General equation describing viscosity of metallic melts under horizontal magnetic field*

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(Received 6 May 2016; revised manuscript received 20 December 2016; published online 18 January 2017)

Viscosities of pure Ga, $Ga_{80}Ni_{20}$, and $Ga_{80}Cr_{20}$ metallic melts under a horizontal magnetic field were investigated by a torsional oscillation viscometer. A mathematical physical model was established to quantitatively describe the viscosity of single and binary metallic melts under a horizontal magnetic field. The relationship between the viscosity and the electrical resistivity under the horizontal magnetic field was studied, which can be described as $\eta_B = \eta + \frac{2H}{\pi\Omega}B^2$ (η_B is the viscosity under the horizontal magnetic field, η is the viscosity without the magnetic field, H is the height of the sample, Ω is the electrical resistivity, and B is the intensity of magnetic field). The viscosity under the horizontal magnetic field is proportional to the square of the intensity of the magnetic field, which is in very good agreement with the experimental results. In addition, the proportionality coefficient of η_B and quadratic B, which is related to the electrical resistivity, conforms to the law established that increasing the temperature of the completely mixed melts is accompanied by an increase of the electrical resistivity. We can predict the viscosity of metallic melts under magnetic field by measuring the electrical resistivity based on our equation, and vice versa. This discovery is important for understanding condensed-matter physics under external magnetic field.

Keywords: viscosity, horizontal magnetic field, metallic melts, electrical resistivity

PACS: 66.20.Ej, 64.30.Ef, 75.20.En **DOI:** 10.1088/1674-1056/26/3/036601

1. Introduction

Metallic melts are of great importance in innovative applications.[1–3] Viscosity is one of the important physical properties of metallic melts and is closely related to the melt structure.[4–6] Electrical resistivity is another property that plays a prominent role in industrial applications such as mortar and concrete.[7] Investigations of the viscosity and electrical resistivity of metallic melts remain an active domain of research in both technical and theoretical fields of condensed matter.[8–10] But little research has been focused on the relationship between viscosity and electrical resistivity;[11] there is no quantitative report about that relationship.

Studies of the viscosity of melts under traditional conditions have achieved much progress.[12–15] But investigations of viscosity under extraordinary conditions are still challenging because of the limited experimental conditions, such as under external electrical field,[16] pressure field,[17] and external magnetic field.[18] The external field strongly influences physical behaviors such as glass transition[19] and dynamic behaviors including viscosity[20] and diffusion.[21] So much attention is still attracted by the application of different external fields in both technical and theoretical fields.

Magnetic field is one of the important external phenomena that effectively change the properties of metallic melts. In recent decades, the magnetic field has been considered to be a powerful tool in new-materials preparation[22] and crystal growth.[23] In addition, some investigations focus on the effects of magnetic field on controlling directional solidification.[24,25] Meanwhile, the effects of high-intensity magnetic field on alloys have been studied, indicating that the magnetic field can increase the eutectic temperature and shift the eutectic point.[26]

Investigating the viscosity of liquid melts under a magnetic field can help us understand the fundamental physical properties and the structures of the melts, and some investigations in viscosity under magnetic field have been done. Simultaneously, it should be noted that the dynamic viscosity under magnetic field show similar behaviors: a sharp increase in viscosity appears with increasing intensity of magnetic field, and researchers have attempted to explain this surprising phenomenon based on the activation energy, Lorentz force, valence bond, and so on.[27–31] Unfortunately, up to now, the reason for this sharply increased dynamic viscosity in an external magnetic field remains disputable due to the limitations of experimental conditions and the absence of theoretical explanations. So a quantitative understanding of viscosity under a horizontal magnetic field still has not been provided.

In the present work, we have investigated the viscosity of pure Ga, $Ga_{80}Ni_{20}$, and $Ga_{80}Cr_{20}$ metallic melts under a horizontal magnetic field to elucidate how the magnetic field increases the viscosity. Furthermore, we have established a mathematical physical model to describe the viscosity under the magnetic field quantitatively, which is important for studying the effects of magnetic field on the solidification of metal-

*Project supported by the National Natural Science Foundation of China (Grant No. 51371107).

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lic melts. The relationship between the viscosity and the electrical resistivity under the horizontal magnetic field has been studied, which provides a deep understanding for physicists.

