Improved control of distributed parameter systems using wireless sensor and actuator networks: An observer-based method*

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In this paper, the control problem of distributed parameter systems is investigated by using wireless sensor and actuator networks with the observer-based method. Firstly, a centralized observer which makes use of the measurement information provided by the fixed sensors is designed to estimate the distributed parameter systems. The mobile agents, each of which is affixed with a controller and an actuator, can provide the observer-based control for the target systems. By using Lyapunov stability arguments, the stability for the estimation error system and distributed parameter control system is proved, meanwhile a guidance scheme for each mobile actuator is provided to improve the control performance. A numerical example is finally used to demonstrate the effectiveness and the advantages of the proposed approaches.

Keywords: distributed parameter systems, wireless sensor and actuator networks, mobile actuator, observer

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1. Introduction

The wireless sensor and actuator networks (WSAN) have been receiving increased research interest due to their numerous applications in the fields of cyber-physical systems, forest fire fighting, environmental and industrial monitoring, and so on.[1–3] The actuators which have much stronger power than sensors can provide the appropriate actuation on the physical systems with the help of the networked sensors. In many scenarios, mobile actuators which can be attached to the mobile agents (terrain robots, underwater vehicles, UAVs) can improve the performance in different tasks.[4–6] A series of research results have been reported in applications of static and mobile sensor and actuator networks for finite dimensional systems, such as for coverage,[4] localization,[5] and optimal control, etc.[6]

On the other hand, most of the practical systems whose states vary both spatially and temporally are infinite dimensional systems, such as many thermal, chemical processes, and fluid flow, etc. These systems are commonly termed the distributed parameter systems (DPS). The past few years have witnessed increasing interest in the distributed parameter systems.[7–13] Especially, when the wireless sensor and actuator networks are used in distributed parameter systems, the estimation or control performance of the distributed parameter systems can be effectively improved. To enhance the system performance, how to navigate the wireless sensor and actuator networks and how to choose the control architecture are the main concern to be considered. The focus for these problems has motivated recent efforts in Refs. [14]–[18] and the references therein. For example, in Ref. [15], Chao and Chen considered the sensing and distributed control of a distributed parameter system by using Centroidal Voronoi tessellations and consensus strategy in the mobile sensor and actuator networks. The static output feedback control problem and adaptive control problem for the distributed parameter systems have been studied by Demetriou in Refs. [16] and [17], respectively, meanwhile the guidance of the mobile collocated sensor and actuator networks was designed.

In practical applications, it is impossible to have all the reliable information of system states, because some state variables cannot be measured or the number of the sensors is usually finite. Hence, one needs to estimate the physical systems using the finite measurement outputs in order to provide the feedback information to controllers. Then the observer-based control method is provided to obtain certain control objectives, which is attracting many researchers.[19–21] However, to the best of our knowledge, very little research attention has been paid to the combination of the guidance design of the mobile sensor and actuator networks and the control for the distributed parameter systems using the observer-based control method. Therefore, the main purpose of this work is to shorten such a gap by the study of how to navigate the networked actuators based on the observer-based control for the distributed parameter systems.

In this article, we investigate the observer-based control for the distributed parameter systems and the guidance scheme of the wireless sensor and actuator networks. Here, it is assumed that each sensor node is fixed and each actuator node

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with a controller can move freely in the spatial domain. Based on the measurement information provided by the fixed sensors, a centralized observer is designed to estimate the distributed parameter control system. Then the observer-based controllers are established to control the distributed parameter systems, meanwhile a motion strategy of the mobile actuators is provided to improve the control performance by using Lyapunov stability arguments.

The rest of this paper is organized as follows. In Section 2, a distributed parameter system and a network consisting of fixed sensors and mobile actuators are introduced, and a centralized observer and the observer-based controllers are presented. In Section 3, we present a guidance scheme for every mobile actuator and prove that the distributed parameter system controlled by the mobile actuators with the observer-based controllers is stable by using Lyapunov stability theory. A numerical example is given to verify the effects and advantages of our results in Section 4. Finally, conclusions are drawn in Section 5.

2. Problem formulation

Consider the following distributed parameter system given by

$$\frac{\partial w(t,x)}{\partial t} = \frac{\partial}{\partial x}(a(x)\frac{\partial w(t,x)}{\partial x}) + \sum_{i=1}^{m} b(x;\theta_i(t))u_i(t), \qquad (1)$$

with the Dirichlet boundary condition $w(t,0) = w(t,l) = 0$ and the initial condition $w(0,x) = w_0(x)$. $t \in [0,+\infty)$ and $x \in \Omega = [0,l]$ represent the time variable and spatial variable, respectively. $a(x) > a_0 > 0$ denotes the diffusing coefficient and $w(t,x)$ denotes the state of the system (1). The function $b(x;\theta_i(t))$ denotes the spatial distribution of the i-th moving actuator and $u_i(t)$ denotes the associated control signal. $\theta_i(t)$ denotes the time varying centroid of the i-th moving actuator.

