Bidirectional multi-qubit quantum teleportation in noisy channel aided with weak measurement*

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(Received 7 November 2016; revised manuscript received 23 December 2016; published online 20 February 2017)

Recently, bidirectional quantum teleportation has attracted a great deal of research attention. However, existing bidirectional teleportation schemes are normally discussed on the basis of perfect quantum environments. In this paper, we first put forward a bidirectional teleportation scheme to transport three-qubit Greenberger–Horne–Zeilinger (GHZ) states based on controled-not (CNOT) operation and single-qubit measurement. Then, we generalize it to the teleportation of multi-qubit GHZ states. Further, we discuss the influence of quantum noise on our scheme by the example of an amplitude damping channel, then we obtain the fidelity of the teleportation. Finally, we utilize the weak measurement and the corresponding reversing measurement to protect the quantum entanglement, which shows an effective enhancement of the teleportation fidelity.

Keywords: quantum teleportation, quantum decoherence, amplitude damping, weak measurement

PACS: 03.67.Pp, 03.65.Yz, 03.67.Hk **DOI:** 10.1088/1674-1056/26/4/040305

1. Introduction

Quantum teleportation (QT) plays an important role in quantum computing and quantum communication, which utilizes pre-shared quantum entanglement and classical communication to transmit an unknown quantum state from one location to another. Since the first QT protocol was introduced by Bennnet *et al.* in 1993,[1] it has become a research hotspot. Many modified QT schemes have been proposed, such as controlled teleportation,[2–5] probabilistic teleportation,[6–9] and bidirectional quantum teleportation (BQT).[10–13]

Having the capability of transmitting quantum information between Alice and Bob in two directions simultaneously, BQT has drawn more and more attention in recent years. Huelga *et al.* discussed the possibility of using BQT to implement the nonlocal quantum gates.[10,11] In 2013, Zha *et al.* proposed a controlled BQT scheme by using a five-qubit cluster state to transmit arbitrary single-qubit states.[12] Other similar BQT protocols have been presented, by using different types of entanglement states as the teleportation channels, such as five-qubit composite GHZ–Bell state,[13] six-qubit cluster state,[14] and genuine five-qubit entangled state.[15] After that, asymmetric controlled BQT schemes were devised to transmit a single-qubit state and a two-qubit entanglement state.[16–18] Binayak and Arpan proposed a BQT protocol to transmit arbitrary two-qubit entanglement states via a ten-qubit entangled state.[19] Hassanpour and Houshmand presented a BQT scheme to transmit pure EPR states by using a composite GHZ state.[20]

However, current research work relating to the above BQT seldom thinks about the problem of quantum noise. In a practical situation, a quantum state will couple with the environment inevitably, which breaks the coherence of the quantum entanglement state and gives rise to quantum noise. This does occur during the entanglement distribution in BQT. So, it is very important to analyze the influence of the quantum noise in the process of BQT, thus to determine whether a BQT is successful in a certain circumstance.

On the other hand, to overcome the influence of decoherence on quantum entanglement, several methods have been proposed, including quantum entanglement purification,[21–23] quantum error coding,[24–26] and decoherence-free subspace.[27–29] In recent years, weak measurement and its reversing measurement were widely studied to protect quantum state and quantum entanglement. Korotkov and Jordan demonstrated that the weak measurement of a solid-state qubit could be recovered with a reversal procedure.[30] The similar recovery methods of a superconducting phase qubit and an arbitrary cavity field with finite photon number were discussed and demonstrated in Refs. [31] and [32]. Liao *et al.* proposed an alternative way based on a Hadamard gate and CNOT gate to recover an arbitrary pure two-qubit entanglement state that has undergone a weak measurement.[33] Korotkov and Keane,[34] Lee *et al.*,[35] and Kim *et al.*[36] demonstrated weak measurement and the reversing measurement can protect a single-qubit state and two-qubit entanglement state from decohering the amplitude damping (AD). In Refs. [37] and [38], it was demonstrated that the weak measurement

*Project supported by the National Natural Science Foundation of China (Grant No. 61172071), the Scientific Research Program Funded by Shaanxi Provincial Education Department, China (Grant No. 16JK1711), the International Scientific Cooperation Program of Shaanxi Province, China (Grant No. 2015KW-013), and the Natural Science Foundation Research Project of Shaanxi Province, China (Grant No. 2016JQ6033).

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method can increase the fidelities of a three-qubit GHZ state and a W-like state in an AD channel.