2. Experimental procedure

The samples of $Ga_{80}Ni_{20}$ and $Ga_{80}Cr_{20}$ alloys used in the present study were prepared from pure Ga, Ni, and Cr of 99.99% purity in a high frequency induction furnace under Ar atmosphere.

Viscosity measurements were completed with a torsional OSVM oscillation viscometer. By measuring logarithmic decrement, the kinetic viscosity is calculated using the Shvidkovskiy equation[32]

$$\nu = \frac{I^2(\delta - T\delta_0/T_0)^2}{\pi(MR_v)^2TW^2}, \tag{1}$$

where

$$W = 1 - \frac{3}{2}\Delta - \frac{3}{8}\Delta^2 - a + (b - c\Delta)\frac{2nR_v}{H}. \tag{2}$$

In these equations, $\Delta = \delta/2\pi$; I is the inertial momentum of the suspended system, δ is the logarithmic damping decrement, and δ_0 is the logarithmic damping decrement for an empty vessel. T is the period time of the oscillations, while T_0 is the period time of the oscillations for an empty vessel. M represents the mass of the liquid sample, R_v is the radius of the vessel, H is the height of the liquid sample in the vessel, a, b, c are constants, and n is the number of solid planes contacted horizontally by the liquid sample (i.e., in the case of a vessel having its lower end closed and its upper surface free, $n = 1$; if the vessel encloses the fluid at its top and bottom, $n = 2$).

After careful surface treatment of the specimens with a file, the specimens were put into an alumina crucible, 30 mm in diameter and 60 mm in height. The samples were measured during the cooling process from elevated temperature above liquidus to a supercooling temperature. Before the viscous experiment, the viscosity of the empty alumina crucible used in the experiment should be measured in the same condition during the process of the experiment in order to eliminate the influences of the atmosphere, ensuring the precision of the experiment. During the process of the viscosity experiment, the samples were homogenized for 1 h at the highest temperature and were held for 40 min at each temperature at which viscosity was measured. The data error of different measurements at the same temperature is less than 3%. The relative accuracy of the viscometer is better than 5%. The viscosity experiments were repeated four times and the values were averaged. The intensities of the magnetic field were set at 0 Gs, 95 Gs, 245 Gs, 475 Gs, 650 Gs, and 935 Gs respectively.

3. Results

The viscosities of $Ga_{80}Cr_{20}$ and $Ga_{80}Ni_{20}$ alloy melts under different magnetic fields during the cooling progress are shown in Fig. 1. Some researchers have discovered that the temperature-dependence of viscosity of melts under magnetic fields of different intensities in a wide temperature range follows the Arrhenius law[27,28]

$$\eta = A\exp(E/RT), \tag{3}$$

where η is the viscosity (mPa·s), A is the pre-exponential constant accounting (mPa·s), T is the absolute temperature (K), E (J/mol) and R (J/mol·K) are the flow activation energy and the gas constant, respectively. So the Arrhenius equation is used to fit the experimental data, and the solid curves are the fitting curves under magnetic fields of different intensities as shown in Fig. 1. The values of fitting parameters A and E for the cooling process at different intensities of magnetic fields are listed in Table 1. We can see that the curves fit the experimental data well.