It is assumed that the spatial distributions of the m actuators are the same and given by

$$b(x;\theta_i(t)) = \begin{cases} \dfrac{1}{2\varepsilon} & \text{if } x \in [\theta_i(t)-\varepsilon,\ \theta_i(t)+\varepsilon], \\ 0 & \text{otherwise,} \end{cases} \qquad (2)$$

with $i = 1,2,\ldots,m$, where ε is the spatial support of the actuators. The first-order dynamics for the i-th mobile actuator is governed by

$$\dot{\theta}_i(t) = v_i(t), \quad \theta_i(0) = \theta_{i0}, \qquad (3)$$

where the velocity $v_i(t)$ is to be designed.

In the wireless sensor and actuator network, n distributed sensors are used to provide the following measurement outputs on the distributed parameter system (1)

$$y(t) = \begin{pmatrix} y_1(t) \\ \vdots \\ y_n(t) \end{pmatrix} = \begin{pmatrix} \int_0^l c(x;\eta_1)w(t,x)\,\mathrm{d}x \\ \vdots \\ \int_0^l c(x;\eta_n)w(t,x)\,\mathrm{d}x \end{pmatrix}, \qquad (4)$$

where $y_i(t)$ is the measurement output by the i-th sensor, and $c(x;\eta_i)$ denotes the spatial distribution of the i-th sensor whose position η_i is fixed. It is also assumed that the spatial distributions of the n fixed sensors are the same and given by

$$c(x;\eta_i) = \begin{cases} 1 & \text{if } x \in [\eta_i - \nu,\ \eta_i + \nu], \\ 0 & \text{otherwise,} \end{cases} \quad i = 1,2,\ldots,n, \qquad (5)$$

where ν is the spatial support of the sensor.

Based on the above measurement information by the fixed sensors, we consider the following Luenberger observer:

$$\frac{\partial \hat{w}(t,x)}{\partial t} = \frac{\partial}{\partial x}(a(x)\frac{\partial \hat{w}(t,x)}{\partial x}) + \sum_{i=1}^{m} b(x;\theta_i(t))u_i(t) + \sum_{i=1}^{n} l_i(x)(y_i(t) - \hat{y}_i(t)), \qquad (6)$$

where

$$\hat{y}_i(t) = \int_0^l c(x;\eta_i)\hat{w}(t,x)\,\mathrm{d}x$$

and $l_i(x)$ is the observer gain. For simplicity, $l_i(x)$ is chosen as $l_i(x) = r_i c(x;\eta_i)$ with $r_i > 0$.

According to the Luenberger observer (6), then the observer-based controllers are designed as

$$u_i(t) = -k_i \int_0^l b(x;\theta_i(t))\hat{w}(t,x)\,\mathrm{d}x, \ \ i = 1,2,\ldots,m, \qquad (7)$$

where $k_i > 0$ is the controller gain.

Remark 1 Usually, for the distributed parameter system, the full information of the process at all positions and times cannot be obtained by distributed sensors. Therefore, the estimation for the distributed parameter system from available measurements is needed before providing a better control. In this paper, the Luenberger observer (6) is used to make the estimation. Based on the estimation information, each actuator with the observer-based controller (7) gives the corresponding control signals on the target system. Besides the choice of the controllers, the control performance of the distributed parameter system also depends on how the actuators positions are chosen. So another task of this paper is to design the motion trajectory for each mobile actuator based on the estimation information to enhance the control performance of the distributed parameter system.

Remark 2 Notice that each distributed controller given by system (7) will receive the estimated state from the Luenberger observer (6) and yield the corresponding control signal.

Using the first mean value theorem for integration, the control signal $u_i(t)$ can be computed by

$$u_i(t) = -k_i \int_{\theta_i(t)-\varepsilon}^{\theta_i(t)+\varepsilon} \frac{1}{2\varepsilon}\hat{w}(t,x)\,\mathrm{d}x = -k_i\hat{w}(t,\bar{\theta}_i),$$

where $\bar{\theta}_i \in [\theta_i(t)-\varepsilon, \theta_i(t)+\varepsilon]$, $i = 1,2,\ldots,m$. $\hat{w}(t,\bar{\theta}_i)$ is the mean value of $\hat{w}(t,x)$ on $[\theta_i(t)-\varepsilon, \theta_i(t)+\varepsilon]$.

Remark 3 Based on the assumption of each collocated mobile sensor/actuator pair, the static output feedback control problem for the distributed parameter system was studied by Demetriou in Ref. [16]. However, in the wireless sensor and actuator networks, sensor nodes are usually low-power, low-cost devices and actuator nodes have more energy and higher processing capabilities, so it is more practical to assume that each sensor node is fixed and each actuator node can move freely in the spatial domain. The problem of this paper, where n fixed sensors and m mobile actuators are utilized to make the observer-based control for distributed parameter systems, is more practically significant.