In this paper, we first propose a BQT scheme to transmit three-qubit GHZ states based on the work of Ref. [20], then we generalize this scheme to teleport multi-qubit GHZ states. As an example, we use the AD channel model to analyze the influence of quantum noise on this BQT scheme and give the fidelity of the result of the teleportation. Finally, we use the weak measurement and the reversing measurement on part of the qubits of the teleportation channel state before and after the entanglement distribution. The comparison between the results shows the weak measurement method is helpful for enhancing the fidelity of the bidirectional teleportation in an AD channel.

2. Bidirectional multi-qubit quantum teleportation in ideal environment

2.1. Bidirectional quantum teleportation of three-qubit GHZ states

In this section, we first discuss the bidirectional teleportation scheme to transport three-qubit GHZ states in a noiseless quantum channel. Suppose that Alice and Bob have the following three-qubit GHZ states to be teleported, respectively, which are described as

$$|\psi_1\rangle = (\alpha_0|000\rangle + \alpha_1|111\rangle)_{A_1A_2A_3}, \tag{1}$$

$$|\psi_2\rangle = (\beta_0|000\rangle + \beta_1|111\rangle)_{B_1B_2B_3}, \tag{2}$$

where α_0, α_1, β_0, and β_1 are the arbitrary complex numbers that satisfy $|\alpha_0|^2+|\alpha_1|^2 = 1$, $|\beta_0|^2+|\beta_1|^2 = 1$, respectively. The bidirectional teleportation procedure is composed of the following five steps.

Step 1 An eight-qubit composite GHZ state should be shared by Alice and Bob as the teleportation channel, which is generated as follows:

$$\begin{aligned}|\omega\rangle &= |\omega_1\rangle_{a_1b_2b_3b_4} \otimes |\omega_2\rangle_{a_2a_3a_4b_1}\\ &= \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)_{a_1b_2b_3b_4}\\ &\quad \otimes \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)_{a_2a_3a_4b_1}\\ &= \frac{1}{2}(|00000000\rangle + |00001111\rangle + |11110000\rangle\\ &\quad + |11111111\rangle)_{a_1b_2b_3b_4a_2a_3a_4b_1}. \end{aligned} \tag{3}$$

Here, qubits a_1, a_2, a_3, and a_4 are held by Alice, and qubits b_1, b_2, b_3, and b_4 are held by Bob. The total state of $|\omega\rangle$, $|\psi_1\rangle$, and $|\psi_2\rangle$ can be described as

$$\begin{aligned}&|\chi_{\text{total}}\rangle_{a_1b_2b_3b_4a_2a_3a_4b_1A_1A_2A_3B_1B_2B_3}\\ &= \frac{1}{2}(|00000000\rangle + |00001111\rangle + |11110000\rangle\\ &\quad + |11111111\rangle)_{a_1b_2b_3b_4a_2a_3a_4b_1}\\ &\quad \otimes(\alpha_0\beta_0|000000\rangle + \alpha_0\beta_1|000111\rangle + \alpha_1\beta_0|111000\rangle\\ &\quad + \alpha_1\beta_1|111111\rangle)_{A_1A_2A_3B_1B_2B_3}. \end{aligned} \tag{4}$$

Step 2 Alice performs a controlled-NOT (CNOT) operation on qubits A_1 and a_1, and Bob performs a CNOT operation on qubits B_1 and b_1, where A_1 and B_1 are control qubits and a_1 and b_1 are target qubits. Then, the state in Eq. (4) is changed into the following form:

$$\begin{aligned}&|\chi_{\text{total-CN}}\rangle_{a_1b_2b_3b_4a_2a_3a_4b_1A_1A_2A_3B_1B_2B_3}\\ &= \frac{1}{2}[(|00000000\rangle + |00001111\rangle + |11110000\rangle\\ &\quad + |11111111\rangle)_{a_1b_2b_3b_4a_2a_3a_4b_1}\\ &\quad \otimes(\alpha_0\beta_0|000000\rangle)_{A_1A_2A_3B_1B_2B_3}\\ &\quad + (|00000001\rangle + |00001110\rangle + |11110001\rangle\\ &\quad + |11111110\rangle)_{a_1b_2b_3b_4a_2a_3a_4b_1}\\ &\quad \otimes(\alpha_0\beta_1|000111\rangle)_{A_1A_2A_3B_1B_2B_3}\\ &\quad + (|10000000\rangle + |10001111\rangle + |01110000\rangle\\ &\quad + |01111111\rangle)_{a_1b_2b_3b_4a_2a_3a_4b_1}\\ &\quad \otimes(\alpha_1\beta_0|111000\rangle)_{A_1A_2A_3B_1B_2B_3}\\ &\quad + (|10000001\rangle + |10001110\rangle + |01110001\rangle\\ &\quad + |01111110\rangle)_{a_1b_2b_3b_4a_2a_3a_4b_1}\\ &\quad \otimes(\alpha_1\beta_1|111111\rangle)]_{A_1A_2A_3B_1B_2B_3}. \end{aligned} \tag{5}$$