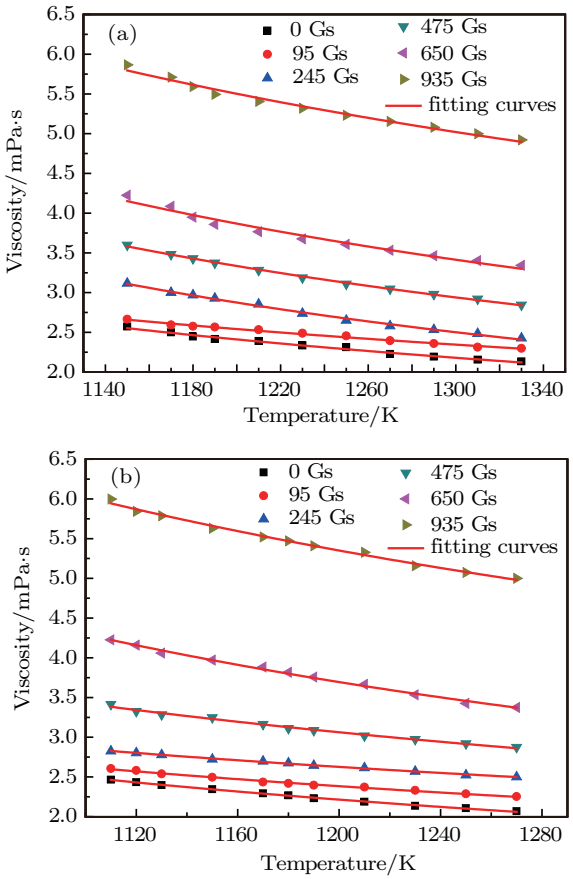


Fig. 1. (color online) The viscosities of (a) $Ga_{80}Cr_{20}$ and (b) $Ga_{80}Ni_{20}$ alloy melts under magnetic fields of intensities 0 Gs, 95 Gs, 245 Gs, 475 Gs, 650 Gs, and 935 Gs. The solid lines are the fitting curves according to the Arrhenius law.

Table 1. The viscous parameters of $Ga_{80}Cr_{20}$ and $Ga_{80}Ni_{20}$ melts under magnetic fields of different intensities.

Sample	Parameter	Gs	95 Gs	245 Gs	475 Gs	650 Gs	935 Gs
$Ga_{80}Cr_{20}$	A	0.4712	0.8954	0.5471	0.6499	0.7619	1.6721
$Ga_{80}Cr_{20}$	E	1573	1251	1429	1949	1967	2170
$Ga_{80}Ni_{20}$	A	0.6001	0.8295	0.8961	1.0599	1.0048	1.4621
$Ga_{80}Ni_{20}$	E	1089	1266	1474	1566	1556	1987

Figures 2(a) and 2(b) are schematics of the torsional oscillation viscometer and the simplified optical measuring system. At first, the driving motor activates the lever, which makes the sample rotate with the lever. A mirror is placed on the lever; when the lever rotates, the beam transmitted by the laser transmitter is reflected by the mirror. The reflected beam will pass receivers A and B because of the rotation of the mirror. When the motor stops working, the velocity of rotation decreases because the viscosity of the melt impedes the rotation of the samples. When measuring, the shape of the samples is shown as Fig. 2(c). Besides the external effects, which have been ruled out by measuring the empty alumina crucible, the viscous force whose direction is opposite to the direction of the velocity is the only factor resulting in the decrease of the speed of rotation. In this ideal state, the stress analysis can be simplified as Fig. 2(d). Based on this mathematical physical model, we can obtain the equation

$$J\alpha = Fr. \tag{4}$$

In the above equation, J is the inertial momentum of the sample, which can be described as

$$\begin{aligned} J &= \int_0^{R_s} \rho 2\pi r H \,\mathrm{d}r \times r^2 \\ &= \int_0^{R_s} 2\pi r^3 \rho H \,\mathrm{d}r = \frac{1}{2}\pi R_s^4 \rho H, \end{aligned} \tag{5}$$

where R_s is the radius of the sample, ρ is the density of the sample, and H is the height of the sample.

In Eq. (4), α is the angular acceleration, r is the radius, and F is the force applied to the sample resulting from the wall of the alumina crucible, which can be described as

$$F = \eta S \frac{\mathrm{d}\upsilon}{\mathrm{d}r} = \eta S\omega = \eta 2\pi R_s H(\omega_0 - \int_0^t \alpha \,\mathrm{d}t), \tag{6}$$

where η is the viscosity, S is the area, υ is the velocity, ω is the angular velocity, ω_0 is the initial angular velocity, and t is time.

