Degree of polarization based on the three-component pBRDF model for metallic materials

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An expression of degree of polarization (DOP) for metallic material is presented based on the three-component polarized bidirectional reflectance distribution function (pBRDF) model with considering specular reflection, directional diffuse reflection and ideal diffuse reflection. The three-component pBRDF model with a detailed reflection assumption is validated by comparing simulations with measurements. The DOP expression presented in this paper is related to surface roughness, which makes it more reasonable in physics. Test results for two metallic samples show that the DOP based on the three-component pBRDF model accords well with the measurement and the error of existing DOP expression is significantly reduced by introducing the diffuse reflection. It indicates that our DOP expression describes the polarized reflection properties of metallic surfaces more accurately.

Keywords: degree of polarization (DOP), polarized bidirectional reflectance distribution function (pBRDF), metallic materials

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1. Introduction

Polarization is a characteristic of light or electromagnetic radiation that conveys information about the orientation of the transverse electric and magnetic fields. It tends to provide information that is largely uncorrelated with spectral and intensity images, and thus has the potential to enhance many fields of optical metrology.[1]

Degree of polarization (DOP) is the most important and widely used feature parameter in the polarization detection field at present. In target detection applications,[2,3] the DOP is often used as a contrast-enhancing technique. It is also considered to be a crucial technique in atmospheric remote sensing[4] and turbid media.[5] In biomedical diagnostics,[6] the DOP associated with a polarimetric difference measurement[7] is a diagnostic for tissue properties, and can be used for early cancer detection. So the DOP expression is particularly critical for the studies of polarization. Vimal and Charles[8] derived the DOP expression based on Priest–Germer (P-G) pBRDF model[9] from passive polarimetric imagery for the case of scattering in the plane of incidence Unfortunately, P-G pBRDF model ignores diffuse reflection which makes DOP unrelated to surface roughness and caused large errors of DOP values compared with our measurements which shows significant defects in physics.

Focusing on the problem above, we present a DOP expression based on the three-component pBRDF model in this paper for metallic material to increase the physical rationality and simulation accuracy for describing polarized reflected properties.

2. Theory

2.1. Three-component pBRDF model

The fundamental description of optical scattering is the bidirectional reflectance distribution function (BRDF)[10] as shown in Fig. 1. The BRDF expression is given as

$$f(\theta_{\rm i},\varphi_{\rm i};\theta_{\rm r},\varphi_{\rm r})=\frac{{\rm d}L_{\rm r}(\theta_{\rm r},\varphi_{\rm r})}{{\rm d}E_{\rm i}(\theta_{\rm i},\varphi_{\rm i})}({\rm sr}^{-1}). \tag{1}$$

The pBRDF is defined as a 4×4 transformation of the Stokes vectors of incident and reflected light.

$$\begin{pmatrix}S_0'\\S_1'\\S_2'\\S_3'\end{pmatrix}=\begin{pmatrix}f_{00}&f_{01}&f_{02}&f_{03}\\f_{10}&f_{11}&f_{12}&f_{13}\\f_{20}&f_{21}&f_{22}&f_{23}\\f_{30}&f_{31}&f_{32}&f_{33}\end{pmatrix}\begin{pmatrix}S_0\\S_1\\S_2\\S_3\end{pmatrix}. \tag{2}$$

The three-component pBRDF model contains specular reflection $f_{\rm s}$, directional diffuse reflection $f_{\rm dd}$, and ideal diffuse reflection $f_{\rm id}$. The pBRDF expression is the sum of these three components[11]

$$f=k_{\rm s}\cdot f_{\rm s}+k_{\rm dd}\cdot f_{\rm dd}+k_{\rm id}\cdot f_{\rm id}, \tag{3}$$

where $k_{\rm s}$, $k_{\rm dd}$, and $k_{\rm id}$ are the coefficients of these three components, respectively.

