy}&=2R_3\varepsilon_{zy}+K_2\omega_{zy}-d_{15}E_y,\\H_{zx}&=2R_3\varepsilon_{zx}+K_2\omega_{zx}-d_{15}E_x,\end{aligned}\tag{3}$$

$$\begin{aligned}D_x&=2e_{15}\varepsilon_{zx}+d_{15}\omega_{zx}+\lambda_{11}E_x,\\D_y&=2e_{15}\varepsilon_{zy}+d_{15}\omega_{zy}+\lambda_{11}E_y,\\D_z&=e_{31}\varepsilon_{xx}+e_{31}\varepsilon_{yy}+e_{33}\varepsilon_{zz}+d_{33}\omega_{zz}+\lambda_{33}E_z.\end{aligned}\tag{4}$$

The generalized strain-displacement relations are

$$\begin{aligned}\varepsilon_{yz}&=\frac{1}{2}\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right),\ \ \varepsilon_{zx}=\frac{1}{2}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right),\\\varepsilon_{xy}&=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right),\ \ \omega_{zy}=\frac{\partial w_z}{\partial y},\ \ \omega_{zx}=\frac{\partial w_z}{\partial x},\\\omega_{zz}&=\frac{\partial w_z}{\partial z},\quad E_x=-\frac{\partial\phi}{\partial x},\\E_y&=-\frac{\partial\phi}{\partial y},\quad E_z=-\frac{\partial\phi}{\partial z}.\end{aligned}\tag{5}$$

In the absence of body forces and free charges, the equilibrium equations can be expressed as

$$\begin{aligned}&\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}=0,\\&\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}=0,\\&\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}=0,\\&\frac{\partial H_{zx}}{\partial x}+\frac{\partial H_{zy}}{\partial y}+\frac{\partial H_{zz}}{\partial z}=0,\\&\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=0,\end{aligned}\tag{6}$$

where σ_{ij}, ε_{ij}, u_i are the stress, strain, and displacement of the phonon field, respectively; H_{ij}, ω_{ij}, and w_i are the stress, strain, and displacement of the phason field, respectively; D_i, E_i, and ϕ are the electric displacement, the electric field, and the electric potential, respectively; C_{ij}, K_i, and R_i stand for the phonon elastic, phason elastic, and phonon–phason coupling modulus, respectively; e_{ij}, d_{ij}, and λ_{ij} stand for piezoelastic constants and the dielectric permittivity, respectively. All field variables are independent of the longitudinal coordinate z, say

$$\frac{\partial(\ )}{\partial z}=0.\tag{7}$$

With the help of Eq. (7), equations (2)–(6) can be decomposed as two uncoupled problems. The first problem is

$$\begin{aligned}&\sigma_{xx}=C_{11}\varepsilon_{xx}+C_{12}\varepsilon_{yy},\ \ \sigma_{yy}=C_{12}\varepsilon_{xx}+C_{11}\varepsilon_{yy},\\&\sigma_{zz}=C_{13}\varepsilon_{xx}+C_{13}\varepsilon_{yy},\ \ \sigma_{xy}=2C_{66}\varepsilon_{xy},\\&H_{zz}=R_1\varepsilon_{xx}+R_1\varepsilon_{yy},\ \ D_z=e_{31}\varepsilon_{xx}+e_{31}\varepsilon_{yy},\end{aligned}\tag{8}$$

$$\varepsilon_{xx}=\frac{\partial u_x}{\partial x},\ \ \varepsilon_{yy}=\frac{\partial u_y}{\partial y},\ \ \varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right),\tag{9}$$

$$\frac{\partial\sigma_{xx}}{\partial\sigma_x}+\frac{\partial\sigma_{xy}}{\partial\sigma_y}=0,\ \ \frac{\partial\sigma_{yx}}{\partial\sigma_x}+\frac{\partial\sigma_{yy}}{\partial\sigma_y}=0,\tag{10}$$

which is just the classical plane elasticity of conventional hexagonal crystals. The other problem is

$$\begin{aligned}\sigma_{zy}&=2C_{44}\varepsilon_{zy}+R_3\omega_{zy}-e_{15}E_y,\\\sigma_{zx}&=2C_{44}\varepsilon_{zx}+R_3\omega_{zx}-e_{15}E_x,\\H_{zy}&=2R_3\varepsilon_{zy}+K_2\omega_{zy}-d_{15}E_y,\\H_{zx}&=2R_3\varepsilon_{zx}+K_2\omega_{zx}-d_{15}E_x,\\D_y&=2e_{15}\varepsilon_{zy}+d_{15}\omega_{zy}+\lambda_{11}E_y,\end{aligned}$$

$$D_x = 2e_{15}\varepsilon_{zx} + d_{15}\omega_{zx} + \lambda_{11}E_x, \tag{11}$$