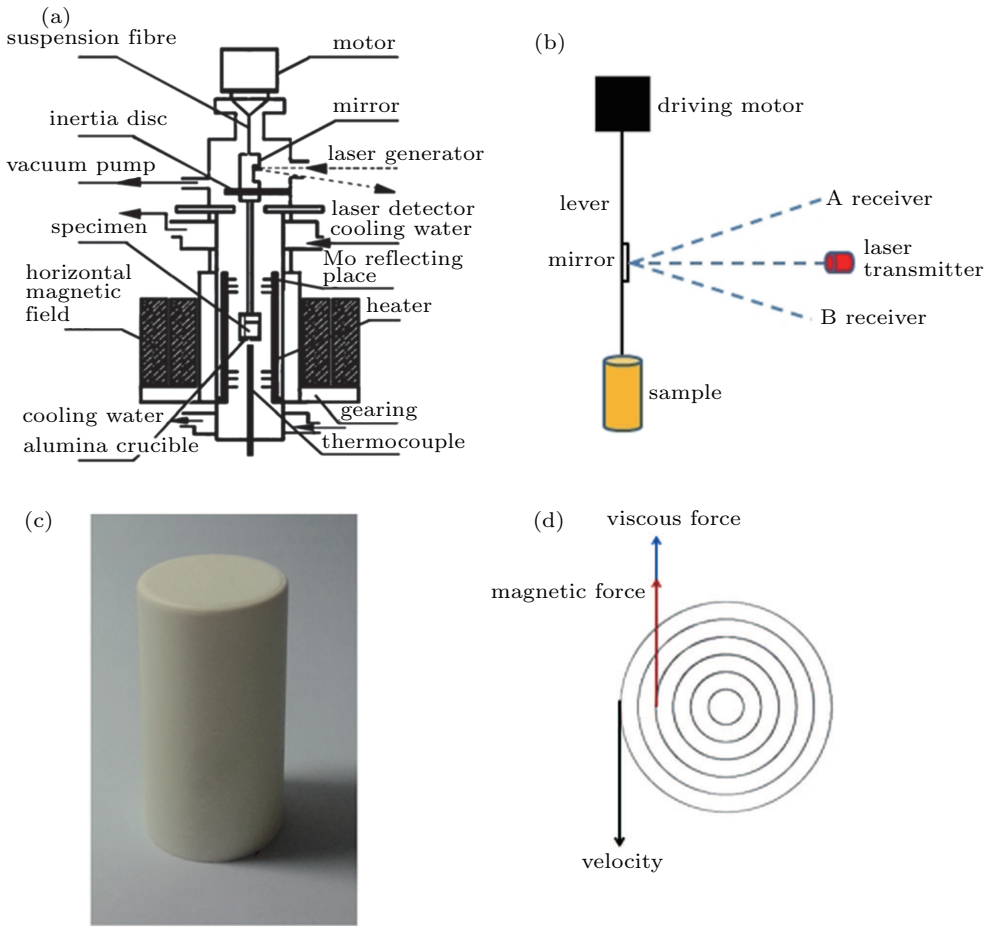


Fig. 2. (color online) (a) Schematic of a torsional oscillation viscometer. (b) Schematic of a simple measuring system. (c) The physical sample. (d) Cross section of the sample.

According to Eqs. (4)–(6), we can solve the angular acceleration α,

$$\alpha = \frac{\omega_0 4\eta}{\rho R_s^2} \exp\left(-\frac{4\eta}{\rho R_s^2} t\right), \tag{7}$$

and angular velocity ω_t, which depends on time,

$$\omega_t = \omega_0 \frac{\rho R_s^2}{4\eta} \exp\left(\frac{4\eta}{\rho R_s^2} t\right). \tag{8}$$

If the angular velocity is measured by the optical system, we can obtain the viscosity of the sample in the ideal state.

When a magnetic field is applied to the sample, the causes

of the decrease of the angular velocity can be divided into two aspects – the viscosity of the sample and the force caused by the magnetic field, as shown in Fig. 2(d). In the macroscopic view, the magnetic force results from the Ampere Force. The samples rotate during the measuring process. When the samples rotate in the magnetic field, the samples cut the magnetic induction line. So an induction current is produced, and the Ampere force is generated. In our experiments, the magnetic induction line is horizontal. For the cylindrical sample, the velocity is not always vertical to the magnetic induction line. The induction current can be obtained as

$$I = \frac{BHv}{\Omega}\sin\theta, \tag{9}$$

where I is the magnetic induction current, θ is the intersection angle between the direction of the magnetic field and the direction of the velocity, B is the intensity of the magnetic field, and Ω is the electrical resistivity of the liquid melt.

For the whole cylindrical sample, the induction current can be described as

$$I = \int_0^{R_s}\int_0^{2\pi}\frac{BHv}{\Omega}|\sin\theta|\,\mathrm{d}\theta\,\mathrm{d}r = \frac{4BH\omega_t R_s}{\Omega}. \tag{10}$$

Then the induced magnetic force F_M can be obtained from

$$F_M = BIH = \frac{4B^2H^2\omega_t R_s}{\Omega}. \tag{11}$$

Then, the forces under magnetic field can be described as

$$F = \eta_B 2\pi rH\omega_t = \eta 2\pi rH\omega_t + \frac{4B^2H^2\omega_t r}{\Omega}, \tag{12}$$

where η_B is the viscosity under the magnetic field, and η is the viscosity without the magnetic field.