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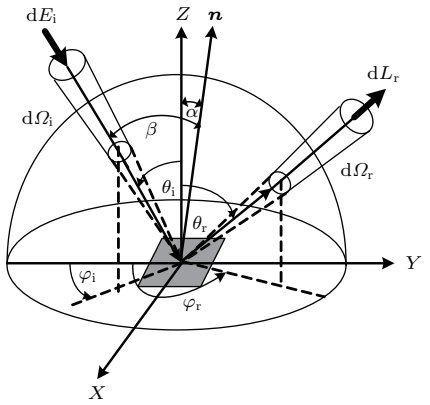


Fig. 1. BRDF coordinate system, where $\mathrm{d}L_\mathrm{r}(\theta_\mathrm{r},\varphi_\mathrm{r})$ is the reflected radiance; $\mathrm{d}E_\mathrm{i}(\theta_\mathrm{i},\varphi_\mathrm{i})$ is the incident irradiance; each of quantities $\mathrm{d}\Omega_\mathrm{i}$ and $\mathrm{d}\Omega_\mathrm{r}$ is commonly called an element of projected solid angle; θ and φ represent the zenith angle and azimuth angle, respectively; the subscripts "i" and "r" represent the incident and reflection rays, respectively; α is the polar angle between the mean surface normal Z and the microfacet normal n; β is the incident angle as measured from the microfacet.

The interaction between the light and surface is shown in Fig. 2.

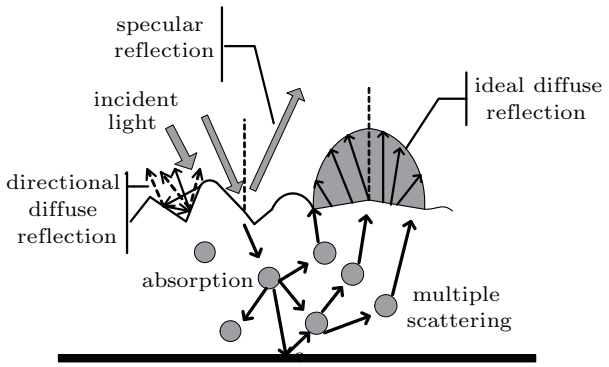


Fig. 2. Light scattering model for materials.

The specular reflection is given based on a microfacet theory. The expression of f_s is shown as

$$f_{jk}(\theta_\mathrm{i},\theta_\mathrm{r},\varphi) = \frac{1}{2\pi}\frac{1}{4\sigma^2}\frac{1}{\cos^4\alpha}\frac{\exp\left(-\dfrac{\tan^2\alpha}{2\sigma^2}\right)}{\cos\theta_\mathrm{r}\cos\theta_\mathrm{i}} \times G(\theta_\mathrm{i},\theta_\mathrm{r},\varphi)M_{jk}(\theta_\mathrm{i},\theta_\mathrm{r},\varphi), \tag{4}$$

where G is the shadowing/masking factor which determines the fraction of an illuminated microfacet which contributes to the scattered radiance, σ is the surface roughness, and M_{jk} is the Mueller matrix element which is given from the form of Jones matrix elements.[9]

The Stokes vector of the specular reflected light S'_s is expressed as follows:

$$S'_S = k_\mathrm{s}\left(\sum_{k=0}^{3} f_{0k}S_k \sum_{k=0}^{3} f_{1k}S_k \sum_{k=0}^{3} f_{2k}S_k \sum_{k=0}^{3} f_{3k}S_k\right)^\mathrm{T}. \tag{5}$$

The directional diffuse reflection is formed by multiple scattering and assumed to be of Gaussian distribution[11]

$$f_\mathrm{dd} = \frac{1}{\sqrt{2\pi}\sigma_m}\mathrm{e}^{-(\theta_\mathrm{r}^2/2\sigma_m^2)}. \tag{6}$$

