Spin transfer torque in the semiconductor/ferromagnetic structure in the presence of Rashba effect

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Spin transfer torque in magnetic structure occurs when the transverse component of the spin current that flows from the nonmagnetic medium to ferromagnetic medium is absorbed by the interface. In this paper, considering the Rashba effect on the semiconductor region, we discuss the spin transfer torque in semiconductor/ferromagnetic structure and obtain the components of spin-current density for two models: (i) single electron and (ii) the distribution of electrons. We show that no matter whether the difference in Fermi surface between semiconductor and Fermi spheres for the up and down spins in ferromagnetic increases, the transmission probability decreases. The obtained results for the values used in this article illustrate that Rashba effect increases the difference in Fermi sphere between semiconductor and Fermi sphere for the up and down spins in ferromagnetic. The results also show that the Rashba effect, brings an additional contribution to the components of spin transfer torque, which does not exist in the absence of the Rashba interaction. Moreover, the Rashba term has also different effects on the transverse components of the spin torque transfer.

Keywords: spin transfer torque, Rashba interaction, ferromagnetic

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1. 1Introduction

The concept of "spin transfer torque" was first introduced in 1996 independently by Slonczewski and Magn[1] and Berger.[2] Spin transfer means that whenever a current of polarized electrons enters the ferromagnetic state, there appears a transfer of angular momentum between the electrons and the magnetic moment of the ferromagnetic area. In other words, the spin torque transfer occurs when the flow of transfer of spin angular momentum is not constant. For example, a spin current, which is polarized by the filtration of a thin magnetic layer, faces a second polarization by another thin magnetic layer, in which its magnetic moment direction is not the same as the first one. In this process the second magnetic layer will definitely absorb part of the spin angular momentum carried by the electron.

In Ref. [3], authors have investigated and reported spin-current densities and spin transfer torques of the general nonmagnetic structure and ferromagnetic structure. The results indicate that only the transverse components of the spin-current play a role in the spin transfer torque, while the longitudinal component of spin current does not show any effect. Here, the longitudinal component of current means that the current whose electrons have spin directions the same as the directions of the magnetizations of the ferromagnetic and the transverse components of the current includes the current of electrons whose spins are normal to the magnetization direction (see Fig. 1).

The spin–orbit coupling in semiconductor low-dimensional system plays an important role in developing the spintronics, which exploits electron spin in order to store information.[4–10] The prediction theory of spin transfer torque is created with harnessing the spin–orbit coupling first reported in Ref. [11]. Then experimentally for the first time they were observed and reported in Ref. [12]. In Ref. [13] authors have shown the effect of Rashba interaction using the Boltzmann transport theory that a maximum efficiency of spin torque transfer can be achieved by optimally changing the parameter of the Rashba interaction. Also, authors of Ref. [14] have reported spin transfer torque due to the non-uniform Rashba interaction. In the present paper, considering the effect of Rashba in the semiconductor, we investigate spin-current density and spin transfer torque in semiconductor Rashba/ferromagnetic junction for two free single electron and distribution of electrons models.

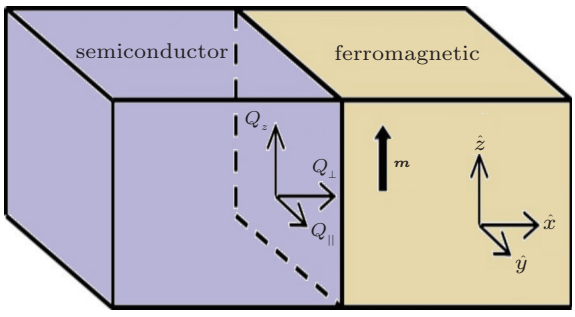


Fig. 1. (color online) The sketch of the transverse and longitudinal components of spin-current density. In this figure, $Q_\perp$ and $Q_\parallel$ spin currents that are Q_{xx} and Q_{yx}, respectively, are called the transverse components and Q_z the longitudinal spin current component.

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The rest of the present paper is organized as follows. In Section 2, we summarize the model, Hamiltonian and formalism. More details are given in the supplementary material. In Section 3, we present our numerical calculations. Finally, conclusion is given in Section 4.

2. Theory and model

The model considered here, which also was used in Ref. [15], is the Rashba semiconductor/ferromagnetic junction, in which the magnetization in the ferromagnetic layer is considered in $\hat{z}$ direction. The Hamiltonian is given as follows:

$$H_{\text{Ferro}} = \begin{pmatrix} \frac{\hbar^2}{2m}k^2 & 0 \\ 0 & \frac{\hbar^2}{2m}k^2 \end{pmatrix} + \Delta\sigma_z, \tag{1}$$

where 2Δ is the exchange split and Hamiltonian in the Rashba semiconductor can be described as follows:

$$H = -\frac{\hbar^2\nabla^2}{2m} + \alpha(-\mathrm{i}\nabla\times\boldsymbol{E})\cdot\boldsymbol{\sigma}, \tag{2}$$

where α is the effective mass and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli vector matrix. Assuming that the electrical field is $\boldsymbol{E} = (0, 0, E_z)$, we will have