According to Eq. (12), we can easily obtain the equation

$$\eta_B = \eta + \frac{2H}{\pi\Omega}B^2 = \eta + \beta B^2. \tag{13}$$

Figure 3 shows the dependence of the viscosity of pure Ga metallic melt, $Ga_{80}Cr_{20}$ metallic melt, and $Ga_{80}Ni_{20}$ metallic melt upon the intensity of the magnetic field at various temperatures. The solid lines are the fitting curves according to Eq. (13). It is obvious that the experimental viscous data fit well with Eq. (13). The fitting parameters of β which can be described as $2H/\pi\Omega$, are shown in Fig. 4. It can be seen that β decreases with increasing temperature. During the measuring process, the temperature is changed, which results in a change of the electrical resistivity of the liquid melt, but the height of the sample is unchanged, so we can conclude that the change of β is the same as the reciprocal electrical resistivity. The electrical resistivity increases with increasing temperature,[33] indicating that equation (13) conforms to the well known fact that electrical resistivity increases with increasing temperature. According to Eq. (13), we can test electrical resistivity by measuring the viscosity under horizontal magnetic field, and vice versa.

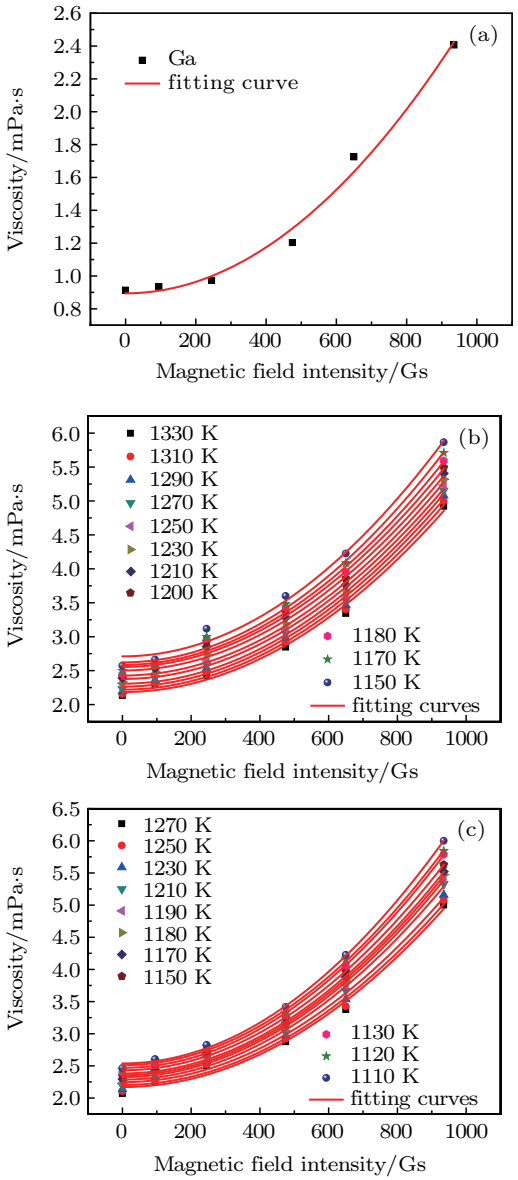


Fig. 3. (color online) Dependence of the viscosity of (a) pure Ga melt, (b) $Ga_{80}Cr_{20}$ melt, and (c) $Ga_{80}Ni_{20}$ melt upon the intensity of the magnetic field at various temperatures. The red solid lines are the fitting curves.

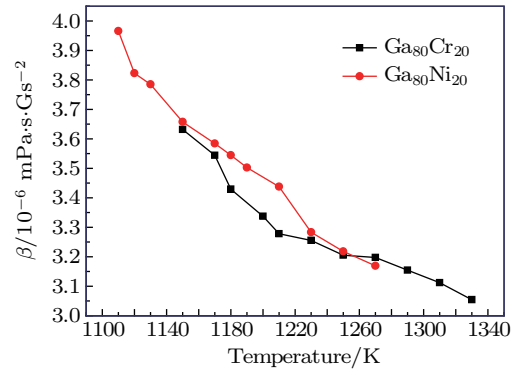


Fig. 4. (color online) β of $Ga_{80}Cr_{20}$ and $Ga_{80}Ni_{20}$ melts dependence upon temperature.