The spatial distribution of the ideal diffuse reflection is assumed to be homogeneous in hemisphere space[11]

$$f_\mathrm{id} = 1. \tag{7}$$

The directional diffuse reflection and ideal diffuse reflection show strong depolarized property, so their Stokes vectors S'_dd and S'_id can be expressed as

$$S'_\mathrm{dd} = k_\mathrm{dd}\begin{pmatrix} f_\mathrm{dd}S_0 & 0 & 0 & 0\end{pmatrix}^\mathrm{T} = \begin{pmatrix} k_\mathrm{dd}\cdot f_\mathrm{dd}S_0 & 0 & 0 & 0\end{pmatrix}^\mathrm{T}, \tag{8}$$

$$S'_\mathrm{id} = k_\mathrm{id}\begin{pmatrix} f_\mathrm{id}S_0 & 0 & 0 & 0\end{pmatrix}^\mathrm{T} = \begin{pmatrix} k_\mathrm{id}\cdot f_\mathrm{id}S_0 & 0 & 0 & 0\end{pmatrix}^\mathrm{T}. \tag{9}$$

The total Stokes vector of the reflected light is shown below

$$\begin{aligned} S' &= S'_\mathrm{s} + S'_\mathrm{dd} + S'_\mathrm{id} \\ &= \begin{pmatrix} k_\mathrm{s}\cdot\sum\limits_{j=0}^{3} f_{0j}S_j + k_\mathrm{dd}\cdot f_\mathrm{dd}S_0 + k_\mathrm{id}\cdot f_\mathrm{id}S_0 \\ k_\mathrm{s}\cdot\sum\limits_{j=0}^{3} f_{1j}S_j \\ k_\mathrm{s}\cdot\sum\limits_{j=0}^{3} f_{2j}S_j \\ k_\mathrm{s}\cdot\sum\limits_{j=0}^{3} f_{3j}S_j \end{pmatrix}. \end{aligned} \tag{10}$$

The pBRDF matrix F and its elements F_{jk} can be derived from Eqs. (2) and (10) as follows:

$$\begin{aligned} &\begin{pmatrix} F_{00} & F_{01} & F_{02} & F_{03} \\ F_{10} & F_{11} & F_{12} & F_{13} \\ F_{20} & F_{21} & F_{22} & F_{23} \\ F_{30} & F_{31} & F_{32} & F_{33} \end{pmatrix} \\ &= \begin{pmatrix} k_\mathrm{s}f_{00} + k_\mathrm{dd}f_\mathrm{dd} + k_\mathrm{id}f_\mathrm{id} & k_\mathrm{s}f_{01} & k_\mathrm{s}f_{02} & k_\mathrm{s}f_{03} \\ k_\mathrm{s}f_{10} & k_\mathrm{s}f_{11} & k_\mathrm{s}f_{12} & k_\mathrm{s}f_{13} \\ k_\mathrm{s}f_{20} & k_\mathrm{s}f_{21} & k_\mathrm{s}f_{22} & k_\mathrm{s}f_{23} \\ k_\mathrm{s}f_{30} & k_\mathrm{s}f_{31} & k_\mathrm{s}f_{32} & k_\mathrm{s}f_{33} \end{pmatrix}. \end{aligned} \tag{11}$$

The general expression of F_{jk} is shown below

$$\begin{aligned} F_{00} &= k_\mathrm{s}f_{00}(\theta_\mathrm{i},\theta_\mathrm{r},\varphi) + k_\mathrm{dd}f_\mathrm{dd} + k_\mathrm{id}f_\mathrm{id}, \\ F_{jk} &= k_\mathrm{s}f_{jk}(\theta_\mathrm{i},\theta_\mathrm{r},\varphi), \qquad j,k \neq 0. \end{aligned} \tag{12}$$

In this paper, circular polarization in the reflection is assumed to be insignificant for a general understanding of most naturally illuminated surfaces.[12] This assumption reduces the 4×4 pBRDF Mueller matrix to a 3×3 matrix.