Now we can state the problem under consideration: Given the distributed parameter system (1) with the proposed observer-based controllers (7), implemented by the mobile agents, each of which has a controller and an actuator, we aim to derive a guidance scheme for each mobile agent to improve the control performance based on the state estimation (6).

In order to employ Lyapunov methods for the stability analysis of distributed parameter control system and the estimation error system, meanwhile to derive the guidance scheme for each mobile actuator, it is convenient to bring the above systems (1) into an abstract framework.

Using standard results from abstract theory, let $\mathcal{W}$ be a Hilbert space with the inner product $\langle\cdot,\cdot\rangle$ and corresponding induced norm $|\cdot|$. $\mathcal{V}$ is a reflexive Banach space with norm denoted by $\|\cdot\|$ and $\mathcal{V}^*$ denotes the conjugate dual of $\mathcal{V}$ with induced norm $\|\cdot\|_*$. It follows $\mathcal{V} \hookrightarrow \mathcal{W} \hookrightarrow \mathcal{V}^*$ with both embedding dense and continuously, and as a consequence we have $|\phi| \le c\|\phi\|$, $\phi \in \mathcal{V}$, for some positive constant c.[16]

In this paper, $\mathcal{W} = L_2(\Omega)$ is the state space, where $w(t,\cdot) = \{w(t,x), 0 \le x \le l\}$ denotes the state of the distributed parameter system (1). The Sobolev space $\mathcal{V}$ is given by $\mathcal{V} = H_0^1 = \{\psi \in H^1(\Omega) | \psi(0) = \psi(l) = 0\}$ and its conjugate dual space $\mathcal{V}^*$ is $H^{-1}(\Omega)$.

The second-order operator $\mathcal{A}$ of the system is given by

$$\mathcal{A}\phi = \frac{\mathrm{d}}{\mathrm{d}x}(a(x)\frac{\mathrm{d}\phi}{\mathrm{d}x}),\ a(x) > 0,\ \phi \in \mathrm{Dom}(\mathcal{A}), \tag{8}$$

where $\mathrm{Dom}(\mathcal{A}) = \{\psi \in L_2(\Omega)\ |\psi, \psi' \text{ abs. continuous}, \psi'' \in L_2(\Omega) \text{ and } \psi(0) = \psi(l) = 0\}$. Following Ref. [16], the operator $\mathcal{A}$ is symmetric and satisfies

$$|\langle \mathcal{A}\phi, \varphi\rangle| \le \kappa\|\phi\|\|\varphi\|, \text{ for } \phi, \varphi \in \mathcal{V} \text{ and some positive } \kappa,$$

$$\langle -\mathcal{A}\phi, \phi\rangle \ge \tau\|\phi\|^2, \text{ for } \phi \in \mathcal{V} \text{ and some positive } \tau. \tag{9}$$

The input operator $\mathcal{B}(\theta(t)) : \boldsymbol{R}^m \longrightarrow \mathcal{V}^*$ is defined by

$$\mathcal{B}(\theta(t))u(t) = [\mathcal{B}_1(\theta_1(t)) \cdots \mathcal{B}_m(\theta_m(t))] \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix},$$

where $u(t) \in \boldsymbol{R}^m$ denotes the control signals by the m actuators and $\mathcal{B}_i(\theta_i(t))u_i(t) = b(x;\theta_i(t))u_i(t)$. From Eq. (7), the observer-based control input $u(t)$ can be represented by $u(t) = -K\mathcal{B}^*(\theta(t))\hat{w}(t)$ with the feedback gain matrix $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$. The output operator $\mathcal{C} : \mathcal{V} \longrightarrow \boldsymbol{R}^n$ is defined by

$$\mathcal{C}w(t) = \begin{bmatrix} \mathcal{C}_1(t) \\ \vdots \\ \mathcal{C}_n(t) \end{bmatrix} w(t) = \begin{pmatrix} \int_0^l c(x;\eta_1)w(t,x)\,\mathrm{d}x \\ \vdots \\ \int_0^l c(x;\eta_n)w(t,x)\,\mathrm{d}x \end{pmatrix}.$$

Then the distributed parameter control system (1) can be rewritten as

$$\begin{aligned} &\dot{w}(t) = \mathcal{A}w(t) + \mathcal{B}u(t), \\ &u(t) = -K\mathcal{B}^*(\theta(t))\hat{w}(t), \\ &w(0) = w_0, \end{aligned} \tag{10}$$

and the state estimation system (6) can be represented as

$$\begin{aligned} &\dot{\hat{w}}(t) = \mathcal{A}\hat{w}(t) + \mathcal{B}u(t) + \mathcal{C}^*\Gamma\mathcal{C}(w(t) - \hat{w}(t)), \\ &\hat{w}(0) \ne w_0, \end{aligned} \tag{11}$$

where $\Gamma = \mathrm{diag}(r_1, r_2, \ldots, r_n) > 0$.