Step 3 Alice and Bob perform a single-qubit $|Z\rangle$-basis measurement on qubits a_1 and b_1 respectively, and they perform a single-qubit $|X\rangle$-basis measurement on qubits A_2 and B_2 respectively. After these measurements, one of the following sixteen results can be obtained with the same probability:

$$\begin{aligned}&|++00\rangle_{A_2B_2a_1b_1}, \quad |++01\rangle_{A_2B_2a_1b_1},\\ &|++10\rangle_{A_2B_2a_1b_1}, \quad |++11\rangle_{A_2B_2a_1b_1},\\ &|+-00\rangle_{A_2B_2a_1b_1}, \quad |+-01\rangle_{A_2B_2a_1b_1},\\ &|+-10\rangle_{A_2B_2a_1b_1}, \quad |+-11\rangle_{A_2B_2a_1b_1},\\ &|-+00\rangle_{A_2B_2a_1b_1}, \quad |-+01\rangle_{A_2B_2a_1b_1},\\ &|-+10\rangle_{A_2B_2a_1b_1}, \quad |-+11\rangle_{A_2B_2a_1b_1},\\ &|--00\rangle_{A_2B_2a_1b_1}, \quad |--01\rangle_{A_2B_2a_1b_1},\\ &|--10\rangle_{A_2B_2a_1b_1}, \quad |--11\rangle_{A_2B_2a_1b_1}. \end{aligned}$$

Alice and Bob announce the measurement results to each other in a classic channel. Suppose that the results are $|++00\rangle_{A_2B_2a_1b_1}$, then the quantum state of the remaining ten qubits will collapse into the following state:

$$\begin{aligned}&|\chi_{10}\rangle_{A_1A_3B_1B_3b_2b_3b_4a_2a_3a_4}\\ &= (\alpha_0\beta_0|0000000000\rangle + \alpha_0\beta_1|0011000111\rangle\\ &\quad + \alpha_1\beta_0|1100111000\rangle\\ &\quad + \alpha_1\beta_1|1111111111\rangle)_{A_1A_3B_1B_3b_2b_3b_4a_2a_3a_4}. \end{aligned} \tag{6}$$

Step 4 Alice and Bob perform a single-qubit $|X\rangle$-basis measurement on qubits A_1 and B_1 respectively, then they announce the measurement results. The results may be $|++\rangle_{A_1B_1}$, $|+-\rangle_{A_1B_1}$, $|-+\rangle_{A_1B_1}$ or $|--\rangle_{A_1B_1}$. Suppose that the

results are $|{+}{+}\rangle_{A_1B_1}$, then the quantum state of the remaining eight qubits will collapse into

$$\begin{aligned}&|\chi_8\rangle_{A_3B_3b_2b_3b_4a_2a_3a_4}\\ =\ &(\alpha_0\beta_0\,|00000000\rangle+\alpha_0\beta_1\,|01000111\rangle+\alpha_1\beta_0\,|10111000\rangle\\ &+\alpha_1\beta_1\,|11111111\rangle)_{A_3B_3b_2b_3b_4a_2a_3a_4}.\end{aligned}\tag{7}$$

Step 5 Alice and Bob perform a single-qubit $|X\rangle$-basis measurement on qubits A_3 and B_3 respectively, then they announce the measurement results. Suppose that the results are $|{+}{+}\rangle_{A_3B_3}$, then the quantum state of the remaining six qubits will collapse into

$$\begin{aligned}&|\chi_6\rangle_{b_2b_3b_4a_2a_3a_4}\\ =\ &(\alpha_0\beta_0\,|000000\rangle+\alpha_0\beta_1\,|000111\rangle\\ &+\alpha_1\beta_0\,|111000\rangle+\alpha_1\beta_1\,|111111\rangle)_{b_2b_3b_4a_2a_3a_4}\\ =\ &(\alpha_0\,|000\rangle+\alpha_1\,|111\rangle)_{b_2b_3b_4}\\ &\otimes(\beta_0\,|000\rangle+\beta_1\,|111\rangle)_{a_2a_3a_4}.\end{aligned}\tag{8}$$

Equation (8) shows that after the above steps, Alice has reconstructed the initial state $|\psi_2\rangle$ on her qubits a_2, a_3, and a_4, and Bob has reconstructed the initial state $|\psi_1\rangle$ on his qubits b_2, b_3, and b_4. The bidirectional quantum teleportation of three-qubit GHZ state is completed successfully. If Alice and Bob obtain other results in Step 5, they can perform the corresponding unitary operations in Table 1 to reconstruct the quantum states $|\psi_2\rangle$ and $|\psi_1\rangle$.