2.2. DOP expression

The relationship between the Stokes vectors and pBRDF Mueller matrix is shown as follows:

$$\begin{pmatrix} S'_0 \\ S'_1 \\ S'_2 \end{pmatrix} = \begin{pmatrix} F_{00} & F_{01} & F_{02} \\ F_{10} & F_{11} & F_{12} \\ F_{20} & F_{21} & F_{22} \end{pmatrix}\begin{pmatrix} S_0 \\ S_1 \\ S_2 \end{pmatrix}. \tag{13}$$

So the DOP expression is expressed as follows:

$$\mathrm{DOP} = \frac{\sqrt{S_1'+S_2'}}{S_0'} = \frac{\sqrt{(S_0F_{10}+S_1F_{11}+S_2F_{12})^2+(S_0F_{20}+S_1F_{21}+S_2F_{22})^2}}{S_0F_{00}+S_1F_{01}+S_2F_{02}}. \tag{14}$$

For the case of in-plane scattering ($\varphi_\mathrm{r}-\varphi_\mathrm{i}=\pi$), $\eta_\mathrm{i}=\eta_\mathrm{r}=0$ (η_i is the angle between the macroscopic plane of incidence and the scattering plane of the microfacet, η_r is the angle between the macroscopic plane of reflection and the scattering plane of the microfacet). So the off diagonal elements of Jones matrix in Eq. (15) are zero

$$\begin{aligned}\begin{pmatrix} E_\mathrm{r}^\mathrm{s} \\ E_\mathrm{r}^\mathrm{p} \end{pmatrix} &= \begin{pmatrix} \cos\eta_\mathrm{r} & \sin\eta_\mathrm{r} \\ -\sin\eta_\mathrm{r} & \cos\eta_\mathrm{r} \end{pmatrix} \begin{pmatrix} r_\mathrm{s} & 0 \\ 0 & r_p \end{pmatrix} \\ &\quad \times \begin{pmatrix} \cos\eta_\mathrm{i} & -\sin\eta_\mathrm{i} \\ \sin\eta_\mathrm{i} & \cos\eta_\mathrm{i} \end{pmatrix} \begin{pmatrix} E_\mathrm{i}^\mathrm{s} \\ E_\mathrm{i}^\mathrm{p} \end{pmatrix}, \\ \begin{pmatrix} E_\mathrm{r}^\mathrm{s} \\ E_\mathrm{r}^\mathrm{p} \end{pmatrix} &= \begin{pmatrix} T_{ss} & T_{ps} \\ T_{sp} & T_{pp} \end{pmatrix} \begin{pmatrix} E_\mathrm{i}^\mathrm{s} \\ E_\mathrm{i}^\mathrm{p} \end{pmatrix}, \end{aligned} \tag{15}$$

where E_i^s and E_i^p are the perpendicular polarization, incident and reflected, complex electric field components; E_r^s and E_r^p are the parallel polarization,incident and reflected, complex electric field components; and r_s and r_p are the complex Fresnel field reflection coefficients for the perpendicular and parallel polarization respectively.

Under in-plane scattering condition, $M_{02}=M_{20}=M_{12}=M_{21}=0$, so we can obtain $F_{02}=F_{20}=F_{12}=F_{21}=0$. Thus the pBRDF matrix F can be shown as

$$F = \begin{pmatrix} F_{00} & F_{01} & 0 \\ F_{10} & F_{11} & 0 \\ 0 & 0 & F_{22} \end{pmatrix}. \tag{16}$$

2.2.1. DOP expression for unpolarized illumination

The light source is assumed to be unpolarized in the case of passive detection where the Stokes vector of incident light is assumed to be $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^\mathrm{T}$. The Stokes vector of reflected light $\begin{pmatrix} S_0' & S_1' & S_2' \end{pmatrix}^\mathrm{T}$ can be given as

$$\begin{pmatrix} S_0' \\ S_1' \\ S_2' \end{pmatrix} = \begin{pmatrix} F_{00} & F_{01} & 0 \\ F_{10} & F_{11} & 0 \\ 0 & 0 & F_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{17}$$

