Correction of cosine oscillation to the improved correlation method of estimating the amplitude of gravitational background signal*

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In the measurement of G with the angular acceleration method, the improved correlation method developed by Wu *et al*. (Wu W H, Tian Y, Luo J, Shao C G, Xu J H and Wang D H 2016 *Rev. Sci. Instrum.* **87** 094501) is used to accurately estimate the amplitudes of the prominent harmonic components of the gravitational background signal with time-varying frequency. Except the quadratic slow drift, the angular frequency of the gravitational background signal also includes a cosine oscillation coming from the useful angular acceleration signal, which leads to a deviation from the estimated amplitude. We calculate the correction of the cosine oscillation to the amplitude estimation. The result shows that the corrections of the cosine oscillation to the amplitudes of the fundamental frequency and second harmonic components obtained by the improved correlation method are within respective errors.

Keywords: gravitational background signal, improved correlation method, correction, cosine oscillation

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1. Introduction

The accurate amplitude estimation of the useful gravitational angular acceleration signal that is a sinusoidal signal is of great importance for measuring the gravitational constant G with angular acceleration method.[1–5] The amplitude of the sinusoidal signal is determined with high precision by the correlation method,[6] which is highly efficient for subtle signal analysis,[7–10] but only on condition that the frequency of the sinusoidal signal is constant and known exactly. Since the gravitational force cannot be shielded in the experiment,[11] the room fixed background mass causes a spurious gravitational angular acceleration signal on torsion pendulum inevitably. That is to say, the angular acceleration signal of the torsion balance turntable equals the sum of the useful angular acceleration signal and the spurious gravitational background signal. Besides, the useful angular acceleration signal obviously consists of the fundamental frequency and second harmonic components, and the spurious gravitational background signal mainly contains the prominent fundamental frequency and second harmonic components.[12] The coupling effect of the spurious gravitational background signal and the linear slow drift of torsion balance's equilibrium position[13–16] leads to the fact that the angular frequency of the spurious gravitational background signal undergoes a quadratic slow drift.[17] Then, the accurate amplitude determination of the useful angular acceleration signal depends on the effective subtraction of the spurious gravitational background signal.[18]

In order to subtract the gravitational background signal from the angular acceleration signal of the torsion balance turntable effectively, we need to determine the amplitudes of the fundamental frequency and second harmonic components of the gravitational background signal accurately. Based on the basic principle of the measurement of G by the angular acceleration method with PID feedback control unit,[5,12] the useful angular acceleration signal that equals the useful gravitational signal generated by the four source masses is a standard sinusoidal function. Since the angular velocity of the torsion balance turntable, which is also the angular frequency of the gravitational background signal, is the integral of its angular acceleration over time, besides the quadratic slow drift, the time-varying frequency of the gravitational background signal also contains a cosine oscillation coming from the useful angular acceleration signal.[17] The improved correlation method[18] with stretch processing of time is used to accurately determine the amplitudes of the prominent fundamental frequency and second harmonic components of the gravitational background signal with time-varying frequency in the meantime, and the cosine oscillation term is considered to be negligible with respect to the quadratic slow drift term. To

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judge whether the influence of the cosine oscillation is negligible compared with the error of the estimated amplitude, one needs to calculate the correction caused by the cosine oscillation.

In this paper, the cosine oscillation of the angular acceleration signal of the torsion balance turntable is analyzed, and the influence of the cosine oscillation is isolated alone by Taylor expansion. The gravitational background signal is divided into some small parts. In each part, the amplitudes of the prominent fundamental frequency and second harmonic components of the gravitational background signal without correcting the influence of the cosine oscillation is calculated by the improved correlation method with high precision, and the estimated amplitudes with the cosine oscillation influence corrected are also given by the same way. The bias of the cosine oscillation is obtained as the difference between the corrected amplitude and the uncorrected amplitude. Finally, we process a typical experimental data set to give the correction of the cosine oscillation to the estimated amplitude, then lay the foundation of the effective subtraction of the gravitational background signal, and further make contribution to estimating the amplitude of the useful gravitational signal.

2. Cosine oscillation of the gravitational background signal

In the measurement of the gravitational constant G with the angular acceleration method, the angular acceleration of the torsion balance turntable can be expressed as[17,18]

$$\begin{aligned}\alpha_{\rm t}(t) &= \ddot{\Theta}_{\rm t}(t)\\ &= \alpha_{\rm g}(t)+\alpha_{\rm b}(t)=\sum_{n=1}^{2}A_n\sin(n\omega_{\rm s}t+\phi_n)\\ &\quad+\sum_{m=1}^{2}\left\{\alpha_{{\rm b},m}\sin[m\Theta_{\rm t}(t)+\varphi_n]\right\}\\ &=\sum_{n=1}^{2}[a_n\cos(n\omega_{\rm s}t)+b_n\sin(n\omega_{\rm s}t)]\\ &\quad+\sum_{m=1}^{2}\{a_{{\rm b}m}\cos[m\Theta_{\rm t}(t)]+b_{{\rm b}m}\sin[m\Theta_{\rm t}(t)]\},\end{aligned}\tag{1}$$

where $\alpha_{\rm t}(t)$ and $\Theta_{\rm t}(t)$ denote the angular acceleration signal and rotation angle of the torsion balance turntable, respectively; $\alpha_{\rm g}(t)$ and $\alpha_{\rm b}(t)$ represent the useful angular acceleration signals generated by the four source masses and the gravitational background signal caused by the room fixed background masses respectively; $\omega_{\rm s}$ is the angular frequency of the useful signal; $\varphi_n=\tan^{-1}(b_n/a_n)$ and $\phi_m=\tan^{-1}(b_{{\rm b}m}/a_{{\rm b}m})$ denote the initial phases of the n-th order harmonic component of the useful signal and the m-th order harmonic component of the gravitational background signal, respectively; $A_n=\sqrt{a_n^2+b_n^2}$ and $\alpha_{{\rm b},m}=\sqrt{a_{{\rm b}m}^2+b_{{\rm b}m}^2}$ are the amplitudes of the n-th order harmonic component of the useful signal $\alpha_{\rm g}(t)$ and the m-th order harmonic component of the gravitational background signal $\alpha_{\rm b}(t)$, respectively; $\Theta_{\rm t}(t)$ is the rotation angle and equals the integral of the angular velocity with time-varying frequency, and it is expressed as

$$\begin{aligned}\Theta_{\rm t}(t) &= \int_{t_0}^{t}\omega_{\rm t}(t)\,{\rm d}t\\ &=\int_{t_0}^{t}\left[p_3+p_2t+p_1t^2-p_{\rm c}\cos(\omega_{\rm s}t)\right]{\rm d}t,\end{aligned}\tag{2}$$

where t_0 is the initial time, $\omega_{\rm t}(t)$ is the time-varying angular frequency of the gravitational background signal $\alpha_{\rm b}(t)$, p_3 is the initial angular velocity of the turntable, p_2 and p_1 are the coefficients of the quadratic slow drift, and $p_{\rm c}$ is the amplitude of the cosine oscillation.