$$\begin{aligned}
&\varepsilon_{yz} = \frac{1}{2}\frac{\partial u_z}{\partial y}, \quad \varepsilon_{zx} = \frac{1}{2}\frac{\partial u_z}{\partial x},\\
&\omega_{zy} = \frac{\partial w_z}{\partial y}, \quad \omega_{zx} = \frac{\partial w_z}{\partial x},\\
&E_y = -\frac{\partial \phi}{\partial y}, \quad E_x = -\frac{\partial \phi}{\partial x},
\end{aligned} \tag{12}$$

$$\begin{aligned}
&\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} = 0,\\
&\frac{\partial H_{zx}}{\partial x} + \frac{\partial H_{zy}}{\partial y} = 0,\\
&\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0,
\end{aligned} \tag{13}$$

which is a phonon–phason–electric coupling anti-plane piezoelasticity problem of 1D hexagonal quasicrystals. Since the first problem described by Eqs. (8)–(10) has been studied extensively, we only need to consider the second problem.

Substituting Eq. (12) into Eq. (11) and then into Eq. (13), we have

$$\begin{aligned}
&C_{44}\nabla^2 u_z + R_3\nabla^2 w_z + e_{15}\nabla^2\phi = 0,\\
&R_3\nabla^2 u_z + K_2\nabla^2 w_z + d_{15}\nabla^2\phi = 0,\\
&e_{15}\nabla^2 u_z + d_{15}\nabla^2 w_z - \lambda_{11}\nabla^2\phi = 0,
\end{aligned} \tag{14}$$

where ∇^2 is the two-dimensional Laplace operator. The general solution for the displacement vector $\boldsymbol{u}$ obtained from Eq. (13) can be written as

$$\begin{aligned}
\boldsymbol{u} &= \{u_z, w_z, \phi\}^{\mathrm{T}} = \mathrm{Re}\{f_u(z), f_w(z), f_\phi(z)\}^{\mathrm{T}}\\
&= \mathrm{Re}[\boldsymbol{f}(z)],
\end{aligned} \tag{15}$$

where Re is the real part of the complex function, and $f_u(z)$, $f_w(z)$, and $f_\phi(z)$ are arbitrary analytic functions of z. By using Eqs.(1), the complex function vector $\boldsymbol{f}(\boldsymbol{z})$ can be written as

$$\boldsymbol{f}(\boldsymbol{z}) = \boldsymbol{f}_0(\boldsymbol{z}) + \boldsymbol{f}_1(\boldsymbol{z}), \tag{16}$$

where $\boldsymbol{f}_0(\boldsymbol{z})$ is associated with the unperturbed field without the wedge crack and can be chosen as

$$\boldsymbol{f}_0(\boldsymbol{z}) = \boldsymbol{A}\ln(z - z_{\mathrm{d}}) + \boldsymbol{B}\boldsymbol{z}, \tag{17}$$

in which

$$\begin{aligned}
&z_{\mathrm{d}} = r_{\mathrm{d}}\mathrm{e}^{\mathrm{i}i}\theta_{\mathrm{d}} \left(\theta_{\mathrm{d}} \in \left(-\pi + \frac{\alpha_0}{2}, \pi - \frac{\alpha_0}{2}\right)\right),\\
&\boldsymbol{A} = \{A_1\mathrm{i}, A_2\mathrm{i}, A_3\mathrm{i}\}^{\mathrm{T}},\\
&\boldsymbol{B} = \{B_1\mathrm{i}, B_2\mathrm{i}, B_3\mathrm{i}\}^{\mathrm{T}},
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
&A_1 = -\frac{b_3^{\parallel}}{2\pi}, \quad A_2 = -\frac{b_3^{\perp}}{2\pi}, \quad A_3 = -\frac{b_3^{\phi}}{2\pi},\\
&B_1 = \frac{e_{15}K_2D_0 + d_{15}^2\tau_1 + \lambda_{11}K_2\tau_1 - R_3\lambda_{11}\tau_2 - R_3d_{15}D_0 - e_{15}d_{15}\tau_2}{C_{44}d_{15}^2 - 2e_{15}d_{15}R_3 + K_2e_{15}^2 - R_3^2\lambda_{11} + C_{44}\lambda_{11}K_2},\\
&B_2 = \frac{e_{15}^2\tau_2 + C_{44}\lambda_{11}\tau_2 - R_3\lambda_{11}\tau_1 + C_{44}d_{15}D_0 - e_{15}d_{15}\tau_1 - e_{15}R_3D_0}{C_{44}d_{15}^2 - 2e_{15}d_{15}R_3 + K_2e_{15}^2 - R_3^2\lambda_{11} + C_{44}\lambda_{11}K_2},\\
&B_3 = \frac{R_3^2D_0 + e_{15}K_2\tau_1 - R_3e_{15}\tau_2 - R_3d_{15}\tau_1 + C_{44}d_{15}\tau_2 - C_{44}K_2D_0}{C_{44}d_{15}^2 - 2e_{15}d_{15}R_3 + K_2e_{15}^2 - R_3^2\lambda_{11} + C_{44}\lambda_{11}K_2}.
\end{aligned} \tag{19}$$