Table 1. Unitary operations of Alice and Bob.

Alice's and Bob's measurement results	Alice's operation	Bob's operation
$\|+\rangle_{A_3}\|+\rangle_{B_3}$	$I_{a_2}\otimes I_{a_3}\otimes I_{a_4}$	$I_{b_2}\otimes I_{b_3}\otimes I_{b_4}$
$\|+\rangle_{A_3}\|-\rangle_{B_3}$	$I_{a_2}\otimes I_{a_3}\otimes \sigma^Z_{a_4}$	$I_{b_2}\otimes I_{b_3}\otimes I_{b_4}$
$\|-\rangle_{A_3}\|+\rangle_{B_3}$	$I_{a_2}\otimes I_{a_3}\otimes I_{a_4}$	$I_{b_2}\otimes I_{b_3}\otimes \sigma^Z_{b_4}$
$\|-\rangle_{A_3}\|-\rangle_{B_3}$	$I_{a_2}\otimes I_{a_3}\otimes \sigma^Z_{a_4}$	$I_{b_2}\otimes I_{b_3}\otimes \sigma^Z_{b_4}$

2.2. Bidirectional quantum teleportation of multi-qubit GHZ states

The scheme in Subsection 2.1 could be extended to teleporting multi-qubit GHZ states. Suppose that Alice and Bob have the following n-qubit GHZ states to be teleported, respectively:

$$|\psi_{n1}\rangle=(\alpha_0\,|000\cdots0\rangle+\alpha_1\,|111\cdots1\rangle)_{A_1A_2A_3\cdots A_n},\tag{9}$$

$$|\psi_{n2}\rangle=(\beta_0\,|000\cdots0\rangle+\beta_1\,|111\cdots1\rangle)_{B_1B_2B_3\cdots B_n}.\tag{10}$$

Before the teleportation, Alice and Bob must share a $2N$-qubit composite GHZ state described as

$$\begin{aligned}|\omega_{2N}\rangle\ =\ &\frac{1}{\sqrt{2}}(|000\ldots0\rangle+|111\ldots1\rangle)_{a_1b_2b_3\ldots b_N}\\ &\otimes\frac{1}{\sqrt{2}}(|000\ldots0\rangle+|111\ldots1\rangle)_{a_2a_3\ldots a_Nb_1}.\end{aligned}\tag{11}$$

In Eq. (11), $N=n+1$. Qubit a_i $(i=1,2,\ldots,N)$ belongs to Alice and qubit b_i $(i=1,2,\ldots,N)$ belongs to Bob. Using a similar method to that in Subsection 2.1, the bidirectional n-qubit GHZ state teleportation can be completed successfully. In this scheme, Alice and Bob each need to perform the CNOT operation once, the single-qubit $|Z\rangle$-basis measurement once, the single-qubit $|X\rangle$-basis measurements $N-1$ times and the $(N-1)$-qubit unitary operation once. The $(N-1)$-qubit unitary operation is described as

$$U_1\otimes U_2\otimes U_3\cdots\cdots\otimes U_{N-1},\tag{12}$$

where U_i $(i=1,2,\ldots,N-1)$ is an appropriate Pauli gate.

3. Bidirectional quantum teleportation in noisy channel

In this section, we will discuss how the noise in a quantum channel influences the procedure of bidirectional teleportation. As an example, we assume that the channel model is an AD channel. It can model many practical physical processes, such as spontaneous emission and super-conduction with zero-temperature energy relaxation. In this model, the evolutions of a single-qubit system state and the environment can be described as the following quantum transformation:[39]

$$U_{\rm AD}=\begin{cases}|0\rangle_S|0\rangle_E\to|0\rangle_S|0\rangle_E,\\ |1\rangle_S|0\rangle_E\to\sqrt{1-p}\,|1\rangle_S|0\rangle_E+\sqrt{p}\,|0\rangle_S|1\rangle_E,\end{cases}\tag{13}$$

where subscripts S and E denote the system and the environment respectively, and p is the amplitude damping factor. The Kraus operators of this channel are

$$E_0=\begin{bmatrix}1&0\\0&\sqrt{1-p}\end{bmatrix},\quad E_1=\begin{bmatrix}0&\sqrt{p}\\0&0\end{bmatrix}.\tag{14}$$