When the cosine oscillation term is not considered at first, based on the stretch and non-dimensionalized processing of time, after each small part is shifted forward t_0 to start from $t=0$, the rotation angle $\Theta_{\rm t}(t)$ is approximated as

$$\Theta_{\rm t}(t)\approx\Theta(t)=\omega_1 t\left[1+\frac{d_{1\omega{\rm Tp}}}{2}\left(\frac{t}{T_{\rm p}}\right)+\frac{d_{2\omega{\rm Tp}}}{3}\left(\frac{t}{T_{\rm p}}\right)^2\right],\tag{3}$$

where $\omega_1=\omega_{\rm t}(t_0)$ denotes the initial angular velocity of each small part and the constants $T_{\rm p}$, $d_{1\omega{\rm Tp}}$, and $d_{2\omega{\rm Tp}}$ are the same as those in Ref. [18]. Then equations (1) is rewritten as

$$\begin{aligned}\ddot{\Theta}_{\rm t}(t) &= \sum_{n=1}^{2}[a_n\cos(n\omega_{\rm s}t)+b_n\sin(n\omega_{\rm s}t)]\\ &\quad+\sum_{m=1}^{2}\{a_{{\rm b}m}\cos[m\Theta(t)]+b_{{\rm b}m}\sin[m\Theta(t)]\}.\end{aligned}\tag{4}$$

Supposing that the drive signal is 0, the homogeneous solution of Eq. (4) is Eq. (3). To obtain the particular solution of Eq. (4), we perform twice successive integral on the right drive signal. After the uniform processing of Ref. [18], the high order term of t in Eq. (3) is negligible, and then $\Theta(t)$ is approximated as $\omega_1 t$, so equation (4) is further expressed as

$$\begin{aligned}\ddot{\Theta}_{\rm t}(t) &= \sum_{n=1}^{2}[a_n\cos(n\omega_{\rm s}t)+b_n\sin(n\omega_{\rm s}t)]\\ &\quad+\sum_{m=1}^{2}[a_{{\rm b}m}\cos(m\omega_1 t)+b_{{\rm b}m}\sin(m\omega_1 t)].\end{aligned}\tag{5}$$

Then the particular solution is given approximately by

$$\begin{aligned}-\Delta\Theta_{\rm t}=&-\sum_{n=1}^{2}\left[\frac{a_n}{n^2\omega^2}\cos(n\omega t)+\frac{b_n}{n^2\omega^2}\sin(n\omega t)\right]\\ &-\sum_{m=1}^{2}\left[\frac{a_{{\rm b}m}}{m^2\omega_1^2}\cos(m\omega_1 t)+\frac{b_{{\rm b}m}}{m^2\omega_1^2}\sin(m\omega_1 t)\right].\end{aligned}\tag{6}$$

Therefore, the complete solution of Eq. (1) is approximated as

$$\Theta_{\rm t}(t)=\Theta(t)-\Delta\Theta_{\rm t},\tag{7}$$

where $\Theta(t)$ is the main term caused by the quadratic slow drift and $\Delta\Theta_{\rm t}$ is the cosine oscillation coming from the useful signal. Substitution of Eq. (7) into Eq. (4) yields

$$\begin{aligned}\alpha_{\rm t}(t) &= \sum_{n=1}^{2}[a_n\cos(n\omega_{\rm s}t)+b_n\sin(n\omega_{\rm s}t)]\\ &\quad+\sum_{m=1}^{2}\{a_{{\rm b}m}\cos[m(\Theta(t)-\Delta\Theta_{\rm t})]\\ &\quad+b_{{\rm b}m}\sin[m(\Theta(t)-\Delta\Theta_{\rm t})]\}.\end{aligned}\tag{8}$$

Based on the Taylor expansion as

$$\begin{cases}\cos[m(\Theta(t)-\Delta\Theta_{\rm t})]\approx\cos[m\Theta(t)]+m\sin[m\Theta(t)]\Delta\Theta_{\rm t},\\ \sin[m(\Theta(t)-\Delta\Theta_{\rm t})]\approx\sin[m\Theta(t)]-m\cos[m\Theta(t)]\Delta\Theta_{\rm t},\end{cases}$$

the angular acceleration signal $\alpha_{\rm t}(t)$ is further expressed as

$$\begin{aligned}\alpha_{\rm t}(t) &= \sum_{n=1}^{2}[a_n\cos(n\omega_{\rm s}t)+b_n\sin(n\omega_{\rm s}t)]\\ &\quad+\sum_{m=1}^{2}\{a_{{\rm b}m}\cos[m(\Theta(t))]\\ &\quad+b_{{\rm b}m}\sin[m(\Theta(t))]\}+\delta\alpha_{\rm t}(t),\end{aligned}\tag{9}$$

where the term

$$\begin{aligned}\delta\alpha_{\rm t}(t) &= \delta\alpha_{\rm g}(t)+\delta\alpha_{\rm b}(t)\\ &=\sum_{m=1}^{2}\{ma_{{\rm b}m}\sin[m\Theta(t)]\Delta\Theta_{\rm t}-mb_{{\rm b}m}\cos[\Theta(t)]\Delta\Theta_{\rm t}\}\end{aligned}$$

is the influence of the cosine oscillation on the $\alpha_{\rm t}(t)$. The $\delta\alpha_{\rm g}(t)$ and $\delta\alpha_{\rm b}(t)$ denote the cosine oscillation influences of the useful signal $\alpha_{\rm g}(t)$ and the gravitational background signal $\alpha_{\rm b}(t)$, respectively. Combined with Eq. (6), the oscillation term $\delta\alpha_{\rm t}(t)$ is rewritten as

$$\begin{aligned}\delta\alpha_{\rm t}(t) &= \sum_{m=1}^{2}\{ma_{{\rm b}m}\sin[m\Theta(t)]-mb_{{\rm b}m}\cos[\Theta(t)]\}\\ &\quad\times\sum_{m=1}^{2}\left[\frac{a_m\cos(m\omega t)+b_m\sin(m\omega t)}{m^2\omega^2}\right.\\ &\quad\left.+\frac{a_{{\rm b}m}\cos(m\omega_1 t)+b_{{\rm b}m}\sin(m\omega_1 t)}{m^2\omega_1^2}\right].\end{aligned}\tag{10}$$

Hence, the oscillation term $\delta\alpha_{\rm b}(t)$ can be further divided into 6 parts,i.e.,

$$\begin{aligned}\delta\alpha_{\rm b}(t) &= \delta_1\alpha_{\rm b}(t)+\delta_2\alpha_{\rm b}(t)+\delta_3\alpha_{\rm b}(t)\\ &\quad+\delta_4\alpha_{\rm b}(t)+\delta_5\alpha_{\rm b}(t)+\delta_6\alpha_{\rm b}(t),\end{aligned}\tag{11}$$

where

$$\begin{aligned}\delta_1\alpha_{\rm b}(t) &= \frac{a_1\cos(\omega t)+b_1\sin(\omega t)}{\omega^2}\\ &\quad\times\{2a_{\rm b2}\sin[2\Theta(t)]-2b_{\rm b2}\cos[2\Theta(t)]\},\\ \delta_2\alpha_{\rm b}(t) &= \frac{a_2\cos(2\omega t)+b_2\sin(2\omega t)}{4\omega^2}\\ &\quad\times\{2a_{\rm b2}\sin[2\Theta(t)]-2b_{\rm b2}\cos[2\Theta(t)]\},\\ \delta_3\alpha_{\rm b}(t) &= \frac{a_{\rm b2}\cos(2\omega_1 t)+b_{\rm b2}\sin(2\omega_1 t)}{4\omega_1^2}\\ &\quad\times\{2a_{\rm b2}\sin[2\Theta(t)]-2b_{\rm b2}\cos[2\Theta(t)]\},\\ \delta_4\alpha_{\rm b}(t) &= \frac{a_1\cos(\omega t)+b_1\sin(\omega t)}{\omega^2}\\ &\quad\times\{a_{\rm b1}\sin[\Theta(t)]-b_{\rm b1}\cos[\Theta(t)]\},\\ \delta_5\alpha_{\rm b}(t) &= \frac{a_2\cos(2\omega t)+b_2\sin(2\omega t)}{4\omega^2}\\ &\quad\times\{a_{\rm b1}\sin[\Theta(t)]-b_{\rm b1}\cos[\Theta(t)]\},\\ \delta_6\alpha_{\rm b}(t) &= \frac{a_{\rm b1}\cos(\omega_1 t)+b_{\rm b1}\sin(\omega_1 t)}{\omega_1^2}\\ &\quad\times\{2a_{\rm b1}\sin[2\Theta(t)]-2b_{\rm b1}\cos[2\Theta(t)]\}\\ &\quad+\frac{a_{\rm b2}\cos(2\omega_1 t)+b_{\rm b2}\sin(2\omega_1 t)}{4\omega_1^2}\\ &\quad\times\{a_{\rm b1}\sin[\Theta(t)]-b_{\rm b1}\cos[\Theta(t)]\}.\end{aligned}$$