$$\begin{aligned} H_{\text{R}} &= \begin{pmatrix} E_0 + \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) & \langle\alpha E_z\rangle\left(\frac{\partial}{\partial x} + \mathrm{i}\frac{\partial}{\partial y}\right) \\ -\langle\alpha E_z\langle\left(\frac{\partial}{\partial x} + \mathrm{i}\frac{\partial}{\partial y}\right) & E_0 + \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \end{pmatrix} \\ &= \begin{pmatrix} \frac{\hbar^2}{2m}k^2 & \mathrm{i}\Delta R \\ -\mathrm{i}\Delta R^* & \frac{\hbar^2}{2m}k^2 \end{pmatrix}, \end{aligned} \tag{3}$$

where $\Delta R = \langle\alpha E_z\rangle(k_x - \mathrm{i}k_y)$. In the above equation $\langle\alpha E_z\rangle$ is the value of expectation on the smallest subband with positive E_0 energy and normally, its observed value is of the order of 10^{-11} eV·m.[6,16] Eigenstates for moving on the surface with wave vector of $\boldsymbol{k} = (k_x, k_y)$ and $s = \pm 1$ are given as

$$\phi_{\boldsymbol{k}s}(\boldsymbol{r}) = N_{\boldsymbol{k}s}\,\mathrm{e}^{\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{r}}\begin{pmatrix} \mathrm{i}s\left(\frac{k_x}{k} - \mathrm{i}\frac{k_y}{k}\right) \\ 1 \end{pmatrix}, \tag{4}$$

where $N_{\boldsymbol{k}s}$ is the normalization factor. Regarding the above equation for eigenstates, it is obvious that split subband by Rashba does not have spin polarization and then the electron energy dispersion can read as

$$E_{\boldsymbol{k}s} = E_0 + \frac{\hbar^2}{2m}[(k + sk_{\text{R}})^2 - k_{\text{R}}^2], \tag{5}$$

in which $k = \sqrt{k_x^2 + k_y^2}$ and $k_{\text{R}} = \langle\alpha E_z\rangle m/\hbar^2$ is Rashba-dependent wave vectors. In the following, by using a rotation matrix

$$U(\theta,\phi) = \begin{pmatrix} \cos\theta/2 & \mathrm{e}^{-\mathrm{i}\phi}\sin\theta/2 \\ \mathrm{e}^{\mathrm{i}\phi}\sin\theta/2 & -\cos\theta/2 \end{pmatrix} \tag{6}$$

with $\phi = -\pi/2$, $\theta = \pi/2$ and applying it to Eq. (3), we will have new following Hamiltonian in the Rashba semiconductor

$$U^{\dagger}(\theta,\phi)H_{\text{R}}U(\theta,\phi) = \begin{pmatrix} \frac{p_x^2}{2m} + \langle\alpha E_z\rangle k_x & 0 \\ 0 & \frac{p_x^2}{2m} - \langle\alpha E_z\rangle k_x \end{pmatrix}. \tag{7}$$

Comparing Eq. (7) and Eq. (1), the Rashba semiconductor can be considered to be the same as a pseudo-ferromagnet in which its magnetization is along the $\hat{z}$ direction. Now, the wave vector in a Rashba semiconductor[12] can be recast as follows:

$$k_{\text{R}}^s(q) = -sk_{\text{R}} + \sqrt{k_{\text{R}}^2 + k_{\text{F}}^2 - q^2};\ (s = \pm 1). \tag{8}$$

The wave vector in ferromagnetic layer can be defined as follows:[11]

$$k_{x,\text{Ferro}}^{\sigma}(q) = \sqrt{(k_{\text{F}}^{\sigma})^2 - q^2}. \tag{9}$$

In the above two equations, k_R^s is the Rashba spin wave vector which shows that up and down spins in semiconductor region are different as much as k_R. Also the wave vector in this region in the presence and absence of Rashba spin wave vector is defined as follows:

$$k_x^{\text{R}}(q) = \sqrt{k_R^2 + k_{\text{F}}^2 - q^2},\ \ k_x(q) = \sqrt{k_{\text{F}}^2 - q^2},$$

respectively. $k_{x,\text{Ferro}}^{\sigma}$ and k_{F}^{σ} are the spin-dependent wave vector and Fermi wave vector in ferromagnetic layer and q is the $\hat{y}$ or $\hat{z}$ component of the wave vector at surface of the junction. In order to calculate the charge current density and spin current density at the interface between ferromagnetic and Rashba semiconductor, it is necessary to introduce velocity operator in two areas. Assuming the propagation direction along the $\hat{x}$ axis, the velocity operator in this direction is defined as follows:

$$v_x = \frac{\partial H_{\text{R}}}{\partial p_x} = \begin{pmatrix} \frac{p_x}{m} + \frac{\hbar}{m}k_{\text{R}} & 0 \\ 0 & \frac{p_x}{m} - \frac{\hbar}{m}k_{\text{R}} \end{pmatrix}. \tag{10}$$