We still consider the bidirectional teleportations of three-qubit GHZ states. Assume the noise mainly exists in the process of entanglement distribution in the quantum channel. To reduce the influence of quantum noise as much as possible, Bob should prepare the four-qubit GHZ state $|\omega_1\rangle_{a_1b_2b_3b_4}$ and distribute qubit a_1 to Alice. Similarly, Alice prepares the four-qubit GHZ state $|\omega_2\rangle_{a_2a_3a_4b_1}$ and distribute qubit b_1 to Bob. During the distribution, only qubits a_1 and b_1 undergo amplitude damping, the eight-qubit channel state in Eq. (3) evolves into the following form:

$$\begin{aligned}\left|\omega^{\rm AD}\right\rangle\ =\ &\frac{1}{\sqrt{2}}(|00000\rangle+\sqrt{1-p}\,|11110\rangle\\ &+\sqrt{p}\,|01111\rangle)_{a_1b_2b_3b_4E}\\ &\otimes\frac{1}{\sqrt{2}}(|00000\rangle+\sqrt{1-p}\,|11110\rangle\\ &+\sqrt{p}\,|11101\rangle)_{a_2a_3a_4b_1E}.\end{aligned}\tag{15}$$

After the bidirectional teleportation described in Subsection 2.1, Alice can reconstruct the quantum state $\left|\psi_2^{\rm AD}\right\rangle$ on

her qubits a_2, a_3, and a_4, and Bob can reconstruct the quantum state $|\psi_1^{\rm AD}\rangle$ on his qubits b_2, b_3, and b_4, which can be expressed as

$$|\psi_2^{\rm AD}\rangle = \frac{1}{\sqrt{L_2^{\rm AD}}}(\beta_0|0000\rangle+\beta_1\sqrt{1-p}|1110\rangle + \beta_0\sqrt{p}|1111\rangle)_{a_2a_3a_4E}, \quad (16)$$

$$|\psi_1^{\rm AD}\rangle = \frac{1}{\sqrt{L_1^{\rm AD}}}(\alpha_0|0000\rangle+\alpha_1\sqrt{1-p}|1110\rangle + \alpha_0\sqrt{p}|1111\rangle)_{b_2b_3b_4E}, \quad (17)$$

where $L_2^{\rm AD}$ and $L_1^{\rm AD}$ are the normalization factors. Performing a partial trace over the environment in Eq. (16), we can obtain the following reduced density matrix of $|\psi_2^{\rm AD}\rangle$:

$$\rho_2^{\rm AD} = \frac{1}{L_2^{\rm AD}}\begin{bmatrix} |\beta_0|^2 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0\beta_1^*\sqrt{(1-p)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0^*\beta_1\sqrt{(1-p)} & 0 & 0 & 0 & 0 & 0 & 0 & |\beta_1|^2(1-p)+|\beta_0|^2 p \end{bmatrix}, \quad (18)$$

where $L_2^{\rm AD}$ is described as

$$L_2^{\rm AD} = |\beta_0|^2(1+p)+|\beta_1|^2(1-p). \quad (19)$$

4. Bidirectional quantum teleportation aided with weak measurement and the reversing measurement

According to Ref. [35], the weak measurement and the corresponding reversing measurement on a single-qubit system can be described as the following transformations:

$$\begin{cases} |0\rangle_S|0\rangle_{E_{\rm w}} \to |0\rangle_S|0\rangle_{E_{\rm w}}, \\ |1\rangle_S|0\rangle_{E_{\rm w}} \to \sqrt{1-p_{\rm w}}|1\rangle_S|0\rangle_{E_{\rm w}}, \end{cases} \text{(weak measurement)},$$
$$\begin{cases} |0\rangle_S|0\rangle_{E_{\rm r}} \to \sqrt{1-p_{\rm r}}|0\rangle_S|0\rangle_{E_{\rm r}}, \\ |1\rangle_S|0\rangle_{E_{\rm r}} \to |1\rangle_S|0\rangle_{E_{\rm r}}, \end{cases} \text{(reversing measurement)}, \quad (20)$$

where $p_{\rm w}$ and $p_{\rm r}$ are the strengths of the weak measurement and the reversing measurement, respectively; $E_{\rm w}$ denotes the weak measurement environment; $E_{\rm r}$ denotes the reversing measurement environment.

During the entanglement distribution, only qubit a_1 of the four-qubit GHZ state $|\omega_1\rangle_{a_1b_2b_3b_4}$ suffers amplitude damping, so Bob only needs to perform the weak measurement on qubit a_1 before it is distributed, then $|\omega_1\rangle_{a_1b_2b_3b_4}$ is changed into the following form:

$$|\omega_1^{\rm W}\rangle = \frac{1}{\sqrt{2-p_{\rm w}}}(|0000\rangle+\sqrt{1-p_{\rm w}}|1111\rangle)_{a_1b_2b_3b_4}. \quad (21)$$

After the weak measurement, qubit a_1 is distributed in the amplitude damping channel, then the quantum state in Eq. (21) evolves into

$$|\omega_1^{\rm W-AD}\rangle = \frac{1}{\sqrt{K_1^{\rm W-AD}}}(|00000\rangle+\sqrt{1-p_{\rm w}}\sqrt{1-p}|11110\rangle + \sqrt{1-p_{\rm w}}\sqrt{p}|01111\rangle)_{a_1b_2b_3b_4E}, \quad (22)$$

where $K_1^{\rm W-AD}$ is the normalization factor.