3. Correction of cosine oscillation to the amplitude estimation of the gravitational background signal

3.1. Corrections of $\delta_1\alpha_{\rm b}(t)$, $\delta_2\alpha_{\rm b}(t)$, and $\delta_3\alpha_{\rm b}(t)$ to amplitude estimation

Based on Ref. [18], the experimental data $\{\alpha_{\rm t}(t)\}$ are divided into some small parts, each of which contains 20 periods of the useful signal $\alpha_{\rm g}(t)$ with frequency exactly known, and $\alpha_{\rm g}(t)$ is removed by subtracting the pairs of the data points $\{\alpha_{\rm t}(t)\}$ separated by 2π phase in the useful signal frequency $\omega_{\rm s}$. For $\delta_1\alpha_{\rm b}(t)$, it yields the filtered gravitational background signal:

$$\begin{aligned}\delta_1\bar{\alpha}_{\rm b}(t) &= \frac{\delta_1\alpha_{\rm b}(t)-\delta_1\alpha_{\rm b}(t+T)}{2}\\ &=\frac{a_1\cos(\omega t)+b_1\sin(\omega t)}{\omega^2}\\ &\quad\times\{a_{\rm b2}[\sin(2\Theta(t))-\sin(2\Theta(t+T))]\\ &\quad-b_{\rm b2}[\cos(2\Theta(t))-\cos(2\Theta(t+T))]\}.\end{aligned}\tag{12}$$

On the basis of Taylor expansion, suppose $\Delta\Theta=\Theta(t)-\Theta(t+T)$, and then we will obtain

$$\begin{cases}\sin(2\Theta(t))-\sin(2\Theta(t+T))\approx[1-\cos(2\Delta\Theta)]\sin[\Theta(t)]+\sin(2\Delta\Theta)\sin[\Theta(t)],\\ \cos(2\Theta(t))-\cos(2\Theta(t+T))\approx[1-\cos(2\Delta\Theta)]\cos[\Theta(t)]+\sin(2\Delta\Theta)\cos[\Theta(t)].\end{cases}\tag{13}$$

Substitution of Eq. (13) into Eq. (12) yields

$$\delta_1\bar{\alpha}_{\rm b}(t) = \frac{a_1\cos(\omega t)+b_1\sin(\omega t)}{\omega^2}\times\{[-a_{\rm b2}\sin 2\Delta\Theta-b_{\rm b2}(1-\cos 2\Delta\Theta)]\cos[2\Theta(t)]$$

$$+[a_{b2}(1-\cos 2\Delta\Theta)-b_{b2}\sin 2\Delta\Theta]\sin[2\Theta(t)]\}. \tag{14}$$

After each small part is shifted forward t_0 to start from $t=0$, according to Eq. (3) and the same processing of Ref. [18], we make the first-order approximation by performing Taylor expansion, and have the following expressions:

$$\begin{cases} \cos(m\Delta\Theta)\approx\cos(m\Phi_0)-\left[\Phi_1\left(\dfrac{t-t_0}{T_p}\right)+\Phi_2\left(\dfrac{t-t_0}{T_p}\right)^2\right]m\sin(m\Phi_0), \\ \sin(m\Delta\Theta)\approx\sin(m\Phi_0)+\left[\Phi_1\left(\dfrac{t-t_0}{T_p}\right)+\Phi_2\left(\dfrac{t-t_0}{T_p}\right)^2\right]m\cos(m\Phi_0), \end{cases} \tag{15}$$

with $m=1,2$, where Φ_1, Φ_2, and Φ_3 are the same as those in Ref. [18]. We need only to analyze the dominant term, namely $\cos(m\Delta\Theta)\approx\cos(m\Theta_0)$ and $\sin(m\Delta\Theta)\approx\sin(m\Theta_0)$, and thus equation (14) is converted into

$$\begin{aligned}\delta_1\bar{\alpha}_b(t)=&\frac{a_1\cos(\omega t)+b_1\sin(\omega t)}{\omega^2}\\&\times\{[-a_{b2}\sin 2\phi_0-b_{b2}(1-\cos 2\phi_0)]\cos[2\Theta(t)]\\&+[a_{b2}(1-\cos 2\phi_0)-b_{b2}\sin 2\phi_0]\sin[2\Theta(t)]\}.\end{aligned} \tag{16}$$

Suppose

$$\bar{t}=t\left[1+\frac{d_{1\omega Tp}}{2}\left(\frac{t}{T_p}\right)+\frac{d_{2\omega Tp}}{3}\left(\frac{t}{T_p}\right)^2\right]$$

that denotes the stretched time after the above Taylor expansion and non-dimensionalized processing with respect to the time t, then equation (3) will be rewritten as $\Theta(t)=\omega_1\bar{t}$. Then we can also obtain the derivative relation between t and $\bar{t}$, expressed as

$$\mathrm{d}\bar{t}=\left[1+d_{1\omega Tp}\left(\frac{t}{T_p}\right)+d_{2\omega Tp}\left(\frac{t}{T_p}\right)^2\right]\mathrm{d}t.$$

After the above processing, the higher order term is negligible with respect to t, and then the t in Eq. (16) can be replaced with $\bar{t}$, and then equation (16) is further converted into

$$\begin{aligned}\delta_1\bar{\alpha}_b(t)=&\frac{a_1\cos(\omega t)+b_1\sin(\omega t)}{\omega^2}\\&\times\{[-a_{b2}\sin 2\phi_0-b_{b2}(1-\cos 2\phi_0)]\cos(2\omega_1\bar{t})\\&+[a_{b2}(1-\cos 2\phi_0)-b_{b2}\sin 2\phi_0]\sin(2\omega_1\bar{t})\}.\end{aligned} \tag{17}$$

According to the correlation method of estimating the amplitude of a standard cosine signal,[6] the amplitude component corrections caused by the cosine oscillation to the amplitudes of the prominent fundamental frequency and second harmonic components of the filtered gravitational background signal in the i-th small part are separately given by

$$\delta_1\bar{a}_{mi}=\frac{2}{\bar{T}}\int_{(i-1)\bar{T}}^{i\bar{T}}\delta_1\bar{\alpha}_b(t)\cos(m\omega_1\bar{t})\,\mathrm{d}\bar{t}, \tag{18}$$

$$\delta_1\bar{b}_{mi}=\frac{2}{\bar{T}}\int_{(i-1)\bar{T}}^{i\bar{T}}\delta_1\bar{\alpha}_b(t)\sin(m\omega_1\bar{t})\,\mathrm{d}\bar{t}, \tag{19}$$

where $\bar{T}=20T$ is the length of the ith small part, and the subscript m denotes the m-th order harmonic component of the filtered gravitational background signal. Substituting Eq. (17) into Eq. (18), then the cosine amplitude component correction $\delta_1\bar{a}_{1i}$ of $\delta_1\bar{\alpha}_b(t)$ will be obtained as

$$\begin{aligned}\delta_1\bar{a}_{1i}=&[-\sin 2\phi_0\times c_{11}+(1-\cos 2\phi_0)\times c_{21}]a_{b2}\\&-[(1-\cos 2\phi_0)\times c_{11}+\sin 2\phi_0\times c_{21}]b_{b2},\end{aligned} \tag{20}$$

where

$$\begin{cases} c_{11}=\dfrac{2}{\omega\bar{T}}\dfrac{\omega^2-5\omega_1^2}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}\times\left\{-a_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]+b_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]\right\}, \\ c_{21}=\dfrac{2}{\omega\bar{T}}\dfrac{-\omega^2+3\omega_1^2}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}\dfrac{2\omega_1}{\omega}\times\left\{a_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]+b_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]\right\}. \end{cases}$$