The function $\boldsymbol{f}_1(\boldsymbol{z})$ corresponds to the perturbed field due to the existence of the wedge crack. By Eqs. (11), (12), and (15), the resultant force along any arbitrary arc AB is

$$\boldsymbol{T} = \int_A^B \boldsymbol{t}_x\mathrm{d}y - \boldsymbol{t}_y\mathrm{d}x = \boldsymbol{C}\,\mathrm{Im}[\boldsymbol{f}(z)]_A^B. \tag{20}$$

In which Im stands for the imaginary part and

$$\begin{aligned}
&\boldsymbol{t}_x = [\sigma_{zx}, H_{zx}, D_x]^{\mathrm{T}},\\
&\boldsymbol{t}_y = [\sigma_{zy}, H_{zy}, D_y]^{\mathrm{T}},\\
&\boldsymbol{C} = \begin{bmatrix} C_{44} & R_3 & e_{15}\\ R_3 & K_2 & d_{15}\\ e_{15} & d_{15} & -\lambda_{11}\end{bmatrix}.
\end{aligned} \tag{21}$$

From Eqs. (1), we have $t_x = t_y = 0$ in the boundary $\theta = \pm(\pi - \alpha_0/2)$, so the boundary condition at the wedge surface $\theta = \pm(\pi - \alpha_0/2)$ is

$$\boldsymbol{T}(z) = \boldsymbol{0}. \tag{22}$$

It is difficult to directly solve the problem in the z plane. Therefore, we introduce the conformal mapping function

$$z = \zeta^{(2-\alpha_0/\pi)} \tag{23}$$

with $\zeta = R\mathrm{e}^{\mathrm{i}\varphi}$ mapping the boundary $\theta = \pm(\pi - \alpha_0/2)$ in the z plane into the boundary conditions $\varphi = \pm(\pi/2)\mathrm{i}$ in the ζ plane and the dislocation located at the point z_{d} in the z plane onto ζ_{d} in the ζ plane. In the ζ plane of the current problem, the state of stress at $\theta = \pm(\pi - \alpha_0/2)$ is free from the stress of the phonon field, the stress of the phason field, and the electric displacement, so the boundary condition becomes

$$\boldsymbol{T}(\zeta_{\mathrm{c}}) = \boldsymbol{0}, \tag{24}$$

where ζ_{c} is along the imaginary axis. From Eq. (20), we know equation (24) can be rewritten as

$$\boldsymbol{f}(\zeta_{\mathrm{c}}) - \overline{\boldsymbol{f}}(\overline{\zeta}_{\mathrm{c}}) = \boldsymbol{0}. \tag{25}$$

Utilizing Eqs. (16) and (25), noting that $\overline{\zeta}_c = -\zeta_c$ holds along the imaginary axis, and using standard analytic continuation arguments,[26] we obtain

$$\boldsymbol{f}_1(\zeta) = \overline{\boldsymbol{f}}_0(-\zeta). \tag{26}$$

From Eqs. (16) and (26), the complex function vector $\boldsymbol{f}(\zeta)$ can be obtained in the ζ plane as

$$\boldsymbol{f}(\zeta) = \underline{\boldsymbol{A}} \ln(\zeta - \zeta_d) + \overline{\boldsymbol{A}} \ln(\zeta + \overline{\zeta}_d) + 2\mathrm{i}\,\mathrm{Im}\,\boldsymbol{B}\zeta. \tag{27}$$

Therefore, in the physical plane, we have

$$\boldsymbol{f}(z) = \boldsymbol{A} \ln(z^\lambda - z_d^\lambda) + \overline{\boldsymbol{A}} \ln(z^\lambda + \overline{z}_d^\lambda) + 2\mathrm{i}\,\mathrm{Im}\,\boldsymbol{B}z^\lambda. \tag{28}$$

3. Stress intensity factors

The entire stress and displacement fields can be calculated by using Eqs. (2)–(5) and (28) (here, we only list the expressions for the stresses in Appendix A). Similar to the classical elasticity theory, we define a complex stress intensity factor vector included by the dislocation as

$$\boldsymbol{K} = \begin{bmatrix} K_{\sigma_{zx}} - \mathrm{i}K_{\sigma_{zy}} \\ K_{H_{zx}} - \mathrm{i}K_{H_{zy}} \\ K_{D_x} - \mathrm{i}K_{D_y} \end{bmatrix}. \tag{29}$$

Since the field variables show $\gamma^{\lambda-1}(1/2 \le \lambda \le 1)$ type of singularity near the wedge crack tip, equation (29) can be rewritten as

$$\begin{aligned} \boldsymbol{K} &= \lim_{z\to 0}(2\pi z)^{(1-\lambda)}(\boldsymbol{t}_x - \mathrm{i}\boldsymbol{t}_y) \\ &= \lim_{z\to 0}(2\pi z)^{(1-\lambda)}[\boldsymbol{C}\boldsymbol{f}'(z)]. \end{aligned} \tag{30}$$