After the processes above, Alice receives qubit a_1 and she performs the reversing measurement, then the quantum state in Eq. (22) is changed into

$$|\omega_1^{\rm W-AD-R}\rangle = \frac{1}{\sqrt{K_1^{\rm W-AD-R}}}(\sqrt{1-p_{\rm r}}|00000\rangle + \sqrt{1-p_{\rm w}}\sqrt{1-p}|11110\rangle + \sqrt{1-p_{\rm r}}\sqrt{1-p_{\rm w}}\sqrt{p}|01111\rangle)_{a_1b_2b_3b_4E}, \quad (23)$$

where $K_1^{\rm W-AD-R}$ is the normalization factor.

With a similar derivation, the four-qubit GHZ state $|\omega_2\rangle_{a_2a_3a_4b_1}$ will be changed into the following state:

$$|\omega_2^{\rm W-AD-R}\rangle = \frac{1}{\sqrt{K_2^{\rm W-AD-R}}}(\sqrt{1-p_{\rm r}}|00000\rangle + \sqrt{1-p_{\rm w}}\sqrt{1-p}|11110\rangle + \sqrt{1-p_{\rm r}}\sqrt{1-p_{\rm w}}\sqrt{p}|11101\rangle)_{a_2a_3a_4b_1E}. \quad (24)$$

Then, the eight-qubit teleportation channel state in Eq. (3) is changed into the following form:

$$|\omega^{\rm W-AD-R}\rangle = |\omega_1^{\rm W-AD-R}\rangle\otimes|\omega_2^{\rm W-AD-R}\rangle. \quad (25)$$

Using the shared channel state in Eq. (25), Alice and Bob perform the bidirectional teleportation described in Subsection 2.1, then Alice can reconstruct the quantum state on her qubits a_2, a_3, and a_4, which has the following form:

$$|\psi_2^{\rm W-AD-R}\rangle = \frac{1}{\sqrt{L_2^{\rm W-AD-R}}}(\beta_0\sqrt{1-p_{\rm r}}|0000\rangle$$

$$+\beta_1\sqrt{1-p_{\rm w}}\sqrt{1-p}\,|1110\rangle$$
$$+\beta_0\sqrt{1-p_{\rm w}}\sqrt{1-p_{\rm r}}\sqrt{p}\,|1111\rangle)_{a_2a_3a_4E}, \quad (26)$$

where $L_2^{\rm W-AD-R}$ is the normalization factor. Similarly, Bob can reconstruct the quantum state on his qubits b_2, b_3, and b_4, which has the following form:

$$\begin{aligned}&|\psi_1^{\rm W-AD-R}\rangle\\ =&\frac{1}{\sqrt{L_1^{\rm W-AD-R}}}(\alpha_0\sqrt{1-p_{\rm r}}\,|0000\rangle\\ &+\alpha_1\sqrt{1-p_{\rm w}}\sqrt{1-p}\,|1110\rangle\\ &+\alpha_0\sqrt{1-p_{\rm w}}\sqrt{1-p_{\rm r}}\sqrt{p}\,|1111\rangle)_{b_2b_3b_4E}. \quad (27)\end{aligned}$$

Performing a partial trace over the environment in Eq. (26), we can obtain the reduced density matrix of $|\psi_2^{\rm W-AD-R}\rangle$ as follows:

$$\rho_2^{\rm W-AD-R}=\frac{1}{L_2^{\rm W-AD-R}}\begin{bmatrix}\mu_1&0&0&0&0&0&0&\mu_2\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\\mu_2^*&0&0&0&0&0&0&\mu_3\end{bmatrix}, \quad (28)$$

where

$$\begin{aligned}&\mu_1=(1-p_{\rm r})\,|\beta_0|^2,\\ &\mu_2=\sqrt{(1-p)(1-p_{\rm w})(1-p_{\rm r})}\beta_0\beta_1^*,\\ &\mu_3=(1-p_{\rm w})(1-p)\,|\beta_1|^2+(1-p_{\rm w})(1-p_{\rm r})p\,|\beta_0|^2,\\ &L_2^{\rm W-AD-R}=(1-p_{\rm r})\,|\beta_0|^2+(1-p_{\rm w})(1-p)\,|\beta_1|^2\\ &\quad+(1-p_{\rm w})(1-p_{\rm r})p\,|\beta_0|^2. \quad (29)\end{aligned}$$