Sun *et al.* measured the viscosity of Sb–Bi melts under magnetic fields of different intensities at various temperatures.[27] Now we use their data to fit with Eq. (13) as shown in Fig. 5. We can find that the viscosities of Sb–Bi

melts under magnetic fields fit well with Eq. (13). It is easily obtained that β increases with decreasing temperature, which means that the electrical resistivity increases with increasing temperature according to Eq. (13), consistent with the well established fact that the electrical resistivity increases with increasing temperature.

Mao *et al.* measured the viscosities of Cu–10% Sn and pure Cu melts under magnetic fields of different intensities at various temperatures.[28] Yang measured the viscosities of some single liquid melts, such as aluminum, indium, tin, and bismuth.[29] Zhang measured the viscosity of Al–5 at% Si liquid melt under a horizontal magnetic field.[30] We can see that the above viscous data all fit well with Eq. (13). The change of coefficient β with the temperature accords with the law that the electrical resistivity increases with increasing temperature.

Based on so many viscous data under different intensities of magnetic fields, such as the same components with different atomic percents, different single metallic melts, and different alloy systems, we can conclude that all the above viscosities are in accordance with Eq. (13).

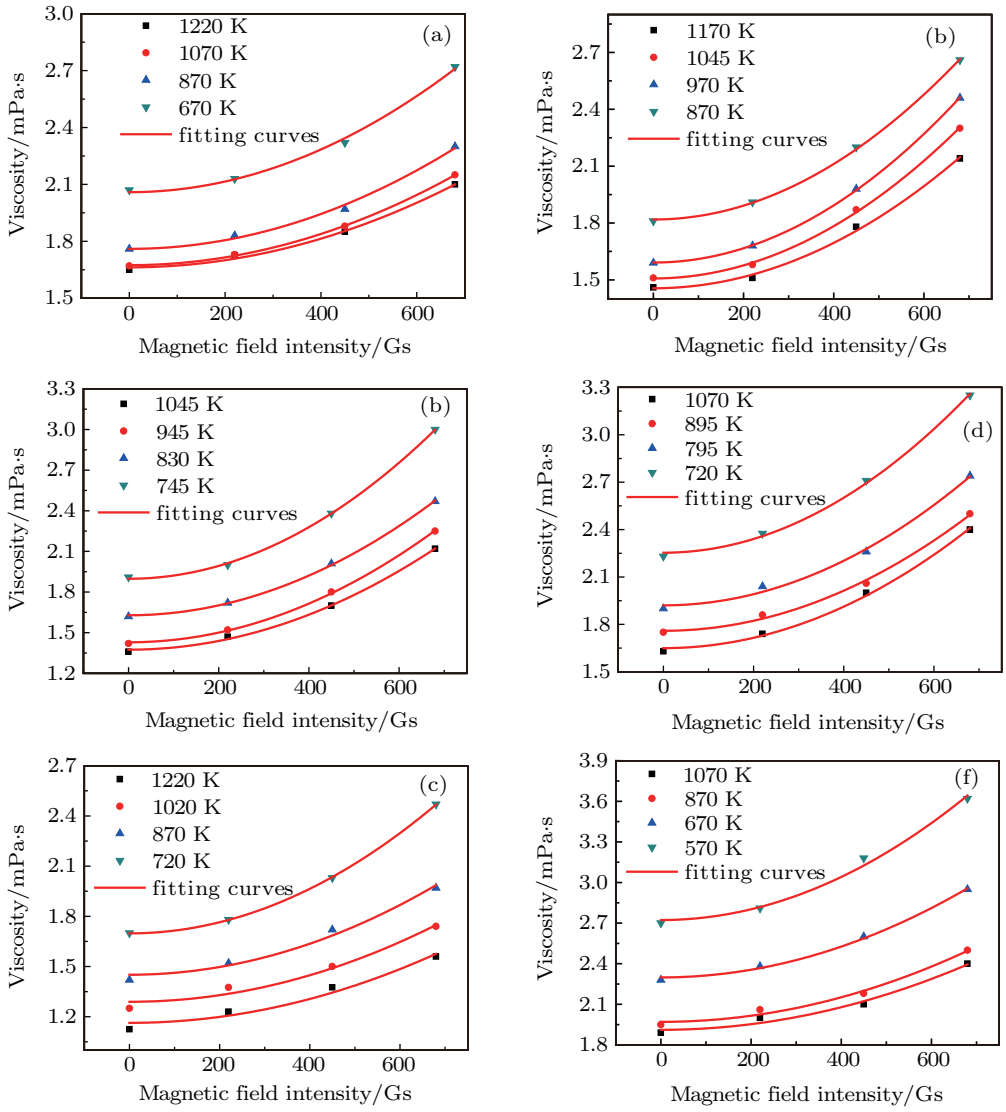


Fig. 5. (color online) Dependence of the viscosity of (a) Bi, (b) Sb, (c) $Bi_{20}Sb_{80}$, (d) $Bi_{40}Sb_{60}$, (e) $Bi_{60}Sb_{40}$, (f) $Bi_{80}Sb_{20}$ melts upon the intensity of the magnetic field at various temperatures.[27] The solid lines are the fitting curves.

However, there is no liquid–liquid transition during the cooling process for the above melts. So we find the viscous data of $Cd_{70}Ga_{30}$, which include the liquid–liquid transition.[31] We know that when the liquid–liquid transition happens, the viscosity presents a turning point. We turn these data into a diagram that shows the relationship between the viscosity and the intensity of the magnetic field, as shown in Figs. 6(a) and 6(b). Figure 6(a) shows the viscosity's dependence upon the intensity of the magnetic field before the liquid–liquid transition appears, and it indicates that the viscosity follows Eq. (13). Figure 6(b) shows the viscosity's dependence upon the intensity of the magnetic field during the liquid–liquid transition process. The fitting curves show that when the liquid–liquid transition starts, the coefficient β decreases sharply, as shown in Table 2. This sharp change may be related to the liquid structure transition. Otherwise, during