This equation indicates that up and down spins in semiconductor move at different velocities in the propagation direction $\hat{x}$. In the ferromagnet the velocity operator is always diagonal in spin space

$$v_x = \partial H_{\text{Ferr}}/\partial p_{x\text{Ferr}} = \frac{p_x}{m}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Now, in order to obtain the transmission probability and eventually the transmitted current density, it is necessary to define wave functions in ferromagnetic and Rashba semiconductor

regions. For the Rashba semiconductor, we need to rotate the wave vector Eq. (4) as $U(\theta,\phi)\varphi_{\boldsymbol{k}s}(\boldsymbol{r})$. Therefore, the wave functions for the ferromagnetic/Rashba semiconductor junction (magnetization in the ferromagnetic area is considered to be in the $\hat{z}$ direction) in three dimensions can be given as follows:

$$S=+1\begin{cases}\psi^{+}_{\text{Rashba}}(x)=\left[\mathrm{e}^{\mathrm{i}k_{+}x}+r_{\uparrow}\mathrm{e}^{-\mathrm{i}k_{-}x}\right]\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{R}}|\uparrow\rangle,\\ \psi^{+}_{\text{Ferr}}x)=t_{\uparrow}\mathrm{e}^{\mathrm{i}k^{\uparrow}_{x}x}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{R}}|\uparrow\rangle,\end{cases}\tag{11}$$

$$S=-1\begin{cases}\psi^{-}_{\text{Rashba}}(x)=\left[\mathrm{e}^{\mathrm{i}k_{-}x}+r_{\uparrow}\mathrm{e}^{-\mathrm{i}k_{+}x}\right]\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{R}}|\downarrow\rangle,\\ \psi^{-}_{\text{Ferr}}(x)=t_{\downarrow}\mathrm{e}^{\mathrm{i}k^{\downarrow}_{x}x}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{R}}|\downarrow\rangle.\end{cases}\tag{12}$$

In these equations $q(k_y,k_z)$ and $R(y,z)$ are the wave vector and position vector, respectively. Reflection and transmission coefficients based on the boundary conditions, that is to say, wave function continuity and its first derivative at the interface:

$$\psi^{s}_{\text{Rashba}}(x=0)=\psi^{\sigma}_{\text{Ferro}}(x=0),\tag{13}$$

$$\left(\frac{\partial}{\partial x}\pm\mathrm{i}k_{\text{R}}\right)\psi^{s}_{\text{Rashba}}(x=0)=\frac{\partial}{\partial x}\psi^{\sigma}_{\text{Ferro}}(x=0)\tag{14}$$

are as follows:

$$t_{\sigma}(q)=\frac{2\sqrt{k_{\text{R}}^2+k_{\text{F}}^2-q^2}}{\sqrt{k_{\text{R}}^2+k_{\text{F}}^2-q^2}+k^{\sigma}_{\text{Ferro}}(q)};\tag{15}$$

$$r_{\sigma}(q)=\begin{cases}\dfrac{\sqrt{k_{\text{R}}^2+k_{\text{F}}^2-q^2}-k^{\sigma}_{\text{Ferro}}(q)}{\sqrt{k_{\text{R}}^2+k_{\text{F}}^2-q^2}+k^{\sigma}_{\text{Ferro}}(q)},\ \text{if } q^2\leqslant\left(k^{\sigma}_{\text{Ferro}}\right)^2,\\ \dfrac{\sqrt{k_{\text{R}}^2+k_{\text{F}}^2-q^2}-\mathrm{i}k^{\sigma}_{\text{Ferro}}(q)}{\sqrt{k_{\text{R}}^2+k_{\text{F}}^2-q^2}+\mathrm{i}k^{\sigma}_{\text{Ferro}}(q)},\ \text{if } q^2>\left(k^{\sigma}_{\text{Ferro}}\right)^2.\end{cases}\tag{16}$$

The possibilities of reflection and transmission are respectively as follows:

$$R_{\sigma}(q)=|r_{\sigma}(q)|^2,\tag{17}$$

$$T_{\sigma}=\frac{k^{\sigma}_{x}(q)}{k^{\text{R}}_{x}(q)}|t_{\sigma}(q)|^2.\tag{18}$$

In general, when a polarized spin current with k_x wave vector in a semiconductor layer enters into the ferromagnetic layer there is a possibility that by the interface it undergoes spin filtering and thus in ferromagnetic layer the up and down spins feel different wave vectors ($k^{\downarrow}_{x},k^{\uparrow}_{x}$ respectively).[3] There are three effective major factors contributing to the torque. The first factor is the spin filtering. This occurs when reflection probabilities are spin dependent. The second factor is the spin rotation. The spin rotation occurs when the quantity of $r^{*}_{\uparrow}r_{\downarrow}$ is not real and positive. So, this quantity can be written as follows:

$$r^{*}_{\uparrow}r_{\downarrow}=\left|r^{*}_{\uparrow}r_{\downarrow}\right|\mathrm{e}^{\mathrm{i}\Delta\phi}.\tag{19}$$

This relationship indicates that during the reflection, $\Delta\phi$ phase is added to azimuthal angleϕ that shows the direction of incident electrons. In other words, the spin direction of electron changes at the interface during reflection. This is completely a quantum phenomenon and cannot imagine a classical counterpart for that. Finally, the third factor in the development of spin transfer torque is the spin precession. In the ferromagnetic layer, as $k^{\uparrow}_{x}\neq k^{\downarrow}_{x}$, the spatially-varying phase factors which appear in the transmitted transverse spin currents is in a way that their outcome creates a spatial precession.