Substituting Eq. (17) into Eq. (19), the sine amplitude component correction $\delta_1\bar{b}_{1i}$ of $\delta_1\bar{\alpha}_b(t)$ is obtained as

$$\delta_1\bar{b}_{1i}=[-\sin 2\phi_0\times c_{31}+(1-\cos 2\phi_0)\times c_{41}]a_{b2}-[(1-\cos 2\phi_0)\times c_{31}+\sin 2\phi_0\times c_{41}]b_{b2}, \tag{21}$$

where

$$\begin{cases} c_{31}=\dfrac{2}{\omega^2\bar{T}}\dfrac{(\omega^2+3\omega_1^2)\omega_1}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}\times\left\{-a_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]-b_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]\right\} \\ c_{41}=\dfrac{2}{\omega\bar{T}}\dfrac{4\omega_1^2}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}\times\left\{a_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]-b_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_1}\right)\right]\right\}. \end{cases}$$

Similarly, the cosine amplitude component correction $\delta_1\bar{a}_{2i}$ of $\delta_1\bar{\alpha}_{\rm b}(t)$ is given as

$$\delta_1\bar{a}_{2i} = [-\sin 2\phi_0 \times c_{51} + (1-\cos 2\phi_0)\times c_{61}]a_{\rm b2} - [(1-\cos 2\phi_0)\times c_{51} + \sin 2\phi_0 \times c_{61}]b_{\rm b2}, \tag{22}$$

where

$$\begin{cases} c_{51} = \dfrac{2}{\omega^2\bar{T}}\dfrac{1}{-\omega^2+16\omega_{\rm l}^2}\dfrac{\omega^2-8\omega_{\rm l}^2}{\omega}\times\left\{a_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]-b_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}, \\ c_{61} = \dfrac{2}{\omega^2\bar{T}}\dfrac{2\omega_{\rm l}}{-\omega^2+16\omega_{\rm l}^2}\times\left\{a_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]+b_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}. \end{cases}$$

The sine amplitude component correction $\delta_1\bar{b}_{2i}$of $\delta_1\bar{\alpha}_{\rm b}(t)$ is given as

$$\delta_1\bar{b}_{2i} = [-\sin 2\phi_0 \times c_{71} + (1-\cos 2\phi_0)\times c_{81}]a_{\rm b2} - [(1-\cos 2\phi_0)\times c_{71} + \sin 2\phi_0 \times c_{81}]b_{\rm b2}, \tag{23}$$

where

$$\begin{cases} c_{71} = \dfrac{2}{\omega^2\bar{T}}\dfrac{2\omega_{\rm l}}{-\omega^2+16\omega_{\rm l}^2}\times\left\{a_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]+b_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}, \\ c_{81} = \dfrac{2}{\omega^2\bar{T}}\dfrac{1}{-\omega^2+16\omega_{\rm l}^2}\dfrac{8\omega_{\rm l}^2}{\omega}\times\left\{-a_1\left[\sin\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]+b_1\left[\cos\left(\dfrac{2(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\dfrac{2i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}. \end{cases}$$

By performing the same processing as the above processing of $\delta_1\alpha_{\rm b}(t)$, $\delta_2\alpha_{\rm b}(t)$ and $\delta_3\alpha_{\rm b}(t)$ can be respectively converted into

$$\begin{aligned} \delta_2\bar{\alpha}_{\rm b}(t) = {} & \frac{a_2\cos(2\omega\bar{t})+b_2\sin(2\omega\bar{t})}{4\omega^2} \\ & \times\big\{[-a_{\rm b2}\sin 2\phi_0 - b_{\rm b2}(1-\cos 2\phi_0)]\cos(2\omega_1\bar{t}) \\ & +[a_{\rm b2}(1-\cos 2\phi_0)-b_{\rm b2}\sin 2\phi_0]\sin(2\omega_1\bar{t})\big\}, \end{aligned} \tag{24}$$

$$\begin{aligned} \delta_3\bar{\alpha}_{\rm b}(t) = {} & \frac{1}{2\omega_2^2}\big\{[(a_{\rm b2}^2-b_{\rm b2}^2)(-\sin 4\phi_0) \\ & -2a_{\rm b2}b_{\rm b2}(1-\cos 4\phi_0)]\cos(4\omega_1 t) \\ & +[(a_{\rm b2}^2-b_{\rm b2}^2)(1-\cos 4\phi_0) \\ & -2a_{\rm b2}b_{\rm b2}\sin 4\phi_0]\sin(4\omega_1 t)\big\}. \end{aligned} \tag{25}$$

Then, the amplitude component corrections of $\delta_2\bar{\alpha}_{\rm b}(t)$ are given by

$$\begin{aligned} \delta_2\bar{a}_{1i} = {} & [-\sin 2\phi_0 \times c_{12} + (1-\cos 2\phi_0)\times c_{22}]a_{\rm b2} \\ & -[(1-\cos 2\phi_0)\times c_{12} + \sin 2\phi_0 \times c_{22}]b_{\rm b2}, \end{aligned} \tag{26}$$

$$\begin{aligned} \delta_2\bar{b}_{1i} = {} & [-\sin 2\phi_0 \times c_{32} + (1-\cos 2\phi_0)\times c_{42}]a_{\rm b2} \\ & -[(1-\cos 2\phi_0)\times c_{32} + \sin 2\phi_0 \times c_{42}]b_{\rm b2}, \end{aligned} \tag{27}$$

$$\begin{aligned} \delta\bar{a}_{2i} = {} & [-\sin 2\phi_0 \times c_{52} + (1-\cos 2\phi_0)\times c_{62}]a_{\rm b2} \\ & -[(1-\cos 2\phi_0)\times c_{52} + \sin 2\phi_0 \times c_{62}]b_{\rm b2}, \end{aligned} \tag{28}$$

$$\begin{aligned} \delta\bar{b}_{2i} = {} & [-\sin 2\phi_0 \times c_{72} + (1-\cos 2\phi_0)\times c_{82}]a_{\rm b2} \\ & -[(1-\cos 2\phi_0)\times c_{72} + \sin 2\phi_0 \times c_{82}]b_{\rm b2}, \end{aligned} \tag{29}$$

where the coefficient c_{12}, c_{22}, c_{32}, c_{42}, c_{52}, c_{62}, c_{72}, and c_{82} respectively denote

$$\begin{aligned} c_{12} = {} & \frac{1}{\omega\bar{T}}\frac{4\omega^2-5\omega_{\rm l}^2}{16\omega^4-40\omega^2\omega_{\rm l}^2+9\omega_{\rm l}^4} \\ & \times\left\{-a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right. \\ & \left.+b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}, \\ c_{22} = {} & \frac{1}{\omega^2\bar{T}}\frac{\omega_{\rm l}(-4\omega^2+3\omega_{\rm l}^2)}{16\omega^4-40\omega^2\omega_{\rm l}^2+9\omega_{\rm l}^4} \\ & \times\left\{a_2[\cos\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right. \\ & \left.+b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}, \\ c_{32} = {} & \frac{1}{2\omega^2\bar{T}}\frac{(4\omega^2+3\omega_{\rm l}^2)\omega_{\rm l}}{16\omega^4-40\omega^2\omega_{\rm l}^2+9\omega_{\rm l}^4} \\ & \times\left\{-a_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right. \\ & \left.-b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}, \\ c_{42} = {} & \frac{1}{\omega\bar{T}}\frac{4\omega_{\rm l}^2}{16\omega^4-40\omega^2\omega_{\rm l}^2+9\omega_{\rm l}^4} \\ & \times\left\{a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right. \\ & -b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)\right. \\ & \left.\left.-\cos\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}, \\ c_{52} = {} & \frac{1}{4\omega^2\bar{T}}\frac{1}{-\omega^2+4\omega_{\rm l}^2}\frac{\omega^2-2\omega_{\rm l}^2}{\omega} \\ & \times\left\{a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\sin\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right. \\ & \left.-b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_{\rm l}}\right)-\cos\left(\frac{4i\pi\omega}{\omega_{\rm l}}\right)\right]\right\}, \end{aligned}$$

$$
\begin{aligned}
c_{62} &= \frac{1}{4\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+4\omega_1^2}\\
&\times\left\{a_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\},\\
c_{72} &= \frac{1}{4\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+4\omega_1^2}\\
&\times\left\{a_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\},\\
c_{82} &= \frac{1}{2\omega^2\bar{T}}\frac{1}{-\omega^2+4\omega_1^2}\frac{\omega_1^2}{\omega}\\
&\times\left\{-a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\}.
\end{aligned}
$$

Likewise, the amplitude component corrections of $\delta_3\bar{\alpha}_{\rm b}(t)$ are given by

$$\delta_3\bar{a}_{1i}=\delta_3\bar{b}_{1i}=\delta_3\bar{a}_{2i}=\delta_3\bar{b}_{2i}=0. \tag{30}$$

It means that the term $\delta_3\bar{\alpha}_{\rm b}(t)$has no correction to the amplitude estimation of the gravitational signal.