the transition process, when the intensity of the magnetic field is 420 Gs, the viscosity is much smaller than the fitting data. During the transition process, the metallic melt is metastable, the external magnetic field has a more obvious effect on the liquid structure, which results in deviation of the viscosity. In a nutshell, the liquid–liquid transition results in a sharp change of the electrical resistivity. The viscosity of single and binary metallic melts under magnetic field may deviate from Eq. (13) because of the metastable liquid structure during the transition process.

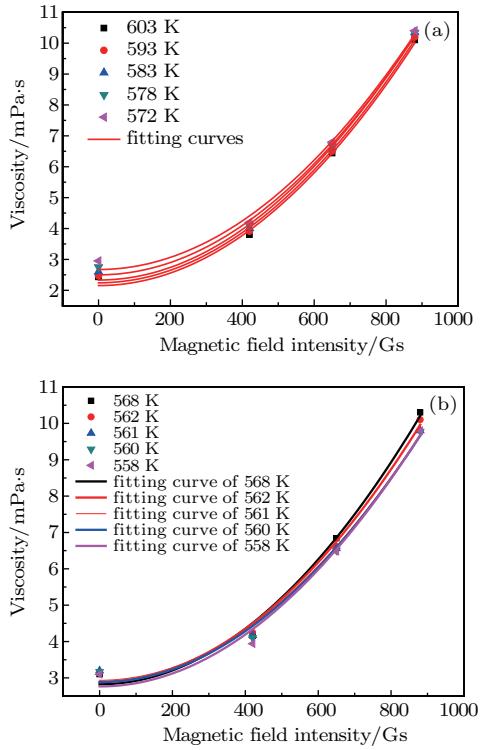


Fig. 6. (color online) (a) Viscosity's dependence upon the intensity of the magnetic field before the liquid–liquid transition appears. (b) Viscosity's dependence upon the intensity of the magnetic field during the liquid–liquid transition process.[31]

Table 2. The values of fitting β during the liquid–liquid transition process.

T/K	$\beta/10^{-6}$ mPa·s·Gs^{-2}
568	9.489
562	8.926
561	8.8816
560	8.7536
558	8.7436

4. Conclusion

In this work, we have studied the viscosities of pure Ga melt, $Ga_{80}Ni_{20}$ melt, and $Ga_{80}Cr_{20}$ melt under a horizontal magnetic field. A quantitative description of this dynamic behavior is given by an explicit analytical equation based on a mathematical physical model that allows us to predict the viscosity of single and binary metallic melts and to control the viscosity under a magnetic field. We can show that the magnetic viscosity is proportional to the square of the intensity of the magnetic field. The relationship between the magnetic viscosity and the electrical resistivity has been established. The change of the proportionality coefficient with temperature in our equation, which is related to the electrical resistivity, shows that our equation is correct. We also use some other data to fit with the equation, which indicates the correctness of our equation when there is no liquid–liquid transition.

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