$$\Delta k=k^{\downarrow}_{x}-k^{\uparrow}_{x}.\tag{20}$$

The angular momentum is conserved in the system. This is true if the FM and electron spin in the Rashba semiconductor are uncoupled (i.e., in the absence of s–d exchange coupling), and the conservation holds only for the FM layer, not for the whole system. So, due to the conservation of angular momentum, the spin transfer torque on the studied material can be achieved which is conducive to the calculation of pure spin current flow from this object's surfaces.[17]

$$\begin{aligned}N_{\text{ST}}&=-\int_{\text{pillbox surfaces}}\mathrm{d}^2R\,\hat{n}\cdot\boldsymbol{Q}\\&=-\int_{\text{pillbox volume}}\mathrm{d}^3r\nabla\cdot\boldsymbol{Q}.\end{aligned}\tag{21}$$

However, it should be noted that the spin-current density Q is a tensor, which includes spatial and spin parts. Point-wise multiplier of $\hat{n}\cdot\boldsymbol{Q}$ operates only on the spatial part and does not act on the spin part. This relationship lies in the fact that if we consider a rectangular surface (Gauss surface) at the interface, the divergence theorem implies that a spin transfer torque of induced current is applied to the interface magnetization.

In the following, the formulation provided above will be investigated for the single-electron model and the distribution model of electrons.

2.1. Spin current for single electron

To investigate spin torque transfer in this model, first we define the spin current density as follows[3]

$$\begin{aligned}Q(r)&=\sum_{i\sigma\sigma'}\text{Re}\left[\psi^{*}_{i\sigma}(r)s\otimes\hat{v}\psi_{i\sigma'}(r)\right]\\&=\frac{\hbar}{2}\sum_{i\sigma\sigma'}\text{Re}\left[\psi^{*}_{i\sigma}(r)\sigma\otimes\hat{v},\psi_{i\sigma'}(r)\right],\end{aligned}\tag{22}$$

where $Q_{ij}(r)$ is a tensor quantity. Left subscript belongs to spin space and right subscript belongs to real space. The incident wave functions, reflection and transmission which are described by Eqs. 11) and (12) can be written as a linear combination of spin up and spin down components, i.e., $\psi=\psi_{\uparrow}+\psi_{\downarrow}$. Now, by inserting the wave functions in Eq. (22),

current components will be as follows. The incident current components are

$$Q_{\perp}^{\text{in}} = Q_{xx}^{\text{in}} = \frac{\hbar^2 k_x}{m}\cos(2k_{\text{R}}x),$$
$$Q_{\parallel}^{\text{in}} = Q_{yx}^{\text{in}}(r) = \frac{\hbar^2 k_x}{m}\sin(2k_{\text{R}}x),$$
$$Q_{zx}^{\text{in}}(r) = \frac{\hbar^2 k_{\text{R}}}{m}, \tag{23}$$

the reflected currents are

$$\begin{aligned} Q_{\perp}^{\text{ref}} &= Q_{xx}^{\text{ref}} = -\left[\frac{\hbar^2 k_x}{m}\right]\text{Re}\left\{r_{\uparrow}^* r_{\downarrow}\,\text{e}^{\,\text{i}\,(2k_{\text{R}})x}\right\} \\ &= -\left[\frac{\hbar^2 k_x}{m}\right]\left|r_{\uparrow}^* r_{\downarrow}\right|\text{Re}\left\{\text{e}^{\,\text{i}\,\Delta\phi}\,\text{e}^{\,\text{i}\,(2k_{\text{R}})x}\right\}, \\ Q_{\parallel}^{ref} &= Q_{yx}^{\text{ref}} = -\left[\frac{\hbar^2 k_x}{m}\right]\text{Im}\left\{r_{\uparrow}^* r_{\downarrow}\,\text{e}^{\,\text{i}\,(2k_{\text{R}})x}\right\} \\ &= -\left[\frac{\hbar^2 k_x}{m}\right]\left|r_{\uparrow}^* r_{\downarrow}\right|\text{Im}\left\{\text{e}^{\,\text{i}\,\Delta\phi}\,\text{e}^{\,\text{i}\,(2k_R)x}\right\}, \\ Q_{zx}^{\text{ref}} &= -\frac{\hbar^2}{2m}\left\{[k_x+k_R]\left|r_{\uparrow}\right|^2 - [k_x-k_{\text{R}}]\left|r_{\downarrow}\right|^2\right\}; \end{aligned} \tag{24}$$

the transmitted currents are

$$\begin{aligned} Q_{\perp}^{\text{tr}} &= Q_{xx}^{\text{tr}} = \frac{\hbar^2}{2m}\text{Re}\left[t_{\uparrow}^* t_{\downarrow}\,\text{e}^{\,\text{i}\,(k_x^{\downarrow}-k_x^{\uparrow})x}\right](k_x^{\uparrow}+k_x^{\downarrow}), \\ Q_{\parallel}^{\text{tr}} &= Q_{yx}^{\text{tr}} = \frac{\hbar^2}{2m}\text{Im}\left[t_{\uparrow}^* t_{\downarrow}\,\text{e}^{\,\text{i}\,(k_x^{\downarrow}-k_x^{\uparrow})x}\right](k_x^{\uparrow}+k_x^{\downarrow}), \\ Q_{zx}^{\text{tr}} &= \frac{\hbar^2}{2m}\left[\left(k_x^{\uparrow}\left|t_{\uparrow}\right|^2 - k_x^{\downarrow}\left|t_{\downarrow}\right|^2\right)\right]. \end{aligned} \tag{25}$$