3.2. Corrections of $\delta_4\alpha_{\rm b}(t)$ and $\delta_5\alpha_{\rm b}(t)$ to the amplitude estimation

By performing the same processing as the above processing of $\delta_1\alpha_{\rm b}(t)$, $\delta_4\alpha_{\rm b}(t)$ and $\delta_5\alpha_{\rm b}(t)$ are converted into

$$
\begin{aligned}
\delta_4\bar{\alpha}_{\rm b}(t) &= \frac{1}{2\omega^2}\{[-a_{\rm b1}\sin\phi_0-b_{\rm b1}(1-\cos\phi_0)]\\
&\times[a_1\cos(\omega\bar{t})\cos(\omega_1\bar{t})+b_1\sin(\omega\bar{t})\cos(\omega_1\bar{t})]\\
&+[a_{\rm b1}(1-\cos\phi_0)-b_{\rm b1}\sin\phi_0]\\
&\times[a_1\cos(\omega\bar{t})\sin(\omega_1\bar{t})\\
&+b_1\sin(\omega\bar{t})\sin(\omega_1\bar{t})]\},
\end{aligned}\tag{31}
$$

$$
\begin{aligned}
\delta_5\bar{\alpha}_{\rm b}(t) &= \frac{1}{8\omega^2}\{[-a_{\rm b1}\sin\phi_0-b_{\rm b1}(1-\cos\phi_0)]\\
&\times[a_2\cos(2\omega\bar{t})\cos(\omega_1\bar{t})+b_2\sin(2\omega\bar{t})\cos(\omega_1\bar{t})]\\
&+[a_{\rm b1}(1-\cos\phi_0)-b_{\rm b1}\sin\phi_0]\\
&\times[a_2\cos(2\omega\bar{t})\sin(\omega_1\bar{t})\\
&+b_2\sin(2\omega\bar{t})\sin(\omega_1\bar{t})]\}.
\end{aligned}\tag{32}
$$

Then, the amplitude component corrections of $\delta_4\bar{\alpha}_{\rm b}(t)$ are given by

$$
\begin{aligned}
\delta_4\bar{a}_{1i} &= [-\sin\phi_0\times d_{11}+(1-\cos\phi_0)\times d_{21}]a_{\rm b1}\\
&-[(1-\cos\phi_0)\times d_{11}+\sin\phi_0\times d_{21}]b_{\rm b1},
\end{aligned}\tag{33}
$$

$$
\begin{aligned}
\delta_4\bar{b}_{1i} &= [-\sin\phi_0\times d_{31}+(1-\cos\phi_0)\times d_{41}]a_{\rm b1}\\
&-[(1-\cos\phi_0)\times d_{31}+\sin\phi_0\times d_{41}]b_{\rm b1},
\end{aligned}\tag{34}
$$

$$
\begin{aligned}
\delta\bar{a}_{2i} &= [-\sin\phi_0\times d_{51}+(1-\cos\phi_0)\times d_{61}]a_{\rm b1}\\
&-[(1-\cos\phi_0)\times d_{51}+\sin\phi_0\times d_{61}]b_{\rm b1},
\end{aligned}\tag{35}
$$

$$
\begin{aligned}
\delta\bar{b}_{2i} &= [-\sin\phi_0\times d_{71}+(1-\cos\phi_0)\times d_{81}]a_{\rm b1}\\
&-[(1-\cos\phi_0)\times d_{71}+\sin\phi_0\times d_{81}]b_{\rm b1},
\end{aligned}\tag{36}
$$

where

$$
\begin{aligned}
d_{11} &= \frac{1}{\omega^2\bar{T}}\frac{1}{-\omega^2+4\omega_1^2}\frac{\omega^2-2\omega_1^2}{\omega}\\
&\times\left\{a_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.-b_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\},\\
d_{21} &= \frac{1}{\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+4\omega_1^2}\\
&\times\left\{a_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\},\\
d_{31} &= \frac{1}{\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+4\omega_1^2}\\
&\times\left\{a_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\},\\
d_{41} &= \frac{1}{\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+4\omega_1^2}\frac{2\omega_1}{\omega}\\
&\times\left\{-a_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\},\\
d_{51} &= \frac{1}{\omega\bar{T}}\frac{\omega^2-5\omega_1^2}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}\\
&\times\left\{-a_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\},\\
d_{61} &= \frac{1}{\omega^2\bar{T}}\frac{\omega_1(\omega^2+3\omega_1^2)}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}\\
&\times\left\{-a_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.-b_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\},\\
d_{71} &= \frac{1}{\omega^2\bar{T}}\frac{2\omega_1(-\omega^2+3\omega_1^2)}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}\\
&\times\left\{a_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.\\
&\left.+b_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\},\\
d_{81} &= \frac{1}{\omega\bar{T}}\frac{r\omega_1^2}{\omega^4-10\omega^2\omega_1^2+9\omega_1^4}
\end{aligned}
$$

$$\times\left\{-a_1\left[\sin\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right.$$
$$\left.-b_1\left[\cos\left(\frac{2(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{2i\pi\omega}{\omega_1}\right)\right]\right\}.$$

The amplitude component corrections of $\delta_5\bar{\alpha}_{\rm b}(t)$ are given by

$$\delta_5\bar{a}_{1i} = [-\sin\phi_0\times d_{12}+(1-\cos\phi_0)\times d_{22}]a_{\rm b1}$$
$$-[(1-\cos\phi_0)\times d_{12}+\sin\phi_0\times d_{22}]b_{\rm b1}, \quad (37)$$
$$\delta_5\bar{b}_{1i} = [-\sin\phi_0\times d_{32}+(1-\cos\phi_0)\times d_{42}]a_{\rm b1}$$
$$-[(1-\cos\phi_0)\times d_{32}+\sin\phi_0\times d_{42}]b_{\rm b1}, \quad (38)$$
$$\delta_5\bar{a}_{2i} = [-\sin\phi_0\times d_{52}+(1-\cos\phi_0)\times d_{62}]a_{\rm b1}$$
$$-[(1-\cos\phi_0)\times d_{52}+\sin\phi_0\times d_{62}]b_{\rm b1}, \quad (39)$$
$$\delta_5\bar{b}_{2i} = [-\sin\phi_0\times d_{72}+(1-\cos\phi_0)\times d_{82}]a_{\rm b1}$$
$$-[(1-\cos\phi_0)\times d_{72}+\sin\phi_0\times d_{82}]b_{\rm b1}, \quad (40)$$