Inserting Eq. (29) into Eq. (30), we have

$$\begin{aligned} K_{\sigma_{zy}} &= -\frac{2\lambda}{(2\pi\gamma_d)^\lambda}[(C_{44}b_3^{\parallel} + R_3 b_3^{\perp} + e_{15}b_3^{\phi})\cos\lambda\theta_d + 2r_d^\lambda \pi(C_{44}B_1 + R_3B_2 + e_{15}B_3)], \\ K_{Hzy} &= -\frac{2\lambda}{(2\pi\gamma_d)^\lambda}[(R_3 b_3^{\parallel} + K_2 b_3^{\perp} + d_{15}b_3^{\phi})\cos\lambda\theta_d + 2r_d^\lambda \pi(R_3B_1 + K_2B_2 + d_{15}B_3)], \\ K_{Dy} &= -\frac{2\lambda}{(2\pi\gamma_d)^\lambda}[(e_{15}b_3^{\parallel} + d_{15}b_3^{\perp} - \lambda_{11}b_3^{\phi})\cos\lambda\theta_d + 2r_d^\lambda \pi(e_{15}B_1 + d_{15}B_2 - \lambda_{11}B_3)]. \end{aligned} \tag{31}$$

We can draw the conclusion from the above results. The stress intensity factor of the wedge-shaped crack is mainly influenced by the screw dislocation along the x axis. It is noted that when $\alpha_0 = 0$, we gain the stress intensity factor of the semi-infinite crack induced by the dislocation. If there is no applied electric loadings at infinity, the above results equations (31) reduce to the corresponding results[24] of one-dimensional hexagonal quasicrystals. If the phason field is neglected, the present solutions (31) can give the corresponding solutions for a wedge-shaped crack interacting with a screw dislocation in piezoelectric materials, which is new and different from the ones in the open literature.[20] The authors discuss the case of $\sigma_{zy} = 0$, $H_{zy} = 0$, $D_y = 0$, and the line force and the line charge at the dislocation core in the literature.[20]

4. Force on the dislocation

According to the generalized Peach–Koehler force formula for quasicrystals derived by Li *et al.*,[22] the image forces acting on the dislocation due to the wedge crack in the piezoelectricity of quasicrystals are

$$\begin{aligned} F_x &= b_3^{\parallel}\sigma_{zy}^{\mathrm{T}} + b_3^{\perp}H_{zy}^{\mathrm{T}} + b_3^{\phi}D_y^{\mathrm{T}}, \\ F_y &= -b_3^{\parallel}\sigma_{zx}^{\mathrm{T}} - b_3^{\perp}H_{zx}^{\mathrm{T}} - b_3^{\phi}D_x^{\mathrm{T}}, \end{aligned} \tag{32}$$

where σ_{zx}^{T}, σ_{zy}^{T}, H_{zx}^{T}, H_{zy}^{T}, D_x^{T}, and D_y^{T} are the stress field produced by the crack in the absence of the dislocation, for which we will give the detailed expressions in Appendix B. For the current problem, equation (32) can be expressed as follows:

$$\begin{aligned} F_x = {} & \frac{1}{4\pi\gamma_d}(C_{44}b_3^{\parallel 2} + K_2 b_3^{\perp 2} - \lambda_{11}b_3^{\phi 2} + 2R_3 b_3^{\parallel}b_3^{\perp} + 2e_{15}b_3^{\parallel}b_3^{\phi} + 2d_{15}b_3^{\perp}b_3^{\phi})\left[(\lambda-1)\cos\theta_d - \frac{\lambda\cos(\lambda-1)\theta_d}{\cos\lambda\theta_d}\right] \\ & -2[(C_{44}B_1 + R_3B_2 + e_{15}B_3)b_3^{\parallel} + (R_3B_1 + K_2B_2 + d_{15}B_3)b_3^{\perp} \\ & + (e_{15}B_1 + d_{15}B_2 - \lambda_{11}B_3)b_3^{\phi}]\lambda\gamma_d^{\lambda-1}\cos(\lambda-1)\theta_d, \\ F_y = {} & \frac{1}{4\pi\gamma_d}(C_{44}b_3^{\parallel 2} + K_2 b_3^{\perp 2} - \lambda_{11}b_3^{\phi 2} + 2R_3 b_3^{\parallel}b_3^{\perp} + 2e_{15}b_3^{\parallel}b_3^{\phi} + 2d_{15}b_3^{\perp}b_3^{\phi})\left[(\lambda-1)\sin\theta_d + \frac{\lambda\sin(\lambda-1)\theta_d}{\cos\lambda\theta_d}\right] \\ & +2[(C_{44}B_1 + R_3B_2 + e_{15}B_3)b_3^{\parallel} + (R_3B_1 + K_2B_2 + d_{15}B_3)b_3^{\perp} \\ & + (e_{15}B_1 + d_{15}B_2 - \lambda_{11}B_3)b_3^{\phi}]\lambda\gamma_d^{\lambda-1}\sin(\lambda-1)\theta_d. \end{aligned} \tag{33}$$