Now, substituting the obtained currents into Eq. (21), for the torque we have

$$\begin{aligned} N_{\text{st}} = &\left\{\frac{\hbar^2 k_x}{m}\cos(2k_R x) - \left[\frac{\hbar^2 k_x}{m}\right]\text{Re}\left[r_{\uparrow}^* r_{\downarrow}\,\text{e}^{\,\text{i}\,(2k_R)x}\right]\right. \\ &\left. -\frac{\hbar^2}{2m}\text{Re}\left[t_{\uparrow}^* t_{\downarrow}\,\text{e}^{\,\text{i}\,(k_x^{\downarrow}-k_x^{\uparrow})x}\right](k_x^{\uparrow}+k_x^{\downarrow})\right\}\hat{x} \\ &+\left\{\frac{\hbar^2 k_x}{m}\sin(2k_R x) - \left[\frac{\hbar^2 k_x}{m}\right]\text{Im}\left[r_{\uparrow}^* r_{\downarrow}\,\text{e}^{\,\text{i}\,(2k_R)x}\right]\right. \\ &\left. -\frac{\hbar^2}{2m}\text{Im}\left[t_{\uparrow}^* t_{\downarrow}\,\text{e}^{\,\text{i}\,(k_x^{\downarrow}-k_x^{\uparrow})x}\right](k_x^{\uparrow}+k_x^{\downarrow})\right\}\hat{y} \\ &+\left\{\frac{\hbar^2 k_R}{m} - \frac{\hbar^2}{2m}k_R(|r_{\uparrow}|^2+|r_{\downarrow}|^2)\right\}\hat{z}. \end{aligned} \tag{26}$$

2.2. Spin current for distribution of electrons

Generally, in the description of transport, it is necessary to consider the effect of quantum mechanics coherence of all the electrons with different special modes. However, for the spin torque transfer model, up to now, experiments have shown[18–20] that it is not necessary to consider the coherence between electrons and different wave vectors of Fermi sources. This is equivalent to the use of semi-classical theory that only note the coherence between up and down spin state at any point of $\boldsymbol{k}$ in the Fermi sea.

In this model, the formula for calculating density of incident, reflected and transmitted currents are defined respectively as follows:[3]

$$Q^{\text{in}} = \frac{\hbar}{2}\int_{v_x>0}\frac{\text{d}\,k^3}{(2\pi)^3}\,\text{Tr}\left[I^{\dagger}f(\boldsymbol{k},\boldsymbol{r})I\sigma\right]\otimes\boldsymbol{V}^{\text{in}}(\boldsymbol{k}), \tag{27}$$

$$Q^{\text{ref}} = \frac{\hbar}{2}\int_{v_x>0}\frac{\text{d}k^3}{(2\pi)^3}\,\text{Tr}\left[R^{\dagger}f(\boldsymbol{k},\boldsymbol{r})R\sigma\right]\otimes\boldsymbol{V}^{\text{ref}}(\boldsymbol{k}), \tag{28}$$

$$Q^{\text{trans}} = \frac{\hbar}{2}\int_{v_x>0}\frac{\text{d}k^3}{(2\pi)^3}\,\text{Tr}\left[T^{\dagger}f(\boldsymbol{k},\boldsymbol{r})T\sigma\right]\otimes\boldsymbol{V}^{\text{trans}}(\boldsymbol{k}), \tag{29}$$

where $f(\boldsymbol{k},\boldsymbol{r})$ is the electron distribution function and can be defined as follows:[3]

$$f(\boldsymbol{k},\boldsymbol{r}) = U(\boldsymbol{k},\boldsymbol{r})\begin{pmatrix} f_{\uparrow}(\boldsymbol{k},\boldsymbol{r}) & 0 \\ 0 & f_{\downarrow}(\boldsymbol{k},\boldsymbol{r})\end{pmatrix}U^{\dagger}(\boldsymbol{k},\boldsymbol{r}), \tag{30}$$

where $f_{\uparrow(\downarrow)}(\boldsymbol{k},\boldsymbol{r})$ is the occupying function for up (down) spin. The operator $U(\boldsymbol{k},\boldsymbol{r})$ is the rotation matrix that has the following form:

$$U(\boldsymbol{k},\boldsymbol{r}) = \begin{pmatrix} \cos(\theta/2)\,\text{e}^{-\text{i}\phi}/2 & -\sin(\theta/2)\,\text{e}^{-\text{i}\phi}/2 \\ \sin(\theta/2)\,\text{e}^{\text{i}\phi}/2 & \cos(\theta/2)\,\text{e}^{\text{i}\phi}/2\end{pmatrix}. \tag{31}$$