where

$$d_{12} = \frac{1}{16\omega^2\bar{T}}\frac{1}{-\omega^2+\omega_1^2}\frac{2\omega^2-\omega_1^2}{\omega}$$
$$\times\left\{a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.$$
$$\left.-b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\},$$
$$d_{22} = \frac{1}{16\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+\omega_1^2}$$
$$\times\left\{a_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.$$
$$\left.+b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\},$$
$$d_{32} = \frac{1}{16\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+\omega_1^2}$$
$$\times\left\{a_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.$$
$$\left.+b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\},$$
$$d_{42} = \frac{1}{16\omega^2\bar{T}}\frac{\omega_1}{-\omega^2+\omega_1^2}\frac{\omega_1}{\omega}$$
$$\times\left\{-a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.$$
$$\left.+b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\},$$
$$d_{52} = \frac{1}{2\omega\bar{T}}\frac{4\omega^2-5\omega_1^2}{16\omega^4-40\omega^2\omega_1^2+9\omega_1^4}$$
$$\times\left\{-a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.$$
$$\left.+b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\},$$
$$d_{62} = \frac{1}{4\omega^2\bar{T}}\frac{\omega_1(4\omega^2+3\omega_1^2)}{16\omega^4-40\omega^2\omega_1^2+9\omega_1^4}$$
$$\times\{-a_2[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)]$$
$$-b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\},$$
$$d_{72} = \frac{1}{2\omega^2\bar{T}}\frac{\omega_1(-4\omega^2+3\omega_1^2)}{16\omega^4-40\omega^2\omega_1^2+9\omega_1^4}$$
$$\times\{a_2[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)]$$
$$+b_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\},$$
$$d_{82} = \frac{1}{\omega\bar{T}}\frac{2\omega_1^2}{16\omega^4-40\omega^2\omega_1^2+9\omega_1^4}$$
$$\times\left\{a_2\left[\sin\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\sin\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right.$$
$$\left.-b_2\left[\cos\left(\frac{4(i-1)\pi\omega}{\omega_1}\right)-\cos\left(\frac{4i\pi\omega}{\omega_1}\right)\right]\right\}.$$

3.3. Correction of $\delta_6\alpha_{\rm b}(t)$ to the amplitude estimation

Performing the same processing as the above processing of $\delta_1\alpha_{\rm b}(t)$, $\delta_6\alpha_{\rm b}(t)$ is converted into

$$\delta_6\alpha_{\rm b}(t) = \left[-\frac{2a_{\rm b1}b_{\rm b2}}{\omega_1^2}-\frac{a_{\rm b2}b_{\rm b1}}{4\omega_1^2}\right]\cos(2\omega_1 t)\cos(\omega_1 t)$$
$$+\left[-\frac{2b_{\rm b1}b_{\rm b2}}{\omega_1^2}+\frac{a_{\rm b1}a_{\rm b2}}{4\omega_1^2}\right]\cos(2\omega_1 t)\sin(\omega_1 t)$$
$$+\left[\frac{2a_{\rm b2}a_{\rm b1}}{\omega_1^2}-\frac{b_{\rm b1}b_{\rm b2}}{4\omega_1^2}\right]\sin(2\omega_1 t)\cos(\omega_1 t)$$
$$+\left[\frac{2a_{\rm b2}b_{\rm b1}}{\omega_1^2}+\frac{a_{\rm b1}b_{\rm b2}}{4\omega_1^2}\right]\sin(2\omega_1 t)\sin(\omega_1 t). \quad (41)$$

Based on the tripling-angle formula, namely

$$\cos(2\omega_1 t)\cos(\omega_1 t) = \frac{1}{2}[\cos(\omega_1 t)+\cos(3\omega_1 t)],$$
$$\sin(2\omega_1 t)\sin(\omega_1 t) = \frac{1}{2}[\cos(\omega_1 t)-\cos(3\omega_1 t)],$$
$$\cos(2\omega_1 t)\sin(\omega_1 t) = \frac{1}{2}[\sin(3\omega_1 t)-\sin(\omega_1 t)],$$
$$\sin(2\omega_1 t)\cos(\omega_1 t) = \frac{1}{2}[\sin(3\omega_1 t)+\sin(\omega_1 t)],$$

equation (41) is further rewritten as

$$\delta_6\bar{\alpha}_{\rm b}(t) = \frac{1}{2}\left\{\left(\frac{2}{\omega_1^2}-\frac{1}{4\omega_1^2}\right)(-a_{\rm b1}b_{\rm b2}+a_{\rm b2}b_{\rm b1})\cos(\omega_1 t)\right.$$
$$+\left(\frac{2}{\omega_1^2}-\frac{1}{4\omega_1^2}\right)(a_{\rm b1}a_{\rm b2}+b_{\rm b1}b_{\rm b2})\sin(\omega_1 t)$$
$$+\left(\frac{2}{\omega_1^2}+\frac{1}{4\omega_1^2}\right)(-a_{\rm b1}b_{\rm b2}-a_{\rm b2}b_{\rm b1})\cos(3\omega_1 t)$$
$$\left.+\left(\frac{2}{\omega_1^2}+\frac{1}{4\omega_1^2}\right)(a_{\rm b1}a_{\rm b2}-b_{\rm b1}b_{\rm b2})\sin(3\omega_1 t)\right\}. \quad (42)$$

By performing the same shift-phase and approximation processing as those of $\delta_1\alpha_{\rm b}(t)$, equation (42) is turned into

$$\delta_6\bar{\alpha}_{\rm b}(t) = \frac{1}{4}\left\{\left[(-a_{\rm b1}b_{\rm b2}+a_{\rm b2}b_{\rm b1})(1-\cos\phi_0)\right.\right.$$
$$\left.\left.-(a_{\rm b1}a_{\rm b2}+b_{\rm b1}b_{\rm b2})\sin\phi_0\right]\left(\frac{2}{\omega_1^2}-\frac{1}{4\omega_1^2}\right)\cos(\omega_1\bar{t})\right.$$

$$
\begin{aligned}
&+[(-a_{\rm b1}b_{\rm b2}+a_{\rm b2}b_{\rm b1})\sin\phi_0+(a_{\rm b1}a_{\rm b2}+b_{\rm b1}b_{\rm b2})\\
&\times(1-\cos\phi_0)]\left(\frac{2}{\omega_1^2}-\frac{1}{4\omega_1^2}\right)\sin(\omega_1\bar{t})\\
&+[(-a_{\rm b1}b_{\rm b2}-a_{\rm b2}b_{\rm b1})(1-\cos 3\phi_0)\\
&-(a_{\rm b1}a_{\rm b2}-b_{\rm b1}b_{\rm b2})\sin 3\phi_0]\\
&\times\left(\frac{2}{\omega_1^2}+\frac{1}{4\omega_1^2}\right)\cos(3\omega_1\bar{t})\\
&+[(-a_{\rm b1}b_{\rm b2}-a_{\rm b2}b_{\rm b1})\sin 3\phi_0+(a_{\rm b1}a_{\rm b2}-b_{\rm b1}b_{\rm b2})\\
&\times(1-\cos 3\phi_0)]\left(\frac{2}{\omega_1^2}+\frac{1}{4\omega_1^2}\right)\sin(3\omega_1\bar{t})\Big\}. \qquad (43)
\end{aligned}
$$

Like Eqs. (18) and (19), the amplitude component corrections of $\delta_6\bar{\alpha}_{\rm b}(t)$ are given by:

$$
\begin{aligned}
\delta_6\bar{a}_{1i} &= \frac{1}{4}\left(\frac{2}{\omega_1^2}-\frac{1}{4\omega_1^2}\right)[(-a_{\rm b1}b_{\rm b2}+a_{\rm b2}b_{\rm b1})\\
&\quad\times(1-\cos\phi_0)-(a_{\rm b1}a_{\rm b2}+b_{\rm b1}b_{\rm b2})\sin\phi_0], \qquad (44)\\
\delta_6\bar{b}_{1i} &= \frac{1}{4}\left(\frac{2}{\omega_1^2}-\frac{1}{4\omega_1^2}\right)[(-a_{\rm b1}b_{\rm b2}+a_{\rm b2}b_{\rm b1})\sin\phi_0\\
&\quad+(a_{\rm b1}a_{\rm b2}+b_{\rm b1}b_{\rm b2})(1-\cos\phi_0)], \qquad (45)\\
\delta_6\bar{a}_{2i} &= \delta_6\bar{b}_{2i}=0. \qquad (46)
\end{aligned}
$$