When the dislocation is in the positive x-axis direction ($z_d = x_d$ with $(x_d > 0)$), the above results can be simplified as

$$F_x = \frac{1}{4\pi\gamma_d}(C_{44}b_3^{\parallel 2} + K_2 b_3^{\perp 2} - \lambda_{11}b_3^{\phi 2} + 2R_3 b_3^{\parallel}b_3^{\perp} + 2e_{15}b_3^{\parallel}b_3^{\phi} + 2d_{15}b_3^{\perp}b_3^{\phi})$$

$$-2[(C_{44}B_1+R_3B_2+e_{15}B_3)b_3^{\parallel}+(R_3B_1+K_2B_2+d_{15}B_3)b_3^{\perp}+(e_{15}B_1+d_{15}B_2-\lambda_{11}B_3)b_3^{\phi}]\lambda x_{\rm d}^{\lambda-1}, \quad (34)$$

$$F_y=0.$$

The results obtained show that the image forces on the dislocation are along the x axis.

5. Numerical examples

Clearly, the image forces on the screw dislocation are given explicitly in Eq. (33). They are functions of $b_3^{\parallel}$, $b_3^{\perp}$, b_3^{φ}, τ_1, τ_2, D_0, $z_{\rm d}$, and material constants. In order to have a deep investigation on how these parameters affect the image force, a particular one-dimensional hexagonal piezoelectric quasicrystals is used as an example to numerically demonstrate the force on the dislocation. The material constants are selected as follows:[27]

$$C_{44}=5.0\times10^{10}\ \text{N/m}^2,\quad R_3=1.2\times10^{9}\ \text{N/m}^2,$$
$$K_3=0.3\times10^{9}\ \text{N/m}^2,\quad e_{15}=-0.318\ \text{C/m}^2,$$
$$d_{15}=-0.160\ \text{C/m}^2,\quad \lambda_{11}=82.6\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2.$$

Other parameters are taken to have the values listed below

$$b_3^{\parallel}=1.6\times10^{-9}\ \text{m},\quad b_3^{\perp}=10.7\times10^{-9}\ \text{m},$$
$$b_3^{\phi}=1.0\times10^{-9}\ \text{V},\quad \tau_1=3.0\times10^{-6}\ \text{N/m}^2,$$
$$\tau_2=5.0\times10^{-6}\ \text{N/m}^2,\quad D_0=3.0\times10^{-8}\ \text{C/m}^2.$$

The forces on the dislocation are normalized with

$$F_0=\frac{C_{44}b_3^{\parallel 2}}{40\pi \text{m}}. \quad (35)$$

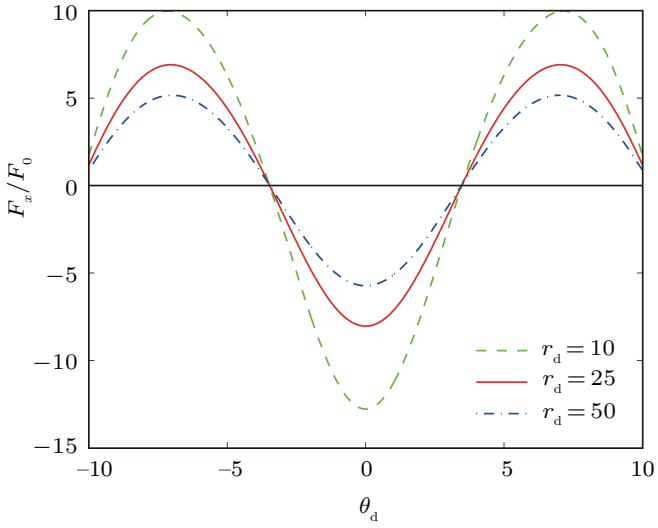


Fig. 2. (color online) The normalized image force on the dislocation along x axis versus angular position when $\alpha_0=\pi/6$.

Figures 2 and 3 show the normalized force on the dislocation versus the angular position of the dislocation when the wedge angle is $\pi/6$. It is found from Figs. 2 and 3 that the magnitudes of the image force on the dislocation increase with decreasing $r_{\rm d}$.

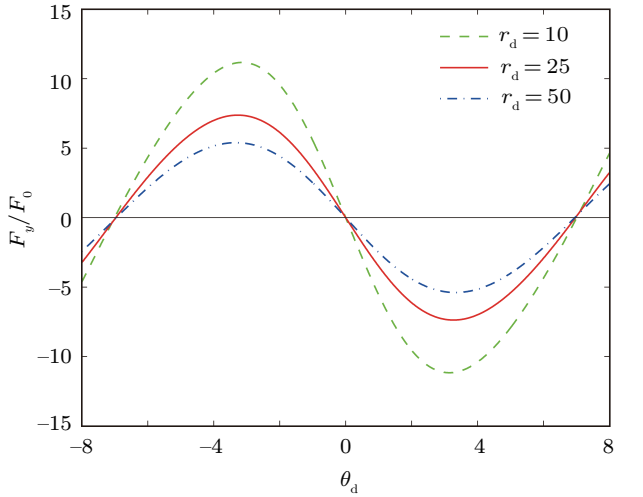


Fig. 3. (color online) The normalized image force on the dislocation along y axis versus angular position when $\alpha_0=\pi/6$.