Remark 4 Combining Eqs. (10) and (11), we obtain the dynamics of the extend state $\begin{pmatrix} w(t) \\ \hat{w}(t) \end{pmatrix}$ as follows:

$$\begin{pmatrix} \dot{w}(t) \\ \dot{\hat{w}}(t) \end{pmatrix} = \begin{pmatrix} \mathcal{A} & -\mathcal{B}K\mathcal{B}^* \\ \mathcal{C}^*\Gamma\mathcal{C} & \mathcal{A} - \mathcal{B}K\mathcal{B}^* - \mathcal{C}^*\Gamma\mathcal{C} \end{pmatrix} \begin{pmatrix} w(t) \\ \hat{w}(t) \end{pmatrix}. \tag{12}$$

Letting

$$\mathbb{A} = \begin{pmatrix} \mathcal{A} & -\mathcal{B}K\mathcal{B}^* \\ \mathcal{C}^*\Gamma\mathcal{C} & \mathcal{A} - \mathcal{B}K\mathcal{B}^* - \mathcal{C}^*\Gamma\mathcal{C} \end{pmatrix},$$

since $\mathbb{A}$ is a bounded perturbation of an infinitesimal generator

$$\begin{pmatrix} \mathcal{A} & 0 \\ 0 & \mathcal{A} \end{pmatrix},$$

$\mathbb{A}$ is the infinitesimal generator of a C_0-semigroup.[22] So the well-posedness of Eq. (12) can be established by using the fact that the operator $\mathbb{A}$ generates a C_0-semigroup.

Setting $e(t) = w(t) - \hat{w}(t)$, then the following estimation error system can be obtained from Eqs. (10) and (11)

$$\begin{aligned} &\dot{e}(t) = (\mathcal{A} - \mathcal{C}^*\Gamma\mathcal{C})e(t), \\ &e(0) = w(0) - \hat{w}(0) \ne 0. \end{aligned} \tag{13}$$

The well-posedness of Eq. (13) can be obtained from the fact that $\mathcal{A}-\mathcal{C}^*\Gamma\mathcal{C}$ generates a $\mathcal{C}_0$-semigroup.

Throughout this paper, λ and μ represent the maximum eigenvalues of the bounded positive operators $\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))$ and $\mathcal{C}^*\Gamma\mathcal{C}$, respectively; ρ and σ represent the minimum eigenvalues of the positive operators $-\mathcal{A}+\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))$ and $-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C}$, respectively.

3. Stability analysis and guidance of mobile actuators

In this section, a Lyapunov-based approach will be utilized to design the trajectories of mobile actuators, meanwhile to prove the stability for distributed parameter control system (10) and the estimation error system (13). The idea on the construction of a Lyapunov functional is inspired from Refs. [16] and [17].

Consider the following Lyapunov functional candidate for the systems (10), (11), and (13)

$$V(t)=V_1(t)+V_2(t)+V_3(t),$$

where

$$V_1(t)=-p_1\langle w(t),\mathcal{A}w(t)\rangle,$$

$$V_2(t)=p_2\langle\hat{w}(t),(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t)\rangle,$$

$$V_3(t)=p_3\langle e(t),(-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C})e(t)\rangle.$$

Here, p_1, p_2, and p_3 are positive constants.

Remark 5 Noticing that the operators $-\mathcal{A}$, $-\mathcal{A}+\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))$, and $-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C}$ are positive, we have the Lyapunov functional candidate $V(t)$ as nonnegative. In fact, $V_1(t)$ and $V_3(t)$ are considered for the stability of distributed parameter control system (10) and the estimation error system (13), respectively, and $V_2(t)$ is incorporated in order to derive the stable motion velocity for each mobile actuator by using the estimation information.

Based on the fact that the operator $\mathcal{A}$ is self-adjoint, the derivative of $V_1(t)$ satisfies

$$\begin{aligned}\dot{V}_1(t)&=-p_1\langle\dot{w}(t),\mathcal{A}w(t)\rangle-p_1\langle w(t),\mathcal{A}\dot{w}(t)\rangle\\&=-2p_1\langle\dot{w}(t),\mathcal{A}w(t)\rangle\\&=-2p_1\langle\mathcal{A}w(t)-\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))\hat{w}(t),\mathcal{A}w(t)\rangle.\end{aligned}$$