Thus, the DOP expression is given by

$$\mathrm{DOP} = \frac{F_{10}}{F_{00}} = \frac{k_\mathrm{s} f_{10}(\theta_\mathrm{i},\theta_\mathrm{r},\varphi)}{k_\mathrm{s} f_{00}(\theta_\mathrm{i},\theta_\mathrm{r},\varphi)+k_\mathrm{dd} f_\mathrm{dd}+k_\mathrm{id} f_\mathrm{id}}. \tag{18}$$

2.2.2. DOP expression for polarized illumination

In this section, the DOP expression for polarized illumination is given. DOP0°, DOP45°, DOP90°, and DOP135° are used to express the DOP values in the four incident polarization directions (0°, 45°, 90°, and 135°) respectively. Take 0° incident polarization state for example. The Stokes vector of incident light is $S=\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^\mathrm{T}$, so the DOP expression for 0° incident polarization state DOP0° is shown as

$$\mathrm{DOP0^\circ} = \frac{\sqrt{S_1'^2+S_2'^2}}{S_0'^2} = \frac{F_{10}+F_{11}}{F_{00}+F_{01}}. \tag{19}$$

Similarly, the DOP expressions for 45°, 90°, and 135° incident polarization states are acquired respectively as follows:

$$\begin{aligned}\mathrm{DOP45^\circ} &= \frac{\sqrt{F_{10}^2+F_{22}^2}}{F_{00}}, \\ \mathrm{DOP90^\circ} &= \frac{F_{10}-F_{11}}{F_{00}-F_{01}}, \\ \mathrm{DOP135^\circ} &= \frac{\sqrt{F_{10}^2+F_{22}^2}}{F_{00}}. \end{aligned} \tag{20}$$

3. Experiment

Two metallic materials: Al and Cu are selected for our measurement. The surface roughness is measured by Dimension ICON piezoresponse force microscope which is produced by BRUCKER Corporation. The surface roughness values of Al and Cu are $\sigma_\mathrm{Al}=0.142$ μm and $\sigma_\mathrm{Cu}=0.070$ μm respectively. The experimental setup is schematically shown in the following Fig. 3. It consists of a light source of 650 nm ± 5 nm, a light power meter and two linear polarized analyzers P1 and P2.

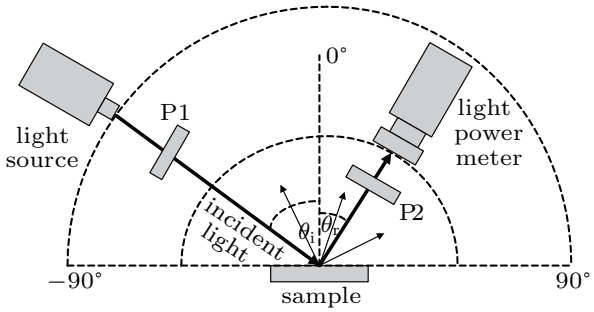


Fig. 3. Schematic diagram of experimental design for pBRDF measurement.

In the experiment, the distance between light sources and the measured sample, and the distance between light power meter and the measured sample are kept at constant values in different angles. Thus the paths of both light source and light power meter are two semicircles with one center of circle. The polarizer 1 in front of the light source is used to generate different polarization states. The polarizer 2 in front of the light power meter is used to measure the intensity of reflected light in 0°, 45°, 90°, and 135° polarization states. Firstly, the light source is fixed to in the direction in which the sample surface is illuminated vertically ($\theta_\mathrm{i}=0^\circ$), the reflected light from sample surface is received by the moving light power meter when the reflected angle ranges from 0° to 80°. Then, the location of light source is changed in steps of 10° and the operation of

the light power meter repeats the above step until the distribution of the reflected light in the whole plane of incidence is acquired for incident angles ranging from 0° to 80°.

The measured results show that for different material surfaces, the coefficients k_{dd} and k_{id} are both the fixed values, while k_s is the function of incident angle θ_i. The parameters in the diffuse reflection are determined by optimization algorithm and we find that $\sigma_m = 0.7$, $k_{dd} = 900$, $k_{id} = 25$ are the best fit for the both samples. The values of k_s are shown in Table 1.