5. Analysis and discussion

5.1. Fidelity of the bidirectional teleportation in AD channel

The fidelity is an important metric to display the distance between quantum states. The fidelity between states ρ and σ is defined as

$$F(\rho,\sigma)={\rm Tr}(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}). \quad (30)$$

If state $|\phi\rangle$ is a pure quantum state, then the fidelity between $|\phi\rangle$ and an arbitrary quantum state ρ can be calculated as

$$F(|\phi\rangle,\rho)=\sqrt{\langle\phi|\,\rho\,|\phi\rangle}. \quad (31)$$

In an AD channel, if Alice and Bob teleport the three-qubit GHZ states $|\psi_1\rangle$ and $|\psi_2\rangle$ by using the scheme in Subsection 2.1, then Alice can obtain a three-qubit quantum state with the density matrix $\rho_2^{\rm AD}$. According to Eqs. (2), (18), and (31), we can calculate the fidelity between $|\psi_2\rangle$ and $\rho_2^{\rm AD}$, which can be expressed as

$$Fd_1=F(|\psi_2\rangle,\rho_2^{\rm AD})=\sqrt{\langle\psi_2|\,\rho_2^{\rm AD}\,|\psi_2\rangle}. \quad (32)$$

The fidelity Fd_1 varying with amplitude damping factor p is shown in Fig. 1 with three different $|\psi_2\rangle$ states denoted as $|\psi_{21}\rangle$, $|\psi_{22}\rangle$, and $|\psi_{23}\rangle$.

In Fig. 1, we choose

$$\begin{aligned}|\psi_{21}\rangle&=\sqrt{0.3}\,|000\rangle+\sqrt{0.7}\,|111\rangle,\\ |\psi_{22}\rangle&=\sqrt{0.5}\,|000\rangle+\sqrt{0.5}\,|111\rangle,\\ |\psi_{23}\rangle&=\sqrt{0.8}\,|000\rangle+\sqrt{0.2}\,|111\rangle.\end{aligned}$$

It can be seen from this figure that Fd_1 decreases with the increase of p. The quality of the teleportation deteriorates when the amplitude damping turns serious.

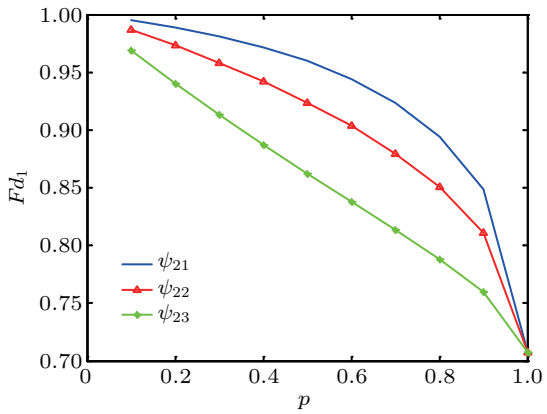


Fig. 1. (color online) Fidelity of the bidirectional teleportation in AD channel.

5.2. Fidelity of the bidirectional teleportation in AD channel aided with weak measurement

According to Section 4, if Alice and Bob each perform a weak measurement before the entanglement distribution of qubits a_1 and b_1, and perform a corresponding reversing measurement after the entanglement distribution, then Alice will obtain the teleportation result $\rho_2^{\rm W-AD-R}$. From Eqs. (2), (28), and (31), we can calculate the fidelity between $|\psi_2\rangle$ and $\rho_2^{\rm W-AD-R}$ as

$$Fd_2=F(|\psi_2\rangle,\rho_2^{\rm W-AD-R})=\sqrt{\langle\psi_2|\,\rho_2^{\rm W-AD-R}\,|\psi_2\rangle}. \quad (33)$$

The comparison between Fd_1 and Fd_2 is given in Fig. 2. Here, we use the optimal reversing condition: $p_{\rm r}=p_{\rm w}+p(1-p_{\rm w})$, and $p_{\rm w}=0.3$. From Fig. 2 we can see that Fd_1 and Fd_2 both decrease with the increase of p. For each $|\psi_2\rangle$ ($|\psi_{21}\rangle$, $|\psi_{22}\rangle$, and $|\psi_{23}\rangle$) and a certain p, Fd_2 is always bigger than Fd_1, showing the method aided with weak measurement and the reversing measurement can increase the fidelity of the bidirectional teleportation in the AD channel effectively.