Using the inequality $2\langle x,y\rangle\le\langle x,x\rangle+\langle y,y\rangle$ and the fact that λ is the maximum eigenvalue of the operator $\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))$, we obtain

$$\begin{aligned}\dot{V}_1(t)&=-2p_1\langle\mathcal{A}w(t),\mathcal{A}w(t)\rangle+2p_1\langle\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))\hat{w}(t),\mathcal{A}w(t)\rangle\\&\le-2p_1|\mathcal{A}w(t)|^2+p_1\langle\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))\hat{w}(t),\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))\hat{w}(t)\rangle+p_1\langle\mathcal{A}w(t),\mathcal{A}w(t)\rangle\\&=-p_1|\mathcal{A}w(t)|^2+p_1|\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))\hat{w}(t)|^2\\&\le-p_1|\mathcal{A}w(t)|^2+p_1\lambda^2|\hat{w}(t)|^2.\end{aligned}\tag{14}$$

On the other hand, the operator $-\mathcal{A}+\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))$ is also self-adjoint, since the operators $\mathcal{A}$ and $\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))$ are self-adjoint. So the derivative of $V_2(t)$ is

$$\begin{aligned}\dot{V}_2(t)&=p_2\langle\dot{\hat{w}}(t),(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t)\rangle+p_2\langle\hat{w}(t),(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\dot{\hat{w}}(t)\rangle\\&\quad+p_2\Big\langle\hat{w}(t),\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\Big\rangle,\\&=2p_2\langle\dot{\hat{w}}(t),(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t)\rangle+p_2\Big\langle\hat{w}(t),\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\Big\rangle.\end{aligned}$$

Noticing that the system (11) can be rewritten as

$$\dot{\hat{w}}(t)=(\mathcal{A}-\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)+\mathcal{C}^*\Gamma\mathcal{C}e(t),$$

we have

$$\begin{aligned}\dot{V}_2(t)&=2p_2\langle(\mathcal{A}-\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)+\mathcal{C}^*\Gamma\mathcal{C}e(t),(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t)\rangle\\&\quad+p_2\Big\langle\hat{w}(t),\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\Big\rangle.\end{aligned}$$

Based on the inequality $2\langle x,y\rangle\le\langle x,x\rangle+\langle y,y\rangle$ and the fact that μ is the maximum eigenvalue of $\mathcal{C}^*\Gamma\mathcal{C}$ and ρ is the minimum eigenvalue of $-\mathcal{A}+\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t))$, we obtain

$$\begin{aligned}\dot{V}_2(t)&=-2p_2\langle(\mathcal{A}-\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t),(\mathcal{A}-\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t)\rangle\\&\quad+2p_2\langle\mathcal{C}^*\Gamma\mathcal{C}e(t),(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t)\rangle+p_2\Big\langle\hat{w}(t),\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\Big\rangle\end{aligned}$$

$$
\begin{aligned}
&\le -2p_2|(\mathcal{A}-\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)|^2+p_2\langle\mathcal{C}^*\Gamma\mathcal{C}e(t),\mathcal{C}^*\Gamma\mathcal{C}e(t)\rangle\\
&\quad+p_2\langle(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t),(-\mathcal{A}+\mathcal{B}(\theta(t))\Gamma\mathcal{B}^*(\theta(t)))\hat{w}(t)\rangle\\
&\quad+p_2\left\langle\hat{w}(t),\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\right\rangle\\
&\le -p_2\rho^2|\hat{w}(t)|^2+p_2\mu^2|e(t)|^2\\
&\quad+p_2\left\langle\hat{w}(t),\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\right\rangle. \qquad (15)
\end{aligned}
$$

Taking an examination of the last term in expression (15), we have

$$
\begin{aligned}
\left\langle\hat{w}(t),\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\right\rangle&=\left\langle\hat{w}(t),\dot{\theta}(t)\frac{\mathrm{d}}{\mathrm{d}\theta(t)}(\mathcal{B}(\theta(t))K\mathcal{B}^*(\theta(t)))\hat{w}(t)\right\rangle\\
&=2\sum_{i=1}^{m}\dot{\theta}_i(t)k_i\left\langle\hat{w}(t),\frac{\mathrm{d}\mathcal{B}_i(\theta_i(t))}{\mathrm{d}\theta_i(t)}\mathcal{B}_i^*(\theta_i(t))\hat{w}(t)\right\rangle\\
&=2\sum_{i=1}^{m}\dot{\theta}_i(t)k_i\frac{1}{2\varepsilon}\int_0^l(\delta(\theta_i(t)-\varepsilon)-\delta(\theta_i(t)+\varepsilon))\hat{w}(t,x)\mathrm{d}x\mathcal{B}_i^*(\theta_i(t))\hat{w}(t)\\
&=2\sum_{i=1}^{m}\dot{\theta}_i(t)k_i\frac{1}{2\varepsilon}(\hat{w}(t,\theta_i(t)-\varepsilon)-\hat{w}(t,\theta_i(t)+\varepsilon))\mathcal{B}_i^*(\theta_i(t))\hat{w}(t). \qquad (16)
\end{aligned}
$$