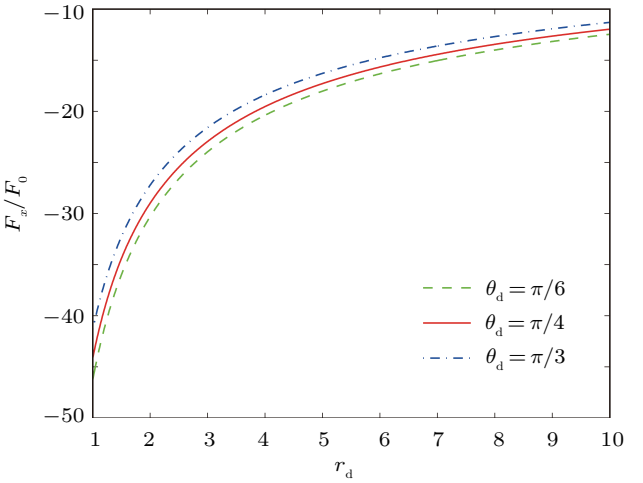


Fig. 4. (color online) The normalized image force on the dislocation along x axis versus $r_{\rm d}$ when $\alpha_0=\pi/6$.

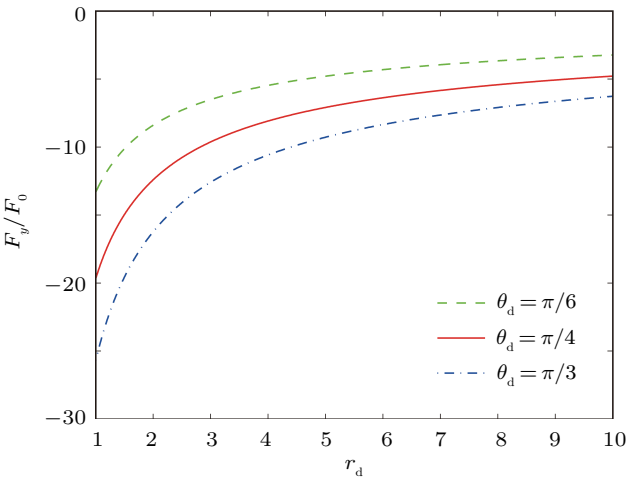


Fig. 5. (color online) The normalized image force on the dislocation along y axis versus $r_{\rm d}$ when $\alpha_0=\pi/6$.

In order to further reveal the variable tendency, figures 4 and 5 depict the variations of the normalized image force ver-

sus $r_{\rm d}$ when the wedge angle is $\pi/6$. It is seen from Figs. 4 and 5 that the crack always attracts the dislocation in the radial direction when it is far away from the dislocation. We further obtain that the forces along the x axis increase with decreasing $\theta_{\rm d}$ while the forces along the y axis increase with increasing $\theta_{\rm d}$.

6. Conclusion

In this paper, a screw dislocation near a wedge crack in one-dimensional hexagonal piezoelectric quasicrystals is analyzed. By using the conformal transformation method and the perturbation technique, the stress and electric displacement intensity factors and the image forces on the dislocation are derived. At the same time the explicit analytical expressions for the stress and displacement are also obtained. The influences of the wedge angle and dislocation location on the image force are also discussed. The results are reduced to the new or previous ones when the wedge angle is taken to be the limiting case or the materials are assumed to be one-dimensional hexagonal quasicrystals.

Appendix A

The expressions for the stress of the phonon, the phason fields, and the electric displacements can be expressed as follows:

$$\begin{aligned}
\sigma_{zx} &= (C_{44}A_1+R_3A_2+e_{15}A_3)(X_2-X_1)\\
&\quad -2(C_{44}B_1+R_3B_2+e_{15}B_3)\lambda\gamma^{\lambda-1}\sin(\lambda-1)\theta,\\
\sigma_{zy} &= (C_{44}A_1+R_3A_2+e_{15}A_3)(Y_2-Y_1)\\
&\quad -2(C_{44}B_1+R_3B_2+e_{15}B_3)\lambda\gamma^{\lambda-1}\cos(\lambda-1)\theta,\\
H_{zx} &= (R_3A_1+K_2A_2+d_{15}A_3)(X_2-X_1)\\
&\quad -2(R_3B_1+K_2B_2+d_{15}B_3)\lambda\gamma^{\lambda-1}\sin(\lambda-1)\theta,\\
H_{zy} &= (R_3A_1+K_2A_2+d_{15}A_3)(Y_2-Y_1)\\
&\quad -2(R_3B_1+K_2B_2+d_{15}B_3)\lambda\gamma^{\lambda-1}\cos(\lambda-1)\theta,\\
D_x &= (e_{15}A_1+d_{15}A_2-\lambda_{11}A_3)(X_2-X_1)\\
&\quad -2(e_{15}B_1+d_{15}B_2-\lambda_{11}B_3)\lambda\gamma^{\lambda-1}\sin(\lambda-1)\theta,\\
D_y &= (e_{15}A_1+d_{15}A_2-\lambda_{11}A_3)(Y_2-Y_1)\\
&\quad -2(e_{15}B_1+d_{15}B_2-\lambda_{11}B_3)\lambda\gamma^{\lambda-1}\cos(\lambda-1)\theta, \qquad \text{(A1)}
\end{aligned}$$