3.4. Total correction of the cosine oscillation $\delta\alpha_{\rm b}(t)$ to the amplitude estimation

In Ref. [18], the cosine oscillation is considered to be negligible, and besides, the relation between the amplitude components ($a_{\rm b1i}$, $b_{\rm b1i}$, $a_{\rm b2i}$, $b_{\rm b2i}$) of the gravitational background signal $\alpha_{\rm b}(t)$ and the amplitude components ($\bar{a}_{1i}$,$\bar{b}_{1i}$,$\bar{a}_{2i}$,$\bar{b}_{2i}$) of the filtered gravitational background signal $\bar{\alpha}_{\rm b}(t)$ for the i-th small part is expressed by the matrix form:

$$
\begin{aligned}
\begin{pmatrix}\bar{a}_{1i}\\ \bar{b}_{1i}\\ \bar{a}_{2i}\\ \bar{b}_{2i}\end{pmatrix} &= \boldsymbol{A}_{4\times4}\cdot\begin{pmatrix}a_{\rm b1i}\\ b_{\rm b1i}\\ a_{\rm b2i}\\ b_{\rm b2i}\end{pmatrix}\\
&= \begin{pmatrix}A_{11}&A_{12}&A_{13}&A_{14}\\ A_{21}&A_{22}&A_{23}&A_{24}\\ A_{31}&A_{32}&A_{33}&A_{34}\\ A_{41}&A_{42}&A_{43}&A_{44}\end{pmatrix}\begin{pmatrix}a_{\rm b1i}\\ b_{\rm b1i}\\ a_{\rm b2i}\\ b_{\rm b2i}\end{pmatrix}, \qquad (47)
\end{aligned}
$$

where all symbols and parameters are the same as those in Ref. [18].

After taking the influence of the cosine oscillation into consideration, the amplitude component corrections of the parts $\delta_1\alpha_{\rm b}(t)$, $\delta_2\alpha_{\rm b}(t)$, $\delta_4\alpha_{\rm b}(t)$, and $\delta_5\alpha_{\rm b}(t)$ are the coupling terms of the gravitational background signal $\alpha_{\rm b}(t)$ and the filtered gravitational background signal $\bar{\alpha}_{\rm b}(t)$, and hence the biases of $\delta_1\alpha_{\rm b}(t)$, $\delta_2\alpha_{\rm b}(t)$, $\delta_4\alpha_{\rm b}(t)$ and $\delta_5\alpha_{\rm b}(t)$ should be corrected in the coefficient matrix $\boldsymbol{A}_{4\times4}$ of Eq. (47). The biases of $\delta_1\alpha_{\rm b}(t)$ and $\delta_2\alpha_{\rm b}(t)$ correct the coefficients of the 3rd and 4th columns in $\boldsymbol{A}_{4\times4}$, and the biases of $\delta_4\alpha_{\rm b}(t)$ and $\delta_5\alpha_{\rm b}(t)$ correct the coefficients of 1st and 2nd columns in $\boldsymbol{A}_{4\times4}$. The parts $\delta_3\alpha_{\rm b}(t)$ has no correction to the amplitude estimation. The correction of $\delta_6\alpha_{\rm b}(t)$ is not a coupling term but an independent term, and thus the bias of $\delta_6\alpha_{\rm b}(t)$ should be corrected directly in the amplitude components ($\bar{a}_{1i}$, $\bar{b}_{1i}$, $\bar{a}_{2i}$, $\bar{b}_{2i}$) of Eq. (47). Then, with the cosine oscillation considered, the relation matrix of Eq. (47) is corrected into

$$
\begin{pmatrix}a_{\rm b1i}\\ b_{\rm b1i}\\ a_{\rm b2i}\\ b_{\rm b2i}\end{pmatrix} = \begin{pmatrix}A'_{11}&A'_{12}&A'_{13}&A'_{14}\\ A'_{21}&A'_{22}&A'_{23}&A'_{24}\\ A'_{31}&A'_{32}&A'_{33}&A'_{34}\\ A'_{41}&A'_{42}&A'_{43}&A'_{44}\end{pmatrix}^{-1}\begin{pmatrix}\bar{a}_{1i}-\delta_6\bar{a}_{1i}\\ \bar{b}_{1i}-\delta_6\bar{b}_{1i}\\ \bar{a}_{2i}\\ \bar{b}_{2i}\end{pmatrix}, \qquad (48)
$$

where

$$
\begin{aligned}
A'_{11} &= A_{11}-\sin\phi_0\times d_{11}+(1-\cos\phi_0)\times d_{21}\\
&\quad-\sin\phi_0\times d_{12}+(1-\cos\phi_0)\times d_{22},\\
A'_{12} &= A_{12}-(1-\cos\phi_0)\times d_1-\sin\phi_0\times d_2\\
&\quad-(1-\cos\phi_0)\times d_{12}-\sin\phi_0\times d_{22},\\
A'_{21} &= A_{21}-\sin\phi_0\times d_{31}+(1-\cos\phi_0)\times d_{41}\\
&\quad-\sin\phi_0\times d_{32}+(1-\cos\phi_0)\times d_{42},\\
A'_{22} &= A_{22}-(1-\cos\phi_0)\times d_{31}-\sin\phi_0\times d_{41}\\
&\quad-(1-\cos\phi_0)\times d_{32}-\sin\phi_0\times d_{42},\\
A'_{31} &= A_{31}-\sin\phi_0\times d_{51}+(1-\cos\phi_0)\times d_{61}\\
&\quad-\sin\phi_0\times d_{52}+(1-\cos\phi_0)\times d_{62},\\
A'_{32} &= A_{32}-(1-\cos\phi_0)\times d_{51}-\sin\phi_0\times d_{61}\\
&\quad-(1-\cos\phi_0)\times d_{52}-\sin\phi_0\times d_{62},\\
A'_{41} &= A_{41}-\sin\phi_0\times d_{71}+(1-\cos\phi_0)\times d_{81}\\
&\quad-\sin\phi_0\times d_{72}+(1-\cos\phi_0)\times d_{82},\\
A'_{42} &= A_{42}-(1-\cos\phi_0)\times d_{71}-\sin\phi_0\times d_{81}\\
&\quad-(1-\cos\phi_0)\times d_{72}-\sin\phi_0\times d_{82},\\
A'_{13} &= A_{13}-\sin 2\phi_0\times c_{11}+(1-\cos 2\phi_0)\times c_{21}\\
&\quad-\sin 2\phi_0\times c_{12}+(1-\cos 2\phi_0)\times c_{22},\\
A'_{14} &= A_{14}-(1-\cos 2\phi_0)\times c_{11}-\sin 2\phi_0\times c_{21}\\
&\quad-(1-\cos 2\phi_0)\times c_{12}-\sin 2\phi_0\times c_{22},\\
A'_{23} &= A_{23}-\sin 2\phi_0\times c_{31}+(1-\cos 2\phi_0)\times c_{41}\\
&\quad-\sin 2\phi_0\times c_{32}+(1-\cos 2\phi_0)\times c_{42},\\
A'_{24} &= A_{24}-(1-\cos 2\phi_0)\times c_{31}-\sin 2\phi_0\times c_{41}\\
&\quad-(1-\cos 2\phi_0)\times c_{32}-\sin 2\phi_0\times c_{42},\\
A'_{33} &= A_{33}-\sin 2\phi_0\times c_{51}+(1-\cos 2\phi_0)\times c_{61}\\
&\quad-\sin 2\phi_0\times c_{52}+(1-\cos 2\phi_0)\times c_{62},\\
A'_{34} &= A_{34}-(1-\cos 2\phi_0)\times c_{51}-\sin 2\phi_0\times c_{61}\\
&\quad-(1-\cos 2\phi_0)\times c_{52}-\sin 2\phi_0\times c_{62},\\
A'_{43} &= A_{43}-\sin 2\phi_0\times c_{71}+(1-\cos 2\phi_0)\times c_{81}\\
&\quad-\sin 2\phi_0\times c_{72}+(1-\cos 2\phi_0)\times c_{82},\\
A'_{44} &= A_{44}-(1-\cos 2\phi_0)\times c_{71}-\sin 2\phi_0\times c_{81}\\
&\quad-(1-\cos 2\phi_0)\times c_{72}-\sin 2\phi_0\times c_{82}.
\end{aligned}
$$