When the velocity $\dot{\theta}_i(t)$ for the i-th actuator is given by

$$
\dot{\theta}_i(t)=-\frac{1}{2\varepsilon}k_i(\hat{w}(t,\theta_i(t)-\varepsilon)-\hat{w}(t,\theta_i(t)+\varepsilon))\mathcal{B}_i^*(\theta_i(t)))\hat{w}(t),
$$

expression (15) becomes

$$
\dot{V}_2(t)\le -p_2\rho^2|\hat{w}(t)|^2+p_2\mu^2|e(t)|^2-2p_2\sum_{i=1}^{m}(\dot{\theta}_i(t))^2. \qquad (17)
$$

Noticing that the self-adjoint operator $-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C}$ satisfies $|(-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C})\phi|\ge\sigma|\phi|$, one has

$$
\begin{aligned}
\dot{V}_3(t)&=p_3\langle\dot{e}(t),(-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C})e(t)\rangle+p_3\langle e(t),(-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C})\dot{e}(t)\rangle\\
&=2p_3\langle\dot{e}(t),(-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C})e(t)\rangle\\
&=2p_3\langle(\mathcal{A}-\mathcal{C}^*\Gamma\mathcal{C})e(t),(-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C})e(t)\rangle\\
&=-2p_3|(-\mathcal{A}+\mathcal{C}^*\Gamma\mathcal{C})e(t)|^2\\
&\le -2p_3\sigma^2|e(t)|^2. \qquad (18)
\end{aligned}
$$

Therefore, combining formulas (14), (17), and (18), we obtain

$$
\begin{aligned}
\dot{V}(t)&\le -p_1|\mathcal{A}w(t)|^2+p_1\lambda^2|\hat{w}(t)|^2-p_2\rho^2|\hat{w}(t)|^2+p_2\mu^2|e(t)|^2\\
&\quad-2p_2\sum_{i=1}^{m}(\dot{\theta}_i(t))^2-2p_3\sigma^2|e(t)|^2\\
&=-p_1|\mathcal{A}w(t)|^2+(p_1\lambda^2-p_2\rho^2)|\hat{w}(t)|^2+(p_2\mu^2-2p_3\sigma^2)|e(t)|^2-2p_2\sum_{i=1}^{m}(\dot{\theta}_i(t))^2. \qquad (19)
\end{aligned}
$$

When the positive constants p_i, $i=1,2,3$ satisfy $p_1\lambda^2-p_2\rho^2\le 0$ and $p_2\mu^2-2p_3\sigma^2\le 0$, we gain

$$
\begin{aligned}
\dot{V}(t)&\le -p_1|\mathcal{A}w(t)|^2-2p_2\sum_{i=1}^{m}(\dot{\theta}_i(t))^2\\
&\le -p_1\tilde{c}|w(t)|^2-2p_2\sum_{i=1}^{m}(\dot{\theta}_i(t))^2\\
&\le -p_1\tilde{c}|w(t)|^2, \qquad (20)
\end{aligned}
$$

where $\tilde{c}>0$ is a function of the coercivity constant τ in Eq. (9) and the embedding constant c.[16]

From the above analysis, the main result is now summarized in the following theorem.

Theorem 1 Consider the distributed parameter system (1) with the observer-based control (7) by mobile actuators, where the state estimation $\hat{w}(t,x)$ is given by Eq. (6) using the measurement outputs (4). If there exist positive constants

p_i, $i = 1,2,3$ satisfying

$$p_1\lambda^2 - p_2\rho^2 \le 0 \ \text{ and } \ p_2\mu^2 - 2p_3\sigma^2 \le 0,$$

then the estimation error $e(t,x)$ and the system state $w(t,x)$ converge to zero. The guidance law for each actuator improves the control performance for the distributed parameter system (1), where the control laws for the guidance of the mobile actuators are designed by

$$\dot{\theta}_i(t) = -\frac{1}{2\varepsilon}k_i(\hat{w}(t,\theta_i(t)-\varepsilon) - \hat{w}(t,\theta_i(t)+\varepsilon))\mathcal{B}_i^*(\theta_i(t))\hat{w}(t), \ \ i = 1,2,\ldots,m. \tag{21}$$

Remark 6 One may notice that the above control law for the i-th actuator depends on the difference of the estimation $\hat{w}(t,\theta_i(t)-\varepsilon) - \hat{w}(t,\theta_i(t)+\varepsilon)$ and $\mathcal{B}_i^*(\theta_i(t))\hat{w}(t)$. On the other hand,

$$\mathcal{B}_i^*(\theta_i(t))\hat{w}(t) = \int_0^l b(x;\theta_i(t))\hat{w}(t,x)\,\mathrm{d}x = \frac{1}{2\varepsilon}\int_{\theta_i(t)-\varepsilon}^{\theta_i(t)+\varepsilon}\hat{w}(t,x)\,\mathrm{d}x = \hat{w}(t,\bar{\theta}),$$

that means $\mathcal{B}_i^*(\theta_i(t))\hat{w}(t)$ is the mean value of $\hat{w}(t,x)$ on $[\theta_i(t)-\varepsilon, \theta_i(t)+\varepsilon]$. Letting $\varepsilon \to 0$ in Eq. (21), the velocity becomes $\dot{\theta}_i(t) = k_i\hat{w}_x(t,x)|_{\theta_i(t)}\hat{w}(t,\theta_i(t))$, that means the guidance scheme for each mobile actuator uses the gradient information $\hat{w}_x(t,x)$ and $\hat{w}(t,x)$ at its position $\theta_i(t)$.