in which

$$\begin{aligned}
X_1 &= \frac{\lambda\gamma^{\lambda-1}[\gamma_{\rm d}^{\lambda}\sin(\lambda\theta_{\rm d}-(\lambda-1)\theta)-\gamma^{\lambda}\sin\theta]}{\gamma^{2\lambda}+\gamma_{\rm d}^{2\lambda}-2\gamma^{\lambda}\gamma_{\rm d}^{\lambda}\cos\lambda(\theta-\theta_{\rm d})},\\
X_2 &= \frac{\lambda\gamma^{\lambda-1}[\gamma_{\rm d}^{\lambda}\sin(\lambda\theta_{\rm d}+(\lambda-1)\theta)-\gamma^{\lambda}\sin\theta]}{\gamma^{2\lambda}+\gamma_{\rm d}^{2\lambda}+2\gamma^{\lambda}\gamma_{\rm d}^{\lambda}\cos\lambda(\theta+\theta_{\rm d})},\\
Y_1 &= \frac{\lambda\gamma^{\lambda-1}[\gamma^{\lambda}\cos\theta-\gamma_{\rm d}^{\lambda}\cos(\lambda\theta_{\rm d}-(\lambda-1)\theta)]}{\gamma^{2\lambda}+\gamma_{\rm d}^{2\lambda}-2\gamma^{\lambda}\gamma_{\rm d}^{\lambda}\cos\lambda(\theta-\theta_{\rm d})},\\
Y_2 &= \frac{\lambda\gamma^{\lambda-1}[\gamma^{\lambda}\cos\theta+\gamma_{\rm d}^{\lambda}\cos(\lambda\theta_{\rm d}+(\lambda-1)\theta)]}{\gamma^{2\lambda}+\gamma_{\rm d}^{2\lambda}+2\gamma^{\lambda}\gamma_{\rm d}^{\lambda}\cos\lambda(\theta+\theta_{\rm d})}. \qquad \text{(A2)}
\end{aligned}$$

Appendix B

The stresses produced by the crack in the absence of the dislocation can be obtained as follows:

$$\begin{aligned}
\sigma_{zx}^{\rm T} &= \frac{1}{4\pi\gamma_{\rm d}}(C_{44}b_3^{\parallel}+R_3b_3^{\perp}+e_{15}b_3^{\phi})\left[(1-\lambda)\sin\theta_{\rm d}-\frac{\lambda\sin(\lambda-1)\theta_{\rm d}}{\cos\lambda\theta_{\rm d}}\right]\\
&\quad -2(C_{44}B_1+R_3B_2+e_{15}B_3)\lambda\gamma_{\rm d}^{\lambda-1}\sin(\lambda-1)\theta_{\rm d},\\
\sigma_{zy}^{\rm T} &= \frac{1}{4\pi\gamma_{\rm d}}(C_{44}b_3^{\parallel}+R_3b_3^{\perp}+e_{15}b_3^{\phi})\left[(\lambda-1)\cos\theta_{\rm d}-\frac{\lambda\cos(\lambda-1)\theta_{\rm d}}{\cos\lambda\theta_{\rm d}}\right]\\
&\quad -2(C_{44}B_1+R_3B_2+e_{15}B_3)\lambda\gamma_{\rm d}^{\lambda-1}\cos(\lambda-1)\theta_{\rm d}, \qquad \text{(B1)}\\
H_{zx}^{\rm T} &= \frac{1}{4\pi\gamma_{\rm d}}(R_3b_3^{\parallel}+K_2b_3^{\perp}+d_{15}b_3^{\phi})\left[(1-\lambda)\sin\theta_{\rm d}-\frac{\lambda\sin(\lambda-1)\theta_{\rm d}}{\cos\lambda\theta_{\rm d}}\right]\\
&\quad -2(R_3B_1+K_2B_2+d_{15}B_3)\lambda\gamma_{\rm d}^{\lambda-1}\sin(\lambda-1)\theta_{\rm d},\\
H_{zy}^{\rm T} &= \frac{1}{4\pi\gamma_{\rm d}}(R_3b_3^{\parallel}+K_2b_3^{\perp}+d_{15}b_3^{\phi})\left[(\lambda-1)\cos\theta_{\rm d}-\frac{\lambda\cos(\lambda-1)\theta_{\rm d}}{\cos\lambda\theta_{\rm d}}\right]\\
&\quad -2(R_3B_1+K_2B_2+d_{15}B_3)\lambda\gamma_{\rm d}^{\lambda-1}\cos(\lambda-1)\theta_{\rm d}, \qquad \text{(B2)}\\
D_x^{\rm T} &= \frac{1}{4\pi\gamma_{\rm d}}(e_{15}b_3^{\parallel}+d_{15}b_3^{\perp}-\lambda_{11}b_3^{\phi})\left[(1-\lambda)\sin\theta_{\rm d}-\frac{\lambda\sin(\lambda-1)\theta_{\rm d}}{\cos\lambda\theta_{\rm d}}\right]\\
&\quad -2(e_{15}B_1+d_{15}B_2-\lambda_{11}B_3)\lambda\gamma_{\rm d}^{\lambda-1}\sin(\lambda-1)\theta_{\rm d},\\
D_y^{\rm T} &= \frac{1}{4\pi\gamma_{\rm d}}(e_{15}b_3^{\parallel}+d_{15}b_3^{\perp}-\lambda_{11}b_3^{\phi})\left[(\lambda-1)\cos\theta_{\rm d}-\frac{\lambda\cos(\lambda-1)\theta_{\rm d}}{\cos\lambda\theta_{\rm d}}\right]
\end{aligned}$$