Table 1. The k_s values of two metallic materials.

Incident angle θ_i	0°	10°	20°	30°	40°	50°	60°	70°	80°
k_s of Al	280	300	350	380	400	400	400	425	475
k_s of Cu	100	135	165	190	215	245	260	300	320

3.1. Measurements and simulations of DOP based on two pBRDF models for unpolarized illumination

The comparisons between the DOP for P-G pBRDF model and the three-component pBRDF model are shown in Fig. 4.

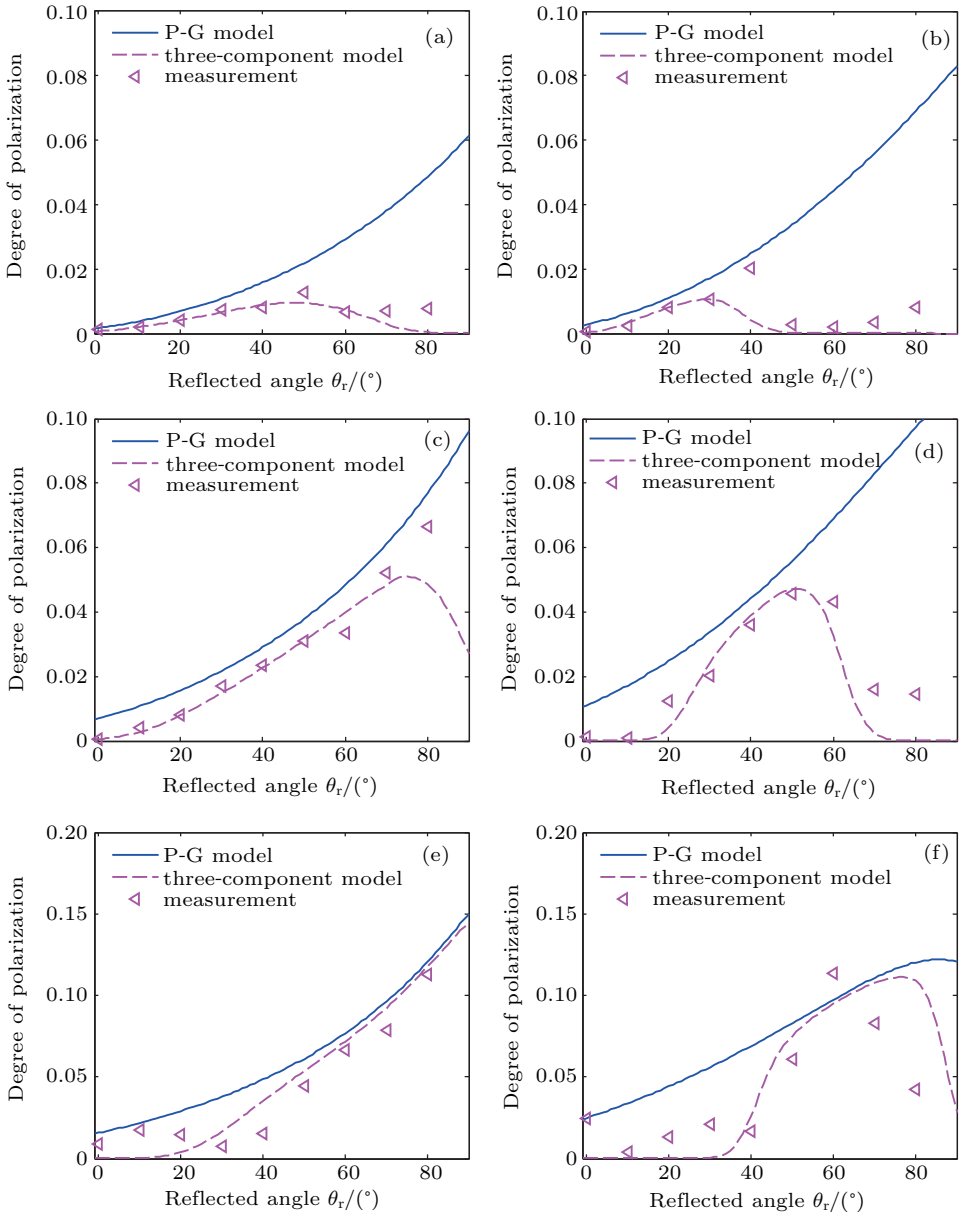


Fig. 4. (color online) Comparisons of DOP between two pBRDF models for different incident angles (a) Al $\theta_i = 20°$, (b) Cu $\theta_i = 20°$, (c) Al $\theta_i = 40°$, (d) Cu $\theta_i = 40°$, (e) Al $\theta_i = 60°$, (f) Cu $\theta_i = 60°$.

In Eq. (18), f_{10} and f_{00} are related to surface roughness σ. So the surfaces with different roughness values show different DOP values. Considering directional diffuse reflection and ideal diffuse reflection, the DOP values based on three-component pBRDF model decrease and approach to the measured data especially for small reflected angles. The reason is that in three-component model, the specular reflection accounts for a higher proportion in large incident angles. And the sum of specular reflection and diffuse reflection over hemisphere space is 1. So diffuse reflection accounts for higher pro-

portion when the incident angles are small. Therefore there are more obvious errors in the existing DOP expression for $\theta_i = 20°$, which is in fact because diffuse reflection in smaller incident angle is not considered in the P-G model. The accuracies of the DOP expression based on the three-component model and the P-G model of the two metallic samples for $\theta_i = 40°$ and $60°$ are compared. The root mean square (RMS) errors of DOP based on P-G model are reduced by 22.6% from 0.02545 to 0.01970 for Al, and by 78.4% from 0.1133 to 0.02454 for Cu when $\theta_i = 40°$, by 35.0% from 0.05516 to 0.03584 for Al and by 28.1% from 0.1156 to 0.08312 for Cu when $\theta_i = 60°$, respectively.

3.2. Measurements and simulations of DOP based on three-component pBRDF model for different incident polarized states

The DOP calculated by the three-component pBRDF model is compared with measurements for two metallic samples for four different incident polarization states, and the results are given in Fig. 5.