For simplicity, the dependence of $\boldsymbol{k}$ and $\boldsymbol{r}$ on angles θ and ϕ have been removed. Now, with the help of Eqs. (27)–(29), the spin-current density components for $\theta=\pi/2$, $\phi=-\pi/2$ at the interface between Rashba ferromagnetic and semiconductor, in the place of $\boldsymbol{r}=0$ are as follows:

$$Q_{xx}^{\text{in}} = -\frac{1}{8\pi^2}\int \text{d}q\,q\Delta\mu\sin(2k_{\text{R}}x+\Delta\phi), \tag{32}$$

$$Q_{xx}^{\text{ref}} = +\frac{1}{8\pi}\int\frac{\text{d}q\,q}{2\pi}\left|r_{\uparrow}^* r_{\downarrow}\right|\Delta\mu\sin(2k_{\text{R}}x+\Delta\phi), \tag{33}$$

$$Q_{xx}^{\text{trans}} = -\frac{1}{4\pi}\int\frac{\text{d}q\,q}{2\pi}\left|t_{\uparrow}^* t_{\downarrow}\right|\Delta\mu\sin[(k_x^{\downarrow}-k_x^{\uparrow})x]\left(\frac{v_x^{\uparrow}+v_x^{\downarrow}}{|2v_x|}\right), \tag{34}$$

$$Q_{yx}^{\text{trans}} = -\frac{1}{4\pi}\int\frac{\text{d}q\,q}{2\pi}\left|t_{\uparrow}^* t_{\downarrow}\right|\Delta\mu\cos[(k_x^{\downarrow}-k_x^{\uparrow})x]\left(\frac{v_x^{\uparrow}+v_x^{\downarrow}}{|2v_x|}\right). \tag{35}$$

More details of calculations are given in the supplementary document.

3. Numerical results and discussion

In this section, we investigate the spin wave vector, reflection, and transmission coefficients for single-electron model, then the current density of the electrons in distribution model will be studied.

Figure 2 shows the spin wave vectors in the semiconductor and ferromagnetic layers. As can be seen, the Rashba effect in semiconductor makes the wave vectors for up and down spin differ, and this difference is such that the up and down spin

wave vectors have the same magnitude, but they, for the non-Rashba, are separated from each other as much as the added amount of Rashba semiconductor. The difference in way of the spin wave vector detachment in Rashba semiconductor and ferromagnetic areas are clearly illustrated in Fig. 2.

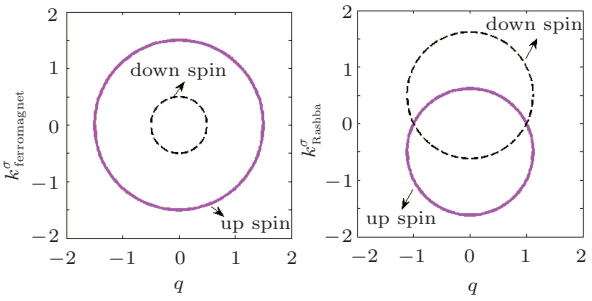


Fig. 2. (color online) $k^{\sigma}_{\rm Ferro}$ spin wave vectors for the up and down spin in ferromagnetic (left panel), and k^{σ}_{R} spin wave vector in Rashba semiconductor (right panel) versus q for $k_{\rm F}=1$, $k_x^{\uparrow}/k_{\rm F}=1.5$, and $k_x^{\downarrow}/k_{\rm F}=0.5$.

Figure 3 displays the plots of transmission and reflection probabilities for up and down spin in the absence and presence of the Rashba interaction versus q. As can be seen in the figure, the possibility of the reflection in the absence of the Rashba interaction for down spin in $q=0$ is close to zero and in $q\simeq 0.5$ the reflection is one. While the possibility of the reflection for up spin in $q=0$ is almost zero and the closer to the Fermi wave vector ($k_{\rm F}=1$) the q is, the more complete the reflection will be. Considering the Rashba interaction the reflection possibilities for up spin and down spin have similar trends to those in the non-Rashba case. The reason is that the wave vector increases in the semiconductor Rashba, with considering the Rashba interaction. In general, as the difference between the Rashba semiconductor wave vectors ($k^{\sigma}_{\rm R}$) and the ferromagnetic spin wave vector ($k^{\sigma}_{\rm Ferro}$) increases the reflection increases as well.