According to Eq. (48), for the i-th small part, we can obtain the amplitude components ($a_{\rm b1i}$, $b_{\rm b1i}$, $a_{\rm b2i}$, $b_{\rm b2i}$) of the

prominent fundamental frequency and second harmonic components of the gravitational background signal with the bias of the cosine oscillation corrected. Then, the amplitudes of the $\alpha_{b1}(t)$ and $\alpha_{b2}(t)$ for the i-th part are respectively obtained by $\alpha_{b,1}(i)=\sqrt{a_{b1i}^2+b_{b1i}^2}$ and $\alpha_{b,2}(i)=\sqrt{a_{b2i}^2+b_{b2i}^2}$. The amplitudes $\alpha_{b,1}$ and $\alpha_{b,2}$ of the gravitational background signal $\alpha_b(t)$, with the cosine oscillation influence corrected, are given as the average values of the sequences $\{\alpha_{b,1}(i)\}$ and $\{\alpha_{b,2}(i)\}$, and their respective uncertainties are given in the same way as that in Ref. [18]. The amplitudes with the cosine oscillation influence uncorrected are also obtained by the improved correlation method in Ref. [18]. Finally, The bias of the cosine oscillation is obtained as the difference between the corrected amplitude and the uncorrected amplitude.

4. Experiment results

For a typical data set[18] of the measurement of the gravitational constant G with angular acceleration method by Huazhong University of Science and Technology group, it has a valid time of about 47 hours. The frequency of the useful signal is set to be 2.5 mHz, the natural angular frequency ω_0 of the torsion pendulum is about 0.015 rad/s, and the initial angular velocity of the torsion balance turntable is 3.032 mrad/s. As figure 1 shows, besides the whole quadratic slow drift, the angular velocity of the torsion balance turntable, which is the angular frequency of the gravitational background signal $\alpha_b(t)$, also undergoes the cosine oscillation fluctuation. After performing the quadratic fit to the angular velocity of the torsion balance turntable, the parameters p_3, p_2, and p_1 in Eq. (2) are obtained to be -2.176×10^{-7}, -8.584×10^{-6}, and 3.032×10^{-3} in the time unit of hour, respectively. Obtained by the improved correlation method, the amplitudes without and with the cosine oscillation influence corrected in each small part are shown in Fig. 2. The amplitude of the fundamental frequency component of the $\alpha_b(t)$ without considering the cosine oscillation influence is given as 1.439(11) nrad/s^2, while the corresponding amplitude with the cosine oscillation influence corrected is given as 1.444(11) nrad/s^2. The correction of the cosine oscillation is the difference between the uncorrected and corrected amplitudes, which is obtained as 0.005 nrad/s^2 within its uncertainty of 0.011 nrad/s^2. Likewise, the amplitude of the second harmonic of the $\alpha_b(t)$ without considering the cosine oscillation influence is given as 45.253(11) nrad/s^2, while the corresponding amplitude with the cosine oscillation influence corrected is given as 45.255(11) nrad/s^2, and hence the correction of the cosine oscillation is 0.002 nrad/s^2 within its uncertainty of 0.011 nrad/s^2. These results prove that the corrections of the cosine oscillation to the estimated amplitudes of the prominent fundamental frequency and second harmonic components of the spurious gravitational background signal are within respective uncertainties, which are small enough to be negligible.

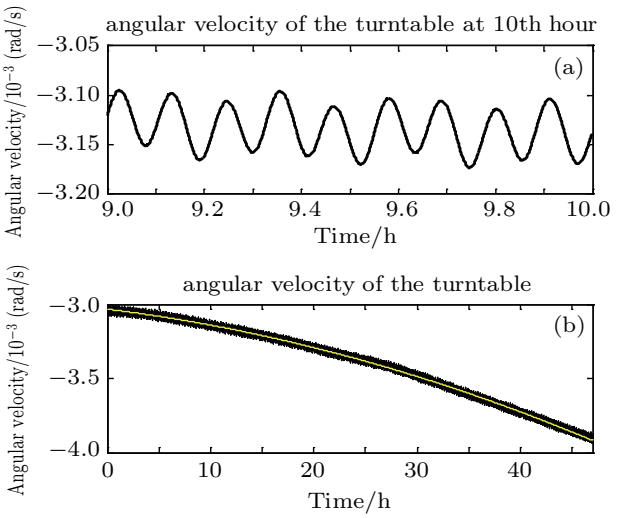


Fig. 1. (color online) (a) The angular velocity of the torsion balance turntable at the 10th hour (black line). (b) The angular velocity of the torsion balance turntable versus time during the whole experiment. The green line is the fitting line of the angular velocity. It is clear that the angular velocity of the torsion balance turntable undergoes not only the whole quadratic slow drift but also the cosine oscillation fluctuation.

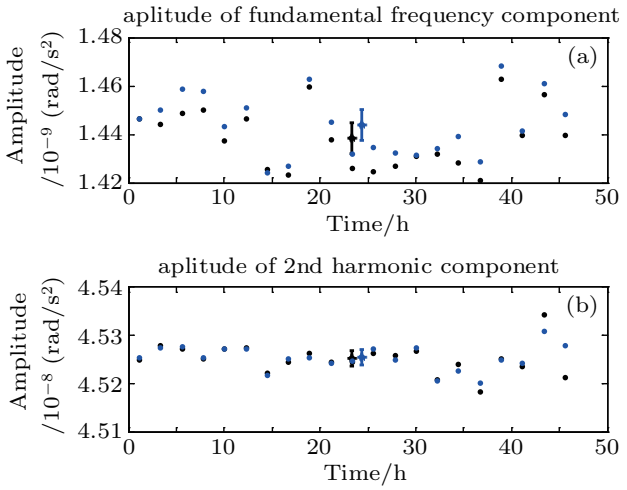


Fig. 2. (color online) (a) The amplitude of the fundamental frequency component of the gravitational background signal $\alpha_b(t)$ in small parts versus time, and (b) the amplitude of second harmonic components of the $\alpha_b(t)$ in small parts versus time. Each blue point denotes the obtained amplitude with the cosine oscillation influence corrected in a small part by the improved correlation method, and each black point denotes the obtained amplitude without correcting the cosine oscillation influence in a small part in the same way. The asterisks and their error bars denote the final values and uncertainties of the amplitudes. Some blue points overlap with corresponding black points, since their differences are too small to distinguish.

5. Conclusions

In the measurement of the gravitational constant G by angular acceleration method with feedback control unit, in order to determine the amplitude of the useful angular acceleration signal accurately, we need to subtract the spurious gravitational background signal effectively. Due to the linear slow drift of equilibrium position of torsion balance and

under the function of the feedback control unit, there is not only the whole quadratic slow drift but also the cosine oscillation fluctuation in the angular velocity of the torsion balance turntable, namely the angular frequency of the spurious gravitational background signal. The amplitudes of the prominent fundamental frequency and second harmonic components of the gravitational background signal with time-varying angular frequency can be estimated by the improved correlation method with high precision in the same time, when the time-varying angular frequency is approximated as the quadratic slow drift. In this paper, we analyze the influence of the cosine oscillation coming from the useful angular acceleration signal on the spurious gravitational background signal, and obtain the corrections of the cosine oscillation influence to the estimated amplitudes of the prominent components of the gravitational background signal. The corrections of the cosine oscillation to the estimated amplitudes have respective uncertainties, which are small enough to be negligible, but the consideration of the cosine oscillation is advisable and important for judging the correctness and completeness of the improved correlation method of estimating the amplitude of the time-varying angular frequency signal subjected to a quadratic slow drift and a cosine oscillation fluctuation.

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