$$-2(e_{15}B_1+d_{15}B_2-\lambda_{11}B_3)\lambda\gamma_{\rm d}^{\lambda-1}\cos(\lambda-1)\theta_{\rm d}. \tag{B3}$$

References

[1] Cui L J, Gao J, Du Y F, Zhang G W, Zhang L, Long Y, Yang S W, Zhan Q and Wan F R 2016 *Acta Phys. Sin.* **65** 066102 (in Chinese)
[2] Zheng S B, Gao Z H, Tang B H, Jiang Y H, Luo Y M and Gao Z H 2016 *Acta Phys. Sin.* **65** 014202 (in Chinese)
[3] Zhao Z G, Tian D X, Zhao J, Liang X B, Ma X Y and Yang D R 2015 *Acta Phys. Sin.* **64** 208101 (in Chinese)
[4] Yu T, Xie H X and Wang C Y 2012 *Chin. Phys. B* **21** 026104
[5] Fang Q H, Song H P and Liu Y W 2010 *Chin. Phys. B* **19** 016102
[6] Ding D H, Wang R H, Yang W G and Hu C Z 1995 *J. Phys.: Condens. Matter* **7** 5423
[7] Zhou W M and Fan T Y 2001 *Chin. Phys.* **10** 743
[8] Liu G T and Fan T Y 2003 *Sci. China E* **46** 326
[9] Li L H and Fan T Y 2006 *J. Phys.: Condens. Matter* **18** 10631
[10] Yu J, Guo J H and Xing Y M 2015 *Chin. J. Aeron.* **28** 1287
[11] Guo J H, Yu J, Xing Y M, Pan E N and Li L H 2016 *Acta Mech.* **227** 2595
[12] Yu J, Guo J H, Xing Y M, Pan E N and Xing Y M 2015 *Acta Mech.* **36** 793
[13] Guo J H and Liu G T 2008 *Chin. Phys. B* **17** 2610
[14] Liu X and Guo J H 2016 *Theor. Appl. Frac. Mech.* **86** 225
[15] Head A K 1953 *Philos. Mag.* **44** 92
[16] Ohr S M, Chang S J and Thomson R 1985 *J. Appl. Phys.* **57** 1839
[17] Pak Y E 1990 *ASME J. Appl. Mech.* **57** 647
[18] Zeng X, Fang Q H, Liu Y W and Wen P H 2013 *Chin. Phys. B* **22** 014601
[19] Lee K Y, Lee W G and Pak Y E 2000 *ASME J. Appl. Mech.* **67** 165
[20] Chen B J, Xiao Z M and Liew K M 2002 *Int. J. Eng. Sci.* **40** 621
[21] Liu J X, Liu A and Jiang Z Q 2004 *Acta Mech. Sin.* **20** 519
[22] Li X F and Fan T Y 1999 *Phys. Stat. Sol* **212** 19
[23] Liu G T, Guo R P and Fan T Y 2003 *Chin. Phys.* **12** 1149
[24] Li L H and Liu G T 2012 *Acta Phys. Sin.* **61** 086103 (in Chinese)
[25] Altay G and Domeci M C 2012 *Int. J. Solids Struct.* **49** 3255
[26] Muskhelishvili N I 1963 *Some Basic Problems of Mathematical Theory of Elasticity* (Noordhoff: Groningen) pp. 123–132
[27] Li X Y, Li P D, Wu T H, Shi M X and Zhu Z W 2014 *Phys. Lett. A* **378** 826