4. Numerical results

The distributed parameter system (1) with $a(x) = 0.006$ is considered in the domain $[0,l] = [0,1]$ and the initial condition is set to $w(0,x) = \sin(\pi x)\,\mathrm{e}^{-2x^2}$. The wireless sensor and actuator network is assumed to have five static sensors and two mobile actuators. The positions for the five static sensors are chosen as $\eta_i = i*0.15$, $i = 1,\ldots,5$. The initial positions and velocities for the two mobile actuators are set to $\theta_1(0) = 0.25$, $\theta_2(0) = 0.75$, $v_1(0) = 0$, $v_2(0) = 0$, respectively. The spatial distributions of each actuator and each sensor are given by Eqs. (2) and (5) with $\varepsilon = 0.06$ and $\nu = 0.08$.

It is assumed that the centralized estimator with $r_i = 50$, $i = 1,\ldots,5$ satisfies the initial condition $\hat{w}(0,x) = 0$. The observer-based controllers (7) are assumed with the controller gains $k_1 = k_2 = 2$. For comparison, two fixed actuators with the same spatial distributions (2) and controller gains as the mobile actuators are considered for the distributed parameter control system (1). In this case, the two actuators are fixed at $\theta_1 = 0.25$, $\theta_2 = 0.75$.

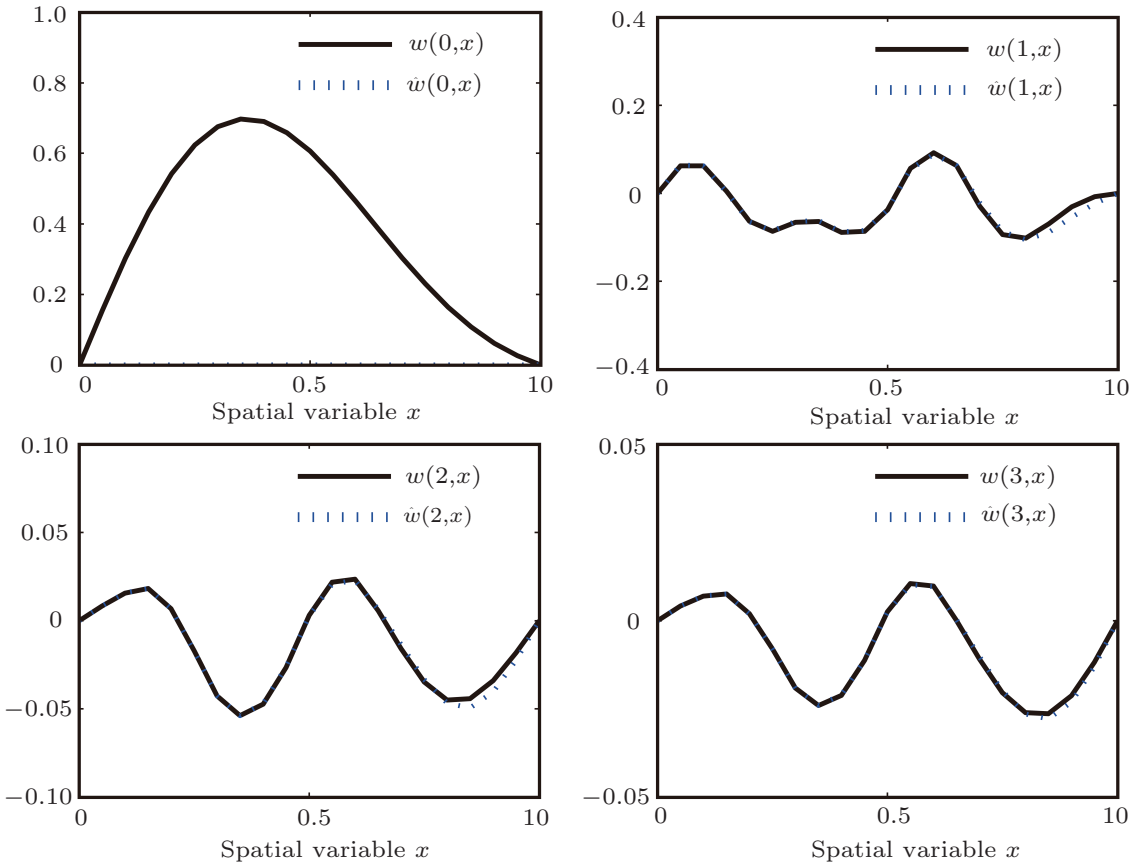


Fig. 1. (color online) System state and its estimate versus spatial variable at four different time instances.