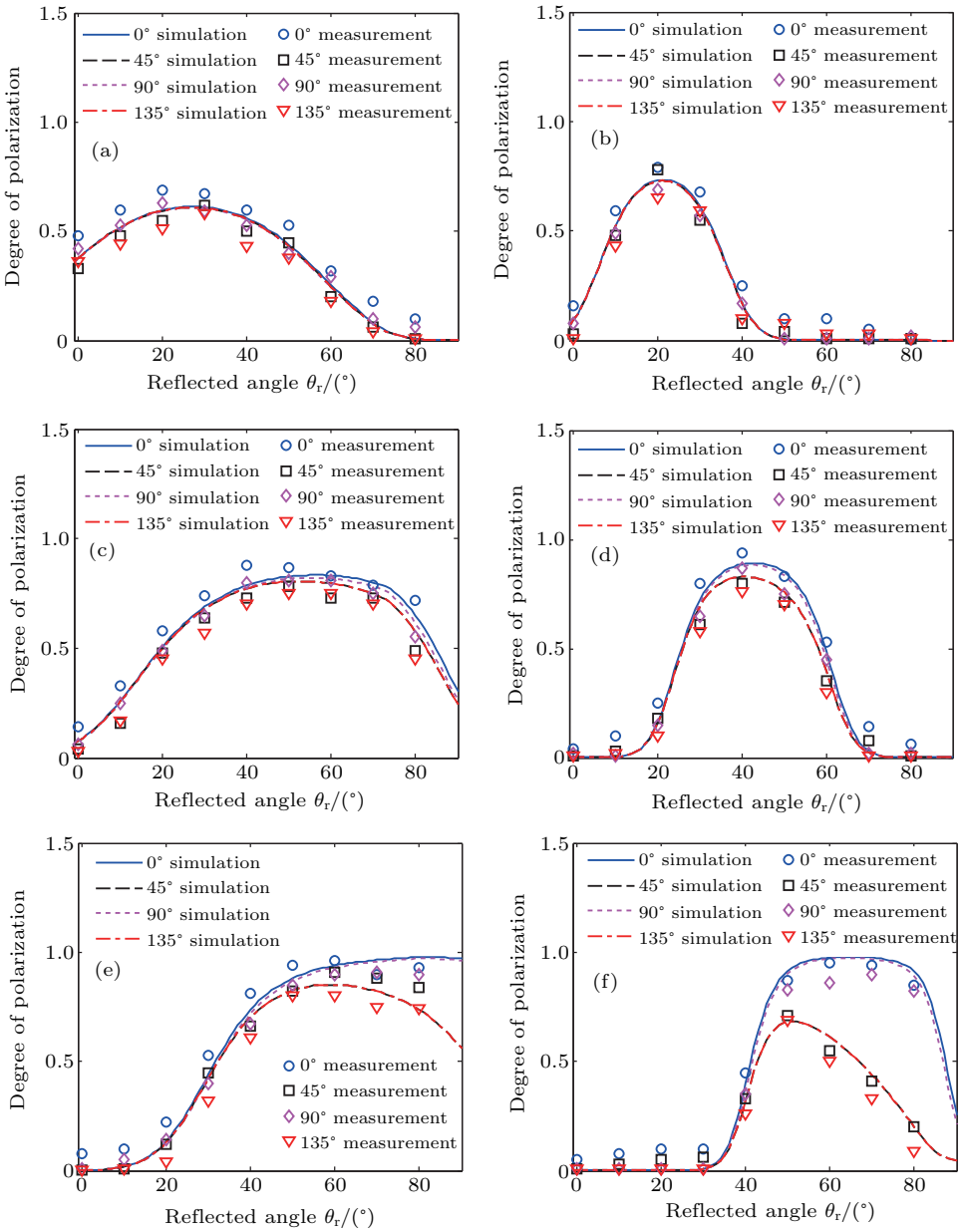


Fig. 5. (color online) Simulated and measured DOPs of two metallic materials at different incident angles: (a) Al $\theta_i = 20°$, (b) Cu $\theta_i = 20°$, (c) Al $\theta_i = 40°$, (d) Cu $\theta_i = 40°$, (e) Al $\theta_i = 60°$, and (f) Cu $\theta_i = 60°$.

The polarization state of incident light has a little effect on the DOP when incident angle is small for metallic material. However, the distinctions between $0°/90°$ incident polarization state and $45°/135°$ incident polarization state are obvious for large incident angles. And the difference of the DOP for each incident polarization direction tends to be large when the incident angle is large. The maximum value of the DOP appears around the direction of specular reflection regardless of incident polarization state. The results show that the DOP based on the three-component model accords well with the ex-

perimental results.

4. Conclusions

A DOP expression is presented based on three-component pBRDF model to improve the existing DOP expression based on the P-G model. Reflected light is divided into specular reflection, directional diffuse reflection, and ideal diffuse reflection. The pBRDF matrix is deduced and DOP expression is given. Compared with the existing DOP expression based on the P-G model, our DOP expression relates to the surface roughness. And the introduction of the diffuse reflection reduces the DOP value and makes simulated DOP closer to the measurement, especially for small incident angle. For unpolarized illumination, the RMS errors of DOP based on the P-G model are reduced by at least 20% for Al and at least 25% for Cu, respectively. For polarized illumination, the simulations match well with the measurements. The results indicate that the incident polarization state has a little effect on the DOP for small incident angle while the distinction is obvious for large incident angle. Therefore, the DOP expression based on the three-component pBRDF model is more reasonable and can depict the polarized reflection characteristics of metallic materials more accurately.

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