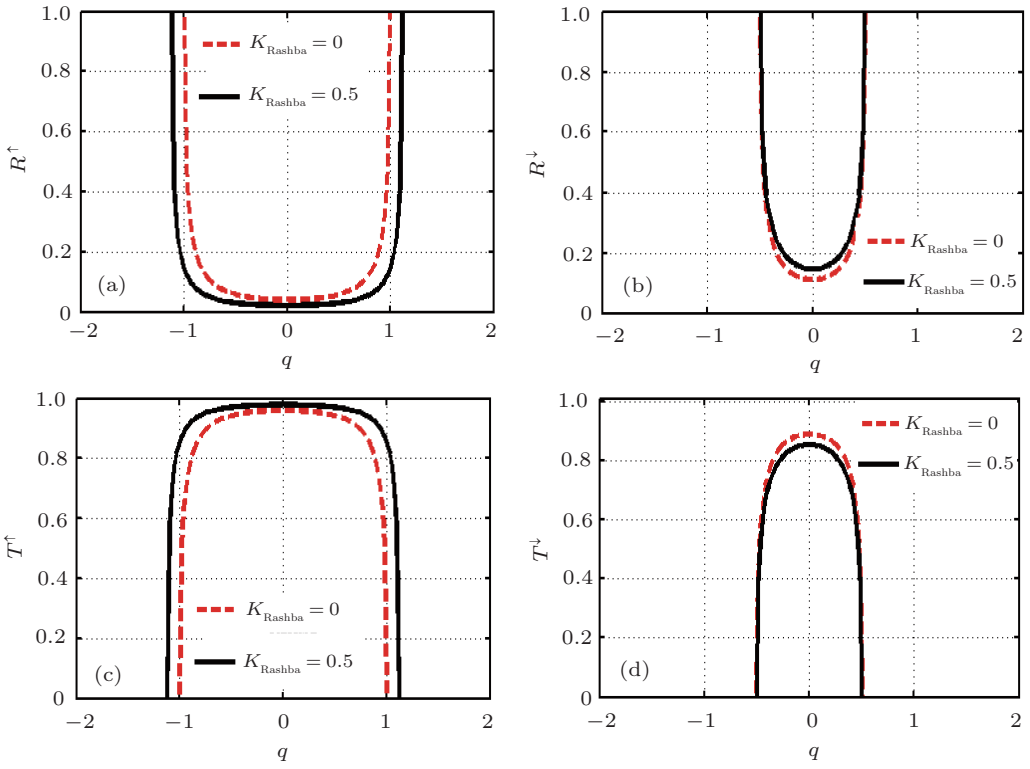


Fig. 3. (color online) Reflection and transmission possibilities for up and down spins as function of q for $k_{\rm F}=1$, $k_x^{\uparrow}/k_{\rm F}=1.5$, and $k_x^{\downarrow}/k_{\rm F}=0.5$.

Figure 4(b) shows the phase difference of an electron due to spin rotation which is experienced at the time of reflection. In Fig. 4(b), at $q=0$ we have $k_x^{\uparrow}=1.5k_{\rm F}$ and $k_x^{\downarrow}=0.5k_{\rm F}$. Thus, the wave vector difference of Δk is $k_{\rm F}$ as can be seen in the figure. Also, the wave vector difference of Δk is 1.3, for $q=0.5k_{\rm F}$. In fact, by reducing q to zero, the Fermi sphere difference for up and down spins increases, as a result of Δk reducing.

Figure 4(a) shows the wave vector difference of a transmitted electron. As can be seen in the figure, by increasing the parameter q, the phase difference between the two cases, that is, in the presence of the Rashba interaction and the absence of Rashba interaction, reduces. In the absence of Rashba interaction for $q=k_{\rm F}=1$, the phase difference becomes zero, which means that the electrons around the Fermi surface with the same incident spin direction are reflected. Meanwhile, in the presence of the Rashba interaction, a non-zero phase difference in $q=k_{\rm F}=1$ is observed, even for a small value of $k_{\rm R}=0.5$. In general, by increasing the Rashba interaction, the phase difference of the reflected electrons compared to that of the incident electrons experience further changes around the Fermi surface.

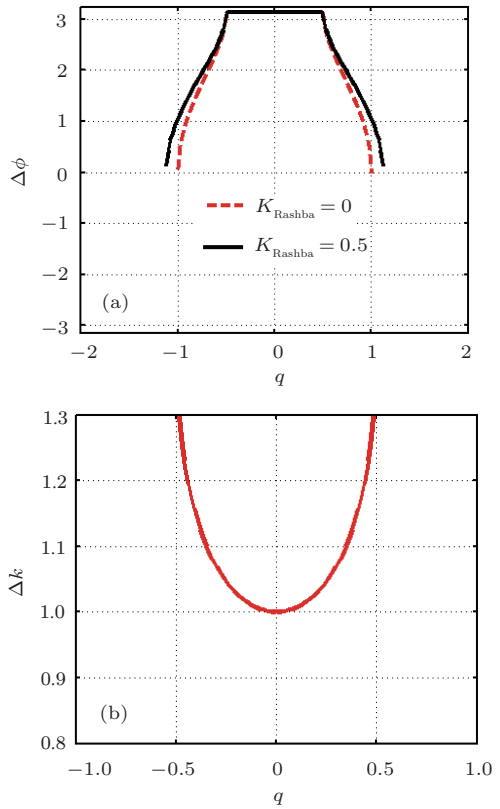


Fig. 4. (color online) Plots of the phase $\Delta\phi$ (a) acquired by an electron because its spin rotates upon reflection versus q, and wave vector difference $\Delta k = k_x^{\downarrow} - k_x^{\uparrow}$ (b) for a transmitted electron, versus q.