Figure 1 depicts the space distribution of the actual system and its estimates at four different time instances. It is observed that the estimator can provide a better estimation of the target system.

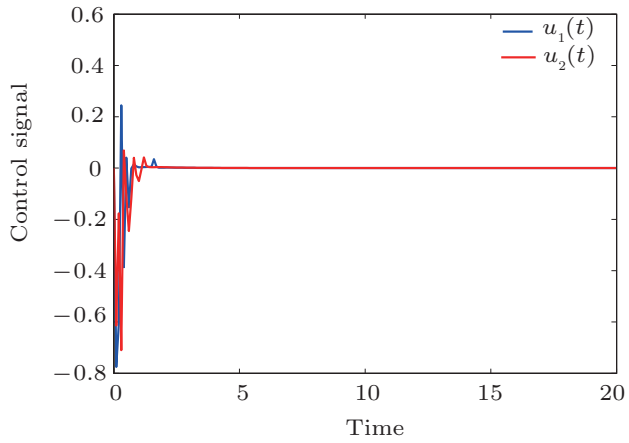


Fig. 2. (color online) The observer-based control signals.

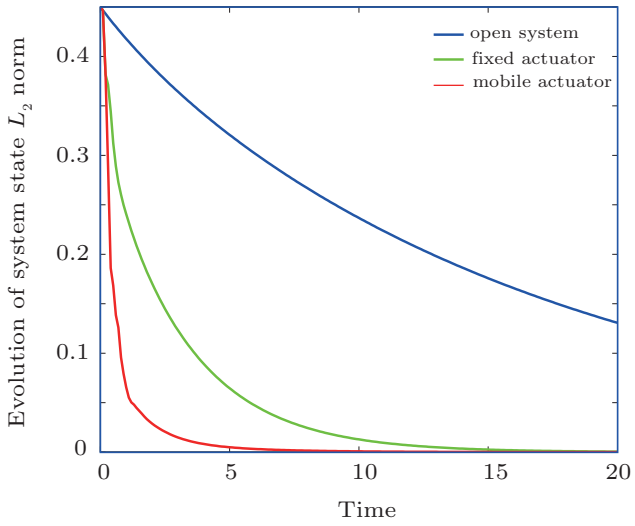


Fig. 3. (color online) Evolution of state L_2 norm.

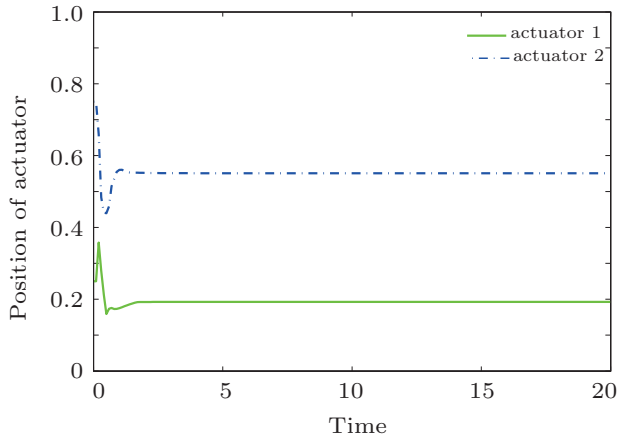


Fig. 4. (color online) Actuator trajectory.

Figure 2 depicts the observer-based control signals which are dispensed to distributed parameter system (1) by the two mobile actuators. Figure 3 presents the state $L_2(0,l)$ norm for the uncontrolled case, the case of the mobile actuators and fixed actuators. One can observe that when the actuators are mobile, the state norm converges to zero much faster than the fixed case. Finally, the trajectory for the two mobile actuators is depicted in Fig. 4.

5. Conclusions

In this paper, the observer-based control problem for the distributed parameter system has been considered by using the wireless sensor and actuator network in which it is assumed that sensors are fixed and actuators can move freely in the spatial domain. Based on the measurement information provided by the fixed sensors, a centralized observer has been designed. Meanwhile the observer-based controllers have been provided to control the distributed parameter system. Furthermore, the stability of the estimation error system and the distributed parameter control system has been analyzed by using Lyapunov stability arguments, meanwhile, a guidance scheme for each mobile actuator has been provided to improve the control performance. Simulation results have demonstrated the effectiveness and the advantages of the proposed approaches. It should be pointed out that our main result can be extended to other distributed parameter systems such as linear or nonlinear reaction–convection–diffusion systems with or without time-delays. This will be our next research work.

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