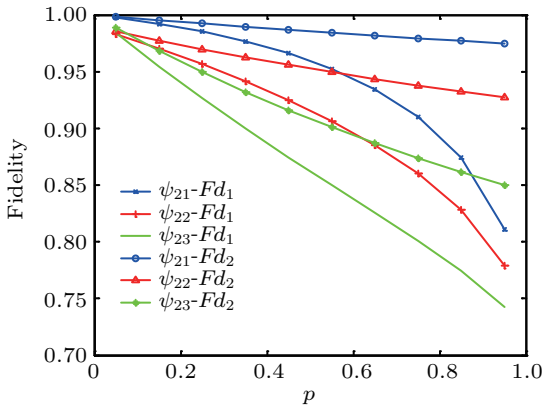


Fig. 2. (color online) Comparison between the BQT fidelities with and without weak measurement.

The fidelities Fd_1 and Fd_2 varying with p and p_w are shown in Fig. 3. Here, we choose $|\psi_2\rangle = |\psi_{22}\rangle = \sqrt{0.5}|000\rangle + \sqrt{0.5}|111\rangle$. It can be seen from this figure that for a certain p, Fd_2 is always bigger than Fd_1, and Fd_2 increases with weak measurement strength p_w increasing.

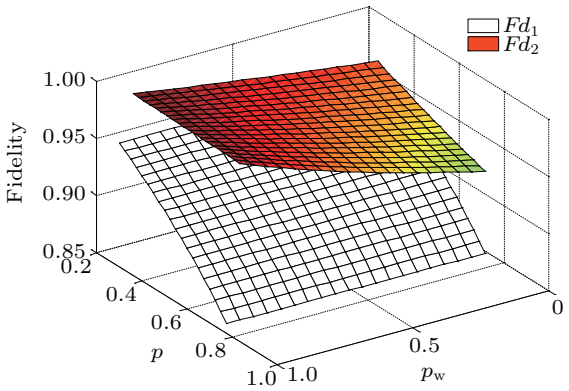


Fig. 3. (color online) 3D comparison between the BQT fidelities with and without weak measurement.

From Eqs. (1), (17), (27), and (31), we can also obtain a similar result that the weak measurement and the reversing measurement can increase the teleportation fidelity of $|\psi_1\rangle$ in the AD channel. Because of the limitation of length, the details of the derivation process are omitted here.

5.3. Success probability of the weak measurement and the reversing measurement

According to Subsection 5.2, we can increase the teleportation fidelity by choosing a larger weak measurement strength p_w. But this is not always the best choice. Based on the procedures described in Section 4, the total success probability of the weak measurement and the reversing measurement on qubits a_1 and b_1 can be calculated from the following form:

$$P_s = \frac{1}{2}(1-p_w)(1-p)[2+(1-p_w)p]. \quad (34)$$

The success probability P_s varying with p and p_w is shown in Fig. 4. It can be seen that P_s decreases with p and p_w increasing. This means if we want to obtain a higher fidelity by choosing a larger p_w, then we will acquire a smaller successful probability. Consequently, how to choose an appropriate p_w needs to be balanced carefully.

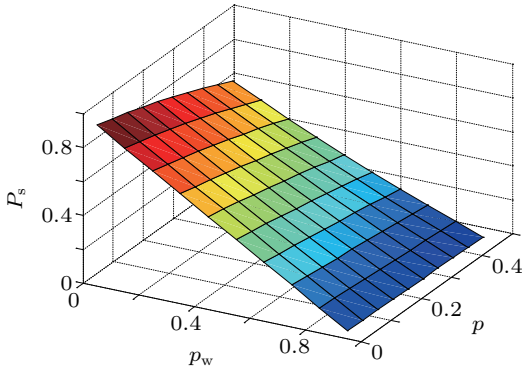


Fig. 4. (color online) Total success probability of the weak measurement and the reversing measurement.

6. Conclusions

In this work, we present a BQT scheme to transmit three-qubit and multi-qubit GHZ states. This scheme mainly utilizes CNOT operation and single-qubit measurement, thus it has a higher practicability. Unlike previous research that usually discuss BQT in an ideal quantum channel, we intended to study the influence of quantum noise on BQT. As an example, we analyze our BQT scheme in an amplitude damping channel and obtain the fidelity of the teleportation, which decreases with the amplitude damping factor increasing. In order to suppress this influence, we use the weak measurement and the reversing measurement before and after the quantum entanglement distribution, demonstrating an effective enhancement of the teleportation fidelity. On the other hand, we find that a higher fidelity needs a stronger weak measurement strength, which leads to a smaller success probability. This is a contradiction we must consider, and the solution is expected to be found in future work.

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