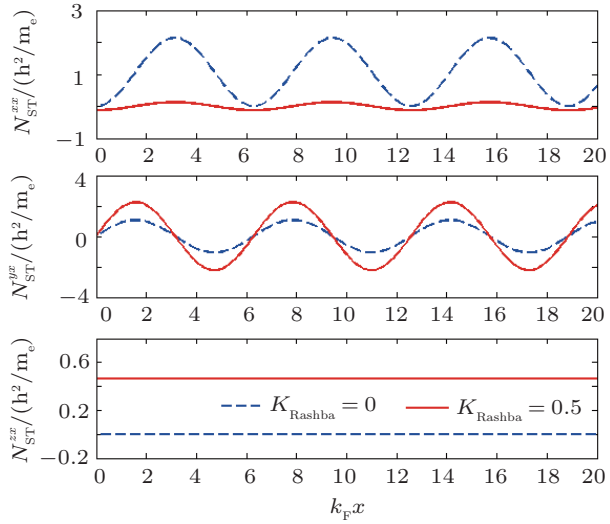


Fig. 5. (color online) Spin torque transfers in the absence and presence of Rashba for a free single electron model. $k_F^{\downarrow}/k_F = 0.5$ and $k_F^{\uparrow}/k_F = 1.5$.

Figure 5 shows the spin torque transfers as a function of distance from the interface of two environments in the presence and absence of Rashba interaction for a single electron model. As can be seen in the figure, in the absence of Rashba interaction, the spin torque only contains the component of transverse spin current. While considering the Rashba interaction, the torque, in addition to the components of the transverse, also will have longitudinal component. Moreover, the influences of Rashba interaction on the transverse components of the spin torque transfer are different.

Figure 6 shows the densities of spin-current for distribution of electrons as a function of distance from the interface of two environments for some parameters. Clearly, the decay of the current density can be seen by getting the distance from the interface. However, this decay is slightly small in the presence of Rashba interaction. In the absence of Rashba interaction Fermi wave vectors for up and down spins are coincided. Thus, the Fermi sphere in the semiconductor is placed between Fermi spheres of up and down spins in ferromagnetic. In Fig. 6(b) Fermi sphere of semiconductor is equal to Fermi sphere of spin majority in ferromagnetic and both are larger than Fermi sphere of spin minority. In Fig. 6(c) Fermi spheres of spin majority and minority are slightly larger and smaller than Fermi sphere in semiconductor, respectively. Consideration of the Rashba effect causes the Fermi sphere and wave vector difference in a semiconductor (k_x^{R}) with wave vector spin (majority and minority) in ferromagnetic (k_x^{σ}) to increase. The difference is less for up spin than for down spin, as a result the chance of pass for that will increase. Thus, it can be concluded that in the transmitted current, in this structure, up spins play a major role in transmitting current.

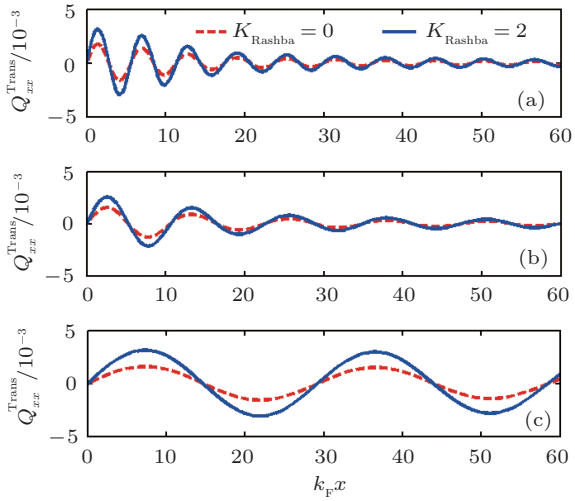


Fig. 6. (color online) Transmitted spin currents as a function of distance from the interface for the Fermi energy of $k_F = 1$ with (a) $k_F^{\downarrow}/k_F = 0.5$, $k_F^{\uparrow}/k_F = 1.5$, (b) $k_F^{\downarrow}/k_F = 0.5$, $k_F^{\uparrow}/k_F = 1.0$, and (c) $k_F^{\downarrow}/k_F = 0.9$, $k_F^{\uparrow}/k_F = 1.1$.

4. Conclusions

In this article we discuss the theory of the spin transfer torque in the Rashba semiconductor/ferromagnetic junction and obtain the components of spin-current density for two models: (i) single electron model and (ii) the distribution of electrons. In the single electron model, the spin torque calculation, Eq. (26), shows that considering the Rashba effect in semiconductor and thus making different the Fermi wave vectors for up and down spins in ferromagnetic region we have all three components of torque. However, in the absence of Rashba interaction and by taking $k_x^{\uparrow} = k_x^{\downarrow} = k_x$, the spin torque only includes the transverse components. In the distribution

model of electrons, it can be concluded from the results of current density that the bigger the difference between the Fermi sphere in semiconductor with the Fermi spheres of spin majority and minority in ferromagnetic, the more the transmitted current oscillation will be. In addition, the consideration of the Rashba effect can lead to an increase in the transmission coefficient for up spins to down spin, while its current has a similar trend to the current in the absence of the Rashba.

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